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Calculation of the Form Factors for ^{7}Li Nucleus with nħω Excitation

حساب عوامل التشكل النووي لنواة الليثيوم -7 المستثارة الى عدة طاقات

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Abstract:

Longitudinal and transverse an electron scattering form factors of ^{7}Li nucleus are calculated with large-basis shell model. This space included the four shells 1s, 1p, 2s-1d, 2p-1f with (0+2)ħω truncation. The large-basis calculation compared with calculations performed with one particle-one hole excitation up to 12$\hbar\omega$. One particle-one hole excitation up to 12$\hbar\omega$ calculation seem to be essential for giving best description to experimental data in a transverse and longitudinal form factors and obtained the second maximum. The large-basis calculation, especially the transverse form factors are improving a good agreement to description the experimental data at momentum transfers up to q= 4 fm^{-1}.

Keywords: P-shell nuclei, Core-polarization effects, Oxbash, Nuclear structure.

الخلاصة:

تم حساب عوامل التشكل للاستطارة الالكترونية الطولية والمستعرضة في نواة الليثوم -7 باستخدام توسيع فضاء نموذج القشرة. هذا الفضاء تضمن اربع قشرات هي 1s, 1p, 2s-1d, 2p-1f بقطع مقداره ħω (2+0). حسابات توسيع الفضاء قورنت بحسابات نفذت باستثارة جسيم واحد – فجوه واحدة لغاية 12ħω. حسابات جسيم واحد – فجوه واحدة لغاية 12ħω تبدو ضرورية للحصول على افضل وصف للقيم العملية في عوامل التشكل الطولية والمستعرضة وكذلك لملاحظة القمة الثانية. حسابات توسيع الفضاء وبشكل خاص لعوامل التشكل المستعرضة حسنت التوافق بشكل جيد لوصف القيم العملية لغاية زخم منتقل 4 فيرمي$^{-1}$.

1. Introduction

Electron scattering has been widely used as a probe of nuclear structure [1]. Shell model calculations are carried out within a model space in which the nucleons are restricted to occupy a few orbits, which are sometime unable to reproduce the experimental data without scaling factors. Thus transition rates or electron scattering form factors in the p-shell are not explained by the simple shell model, when a few nucleons are allowed to be distributed over the p-shell orbits outside a closed ^{4}He core. For p-shell nuclei, Cohen-Kurth[2] derived single-particle energies and an effective interaction by fitting observed energy level. Lichtenstadt et. al. (1990) [3] have been measured the electromagnetic form factors over the momentum transfer range 0.8≤9≤4.2 fm^{-1} at excitation energy 4.63MeV (J^{π} =7\2^{-}) in ^{7}Li nucleus. This form factors compared with the ground-state which doublet indicates that high multiples, not allowed within the 1p-shell, may make significant contributions to the transverse form factors but not in the longitudinal ones. Unkelbach and Hofmann (1991) [4] have calculated electromagnetic form factors for the transitions into the three lowest ^{7}Li states with using meson exchange current (MEC). The calculation reproduces the measured form factor up to 3fm^{-1}. For low and medium moment transfer the MEC give miner modification for all transitions. Booten et al. (1992) [5] calculations have been performed in the

complete (0+2)$\hbar\omega$ model space as well as in the 0p-shell model space for ^{7}Li nucleus. The extended model space gives good agreement with experiment for all calculated form factors up to momentum transfer 3 fm^{-1}. The MEC enhance the transverse form factors only at large momentum transfer. Karataglidis et al. (1997)[6] calculated elastic and inelastic electron scattering form factors from in ^{6,7}Li nuclei, which analyzed using wave functions within the 0$\hbar\omega$, (0+2)$\hbar\omega$, (0+2+4)$\hbar\omega$, and (0+2+4+6)$\hbar\omega$ model spaces. The result obtained in the 0$\hbar\omega$ space using the Cohen-Kurth (CK) wave functions fails to reproduce the magnitude of the data and inclusion of higher $\hbar\omega$ components supplies the necessary strength to reproduce the data.Radhi et. al. (2001) [7] calculated the longitudinal election scattering form factors of 1p-shell nuclei, using the core-polarization(CP) effect up to 6$\hbar\omega$. Their results with inclusion of core-polarization enhances the form factors and brings the results closer to the experimental data. Radhi et. al. (2007) [8] calculated the elastic and inelastic electron scattering form factors for p-shell nuclei using enlarge psd model space. The two body Milliner-Curath interaction were used for psd orbits that included 1p and 2s-1d shells. The calculations with enlarged model space, including core-polarization effects improve the agreement with the experimental data. Al saad et.al.(2012)[9] calculated the longitudinal form factors of C2 transitions in ^{7}Li for higher p-shells configurations from multi-ħω. The results obtained using admixture of the harmonic oscillator shells (1p+2p+3p) and (1p+2p+3p+4p) in the initial and final wave functions of ^{7}Li improve the agreement with experimental data. Al saad.(2014) [10] calculated the longitudinal form factors of the excited states (J^{π} *=1/2*$^{-}$, *T = 1/2*)and (J^{π} *=7/2*$^{-}$, *T = 1/2*) in ^{7}Li nucleus, using the 1p-shell model wave functions taking into account the core-polarization with effects up to 32$\hbar\omega$ through MSDI and M3Y residual interactions. The calculation form factors showed that the MSDI potential enhanced the second lobe, while the realistic potential M3Y fails.

