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تعزيز التقريبات العددية للمعادلات التفاضلية ذات التأخير الخطي العددي مع التأخير المستمر وشروط الدعم الذاتي التلقائي

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المستخلص:

من خلال هذه الدراسة، تم تطوير نهج تقريبي عددي لفئة من المعادلات التفاضلية ذات التأخير العددي التي كانت خاضعة لشروط اندفاعية. وقد تم أخذ كل واحدة من هذه المعادلات بعين الاعتبار. تم استخدام طريقة مبتكرة للتقريب، حيث يتم استخدام المدخلات الثابتة المتعددة التعريف في تطبيقات معادلات التأخير. باستثناء اللحظات المندفعة من الزمن، في كل مرحلة، تم إظهار التقارب النظري لنهج التقريب. علاوة على ذلك، تقدر التقنية الرقمية اللحظات المندفعة من الوقت بشكل جيد إلى حد ما. في حالة أن المعلمة β للمشكلة موجبة؛ فنحن نمتلك القدرة على التوصل إلى نتيجة التقارب من خلال فرض شرط إضافي عن طريق استخدام شرط إضافي. دون افتراض وجود شرط ذاتي الدعم، يضمن هذا الشرط أن الحل سوف يعبر الخط $x = c$ في فترة زمنية مندفعة. وتم عرض استنتاجات التقارب النظري من خلال عرض الأمثلة العددية. ومن خلال استخدام الدراسة العددية تمكنا من التعرف على وجود فترات في الحل. في هذا الحل المحدد، يتم تحديد الفترة الدنيا بكونها أكبر من التأخير τ المرتبط بالنموذج. على الرغم من تقديم متطلبات كافية لضمان وجود الحلول الدورية، إلا أن الأمثلة العددية توضح أن الحلول الدورية تعبر عن نفسها بمعايير أقل صرامة بكثير من تلك التي تضمن وجودها. في ضوء ذلك، هناك حاجة ملحة لإجراء المزيد من الدراسات لتحديد ما إذا كان من الممكن الوصول إلى الحلول الدورية لهذه المجموعة المعينة من المعادلات الاندفاعية أم لا.

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الكلمات المفتاحية: التفاضلية المتأخرة، الاندفاع، المعادلات ذات حجج الثوابت المتعددة التعريف، التقريب العددي.

Enhancing Numerical Approximations for Scalar Linear Delay Differential Equations with Constant Delay and Impulsive Self-Support Conditions

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Abstract:
Through the course of this study, a numerical approximation approach was developed for a category of scalar delay differential equations that were subject to impulsive conditions. Every one of these equations was taken into consideration. An innovative method of approximation is used, it makes use of piecewise constant inputs in delay equation applications. Except for the impulsive instants of time, at each stage, the approximation approach's theoretical convergence was shown. Furthermore, the numerical technique estimates the impulsive instants of time rather well. In the event that the β parameter of the problem is positive; we possess the capability to establish the convergence conclusion by imposing an additional condition via the use of an additional condition. Without making the assumption of a self-supporting condition, this condition guarantees that the solution will cross the line $x = c$ at a temporal interval that is impulsive. The conclusions of the theoretical convergence were shown by the presentation of numerical examples. Through the use of numerical study, we identified periods in the solution. In this particular solution, the minimal period is defined by being bigger than the delay τ that is associated with the model. Though enough requirements are given to ensure that periodic solutions exist, numerical examples show that periodic solutions express themselves through criteria that are far less stringent than those that ensure their existence. In light of this, there is a pressing need for more studies to be conducted to determine

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whether or not periodic solutions are accessible for this particular set of impulsive equations.

Keywords: Delayed differential equation s, Impulses, Equations with piecewise constant arguments, Numerical approximation

1. Introduction

Differential equations that include a delay argument are referred to as delayed differential equations, or DDEs for short (Alaminos et al., 2011, p. 235). The difference between differential equations and ordinary differential equations is that differential equations take into consideration the function at earlier times in addition to its present value and derivatives (Brauer, Castillo-Chavez, 2012, p. 33). The domains of physics and biology are only two of the many areas in which these equations have found specific applications (Hartung 2022, p. 17). Recently, there has been a substantial amount of interest in the process of creating improved numerical approximation techniques for DDEs that include criteria of impulsive self-support (Zhang 2014, p. 340). To achieve solutions that are more dependable and effective, these strategies intend to enhance the accuracy and convergence features of the techniques that are already in use (Koch 2014, p. 302).

