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والعشرون

**P-الجاذبات للمجموعة المغلقة**

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**المستخلص:**

في هذا البحث تمت دراسة مفهوم  $P$  الجاذبات للمجموعة المغلقة وتم تقديم بعض الخصائص الأساسية واثباتها حيث  $P$  هي اي مجموعة شبه تامة في الفضاء الزمني  $T$  و  $K$  تكون اي مجموعة فرعية غير فارغة من  $P$

الكلمات المفتاحية: نظم ديناميكية،  $P$ - لجاذبة، مجموعة مغلقة

**P – Attractors of Closed Sets****Hawraa fadhil Mahdi**

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**Abstract:**

In this paper the concept  $P$  – Attractors of Closed Sets are studied and some basic properties are introduced and proved .where  $P$  is any replete semi group in the time space  $T$  and  $K$  be any a non-empty compact subset of  $P$

**Keywords:** Dynamical System ,  $P$  – attractor , closed set .

**1-Introduction**

Attraction and stability are central to the theories of differential equations and dynamical systems. In 1959, Ura ( Ura ,1959, p143-200) introduced the notion of prolongation sets to characterize some notions of



stability in continuous flows. In (Ahmad, 1970,p 561-574, Ahmad,1971, p157-163), Ahmad used Ura's prolongations to characterize the notion of strong attraction in continuous flows. A systematic use of prolongations in the study of various attraction and stability notions was made by Bhatia and Szego (Bhatia and Szego, 1970, p 56). The theory of topological dynamics has not largely profited from these developments in the last decade, probably due to a major obstacle, namely, the problem of time orientation as it was pointed out by Hajek (Hajek ,1970 , p 79-89) in 1970. In the theory of dynamical systems, most of the dynamical properties are defined for either positive or negative time, whereas in topological dynamics this orientation of time is not available since the phase group of a flow is generally a topological group which lacks any ordering. The first one to tackle the above-mentioned problem was Hajek (Hajek ,1970 , p 79-89). He gave generalizations to some prolongation notions in the setting of flows. Later, Knight (Knight , 1978, p 189-194) applied Hajek's definitions In his study of the flows of characteristic 0. However, Hajek's definitions correspond more or less to the bilateral version of prolongations. In (Elaydi, 1981, p 317-324 ) the author proposed a scheme to lift the unilateral version of all dynamical properties from continuous flows to flows using complete semi-groups ( Elaydi , 1981, p 317-324) of the phase group of a flow.

In this paper,  $P$ -stability is investigated and various notions of  $P$ -attraction and  $P$ -asymptotic stability are introduced. Most of the important results that hold for continuous flows are lifted to flows, under minor restrictions. Let  $(G, X, \pi)$  be a flow, where  $X$  is always assumed to be Banach space,  $T$  is an topological group, and  $\pi: G \times X \rightarrow X$  is a continuous map such that

- 1)  $\pi$  is continuous
- 2)  $\pi(e, x) = x$  ,  $e$  is the identity element of  $G$
- 3)  $\pi(g * h, x) = \pi(g, \pi(h, x))$  ,  $\forall g, h \in G$  ,  $x \in X$

and a replete semi group  $P$  in  $T$

## 2- Definition and Theorem



**Definition(2.1):** A closed set is said to be  $P$  – weakly attractor of the Dynamical System  $\pi$  if there is a  $\delta > 0$  and a net  $\{p_\lambda\}$  in  $P$ ,  $p_\lambda \rightarrow \infty$ , such that

$$\inf_{z \in M} \|\pi(p_\lambda, y) - z\| \rightarrow 0 \text{ whenever } \inf_{z \in M} \|y - z\| < \delta.$$

**Proposition(2.2):** A closed set  $M$  is  $P$  – weak attractor of the Dynamic System  $\pi$  if and only if there is a  $\delta > 0$  such that

$$\Gamma_\pi^P(y) \cap M \neq \emptyset, \text{ whenever } \inf_{z \in M} \|y - z\| < \delta.$$

**Proof:** Let  $M$  is  $P$  –weakly attractor then, there exists a net  $\{p_\lambda\}$  in  $P$  such that  $p_\lambda \rightarrow \infty$  and

$$\inf_{x \in M} \|\pi(p_\lambda, y) - x\| \rightarrow 0 \text{ whenever } \inf_{x \in M} \|y - x\| < \delta.$$

Then there exists  $x_0 \in M$  such that

$$\|\pi(p_\lambda, y) - x_0\| \rightarrow 0.$$

This means that  $x_0 \in \Gamma_\pi^P(y)$ . Hence,  $\Gamma_\pi^P(y) \cap M \neq \emptyset$ .

