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POST BUCKLING BEHAVIOR OF PRISMATIC STRUCTURAL STEEL MEMBERS USING FINITE ELEMENT METHOD

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Abstract:
In this study, a numerical analysis is presented to investigate the post buckling behavior of plane structural steel members subjected to static loads using finite element method. The material of structural steel members is assumed to be elastic. The nonlinear behavior of large displacement formed from buckling has been carried out using arc-length method. The study shows that the numerical results predicted by the nonlinear finite element analysis shows good agreement with results in the literature. Consequently, finite element modeling is found to be reliable and accurate to be used to perform nonlinear analysis for the study of buckling and post buckling behavior. The effect of the most important variable parameters such as cross-sectional area and modulus of elasticity on post buckling behavior of steel trusses, beams and frames were studied.

Keywords: *Post Buckling Behavior, Structural Steel Members & Finite Element Method .*

سلوك مابعد الأنبعاج للاعضاء الأنشائية الحديدية الموشورية بأستخدام طريقة العناصر المحددة

الخلاصة:
في هذه الدراسة ، تم تقديم تحليل عددي لتحري سلوك مابعد الانبعاج للأعضاء الانشائية الحديدية المستوية المعرضة الى احمال ساكنة بأستخدام طريقة العناصر المحددة. تم افتراض أن مادة الاعضاء الانشائية الحديدية مرنة. استخدمت طريقة طول القوس للتنبأ بالسلوك اللاخطي للأزاحات الكبيرة الناشئة من الانبعاج. الدراسة بينت ان النتائج العددية المستحصلة من التحليل اللاخطي للعناصر المحددة كان في تطابق جيد مع نتائج البحوث السابقة. وبناءا على ذلك فقد وجد أن النمذجة بالعناصر المحددة تمتلك الدقة والموثوقية لاستخدامها في انجاز التحليل اللاخطي لدراسة الانبعاج ومابعد الانبعاج. كما تم دراسة العوامل الاكثر اهمية في سلوك مابعد الانبعاج مثل مساحة المقطع العرضي و معامل المرونة للجملونات والعتبات والهياكل الحديدية.

1. Introduction

In slender elements of structures such as frames and trusses ,after a load reaches its buckling value, the load value may remain unchanged or it may decrease, while the deformation continues to increase. For some problems, after a certain amount of

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deformation, the structure may start to take more loading to keep deformation increasing, and a second buckling can occur, the cycle may repeat several times, this phenomena is called post buckling behavior [1]. Estimating post buckling behavior for structures having slender members is very important since post buckling means loss the stability of structure associated with large displacement and may be lead to destruction the structure.

Many researches were presented to study and investigate post buckling behavior. In elastic material analysis of geometrical nonlinear post buckling behavior, Oran [2], in 1973, derived a tangent stiffness matrix for linear elastic in plane prismatic member according to the conventional beam column theory. In 1991, Kassimali and Abasnia [3] presented a method for large deformation and stability analysis of elastic space frames, the method was based on an Eulerian fromulation , proposed by Oran . In 2001 Al Mahdawi [4] presented theoretical analysis to predict pre-and post-buckling behavior of prismatic and non prismatic plane steel frames, the analysis adopted beam-column theory. Al-sarraf et al [5,6] in 2006 and later in 2008 presented nonlinear elastic and elastic-plastic post-buckling analysis on steel frames with prismatic and non-prismatic gusseted plate members using beam column theory. Basaglia et al [7] in 2010 presented post-buckling behavior of restrained thin-walled steel beams using generalized beam theory (GBT). In 2011, Ziolkowski and Imielowski [8] studied buckling and post-buckling behaviour of prismatic aluminium columns submitted to a series of compressive loads. Novoselac et al [9] in 2012 presented Linear and nonlinear buckling and post buckling analysis of a bar with the influence of imperfections , the nonlinear buckling analysis ia acheved using the Riks method. Yang [10] in 2013 presented post-buckling and ultimate strength predication of composite plates using a semi analytical method.

All the presented studies on post buckling behavior give approximate results due to depending on approximate numerical approaches and the accuracy of these approaches depend on efficiency of the used tools. The developments in finite element programs such as ANSYS computer program provide the ability for getting more accurate results and include various strategies to get the solutions and save the effort and time especially for complex problems. This research present an investigation on post buckling behavior of structural steel members using finite element method with ANSYS computer program in order to provide more understanding for post buckling behavior.

2. Aim of The Research

This research aim to present a numerical analysis to investigate the post buckling behavior of plane structural steel members in elastic materials stage subjected to static load using finite element method with benefit from arc-length method.

3. Arc-Length Method

A structure become unstable when a load reaches its buckling value or when nonlinear material becomes unstable. Instability problems usually pose convergence difficulties; therefore, it requires the application of special nonlinear techniques. Arc-length is one of the techniques used in ANSYS to overcome convergence difficulties. The arc-length method is suitable for nonlinear static equilibrium solutions of unstable problems. Applications of the arc-length method involves the tracing of a complex path in the load-displacement response into the buckling/post buckling regimes. The arc-length method uses the explicit spherical iterations to maintain the orthogonality between the arc-length radius and orthogonal directions as described by Forde and Stiemer [11] . It is assumed that all load magnitudes are controlled by a single scalar parameter (i.e., the total load factor). Mathematically, the arc-length method can be viewed as the trace of a single equilibrium curve in a space spanned by the nodal displacement variables and the total load factor. Therefore, all options of the Newton-Raphson method are still the basic method for the arc-length solution.

4. Finite Element Analysis

In order to assess the finite element models proposed in this study , various numerical examples were analyzed with the proposed models, and comparisons were made with previously published results. It is important to mention that all structures in this study is analyzed by discretizing the each bar or member into different number of elements and the mesh that gave a satisfactory solution is implemented.

4.1. Trusses

4.1.1. Two Bar Truss

This structure was analyzed by Noor [12] using a mixed method and a computational procedure formulated in terms of both internal forces and joint displacements. Al-Mahdawi[4] also analyzed this truss using beam-column theory.

The structure load system of the truss is shown in Fig.(1). It is loaded with vertical point load P at the joint B and fixed at joint A and C . The modulus of elasticity of the truss elements is 68.97×10^3 MPa and the cross-sectional area is 645 mm^2 for each one. This truss is represented in ANSYS program using 2-D spar element (LINK1). LINK1is a uniaxial tension-compression element with two degrees of freedom at each node translations in the nodal x and y directions. As in a pin-jointed structure, no bending of the element is considered [13]. Fig.(2) shows LINK1 element and Fig.(3) shows modeling of the truss in ANSYS .

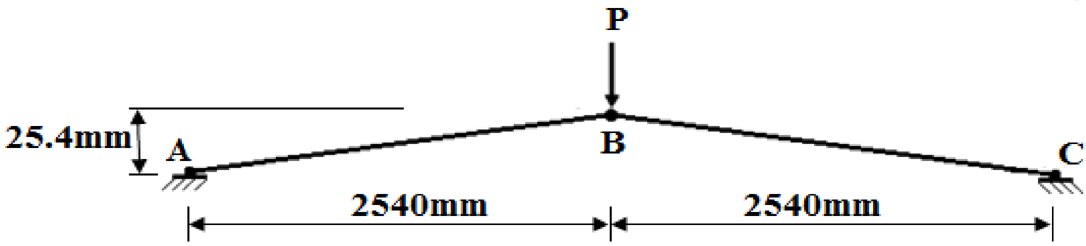

Fig.(1) Geometry and loading of two bar truss

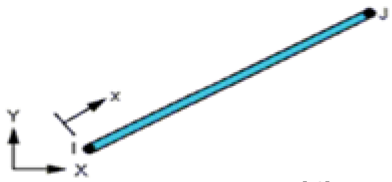

Fig.(2) LINK1 element [13]

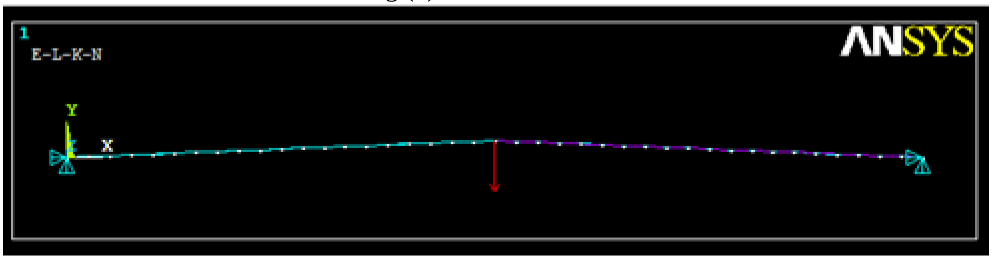

Fig.(3) Modeling the truss in ANSYS

The post-buckling load-vertical displacement curve of joint B, obtained from finite element analysis of this study along with the curve obtained from beam-column theory by Al-Mahdawi [4] study and the curve obtained by Noor [12] study are presented and compared in Fig.(4) . It can be noted that the results of this study show good agreement with the results of Noor [12] and Al-Mahdawi [4]. The good agreement can be attributed to using sufficient number of elements and using arc-length method included in ANSYS with high accuracy to trace post buckling behavior. The Figure reveals that the nonlinear instability range extend from the point in which the truss is stable (point A) to the point in which the truss become stable once again (Point B).The initial peak load at point (A) equal to that at point (B), but the load in the latter point corresponds to a new structural shape. After the initial peak load (Point A), the displacement is still increasing but the load began to decrease until the negative peak load and starts increase again. This behavior which is called snap-through phenomena can be explained by the relationship between the applied load and horizontal reaction force at one of the truss supports as shown in the Fig.(5) which is plotted from the results of this research. The curve in this figure began at point E , beyond this point the reaction value increase with increasing the applied load until point C , after this point the load began decrease with continuation increasing in the reaction value until reach the maximum horizontal reaction force at point D , at this point the vertical reaction force become zero, and after this value the horizontal reaction decreases gradually and the vertical reaction continuous to be in the negative range till it again attains the value of zero at point D. Fig.(6) shows variations in vertical displacement for the truss at ultimate load using ANSYS program.