The purpose of the present work is to observe and describe the contribution of the core nucleon to the model space calculation form factors. This contribution has been analyzed using large-basis shell model calculations (all nucleons are active). The calculated with large space (spsdpf-shell) up to (0+2)$\hbar\omega$ using wbm interaction[11] presented for ^{7}Li nuclei. The transverse and longitudinal electron scattering form factor calculations compared with other calculated using the first order core-polarization effects. In the first order calculations one particle-one hole excitation from the major shell are taken into account up to 12$\hbar\omega$.

2. Theory

The electron scattering longitudinal and transverse form factors for a given multipolarity Λ and momentum transfer q is expressed as[12],

$$\left|F_{\Lambda}^{\eta}(q)\right|^{2}=\frac{4\pi}{z^{2}(2J+1)}\left|\langle J_f\|\|\hat{T}_{\Lambda}^{\eta}\|\|J_i\rangle\right|^{2}|F_{f.s}F_{c.m}|^{2} \quad (1)$$

Where $F_{f.s}(q)=\left[[1+(\frac{q}{4.33}fm^{-1})^{2}\right]^{-2}$[13] is the finite nucleon-size correction, $F_{c.m}=\frac{q^{2}b^{2}}{4A}$ is the center of mass correction, A is the mass number and b is the harmonic oscillator size parameter.

The reduced matrix elements of the electron scattering operator $\hat{T}_{\Lambda}$ can be written as[14]

$$\langle J_f\|\|\hat{T}_{\Lambda}^{\eta}\|\|J_i\rangle=\sum_{\alpha_f,\alpha_i}\chi_{J_fJ_i}^{\Lambda}(\alpha_f\alpha_i)\langle\alpha_f\|\|\hat{T}_{\Lambda}\|\|\alpha_i\rangle \quad (2)$$

where α_f and α_i label single-particle states (isospin is included) for the shell model space.

The One body density matrix (OBDM) is given by

$$\chi_{J_fJ_i}^{\Lambda}(\alpha_f\alpha_i)=\frac{\langle J_f\|\|[a^{+}(\alpha_f\otimes\tilde{a}(\alpha_i)]^{\Lambda}\|\|J_i\rangle}{\sqrt{2\Lambda+1}} \quad (3)$$

The single particle matrix element in the first order perturbation theory can be expressed as[15]

$$\langle\alpha|\hat{T}_J^{\eta}|\beta\rangle = \left\langle\alpha\left|\hat{T}_J^{\eta}\frac{Q}{E-H^{(0)}}V_{res}\right|\beta\right\rangle + \left\langle\alpha\left|V_{res}\frac{Q}{E-H^{(0)}}\hat{T}_J^{\eta}\right|\beta\right\rangle \quad (4)$$

Here V_{res} is a residual nucleon-nucleon interaction, and might be simplified as fallows [15]

$$\left\langle\alpha\left|\hat{T}_J^{\eta}\frac{Q}{E-H^{(0)}}V_{res}\right|\beta\right\rangle = \sum_{\alpha_1\alpha_2\,\Gamma}\frac{(-1)^{\beta+\alpha_2+\Gamma}}{e_\beta - e_\alpha - e_{\alpha_1}+e_{\alpha_2}}(2\Gamma+1)\begin{Bmatrix}\alpha & \beta & \Lambda\\ \alpha_2 & \alpha_1 & \Gamma\end{Bmatrix}\langle\alpha\alpha_1|V_{res}|\beta\alpha_2\rangle$$

$$\times\langle\alpha_2|\hat{T}_\Lambda^{\eta}|\alpha_1\rangle\sqrt{(1+\delta_{\alpha_1\alpha})(1+\delta_{\alpha_2\beta})}$$

$$+ term\ with\ \alpha_1\ and\ \alpha_2\ exchange\ with\ an\ over\ minus\ sign. \quad (5)$$