For a better understanding of DDEs, let's look at some instances below. An illustration of the notion is provided by the following equations:

First, the Equation (1)

$$x^{\wedge\prime}(t) = -2x(t-\pi/2) \quad (1)$$

The following equation may be found in Equation (2):

$$x''(t) = -x'(t) - x'(t-1) - 3sin(x(t)) + \cos(t) \quad (2)$$

$$x^{\wedge\prime}(t) = x(t) - x(t/2) + x^{\wedge\prime}(t-1) \quad (3)$$

according to the Equation (3). With respect to Equation (4),

$$x'(t) = x(t)\,x(t-1) + t^{\wedge}2x(t+2). \quad (4)$$

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Although Equations (1) and (2) are instances of DDEs in this set of equations, Equations (3) and (4) are not examples of DDEs since they do not feature delayed terms. Although equations (3) and (4) both have $x^{\wedge'}(t-1)$ on the right side, these equations are not regarded as DDEs (Zhang 2020, p. 9).

To illustrate how DDEs may be used, let's have a look at the following example:

An example of this would be (1): Consider the case of a barrel that is filled with brine and has a capacity of B gallons overall. In addition to the water that is being poured into the top of the barrel at a pace of q gallons per minute, the salt water that is contained inside the barrel is being stirred on a consistent basis. A steady flow of q gallons of seawater per minute is being expelled from the barrel that is located at the bottom of the container (Zhang 2017, p. 35).

The variable $x(t)$ should be used to represent the amount of salt, measured in pounds of seawater, that is present in the barrel at the moment being signified by the variable. If the saltwater is continuously mixed into the liquid, the remaining salt water in the barrel will have a concentration of $x(t)/B$ pounds of salt per gallon. This is made on the assumption that the saltwater is always being mixed. Therefore, the equation may be represented in the form that is described below (Federson 2021, p. 610).

The expression:

$$x\ (t)\ = -q(x(t))/B$$

In reality, however, it is quite improbable that the seawater that is contained inside the barrel could be churned up in a single second. Since r is a positive constant, the concentration of the saltwater that is still there will be equivalent to the average concentration that was present at some point in the past, which is represented by the symbol $t-r$. A differential equation for x is turned into a differential equation (DDE) in this specific instance (Piper et al., 2013, p. 69):

The $x'\ (t)$ equals $-q(x(t-r))$ divided by B.

We may alternatively write the equation as $x^{\wedge'}\ (t)\ = -cx(t-r)$ if we define c as q/B. This is another possible formula. On the other hand, this is an alternate method of comprehending the equation. A usual term for this specific kind of DDE is either an R-delay, a time-delay, or a slowness equation. All of these names refer to the same thing (Federson et. al, 2020, p. 1675).

Another distinction between delayed differential equations and ordinary differential equations is that delayed differential equations take into consideration the function's previous values. Ordinary differential equations do not take this into account. Delay arguments are included into the equations to create them, and they have a wide range of applications in the domains of physics and biology. It was explained how DDEs may be used in the process of replicating the concentration of saltwater in a barrel over a period of time by the example that was provided (Din et. al, 2015, p. 788, Liu et. al., 2014, p. 4).

This study focuses on scalars linear delays differential equations with constant delays and impulsive self-support conditions. This work aims to develop and investigate more efficient numerical approximation techniques for these equations (Cooke, 1986). To evaluates they accuracy's, stability, and's effectiveness of various techniques based on their unique levels of performance, we want to compare and contrast the levels of performance of the various methods. Furthermore, we will examine the convergence characteristics of different methodologies and assess the degree to which they may be utilized to address a diverse array of real-world issues (Din, 2012, p. 734, Li et. al., 2019, p. 307).

2. Methodology

2.1 Problem Statement

The simple form of delay differential equations is as follows:

$$y'(t) = ay(t) + by(t-\tau), \quad t \geq t_0$$

$$y(t) = \emptyset(t), \quad t \leq t_0 \qquad (5)$$

When a, b are real complex fixed values, $\tau > 0$, and the starting function is provided by $\emptyset$. For a delay differential equation to have a solution, continuity of $\emptyset$ is one of the requirements.