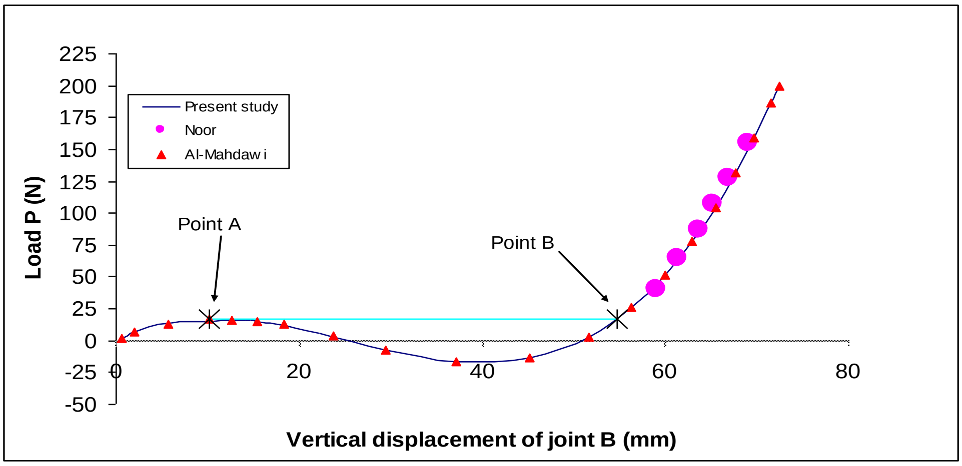

Fig.(4) load-displacement curve of two bar truss

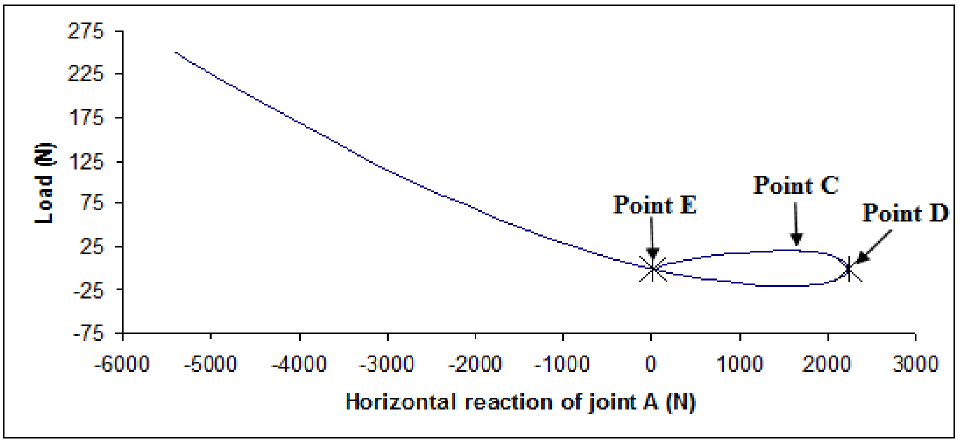

Fig.(5) Load- horizontal reaction of truss support

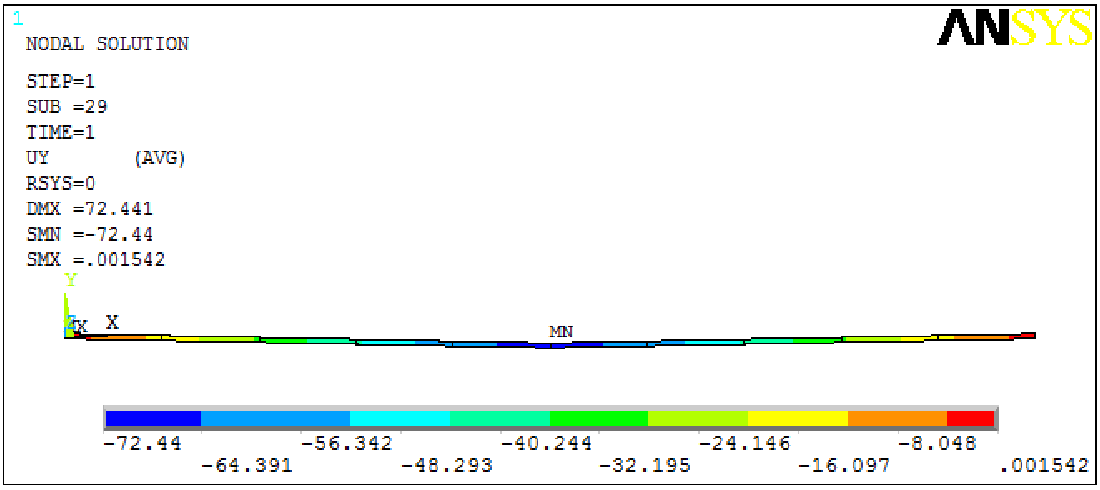

Fig.(6) Variations in vertical displacement for the truss at ultimate load using ANSYS

4.1.2. Effect of Linear Spring on Post Buckling of Two Bar Truss

In order to study the effect of the linear spring on the post buckling behavior of the plane truss in the elastic range, three stiffness values were used (1.5, 3 and 4.5 N/mm) and compared with the case of truss without spring. Fig.(7) shows the two bar truss with linear spring at joint B . The linear spring is modeled in ANSYS using COMBIN14 element, this element has longitudinal or torsional capability in 1-D, 2-D, or 3-D applications and the longitudinal spring-damper option is a uniaxial tension-compression element with up to three degrees of freedom at each node. Fig.(8) shows the geometry of this element [13].

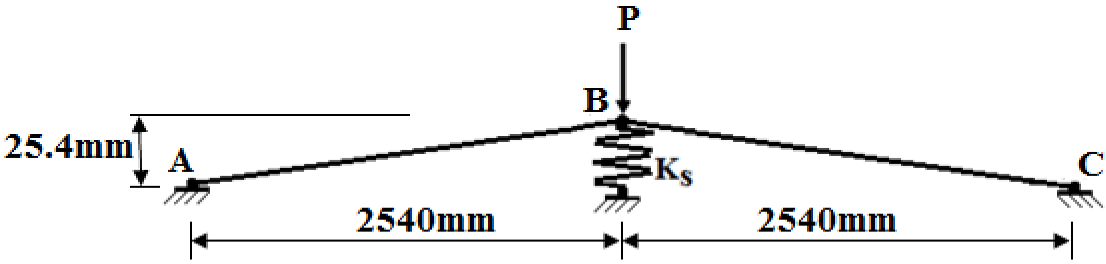

Fig.(7) Geometric and loading of the truss with linear spring

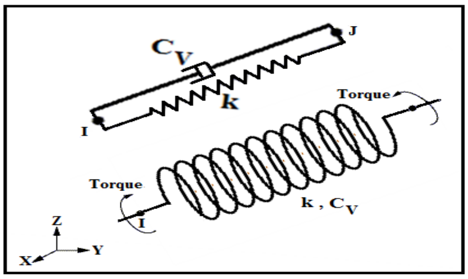

Fig. (8) COMBIN14 element [13]

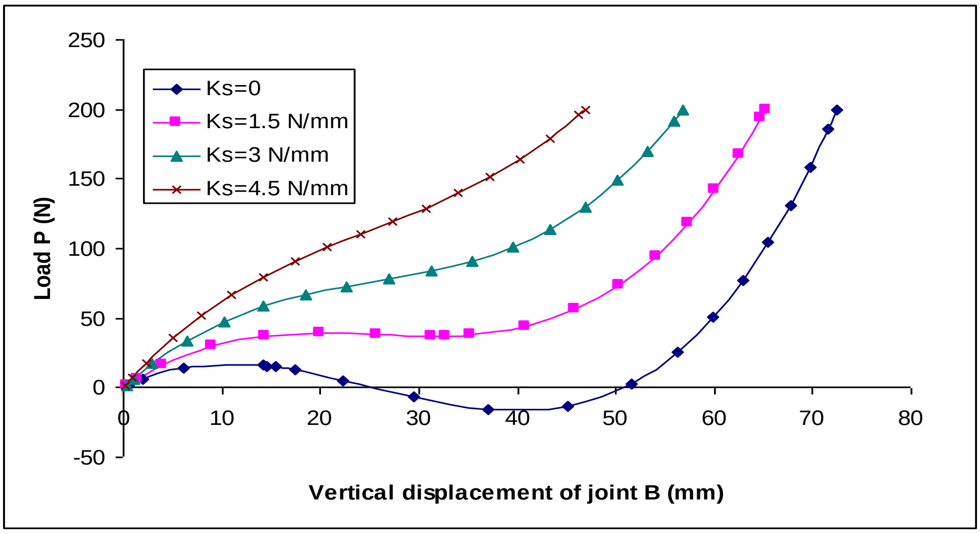

Fig.(9)Effect of linear spring stiffness on load-displacement curve of two bar truss

The relation between the displacement of the joint B and the applied load at that joint are shown in the Fig.(9). This Figure displays the post buckling behavior of the structure. It can be seen that, for Ks=0 curve, after the first peak of load, the displacement is still increasing but the load is reducing until the next negative peak is reached and starts increasing again, this behavior which is called snap through , starts to disappear with increasing stiffness of the spring. However, the load continue in increasing until the ultimate load and no initial peak load appear. This behavior can be attributed to that the force transmitted to the members AB and BC is reduced due to the increase the stiffness of the spring.