Where e_k is the single particle energies k represent α, β, α_1 and α_2 [15]

$$e_{nlj} = \left(2n+\ell-\frac{1}{2}\right)\hbar\omega + \begin{cases}-\frac{1}{2}(\ell+1)\langle f(r)\rangle_{n\ell} & \text{for } j=\ell-\frac{1}{2}\\ \frac{1}{2}\ell\langle f(r)\rangle_{n\ell} & \text{for } j=\ell+\frac{1}{2}\end{cases} \quad (6)$$

With $\langle f(r)\rangle_{n\ell} \approx -20A^{-2/3}$ MeV and $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$

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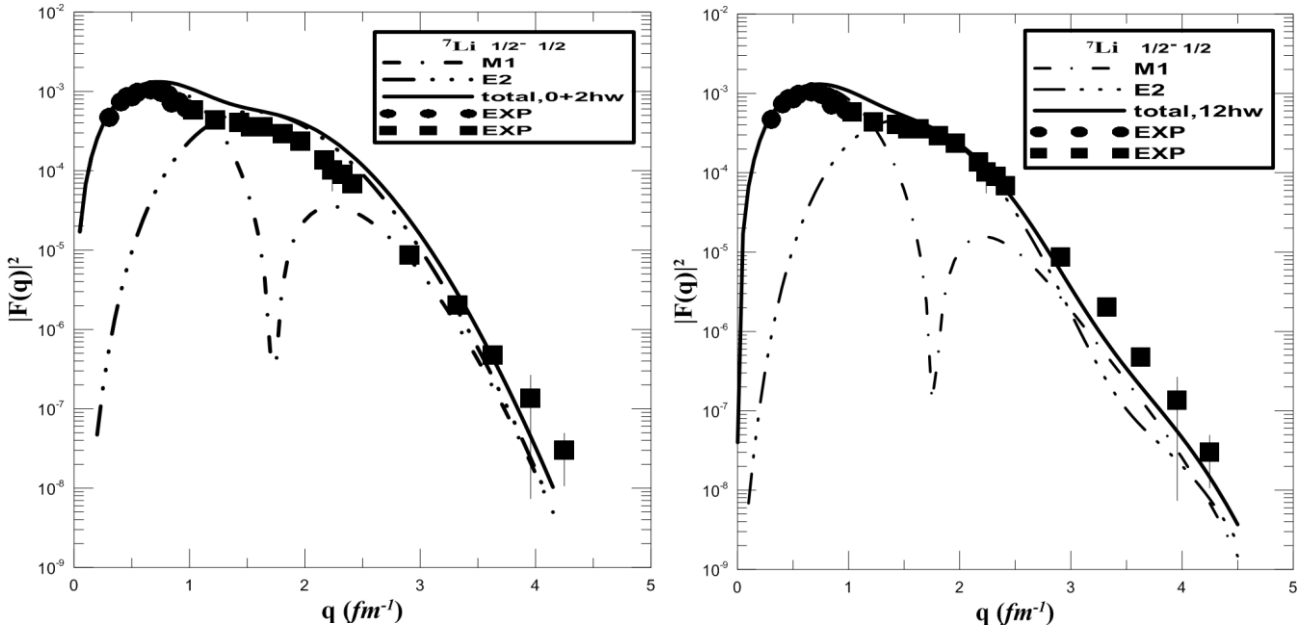
3-Results and Discussion

The longitudinal and transverse form factors for ^{7}Li are calculated in the large-basis model space included the four major shell 1s,1p,2s-1d,2p-1f with (0+2)$\hbar\omega$ truncation.The electron scattering form factors calculated with Warbution and Brown interaction [11] using the shell model code OXBASH[16]. The radial wave function of the single particle matrix element calculated with harmonic oscillator potential. The exacitations from the model space (spsdpf) orbits to higher allowed orbits are also considered up to 12$\hbar\omega$ through first order perturbation theory using the CP-program. The calculated core polarization matrix elements are calculated with MSDI as a residual interaction. The value of MSDI parameters is chosen without any adjusted parameter and equal to $A_0=A_1=B=25/A$ MeV and $C=0$ [15]. The oscillator length parameter $b=1.65fm$[17] is chosen to produce the measured root mean square charge radius. The calculations are presented for the transition from the ground state $J^\pi T= 3/2^- 1/2$ to the states $J^\pi T=1/2^- 1/2$, $3/2^- 1/2$ and $7/2^- 1/2$ with excitation energy 0.478, 0.0, 4.360 MeV respectivelty.