In this thesis, the linear delay equation with self-supporting impulse conditions is generally as follows:

$$\dot{x}(t) = \alpha x(t) + \beta x(t-\tau) \qquad a.e.t \geq 0 \qquad (6)$$

$$x(t) = c + d, \quad if\ x(t-) = c$$

Now, considering the initial conditions, we will have:

$$x(t) = \varphi(t), \ \ t \in [-\tau,\ 0] \qquad (7)$$

In this case, we will keep in mind throughout the above equation:

$$(H)\ \ c, d > 0,\ \alpha + |\beta| < 0, \tau > 0,\ c < \varphi(t)\ for\ t \in [-\tau,\ 0],\ \varphi: [-\tau,\ 0] \to \mathbb{R}$$

The Lipshitz function is continuous. By solving the impulsive initial value problem (6) and (7) on the function, x is continuous on $[0, \infty)$. There are discontinuities only in time values in relation (6). Otherwise, it looks completely continuous in any interval $[0, \infty)$ that does not contain discontinuity points x. Therefore, in this thesis, we seek to achieve the convergence of the existing method and use numerical results to show given the approximation convergence, with a constant delay and an impulsive self-supports condition, wen derive a numerical approximations strategy for a linear scalar delays differential equation.

Delay differential equations with bounded delays and notation, uniqueness and existence

The majority of delay differential equations have limited, constant delays with non-constant delays. Examine the differential equation for delay.

$$x'(\mathrm{t}) = f(t, x(g_1(t)), \dots, x(g_m(t)) \qquad (8)$$

We must assume that $t - r \leq g_j(t) \leq t, \forall t \geq t_0$ and $j = 1, 2, \dots, ms$ for some's constants $r \geq 0$. Then they initial functions is given as follows.

$$x(t) = \emptyset(t), t_0 - r \leq t \leq t_0$$

Note that if $r = 0$, the delay differential equation becomes an ordinary differential equation. Assume that f is on $[t_0, \beta) \times D^m \to R^n$ for some $\beta > t_0$ and the open set $D \subset R^n$ is defined.

$$F(t, x_t) = f(t, x(g_1(t)), \dots, x(g_m(t))) \quad (9)$$

Therefore, Equation (8) briefly becomes the following equation.

$$x'(t) = F(t, x_t) \quad (10)$$

The following definition, which was introduced by Simanov in 1960, is widely used for delay differential equations.

Definition 1. If the function defined on $[t - r, t] \to R^n$, then we define the new function x_l: $[-r, 0] \to R^n$ as follows.

$$x_t(\sigma) = x(t + \sigma) \qquad -r \leq \sigma \leq 0 \quad (11)$$

Note that x_t is obtained by considering $x(s)$ for $t - r \leq s \leq t$ and transferring this part x to the interval $[-r, 0]$.

If x is a continuous function, then x_t is a continuous function on $[-r, 0]$.

Notation: We denote the set $C([-r, 0], R^n$including all continuous functions from $[-r, 0] \to R^n$ by $\mathscr{L}$ and if A is a subset of R^n, we assume do.

$$)\mathscr{L}_A = c([-r, 0], A) \quad (12)$$

So, if x is continuous on $[t - r, t] \to$

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Then: $x_t \in \mathcal{L}_A$Sometimes we use the semi-open interval $[t_0, \beta)$ or the full open interval (α, β) so the symbol J for $[t_0, \beta)$ or (α, β) We use

Delay differential equations with constant coefficients

The delay differential equations with constant coefficients will be as follows.

$$x'(t) = \sum_{j=1}^{m} A_j x\left(t - r_j\right) + h(t) \qquad (13)$$

That A_j are fixed and $0 \le r_j \le r$ for $j = 0, 1, 2, \dots, m$ and $h(t)$ is a continuous function on $[t_0, \beta)$.

We usually consider equation (6) on $[t_0, \beta)$ with the initial function $x_{t0} = \emptyset$, which $\emptyset \in L$.