4.1.3. Effect of cross-sectional area on Post Buckling of Two Bar Truss

Effect of cross-sectional area on the post buckling behavior was studied, two values of cross-sectional area (1290mm^2 and 1935mm^2) were used in addition to the cross-sectional area which are used in the same example of truss with stiffness of spring equal to zero (k=0) . The results of finite element analysis shown in the Fig.(10). It can be noted that increase cross-sectional area from 645 mm^2 to1290mm^2 leads to increase in initial peak load from 15.9N to 32.8N with increasing percentage 106%, while the ultimate displacement decrease from 72.4mm to 65.2mm with decreasing percentage 11%. On the other hand increase cross-sectional area from 645 mm^2 to1935mm^2 lead to increase in initial peak load from 15.9N to 51.4N with increasing percentage223%, and the ultimate displacement decrease from 72.4mm to 61.7mm with decreasing percentage 17%. Also , it can be noted that the deflection related with initial peak still constant for all cross-sectional areas. Thus, increasing cross-sectional areas has no effect on snap-through.

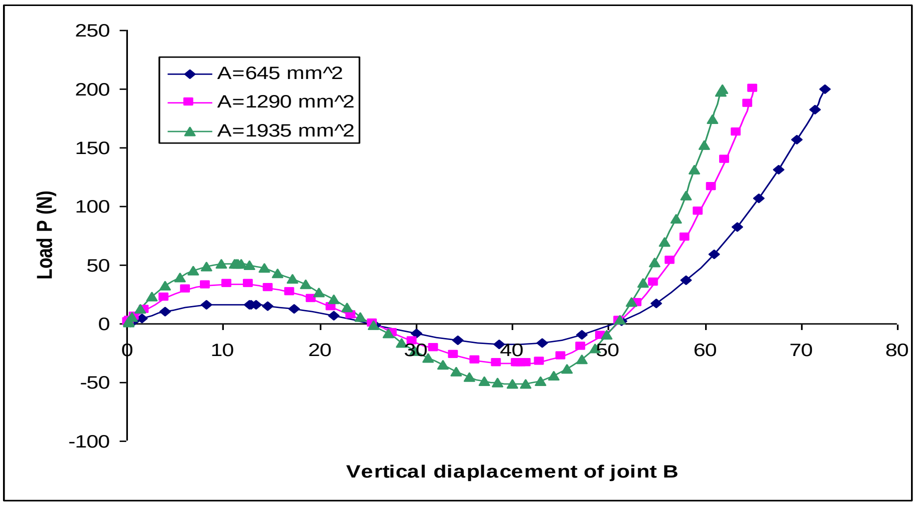

Fig.(10) Effect of cross sectional area on load-displacement curve of two bar truss

4.2. Beams

4.2.1. Cantilever Beam with End Load

The geometry and loading conditions of this beam are shown in the Fig.(11), the modulus of elasticity of the beam is 6.8975 MPa , the cross sectional area is 64516 mm^2 and the moment of inertia is 416231 mm^4. The exact analytical solution , in terms of elliptical integral , is obtained by Bisshopp and Drucker [14] assuming that the beam is perfectly inextensible, in an effort to approximate a similar condition of inextensibility , a relative large value is used for the area of cross section . Al-Mahadawi [4] also analyzed this example using direct load incrementation strategy and arc-length strategy by beam-column theory.

In this study the beam is represent in ANSYS using BEAM188 element. This element is suitable for analyzing slender to moderately stubby/thick beam structures. It is based on Timoshenko beam theory and has six degrees of freedom at each node and it includes stress stiffness terms, this terms enable the elements to analyze flexural, lateral, and torsional stability problems (using Eigen value buckling or collapse studies with arc-length methods) [13]. Fig.(12) shows the geometry of this element and Fig.(13) shows modeling the cantilever beam in ANSYS.

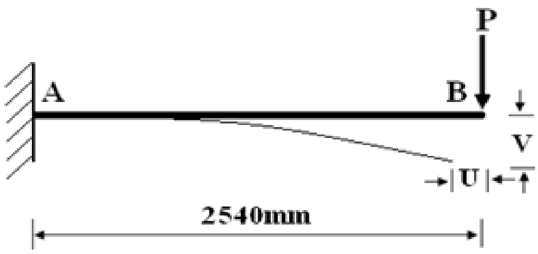

Fig.(11) Geometry and loading of cantilever beam with end load

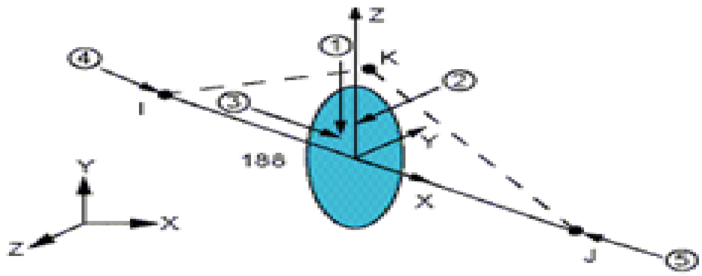

Fig.(12) Geometry of BEAM188 element [13]

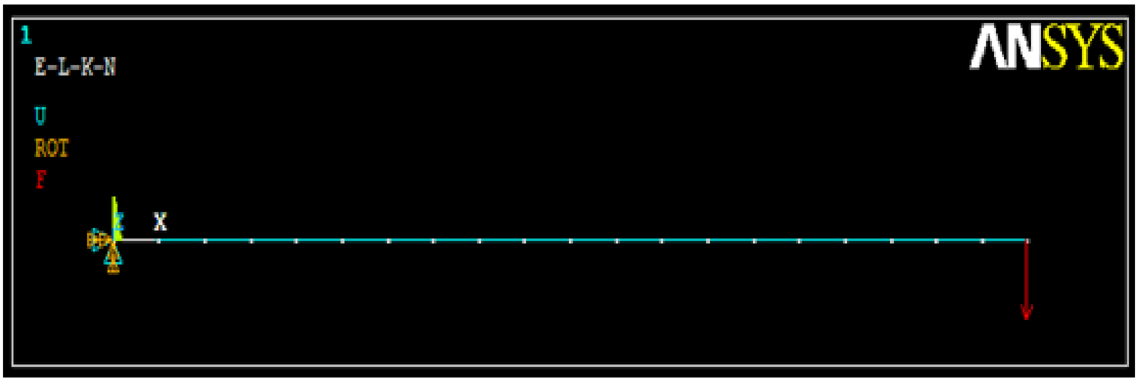

Fig.(13) Modeling the cantilever beam in ANSYS

Fig.(14) shows load-vertical and horizontal displacement ratio of joint B curves in post buckling analysis of this study , Bisshopp and Drucker [14] study and Al-Mahadawi [4] study. It can be seen that the results of this study is with good agreement with previous studies. Fig.(15) shows the beam at original shape and final shape of the buckling and variation of vertical displacement along the beam in ANSYS.

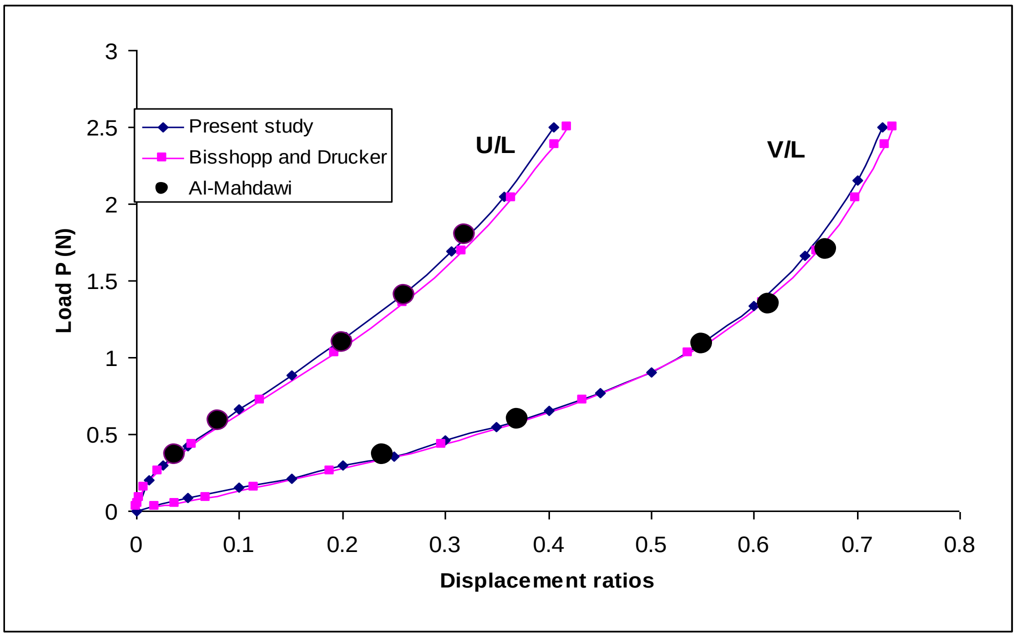

Fig.(14) Load-vertical and horizontal displacement curves of cantilever beam with end load