Figure 1 shows the calculations of total transverse M1+E2 and individual contribution form factors from ground state *(3/2$^-$ 1/2)* to the *(1/2$^-$ 1/2)* state in ^{7}Li at excitation energy 0.478MeV. The large-basis calculation of the total (M1+E2) form factors with (0+2)$\hbar\omega$ truncation figure 1.a and the first order perturbation theory upto 12ħω figure1.b are in good agreement with experimental data. The first order perturbation theory upto 12ħω form factors gave good agreement than that calculated form factors with (0+2)$\hbar\omega$ truncation and that calculation upto 6$\hbar\omega$ as shown in figure 2 especially at high momentum transfer.



(a) (b)

Fig. 1 The calculated (M1+E2) transverse form factor for the $J^{\pi}T$= *1/2*$^-$ *1/2*(0.478 MeV) state in ^{7}Li. a.The calculation with truncated(0+2)$\hbar\omega$ and b. That with perturbation theory upto *12*$\hbar\omega$. The data taken from[3, 4]lastictransverse elastic

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Elastic transverse total (M1 +M3) and individual contribution form factor are displayed in figure 3 and compared with the experimental data [3, 4]. In figure 3.a calculated with(0+2)$\hbar\omega$ truncation fails to describe the experimental data especially at high momentum transfer q > 4 fm^{-1}. The calculated form factors perturbed up to 12$\hbar\omega$ figure 3.b improve the agreement with experimental data in all momentum transfer (q).

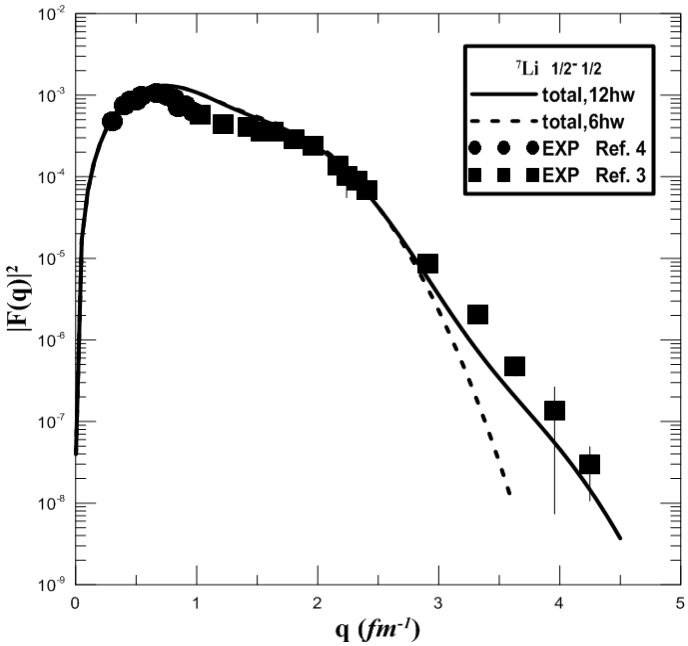
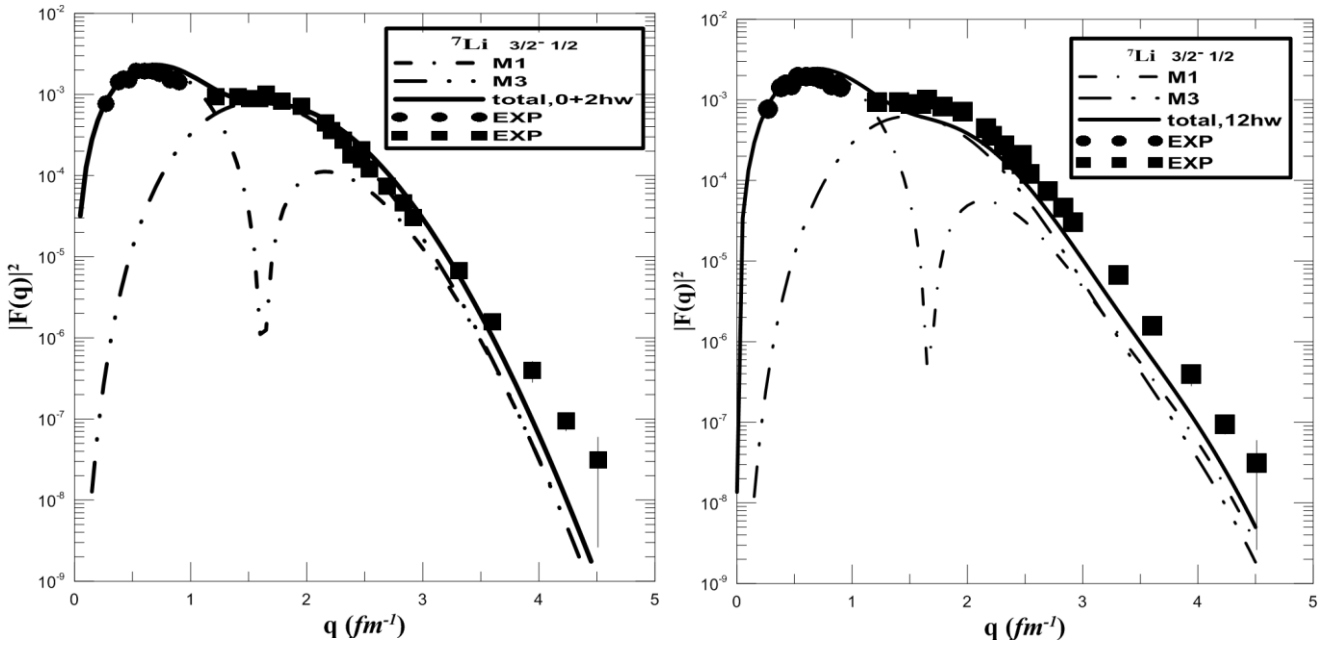