Conversely, let  $\Gamma_\pi^P(y) \cap M \neq \emptyset$ . Thus, there exists  $x \in \Gamma_\pi^P(y) \cap M$ , so  $x \in \Gamma_\pi^P(y)$  and  $x \in M$ . The definition of  $\Gamma_\pi^P(y)$  yield there exists  $\{p_\lambda\}$  in  $P$  such that  $p_\lambda \rightarrow \infty$  and  $\|\pi(p_\lambda, y) - x\| \rightarrow 0$ . Since,  $x \in M$ , then we conclude that

$$\inf_{x \in M} \|\pi(p_\lambda, y) - x\| \rightarrow 0.$$

**Definition(2.3):** Let  $\mathfrak{D}$  be a universal set. A closed set  $M$  in  $\mathfrak{D}$  is said to  $P$  –attracting in  $\mathfrak{D}$  if

$$\sup_{v \in D} \{\inf_{y \in M} \|\pi(p, v) - y\|\} \rightarrow 0, \text{ as } p \rightarrow \infty, \text{ for all } D \in \mathfrak{D}.$$

**Proposition(2.4):** Let  $\mathfrak{D}$  be a universal random set . A closed set  $M(\omega)$  in  $\mathfrak{D}$  is an  $P$  – attractor of the Dynamical System  $\pi$  in  $\mathfrak{D}$  if and only if

$$\Gamma_\pi^P(D) \neq \emptyset \text{ and } \Gamma_\pi^P(D) \subset M, \text{ for all } D \in \mathfrak{D}.$$

**Proof:** Let  $M$  is a  $P$  – attractor then,

$$\sup_{v \in D} \{\inf_{y \in M} \|\pi(p, v) - y\|\} \rightarrow 0, \text{ as } p \rightarrow \infty.$$

So,

$$\inf_{y \in M} \|\pi(p, v) - y\| \rightarrow 0 \text{ as } p \rightarrow \infty.$$

Specially, there exists a sequence  $\{p_\lambda\}$  in  $P$  such that



$$\inf_{y \in M} \|\pi(p_\lambda, x_\lambda) - y\| \rightarrow 0 \text{ as } p_\lambda \rightarrow \infty.$$

Then there exists  $y_0 \in M$  such that

$$\|\pi(p_\lambda, x_\lambda) - y_0\| \rightarrow 0 \text{ as } p_\lambda \rightarrow \infty.$$

Consequently we have  $y_0 \in \Gamma_\pi^P(D)$ . Hence,  $\Gamma_\pi^P(D) \neq \emptyset$ . If  $z \in \Gamma_\pi^P(D)$ , then there exists  $\{p_\lambda\}$  in  $P$  such that

$$p_\lambda \rightarrow \infty \text{ and } \sup_{v \in D} \|\pi(p_\lambda, v) - z\| \rightarrow 0$$

Now,

$$\inf_{m \in M} \|z - m\| \leq \inf_{v \in D} \|z - v\| + \sup_{v \in D} \inf_{m \in M} \|v - m\|$$

Since,

$$\sup_{v \in D} \inf_{m \in M} \|v - m\| \rightarrow 0$$

and

$$\inf_{v \in D} \|z - v\| \rightarrow 0$$

As  $p_\lambda \rightarrow \infty$ . Then,  $\inf_{m \in M} \|z - m\| = 0$ . Since  $M$  is closed, then  $z \in M$ . Therefore,  $\Gamma_\pi^P(D) \subset M$ .

Conversely, let  $\Gamma_\pi^P(D) \neq \emptyset$  and  $\Gamma_\pi^P(D) \subset M$ . Since,  $\Gamma_\pi^P(D) \neq \emptyset$ , then there exists  $y \in \Gamma_\pi^P(D)$ . So there exists  $\{p_\lambda\}$  in  $P$  such that

$$p_\lambda \rightarrow \infty \text{ and } \sup_{v \in D} \|\pi(p_\lambda, v) - y\| \rightarrow 0$$

Now, since  $y \in \Gamma_\pi^P(D)$  and  $\Gamma_\pi^P(D) \subset M$ , then  $y \in M$ , so  $d(y, M(\omega)) = 0$ . Therefore,

$$\sup_{v \in D} \inf_{m \in M} \|\pi(p_\lambda, v) - m\| \rightarrow 0 \text{ as } p_\lambda \rightarrow \infty.$$

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Hence,  $M$  is  $P$  - attractor .