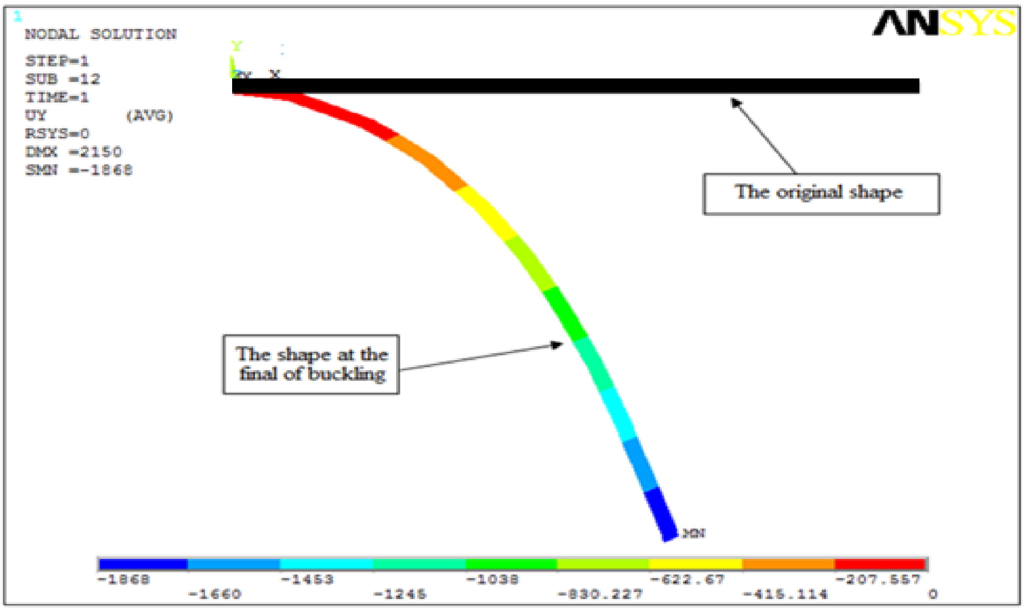

Fig. (15) Variations in vertical and horizontal displacement for the cantilever beam at ultimate load using ANSYS

4.2.1.1. Effect of Cross-Sectional Area on Cantilever Beam with End Load

Fig.(16) and Fig.(17) show effect of cross-sectional area on load-vertical displacement(V) and load-horizontal displacement(U) of joint B respectively. It can be noted that under the same load , increase cross-sectional area from 64516mm2 to 129032mm2 and 193548mm2 lead to decrease in vertical displacement of joint B with percentage 46.5% and 72.7% respectively and decrease in horizontal displacement with percentage 76.5% and 94% respectively. On the other hand Fig.(16) reveals that the relation of load-vertical displacement (V) of cross-sectional area equal to 64516mm2 is concave up and begun to be more linear with increasing cross-sectional area because the beam become more stiffer. On the other hand, Fig.(17) reveals that the relation of load- horizontal displacement (U) of cross-sectional area equal to 64516mm2 begun with concave down curve and turn to concave up with turning load point equal to 0.4N , but for the cross-sectional area129032mm2 the relation consists of two linear parts with turning load point 0.679 N , while for cross-sectional area of 19354mm2 the relation become approximately linear. This behavior can be attributed to the increasing the rigidity of member with increasing cross-sectional area and moment of inertia.

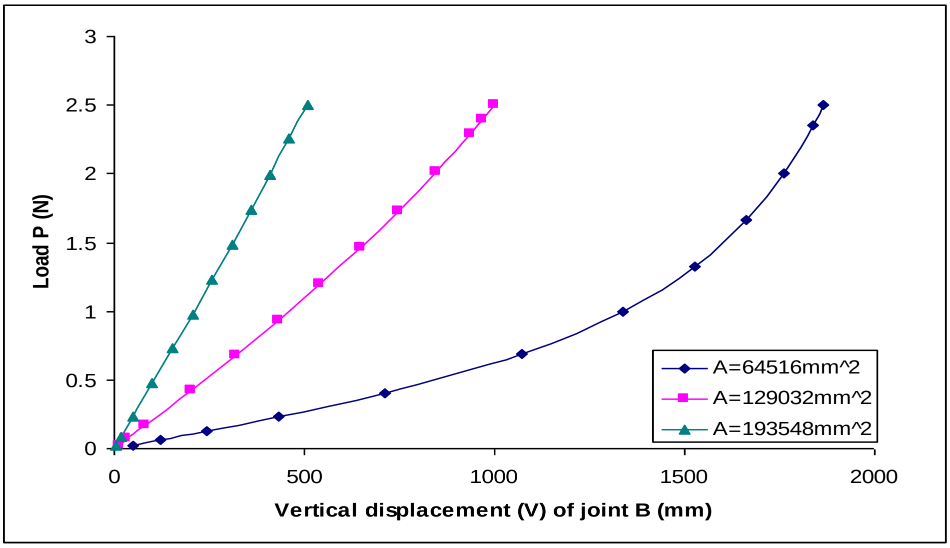

Fig.(16) Effect of cross-sectional area on load-vertical displacement (V) curve

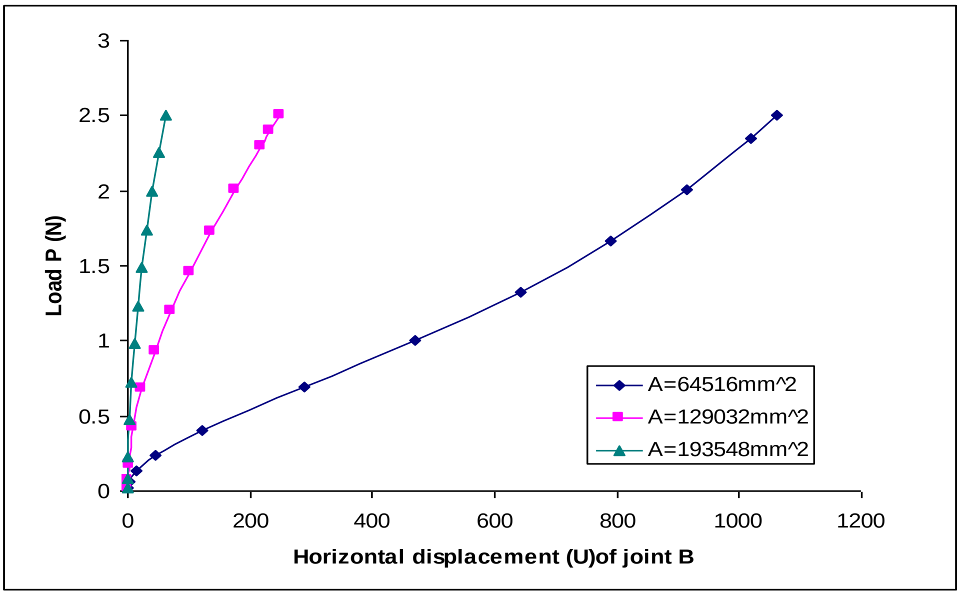

Fig.(17) Effect of cross-sectional area on load-horizontal displacement (U) curve

4.2.1.2. Effect of Modulus of Elasticity on Cantilever Beam with End Load

Fig.(18) and Fig.(19) show effect of modulus of elasticity on load-vertical displacement(V) and load-horizontal displacement(U) of joint B respectively. It can be seen that under the same load , increase modulus of elasticity from 6.8975 MPa to 13.795 MPa and 20.6925 MPa lead to decrease in vertical displacement with percentage 20% and 35.3% and decrease in horizontal displacement of joint B with percentage43% and 64.7%. On the other hand , Fig.(18) reveals that the relation of load-vertical displacement (V) of cross-sectional area equal to 64516mm2 is concave up and begun to be more linear with increasing cross-sectional area. Also, Fig.(19) shows that the relation of load- horizontal displacement (U) of cross-sectional area equal to 64516mm2 begun with concave down curve and turn to concave up with turning load point equal to 0.4N , but for the cross-sectional areas 96774mm2 and 129032mm2 the relation consists of two linear parts with turning load point 0.407Nand 0.589N respectively.