Fig. 2 The calculated (M1+E2) transverse form factor for the $J^{\pi}T$= *1/2*$^-$ 1/2(0.478 MeV) state in ^{7}Li. The calculation with perturbation theory up to *6*$\hbar\omega$(dott curve) and that up to *12*$\hbar\omega$(solid curve). The data taken from[3, 4]lastictransverse elastic



(a) (b)

Fig.3 The calculated total(M1+M3) transverse form factor for the $J^{\pi}T= 3/2^{-}\ 1/2$(0.0 MeV) state in ^{7}Li. a.The calculation with truncated (0+2)ħω and b. That with perturbation theory upto $12\hbar\omega$. The data taken from [3, 4]

From figure 4 the calculation of the perturbed up to $12\hbar\omega$ enhance the form factor incomparison with the calculated up to $6\hbar\omega$ especially at momentum transfer $q> 3$ fm^{-1}.

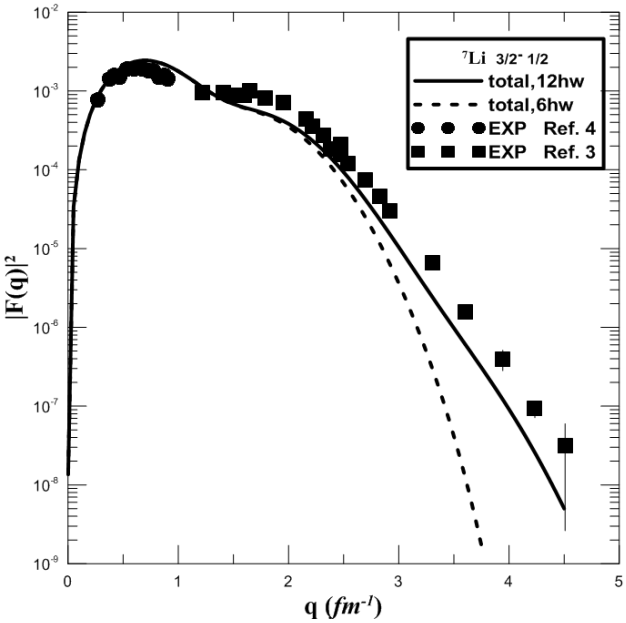
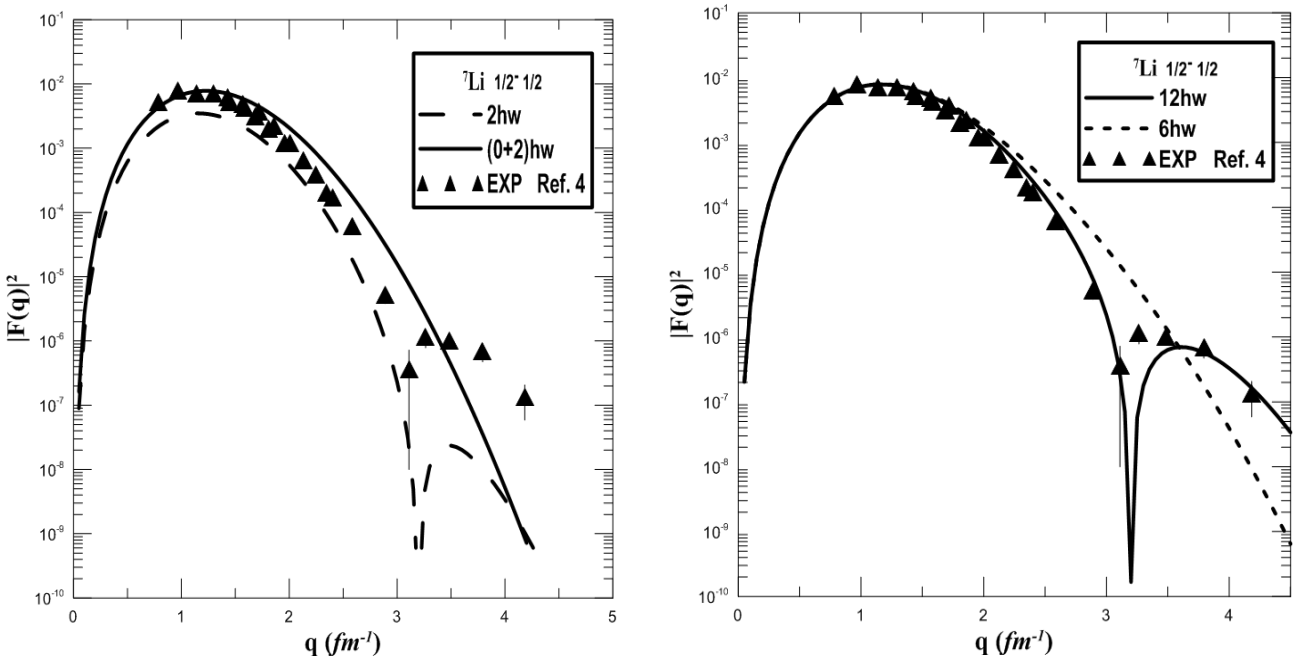


Fig.4 The calculated total(M1+M3) transverse form factor for the $J^{\pi}T= 3/2^{-}\ 1/2$(0.0 MeV) state in ^{7}Li. The calculation with perturbation theory up to $6\hbar\omega$(dott curve) and that up to $12\hbar\omega$ (solid curve). The experimental data taken from[3, 4]l.