**Remark(2.5):** If we replace  $D(\omega) = \{x\}$ , then the Definition (2.3) can be reformulated as follows:

**Definition(2.6):** A closed set  $M(\omega)$  is said to be  $P$  -attractor of the dynamical Systems  $\pi$  if there is a  $\delta > 0$  such that

$$\lim_{p \rightarrow \infty} \inf_{x \in M} \|\pi(p, y) - x\| = 0 \text{ whenever } \inf_{x \in M} \|y - x\| < \delta$$

**Definition(2.7):** A closed set  $M(\omega)$  is said to be a uniform  $P$  -attractor of



the Dynamical System  $\pi$  if for each  $\varepsilon > 0$  there exist  $\delta(\varepsilon)$  such that for every  $p \in P - K$

$$\sup_{v \in \gamma^P(x)} \inf_{m \in M} \|v - m\| < \varepsilon, \text{ for } \inf_{m \in M} \|x - m\| \leq \delta.$$

or equivalently,

$$\inf_{m \in M} \|\pi(p_\lambda, x) - m\| < \varepsilon \text{ for } \inf_{m \in M} \|x - m\| \leq \delta.$$

**Definition(2.8):** Let  $M$  be a set compact set in a locally compact space  $X$

(i) The set

$$A_{M,\psi}^*(\omega) := \{x \in X: \exists \text{ a net } \{p_\lambda\} \text{ in } P \ni p_\lambda \rightarrow \infty \text{ and } \inf_{m \in M} \|\pi(p_\lambda, x) - m\| \rightarrow 0\}$$

is called the region of weak attraction of the random set  $M$ .

(ii) The set

$$A_{M,\psi}(\omega) := \{x \in X: \inf_{m \in M} \|\pi(p, x) - m\| \rightarrow 0 \text{ as } p \rightarrow \infty\}$$

is called the region of attraction of the set random  $M$ .

**Notation**

$$(i) \tilde{A}_\pi^*(M) := \{x \in X: \Gamma_\pi^P(x) \cap M(\omega) \neq \emptyset\}$$

$$(ii) \tilde{A}_\pi(M) := \{x \in X: \Gamma_\pi^P(x) \neq \emptyset \text{ and } \Gamma_{x,\psi}^P(\omega) \subset M(\omega)\}.$$

**Theorem (2.9):** Let  $M$  be a random compact set in a locally compact space  $X$ . Then

$$(i) A_\pi^*(M) = \tilde{A}_\pi^*(M).$$

$$(ii) A_\pi(M) = \tilde{A}_\pi(M).$$

**Proof:** (i) let  $x \in A_\pi^*(M)$ , then  $x \in X$  and there exists  $\{p_\lambda\}$  in  $P$  such that

$$p_\lambda \rightarrow \infty \text{ and } \inf_{y \in M} \|\pi(p_\lambda, x) - y\| \rightarrow 0.$$

Then there exists  $y_0 \in M$  such that



$$\|\pi(p_\lambda, x) - y_0\| \rightarrow 0.$$

Consequently, we have there exists  $\{p_\lambda\}$  in  $P$  such that

$$p_\lambda \rightarrow \infty \text{ and } \|\pi(p_\lambda, x) - y_0\| \rightarrow 0.$$

This means that  $y_0 \in \Gamma_\pi^P(x)$ . Hence  $\Gamma_\pi^P(x) \cap M \neq \emptyset$ . It follows that  $x \in \tilde{A}_\pi^*(M)$ .

Conversely, let  $x \in \tilde{A}_\pi^*(M)$ , then  $x \in X$  such that  $\Gamma_\pi^P(x) \cap M \neq \emptyset$ . Thus there exists  $y \in \Gamma_\pi^P(x) \cap M$ , so  $y \in \Gamma_\pi^P(x)$  and  $y \in M$ . The definition of  $\Gamma_\pi^P(x)$  yield there exists  $\{p_\lambda\}$  in  $P$  such that

$$p_\lambda \rightarrow \infty \text{ and } \|\pi(p_\lambda, x) - y\| \rightarrow 0.$$

Since  $y \in M$ , then we conclude that

$$\inf_{y \in M} \|\pi(p_\lambda, x) - y\| \rightarrow 0.$$

Therefore  $x \in A_\pi^*(M)$ . Thus  $A_\pi^*(M) = \tilde{A}_\pi^*(M)$ .