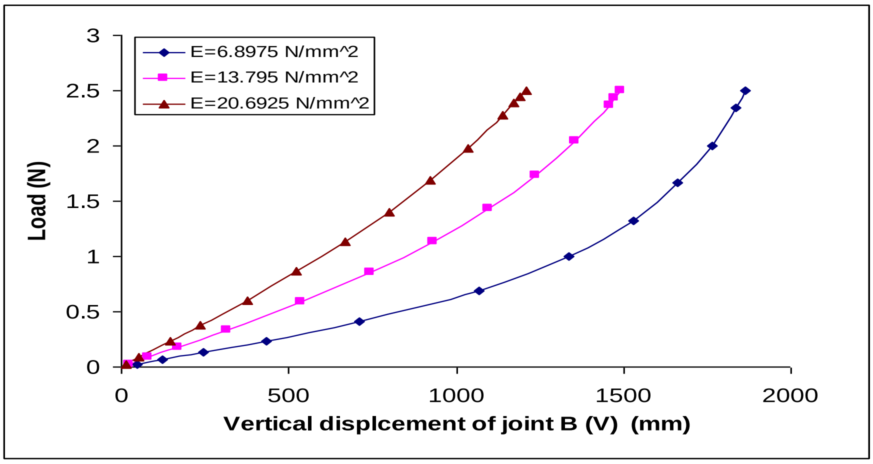

Fig.(18) Effect of modulus of elasticity on vertical displacement (V) curve

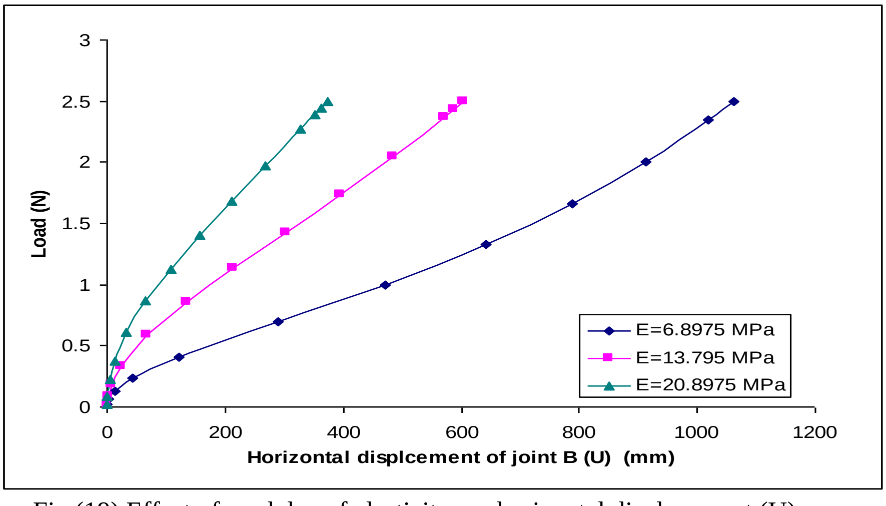

Fig.(19) Effect of modulus of elasticity on horizontal displacement (U) curve

4.2.2. Clamped Beam

The geometry and loading conditions of this example are shown in the Fig.(20). The modulus of elasticity of the beam is 206925 MPa, the cross sectional area is 80.645 mm2 and the moment of inertia is 67.746mm4 . This beam was analyzed by Mondkar and Powell [15] using simple finite element model of the beam consists of five 8-node plane stress elements and the material is assumed to be isotropic and elastic. Al-Mahdawi[4] also analyzed this beam using the direct load incrementation strategy and the constant load iterative strategy.

In this study, clamped beam is modeled in ANSYS using BEAM188 element, the finite element model is shown in the Fig.(21). The load-vertical displacement of joint B curves in post buckling analysis of this study , Mondkar and Powell [15] study and Al-Mahadawi [4] study are shown in the Fig.(22). It can be seen that the results of this study is with good agreement with previous studies. Fig.(23) shows the beam at the original shape and final shape of the buckling and variation of vertical displacement along the beam in ANSYS.

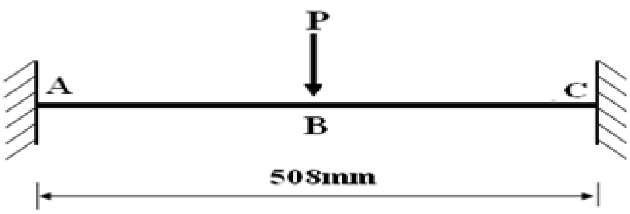

Fig.(20) Geometry and loading of clamped beam

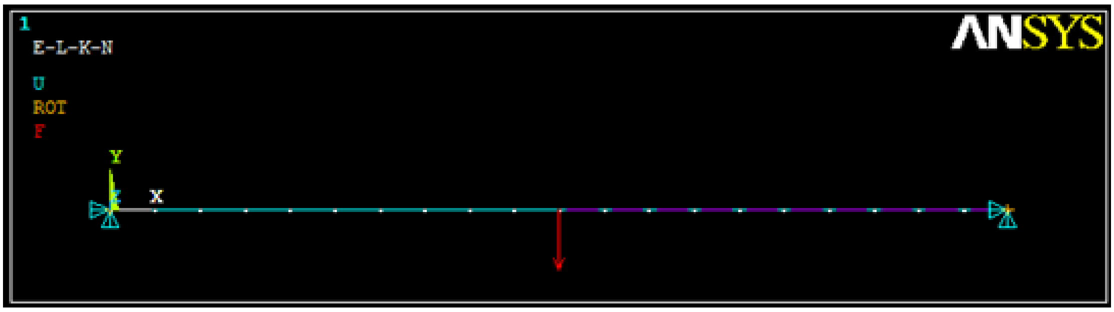

Fig.(21) Modeling the clamped beam in ANSYS

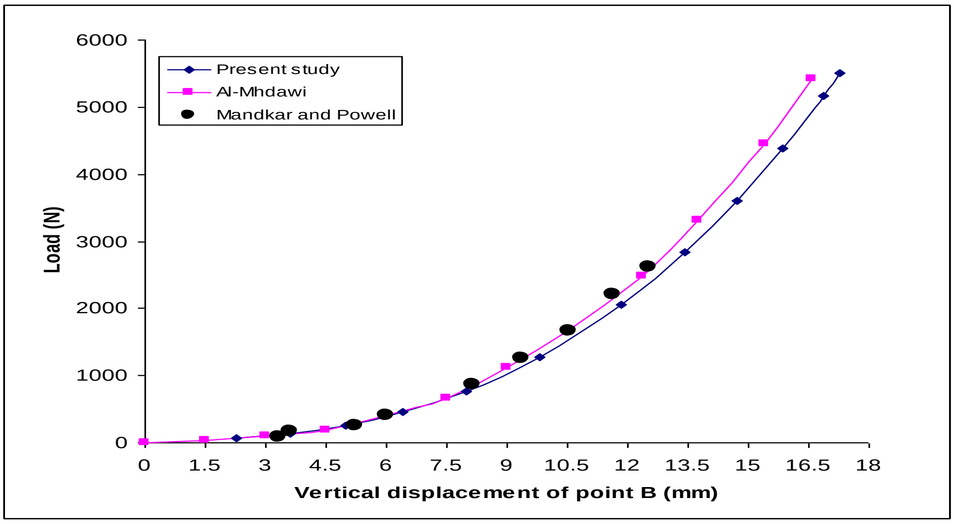

Fig. (22) Load-vertical displacement of joint B of the clamped beam

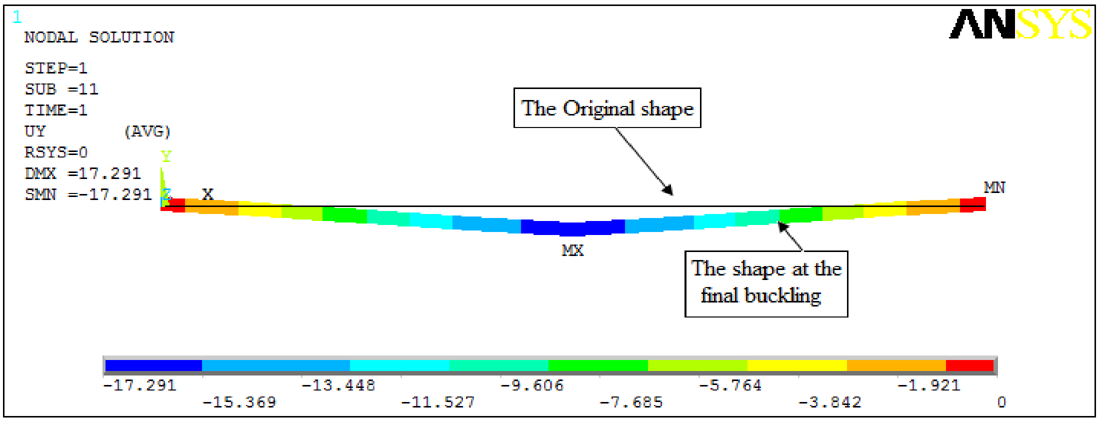

Fig. (23) Variations in vertical displacement for the clamped beam at ultimate load using ANSYS

4.2.2.1 Effect of Cross-Sectional Area on Post Buckling Behavior of Clamped Beam

To investigate the influence of cross-sectional area on post buckling behavior of clamped beam, three values of cross-sectional area (80.645 mm2, 161.29 mm2 and 241.935mm2) were used, Fig.(24) shows the curves of load-vertical displacement (V) of joint B. It can be noted that under the same load , increase cross-sectional area from 80.645 mm2 to 161.29 mm2 and 241.935mm2 lead to decrease in vertical displacement of joint B with percentages 20% and 50.25% respectively.

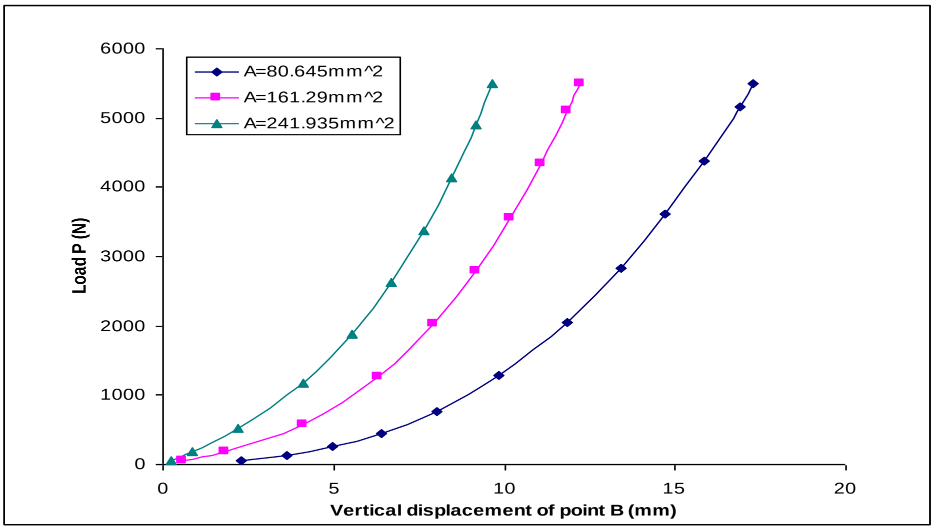

Fig.(24) Effect of cross-sectional area on load-vertical displacement of joint B of clamped beam

4.2.2.2 Effect of Modulus of Elasticity on Post Buckling Behavior of Clamped Beam

Fig.(25) Effect of modulus of elasticity on load-vertical displacement of joint B of clamped beam. It can be noted that under the applied load (5500N) , increase the modulus of elasticity of the beam from 206925 MPa to 413850 MPa and 620775 MPa lead to decrease in vertical displacement of joint B with percentages 23.4% and 34.5% respectively.