If $h(t) = 0$, we call the equation homogeneous, so we emphasize more on homogeneous equations.

$$y'(t) = \sum_{j=1}^{m} A_j y\left(t - r_j\right) \qquad (14)$$

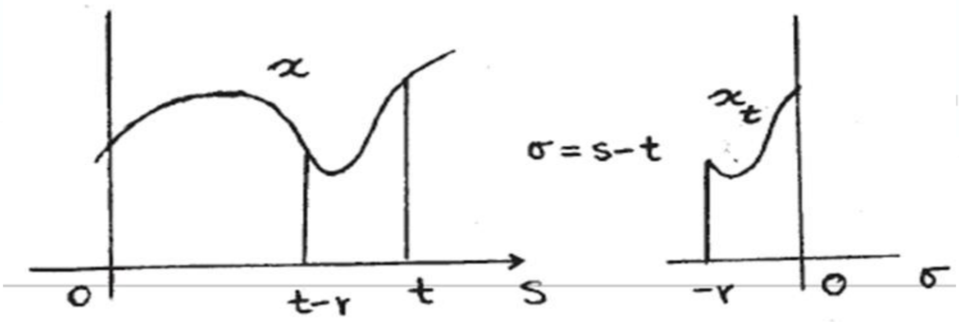

Figure 1. When $D = R^n$ and $\mathscr{L}_{D} = \mathscr{L}$, then $\|\cdot\|_r$

The solutions of equation (15) are

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A numerical approximation scheme. In this section, for a class of linear FDEs, we define a simple numerical approximation of IIVP (15, 16) using EPCAs

$$x(t) = ax(t) + \beta x(t-\tau), a.e. t \geq \quad (15)$$

$$x(t) = \varphi(t), t \in [-\tau,] \quad (16)$$

Prove a discrete parameter $h > 0$ numerical approximation. The set if mesh points $\mathbb{N}h \mid \mathbb{N}$, it exhibits $\mathbb{N}h \mid \mathbb{N}$, h *and* $\mathbb{Z}h$ are defined by $\{kh: k \in \mathbb{N}\} \mid \{kh: k \in \mathbb{N}\}$ and $\{kh: k \in \mathbb{Z}\}$, respectively. We introduce a simplified notation.

$$[t]_h := \left[\frac{h}{t}\right] h \quad (17)$$

There is a piecewise constant function $t \rightarrow [t]h$. At locations $Z\ h$, when it is right-continuous, it exhibits jump discontinuities. The following EPCA is linked to IIVP (15, 16) under self-supporting circumstances:

$$(y\_h)\ (t) = \alpha y\_h\ ([t]\_h\ ) + \beta\_y\ h\ ([t]\_h - [\tau]\_h\ ), a.e. t\ \geq 0 \quad (18)$$

$$y\_h\ (kh\ ) = c + d, if\ y\_h\ (kh -) \leq c, \quad (19)$$

$$y\_h(t) = \varphi(t), t \in [-\tau, 0]. \quad (20)$$

A function $y\_™: [-\tau, \infty) \rightarrow \mathbb{R}$, which is separable $[0, \infty)$, and satisfies (18) omitting possible points, is the solution of IIVP (18) – (20). $N_0\ h$; Its discontinuity point is limited to the mesh locations in $[-\tau, 0]$ where (19) is present. All solutions to (19) for $k \in N$ are in intervals $[kh, (k+1)h)$ since the right side of the equation is in fixed intervals $[kh, (k+1)h)$. We believe that the impulsive condition (19) indicates that there are discontinuities in IIVP (18) and (29) only at some sites.

$\boldsymbol{Nh}$. Some of the attributes $[\cdot]h$ that we will utilize later in the text are summarized in the following. The estimate is provided by the definition of $[t]h$

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$$t - h < [t]_h \leq t,\ t \in \mathbb{R} \qquad (21)$$

From this rule $\lim_{h \to \cdot +} [t]_h = t$ uniform in the entire real line of the above estimate implies

$$-h = t - \tau - t + (\tau - h) < t - \tau - ([t]_h - [\tau]_h) < t - \tau - (t - h) + \tau = h,$$

Therefore

$$|t - \tau - ([t]_h - [\tau]_h)| < h,\ t \in \mathbb{R} \qquad (22)$$

Finally, we mention the relationship

$$[t - jh]_h = [t]_h - jh,\ t \in \mathbb{R},\ j \in \mathbb{Z} \qquad (23)$$

Fix h > 0 and let $t \in [kh, (k + ١)h)$ and $\ell_h := [\tau/h]$. Integrating both sides of (18) from kh to t, and taking the left limit, we get $t \to (k + 1)h$

$$y_h((k+1)h-) = y_h(kh) + h\left(\alpha y_h(kh) + \beta y_h(kh - \ell_h h)\right) \qquad (24)$$

We introduce the sequences

$a(k) := y_h(kh),\ k = -\ell_h,\ -\ell_h + 1,\ \cdots,\ and\ b(k) := y_h(kh-),\ k \in \mathbb{N}.$

Then they impulsive self-supporting conditions (200) yields $a(k) = b(k)$ if $b(k) > c$, otherwise $a(k) = c + d$. Hence, they sequence's $a(k)$ and $b(k)$ are obtained.