The longitudinal C2 transition form factor in ^{7}Li with an excited energy 0.478MeV is displayed in figure 5. The calculation large-basis with truncated (0+2)$\hbar\omega$ form factor fails to describe the experimental data and fail to show the diffraction behavior, you can seen figure 5.a. In figure 5.b the large-basics with truncated 2$\hbar\omega$ are shown the diffraction behavior, but underestimates the data at q >3 fm^{-1}. The calculationof the perturbed up to 12$\hbar\omega$ improve the result and are in agreement with the experimental data. Furthermore, it is shown the minimum diffraction at the correct position in comparison to calculate up to the 6$\hbar\omega$, especially at momentum transfer q > 2.5 fm^{-1}.

Figure 6 shows the calculations longitudinal C2 form factors from ground state *(3/2⁻ 1/2)* to the *(7/2⁻ 1/2)* state in ^{7}Li at excitation energy equal 4.630 MeV. The large-basis calculation form factors with 2, (0+2)$\hbar\omega$ truncation fails to describe the experimental data and diffraction behavior in both curves as shown in figure 6.a. The first order perturbation theory upto *12$\hbar\omega$* figure 6.b are in good agreement with experimental data in all momentum transfer (q). Also, the calculation of the perturbation theory upto *6$\hbar\omega$* fails to describe the experimental data and to reproduce the diffraction structure of the C2 data.