It can be noted the next two parameters post buckling behavior of clamped beam that doubling cross-sectional area has more significant effect than that of doubling modulus of elasticity.

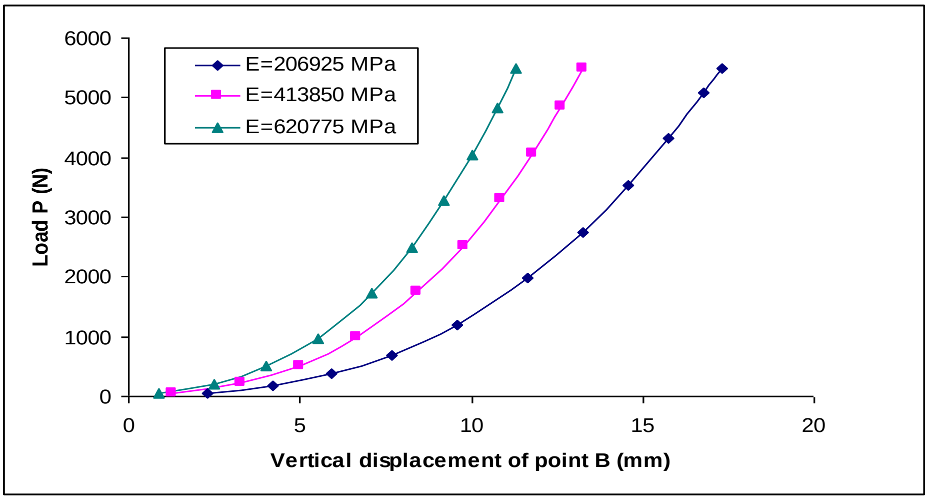

Fig.(25) Effect of modulus of elasticity on load-vertical displacement of joint B of clamped beam

4.3. Frames

4.3.1. Williams' Toggle Frame

Williams' toggle frame has been analyzed by many researchers to verify the numerical accuracy of their studies. The geometry and loading of this frame is shown in the Fig.(26),

The modulus of elasticity of frame parts is 70900 MPa , the cross sectional area is 118 mm2. Williams[16] solved this frame using analytical and experimental tests , in the theoretical work , beam-column model is adopted to describe the behavior of individual members of the frames. Al-Mahadawi [4] also analyzed this example using arc-length strategy by beam-column theory. In this study BEAM188 element was used to modeling this frame in ANSYS.
Fig.(27) shows the load-displacement curves for the vertical displacement of joint B and Fig.(28) shows relation between the external load and horizontal reaction at support . It can be seen that the results of this study is in good agreement with both Williams[16] and Al-Mahadawi [4] results. Fig.(29) shows variations in vertical displacement for the Williams' toggle frame at ultimate load using ANSYS.

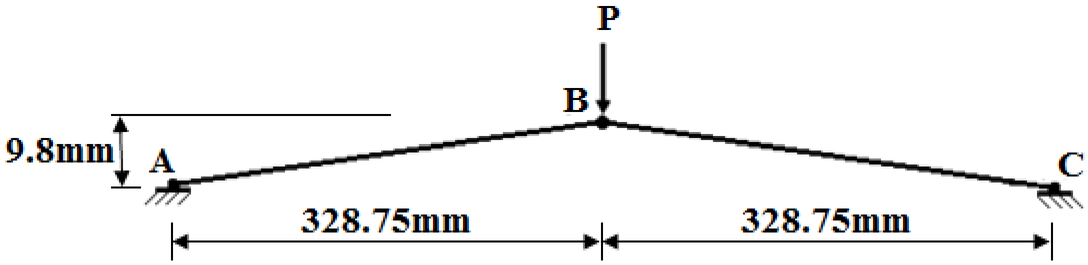

Fig.(26) Geometry and loading Williams' toggle frame

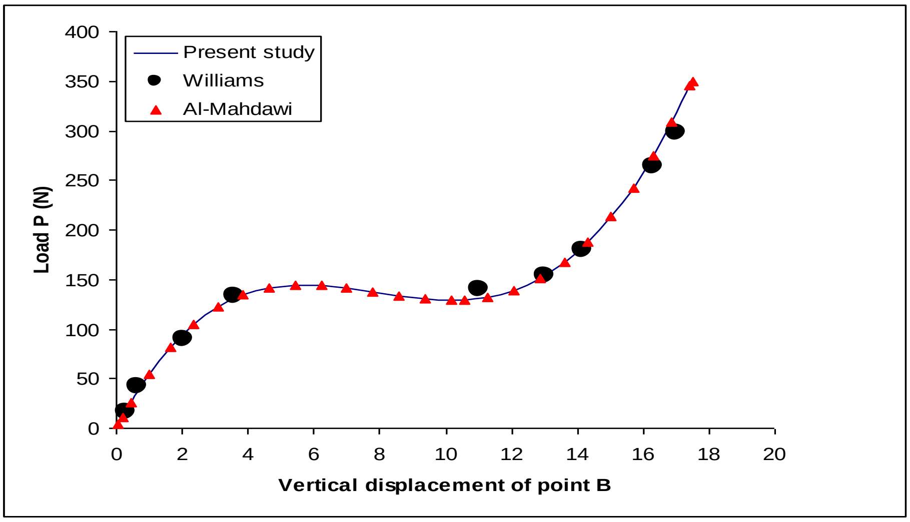

Fig.(27) load - vertical displacement curves at point B for Williams' toggle frame

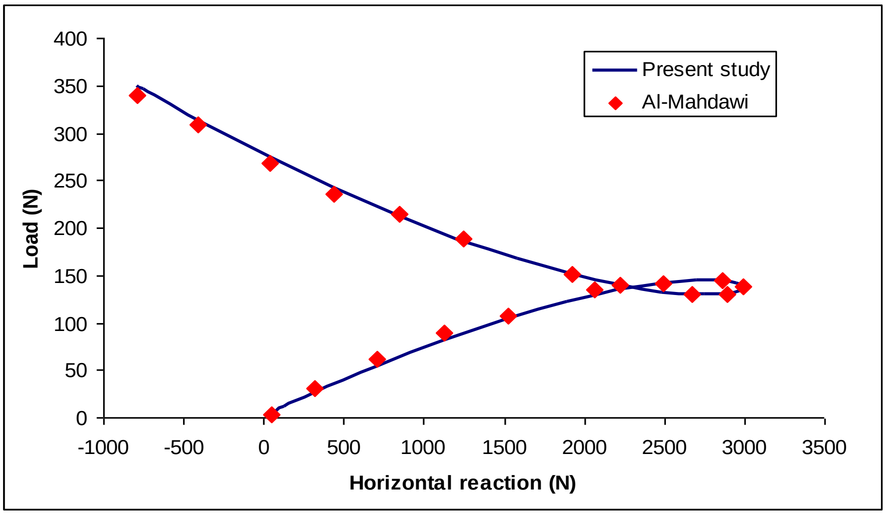

Fig.(28) Load-horizontal reaction curves for Williams' toggle frame

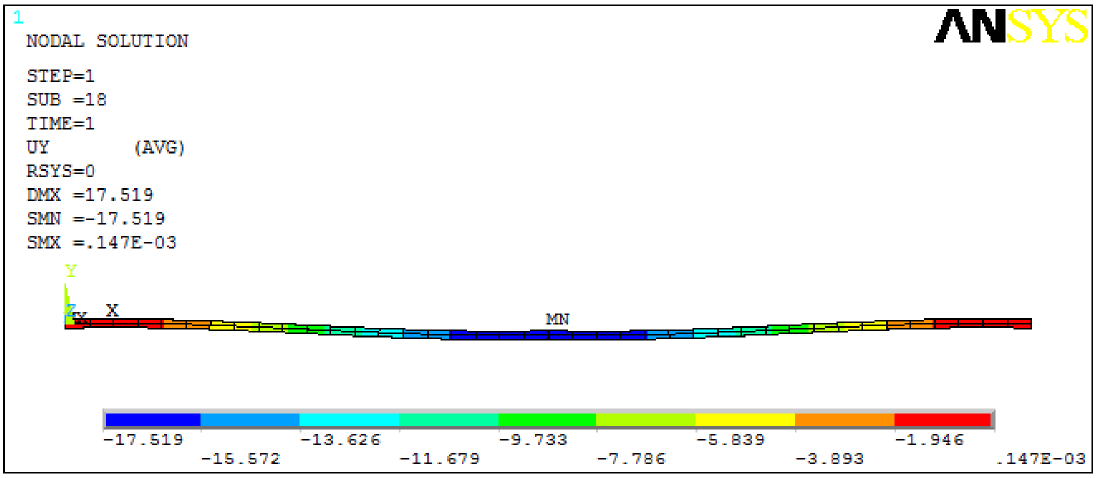

Fig.(29) Variations in vertical displacement for the Williams' toggle frame at ultimate load using ANSYS

4.3.1.1 Effect of Cross-Sectional Area on Post Buckling Behavior of Williams' Toggle Frame

Fig. (30) shows effect of this parameter on load-vertical displacement curves for Williams' toggle frame at joint B, it can be noted that under the same load (2000N) the initial peak load or the snap-through phenomena began to disappear with increasing cross-sectional area , the curves become more stiffer and the ultimate deflection decrease with percentages 25.7% and 37.3% respectively.