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$(k+1) = (1 + h\alpha)\, a(k) + h\beta a(k - \ell_h), k \in \mathbb{N}_0,$

$$a(k+1) = \begin{cases} b(k+1). & b(k+1) > c \\ c + d. & b(k+1) \le c \end{cases} \quad (25)$$

$a(-k) = \varphi(-kh), k = 0, 1, \ldots, \ell_h$

Between kh and $s(k+10)h$, the functions y_h linearly interpolates they values off a (ka) and $b(k+10)$. As a result, we find that IIVP (18) - (20) has a unique solution. Let' $h > 0$ be fixed, and s lets $t_0 \in \mathbb{N}_0 h, \psi \in G$. Wen considers they approximates IVP without impulsive terms:

$$w_h(t) = \alpha w_h([t]_h) + \beta w_h([t]_h - [\tau]_h), t \ge t_0 \quad (26)$$

$$w_h(t) = \psi(t - t_0), t \in [t_0 - \tau, t_0] \quad (27)$$

Introduce the sequence $\bar{a}(k) := w_h(kh + t_0), k = -\ell_h, -\ell_h + 1$ Then, it is easy to see

$$\bar{a}(k+1) = (1 + h\alpha)\bar{a}(k) + h\beta\bar{a}(k - \ell_h), k \in \mathbb{N}_0 \quad (28)$$

$$\bar{a}(-k) = \psi(-kh), k = 0, 1, \ldots, \ell_h. \quad (29)$$

3. Result

Example: In this example, the initial shock value problem of Equation (30) with

$X(t) = ax(t) + Bx(t—T), a, t > 0,$

$x(t) = c + d, if\ x(t-) = c, (2)$

$x(t) = g(t), t \in [-1, 0].$

$a = -0.5, f = 0.4, i = 1.0, c = 0.1, d = 0.4, g(t) = 0.2$

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Consider. Since $B > 0$, the solution is generally not uniform in time intervals

[5 Ent) for n ∈ ℕ. Numerical solution of the impulsive initial value problem

$$Int) = ayn\,(tn) + Byn\,(tn-1 - [t]n), a0.t \geq 0$$

$$yn(kh) = c + d, if\ yn(kh-) < c, (31)$$

$$Ya\,(t) = v(t), t \in [-\tau, 0].$$

With the values of $h = 0.1$ (blue dot s) and $h = 0.001$ (red points), we produced the figures shown in Figure 2. The graphic illustrates the closeness of the numerical approximations for the time span [0,209] for high step size ($h = 0.1$) and small step size ($h = 0.001$). This is obviously present.

In Figure 2, the horizontal green lines denote the levels $x = c$ and $x = c + d$, whereas the blue and purple dots correspond to the numerical solutions for h=0.1 and h=0.001 in the interval [0,300].

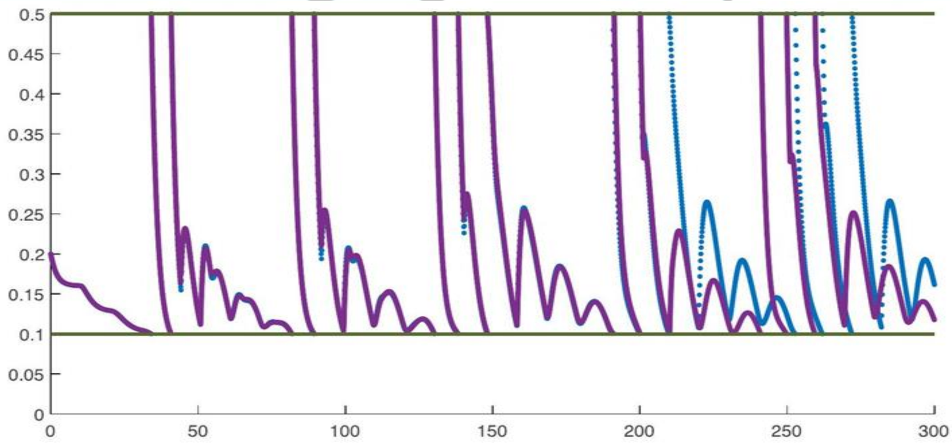

Figure 2. Numerical solution for $h = 0.1$ (blue points) and $h = 0.001$ (red points).