(a) (b)

Fig.5 The inelastic C2 transition form factorto the $J^{\pi}T$= *1/2⁻ 1/2*, E_X=0.478 MeV state in ^{7}Li. a.The calculation with truncated 2, (0+2)ħω and b. That with perturbation theory upto 6, 12$\hbar\omega$.
The data taken from [4].

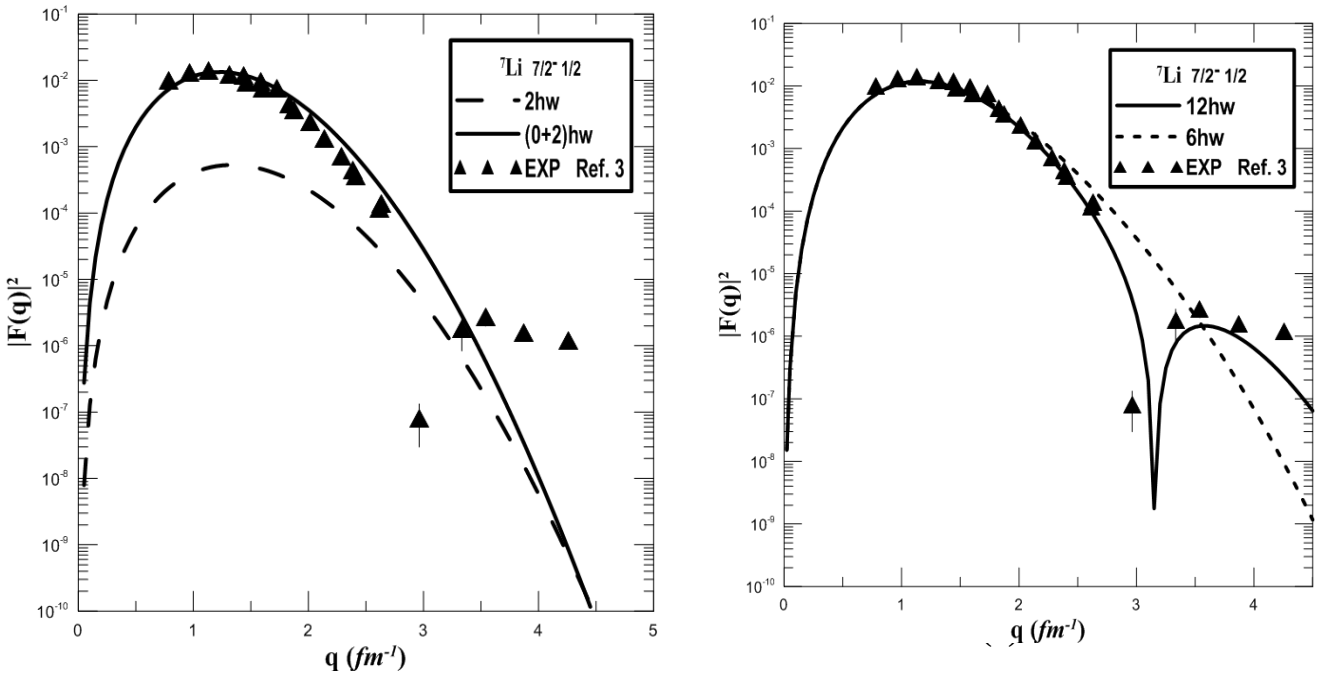


Fig.6 The inelastic C2 transition form factorto the $J^{\pi}T= 7/2^{-}\ 1/2$, E_X=4.630MeV state in ^{7}Li. a.The calculation with truncated 2, (0+2)$\hbar\omega$ and b. That with perturbation theory up to 6, $12\hbar\omega$. The experimental data taken from [3].

4. Conclusions

The large-basis shell model using Warbution and Brown interaction with core polarization effects up to 12$\hbar\omega$ are essential in obtaining a reasonable description of the electron scattering data. The core polarization calculations succeeded in describing the electron scattering data and the locations of the diffraction minimum. The results of the present work show that in general the core polarization effects up to 12$\hbar\omega$ improve the agreement with the experiment data. The calculations of the large-basis shell model to (0+2)$\hbar\omega$ seem to be not sufficient for giving the best description of the form factors data, also can be extended to cover the entire p-shell region.

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