In comparison with two bar truss which has the same structure shape , it can be noted that, increasing cross-sectional area for the truss did not effect on snap through phenomena ,while increasing this parameter in Williams' toggle frame resulting in disappear snap through.

4.3.1.2.Effect of Modulus of Elasticity on Post Buckling Behavior of Williams' toggle frame

Fig.(31) shows effect of modulus of elasticity on load-vertical displacement of point B for Williams' toggle frame. It can be noted that under the same load (1000N) the initial ultimate load increase with percentages 50% and 66.7%, also the corresponding displacement increase with percentages 9.4% and 13.2% as the modulus of elasticity increase from 70900 MPa to 141800 MPa and 212700 MPa, while the ultimate displacement decrease with percentages 16% and 24.5%. Also Fig.(31) reveals that snap-through phenomena did not affect by increasing the modulus of elasticity.

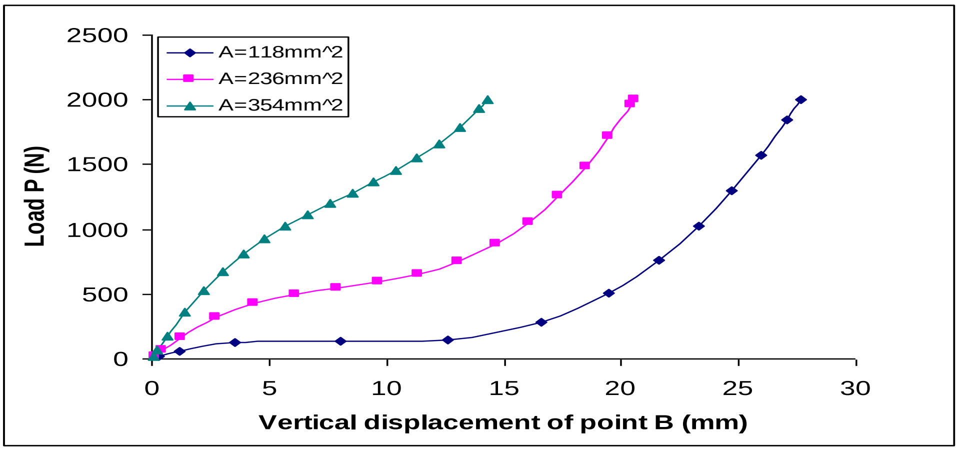

Fig.(30) Effect of cross-sectional area on load-vertical displacement of point B for Williams' toggle Frame

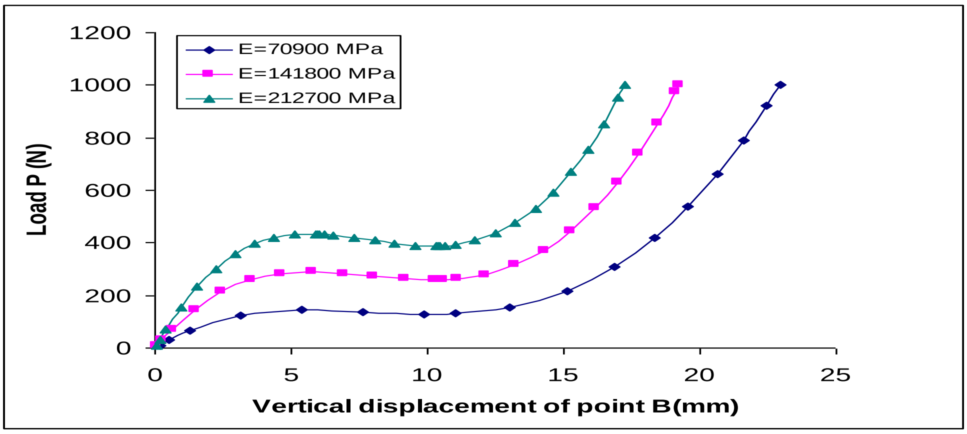

Fig. (31) Effect of modulus of elasticity on load-vertical displacement of point B for Williams' toggle Frame

4.3.2 . Lee's Frame

The geometry and loading of Lee′s frame are shown in the Fig.(32) , The modulus of elasticity of the frame parts is 206925 MPa , the cross sectional area is 7593.53 mm2 and the moment of inertia is 129.0733×106mm4.This frame was analyzed by Fujii et al [17] in 1992 using variable displacement control (modified displacement control) , since its behavior includes load and displacement limit point , the method is based on the way of selecting control displacement. Also, Al-Mahdawi[4] analyzed this frame by beam column theory using the arc-length load incrementation strategy and minimum residual displacement iterative strategy.

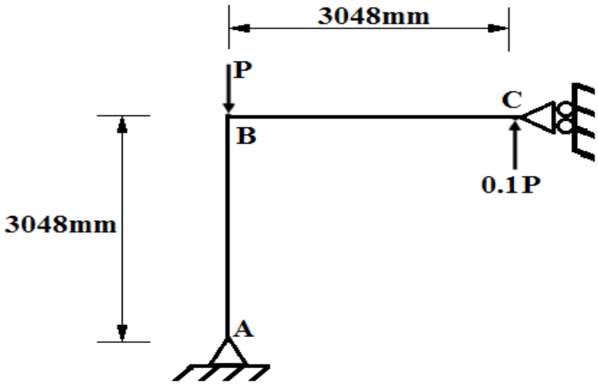

Fig. (32) Geometry and loading of Lee′s frame

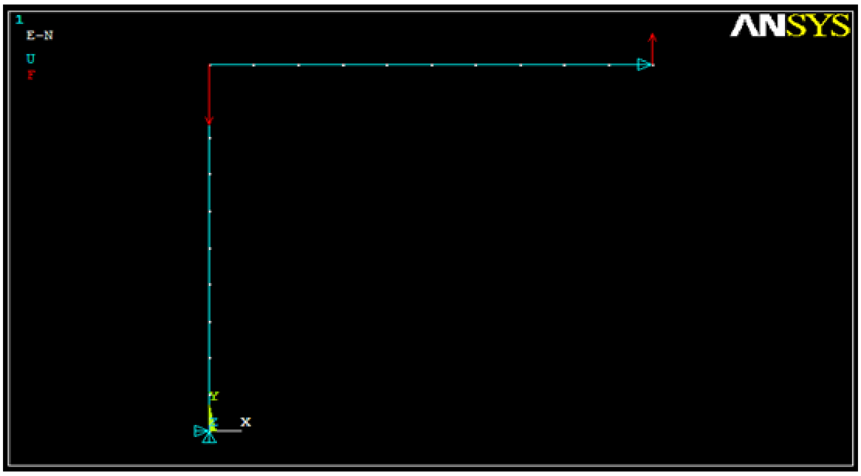

Fig.(33) Modeling the lee′s frame in ANSYS

Fig.(33) shows finite element model of this study using BEAM188 element in ANSYS programs and Fig.(34) shows load-vertical displacement of joint C curves in post buckling analysis of this study , Fujii et al [17] study and Al-Mahadawi [4] . It can be seen that the results of this study is with good agreement with previous studies. Fig.(35) shows the frame at the original shape , the shape at ultimate load, and variation of vertical displacement along the frame.