However, the level c is not reached by the numerical solution corresponding to $h = 0.1$ time, 209.4, so due to the impulsive conditions $y_h(t) =$

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$\varphi(t), t \in [-\tau, \cdot]$ creates a fake jump and for the next time 209.4, these two answers differ significantly.

This is a rough version of this kind of impulsive problem: if the precise answer is really near the lower critical value c, a tiny approximation mistake might result in a false leap that is identical, at which point tracking the solution would fail.

Keep in mind that the resultant graphs match visually if the numerical solutions for $h = 0.01$, $h = 0.001$, and $h = 0.0001$ are found even over a lengthy time period, and the numerical method's convergence is determined as $h \to 0+$.

In the end, we provide the numerical solution for the [0800] interval in Figure 3 at $h = 0.001$.

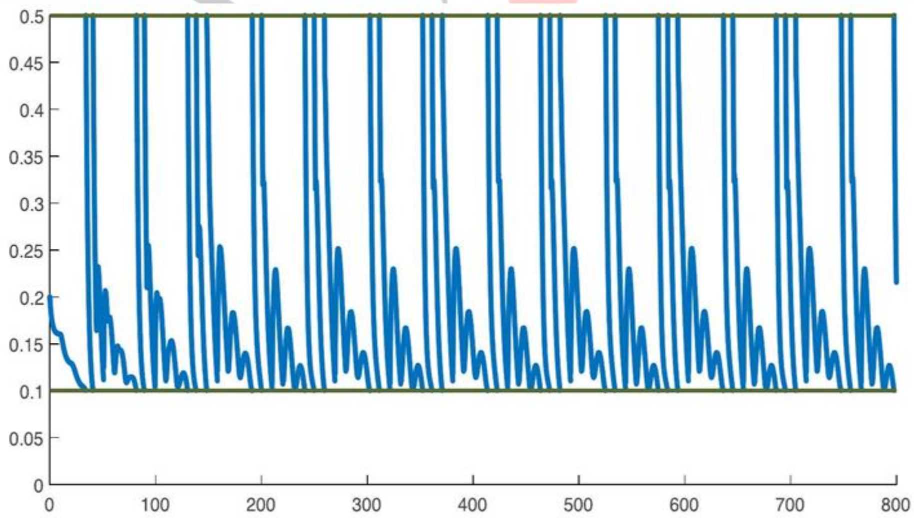

Figure 3. Numerical solution for $h = 0.001$ in [0,800].

For this case, the graphic suggests that there exists a periodic solution with a minimum period around $\tau > 111$, as the numerical solution is asymptotically periodic. As of right now, the existence of a periodic solution for the impulsive starting value problem (30) for $\beta > 0$ has no recognized theoretical conclusion.

4. Conclusion

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An approach to numerical approximation has been developed for the class of scalar delay differential equations that include impulsive elements. This approach has been given specific attention. As its mathematical foundation, the unique technique of approximation uses delay equations with piecewise constant inputs as its implementation. We showed approximation scheme is theoretically convergent at every point, except for impulsive time instants, even though the numerical approach approximates the impulsive time instants. This was accomplished by demonstrating that the approximation scheme is convergent at every point. In situations where the β parameter of the issue is positive, we possess the capability to exhibit the convergence conclusion by applying an additional constraint. It is the responsibility of this condition to guarantee that the solution crosses the line x=c at a moment that is impulsive, in addition to the self-supporting condition that is expected to be there. To illustrate the theoretical convergence results, we gave numerical examples. This was done to illustrate the conclusions. Through the use of numerical research, we were able to demonstrate that there is a periodic solution. This solution in particular has a minimum period that is greater than the model-associated delay τ. Although this thesis provides enough requirements to ensure periodic solutions exist, numerical examples show that periodic solutions can exist with far weaker circumstances. This kind of stuff is something we know exists. Further investigation is required since it is vital to determine whether periodic solutions for this collection of impulsive equations exist.

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