(ii) Let  $x \in A_\pi(M)$ , then  $x \in X$  such that

$$\inf_{y \in M} \|\pi(p, x) - y\| \rightarrow 0 \text{ as } p \rightarrow \infty.$$

So,

$$\|\pi(p, x) - y\| \rightarrow 0 \text{ as } p \rightarrow \infty.$$

Specially, there exists a net  $\{p_\lambda\}$  in  $P$  such that

$$\|\pi(p_\lambda, x) - y\| \rightarrow 0 \text{ as } p_\lambda \rightarrow \infty.$$

Then there exists  $y_0 \in M$  such that

$$\|\pi(p_\lambda, x) - y_0\| \rightarrow 0.$$



Consequently we have there exists  $\{p_\lambda\}$  in  $P$  such that

$$p_\lambda \rightarrow \infty \text{ and } \|\pi(p_\lambda, x) - y_0\| \rightarrow 0.$$

This means that  $y_0 \in \Gamma_\pi^P(x)$ . Hence  $\Gamma_\pi^P(x) \neq \emptyset$ . On the other hand, if  $z \in \Gamma_\pi^P(x)$ , then there exists  $\{p_\lambda\}$  in  $P$  such that

$$p_\lambda \rightarrow \infty \text{ and } \|\pi(p_\lambda, x) - z\| \rightarrow 0.$$

Now, For every  $y \in M$ , we have

$$\|z - y\| \leq \|z - \pi(p_\lambda, x)\| + \|\pi(p_\lambda, x) - y\|.$$

So

$$\inf_{y \in M} \|z - y\| \leq \|z - \pi(p_\lambda, x)\| + \inf_{y \in M} \|\pi(p_\lambda, x) - y\|$$

Since,

$$\|z - \pi(p_\lambda, x)\| \rightarrow 0 \text{ and } \|\pi(p_\lambda, x) - y\| \rightarrow 0 \text{ as } p_\lambda \rightarrow \infty.$$

Then  $\inf_{y \in M} \|z - y\| = 0$ . Since  $M$  is closed, then  $z \in M$ . Therefore

$\Gamma_\pi^P(x) \subset M$ . Thus,  $x \in \tilde{A}_\pi(M)$ .

Conversely, let  $x \in \tilde{A}_\pi(M)$ . Then  $x \in X$  such that  $\Gamma_\pi^P(x) \neq \emptyset$  and  $\Gamma_\pi^P(x) \subset M$ . Since,  $\Gamma_\pi^P(x) \neq \emptyset$ , then there exists  $y \in \Gamma_\pi^P(x)$ . So there exists  $\{p_\lambda\}$  in  $P$  such that

$$p_\lambda \rightarrow \infty \text{ and } \|\pi(p_\lambda, x) - y\| \rightarrow 0.$$

Now, for  $y \in \Gamma_\pi^P(x)$ .

$$\inf_{z \in M} \|\pi(p_\lambda, x) - z\| \leq \|\pi(p_\lambda, x) - y\| + \inf_{z \in M} \|y - z\|.$$



Since,  $\Gamma_{\pi}^P(x) \subset M$ , then  $y \in M$ , so  $\inf_{z \in M} \|y - z\| = 0$ . Therefore

$$\inf_{z \in M} \|\pi(p_{\lambda}, x) - z\| \rightarrow 0 \text{ as } p_{\lambda} \rightarrow \infty.$$

Hence,  $x \in A_{\pi}(M)$ .

**Proposition(2.10):** If  $M$  is  $P$  – attractor, then

$$A_{\pi}^*(M) = A_{\pi}(M) \supset S(M, \delta), \delta > 0.$$

**Proof:** Let  $x \in A_{\pi}(M)$  and let  $M$  is an  $P$  – attractor, then

$$\|\pi(p, x) - y_0\| \rightarrow 0 \text{ and } \Gamma_{\pi}^P(x) \neq \emptyset, \Gamma_{\pi}^P(x) \subset M(\omega).$$

Then, there exists  $y_0 \in \Gamma_{\pi}^P(x)$  such that

$$\|\pi(p_{\lambda}, x) - y_0\| \rightarrow 0, \text{ and } y_0 \in M.$$

Then

$$\inf_{y \in M} \|\pi(p_{\lambda}, x) - y\| \rightarrow 0.$$

Implies that,  $x \in A_{\pi}^*(M)$ .