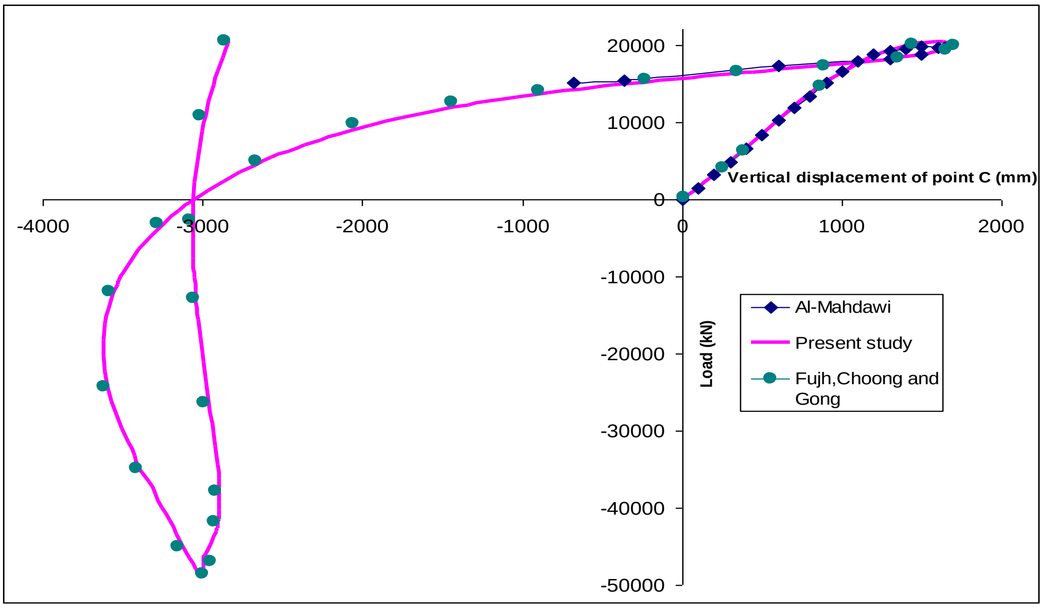

Fig.(34) Load-vertical displacement of point C for lee′s frame

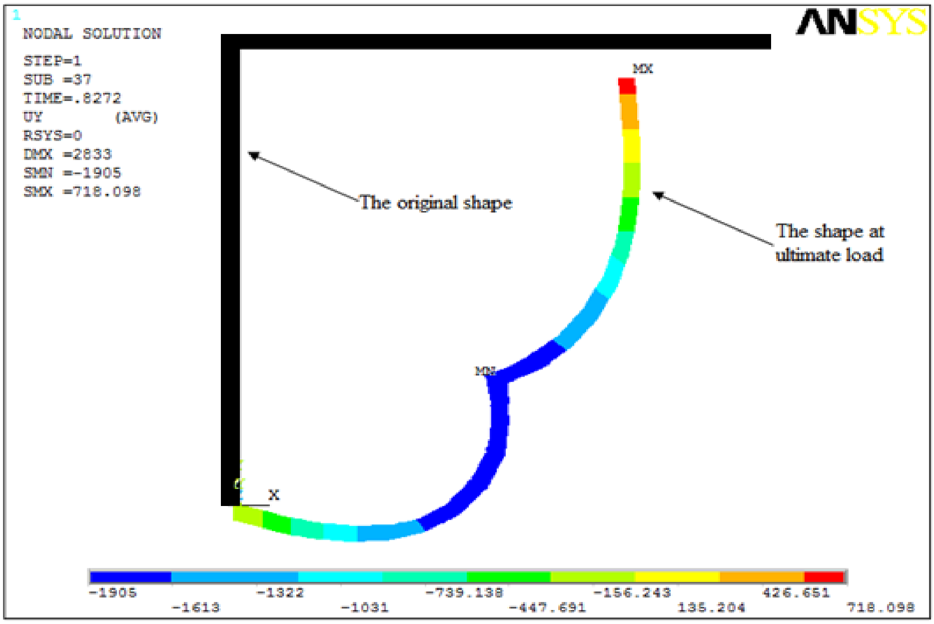

Fig. (35) Variations in vertical displacement for lee′s frame at ultimate load using ANSYS

4.3.2.1 Effect of Cross-Sectional Area on Post Buckling Behavior of Lee's Frame

Fig.(36) shows effect of cross-sectional area on load displacement of point C in post buckling behavior. It can be noted that as cross-sectional area increase from 7593.53 to15187.06 and 22780.59mm2, the initial ultimate load increase with percentages 40.2% and 65% , while the deflection corresponding to the initial ultimate load decrease with percentages 20.3% and 30.5%. On the other hand the ultimate deflection which is in the opposite sense of the deflection corresponding to the initial ultimate load increase with percentages 18.3% and 30.7%.

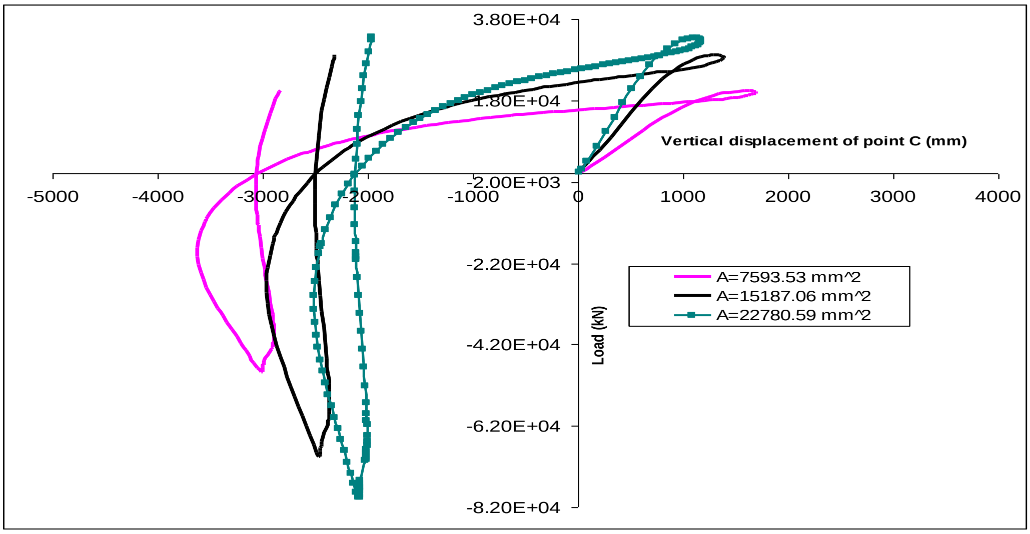

Fig. (36) Effect cross-sectional area on load-vertical displacement of point C for Lee′s frame

4.3.2.2. Effect of Modulus of Elasticity on Post Buckling Behavior of Lee's Frame

Effect of modulus of elasticity on post buckling behavior of lee′s frame was studied. Three values of modulus of elasticity (206925 MPa, 413850 MPa and 620775 MPa) were used. Fig.(37) shows effect of modulus of elasticity on load displacement of point C. It can be noted that as modulus of elasticity increase from 206925MPa to 413850MPa and 620775 MPa , the initial ultimate load increase with percentages 25% and 47% , while the deflection corresponding to the initial peak load decrease with percentages 7% and 19%. On the other hand the peak deflection which is in the opposite sense of the deflection corresponding to the initial peak load increase with percentages 6% and 18%.

Thus , cross-sectional area has more significant effect on post buckling behavior than that of modulus of elasticity.

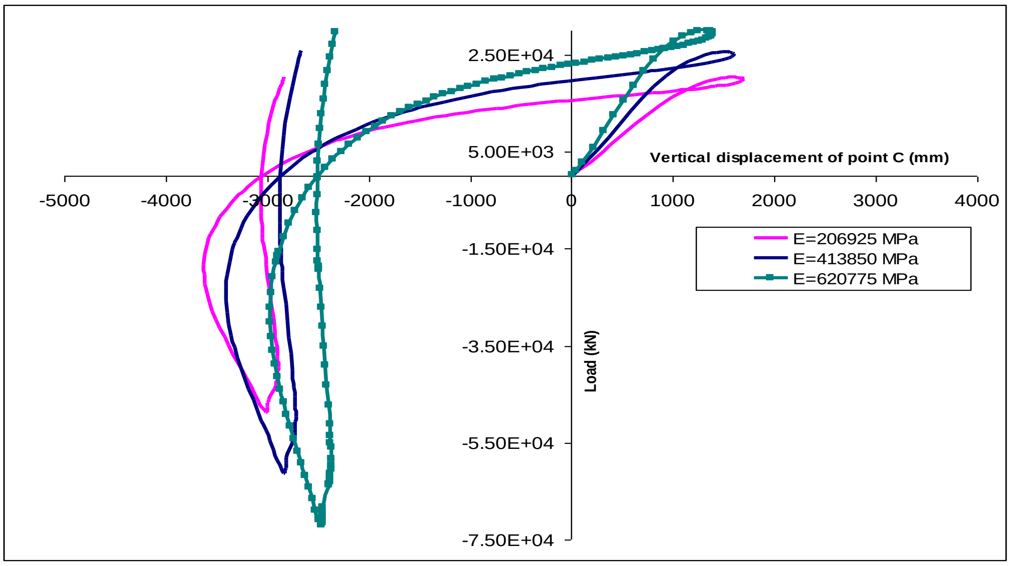

Fig. (37) Effect modulus of elasticity on load-vertical displacement of point C for Lee′s frame

5. Conclusion

The results of the verified examples show good agreement with results in the literature. It was observed that the proposed finite element models using ANSYS computer program is reliable and accurate for studying the post-buckling behavior of deformed plane structure and is applicable to various nonlinear problems.

From the parametric study on truss , beam and frame, it can be observed that

1. For two bar truss with linear spring, it is found that increasing stiffness of the linear spring results in disappear the initial peak load and snap through phenomena, while increasing the cross-sectional area lead to increase the initial peak load with constant deflection corresponding to the initial peak load and decrease the ultimate displacement with maintain snap through phenomena.

2. For cantilever beam with end load it is found that, increasing cross-sectional area from 64516mm2 to 193548 mm2 results in decrease in the vertical displacement of free end with percentage 72.7% and decrease in the horizontal displacement with percentage 94%. While increase modulus of elasticity from 6.8975 to 20.6925 MPa results in decrease in the vertical displacement of free end with percentage 35.3% and decrease in the horizontal displacement with percentage 64.7%.

3. For clamped beam It is found that , increase cross-sectional area from 80.645 mm2 to 241.935mm2 lead to decrease in vertical displacement of midspan with percentage 50.25%. While increase the modulus of elasticity of the beam from 206925 MPa to 620775 MPa lead to decrease in vertical displacement of mid span with percentage 34.5%.

4. For Williams' toggle frame, it is found that the snap-through phenomena began to disappear with increasing cross-sectional area , the curves become more stiffer and the ultimate deflection decrease. While increase modulus of elasticity has no effect on snap-through phenomena and resulting in increase the initial peak load , increase the deflection corresponding with initial peak load and decrease the ultimate deflection.

5. Increasing cross-sectional area for two bar truss which has the same shape of Williams' toggle frame did not effect on snap through phenomena ,while increasing this parameter in Williams' toggle frame resulting in disappear snap through. This behavior can be attributed to increase in moment of inertia of Williams' toggle frame.

6. For Lee′s frame, it is found that, increasing cross-sectional area and increasing modulus of elasticity has the same effect on the shape of load-vertical displacement curve but effect of increasing cross-sectional area is more significant on increase the initial peak load, increase

ultimate load , decrease the deflection corresponding the initial peak load and decrease ultimate deflection than that of modulus of elasticity.

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