Conversely, let  $x \in A_{\pi}^*(M)$ , and  $M$  is  $P$  – attractor .Then,  $\Gamma_{\pi}^P(x) \neq \emptyset, \Gamma_{\pi}^P(x) \subset M$ .

Then, the definition of  $\tilde{A}_{\pi}(M)$  implies that  $x \in \tilde{A}_{\pi}(M)$ . Following theorem (2.9),  $A_{\pi}(M) = \tilde{A}_{\pi}(M)$ , implies that  $x \in A_{\pi}(M)$ .

Now, to prove that  $S(M, \delta) \subset A_{\pi}^*(M)$ . Let  $x \in S(M, \delta)$  then

$$\inf_{y \in M} \|x - y\| < \delta.$$

Hence

$$\inf_{y \in M} \|\pi(p_{\lambda}, x) - y\| < \varepsilon.$$

implies that

$$\inf_{y \in M} \|\pi(p_{\lambda}, x) - y\| \rightarrow 0 \text{ as } p_{\lambda} \rightarrow \infty$$

Hence,  $x \in A_{\pi}(M)$ .





**Theorem (2.11):** For any set  $M$ , the set  $A_\pi(M)$  is always  $P$  –invariant set.

**Proof:** If  $x \in A_{M,\psi}(\omega)$  then

$$\inf_{y \in M} \|\pi(p, x) - y\| \rightarrow 0 \rightarrow 0 \text{ as } p \rightarrow \infty.$$

To show that  $\pi(\tau, x) \in A_\pi(M)$  for every  $\tau \in P - K$ . Note that

$$A_\pi(M) := \{x \in X: \inf_{y \in M} \|\pi(p, x) - y\| \rightarrow 0 \text{ as } p \rightarrow \infty\}$$

$$\inf_{y \in M} \|\pi(p, \pi(\tau, x)) - y\| = \inf_{y \in M} \|\pi(p + \tau, x) - y\|$$

$$= \inf_{y \in M} \|\pi(\tau', x) - y\| \rightarrow 0, \text{ as } p + \tau := \tau' \rightarrow \infty.$$

Thus,  $\pi(\tau', x) \in A_\pi(M)$ , implies that,  $A_\pi(M)$  is  $P$  –invariant set.

**Theorem (2.12):** If  $M$  is  $P$  – attractor set, then  $A_{M,\psi}(\omega)$  is open.

**Proof:** As  $M$  is  $P$  –attractor, there is a  $\delta > 0$  such that

$$\inf_{z \in M} \|\pi(p, y) - z\| = 0 \text{ as } p \rightarrow \infty, \text{ whenever } \inf_{z \in M} \|y - z\| < \delta$$

Now, let  $x \in A_\pi(M)$ , and  $\inf_{z \in M} \|x - z\| \geq \delta$  to show that there exists a  $\mu > 0$  such that

$$\{z \in X: \|z - x\| < \mu\} \subset A_\pi(M).$$

To see this, there is a  $\tau \in P - K$  such that

$$\inf_{z \in M} \|\pi(\tau, x) - z\| \leq \delta/2.$$

Choose  $\varepsilon > 0$  such that

$$\{y \in X: \|\pi(\tau, x) - y\| < \varepsilon\} \subset \{z \in X: \inf_{z \in M} \|y - z\| < \delta\}.$$

So,

$$\text{if } \|\pi(\tau, x) - y\| < \varepsilon \text{ then } \inf_{z \in M} \|\pi(\tau, y) - z\| \rightarrow 0 \text{ as } \tau \rightarrow \infty.$$

Set

$$N = \{\pi(-\tau, y): \|\pi(\tau, x) - y\| < \varepsilon\}.$$



Since,  $\pi(-\tau, \cdot): X \rightarrow X$  is homeomorphism, then  $N$  is a open neighborhood of  $x$ . Note that,

$$z \in N \text{ if and only if } \|\pi(\tau, x) - \pi(\tau, z)\| < \varepsilon.$$

Thus,

$$\inf_{z \in M} \|\pi(p, x) - z\| \rightarrow 0 \text{ as } p \rightarrow \infty.$$

This implies that  $A_\pi(M)$  is open set.

**Theorem (2.13):** If  $M \subset X$  is  $P$  –invariant closed set which is uniformly  $P$  –attracting then it is  $P$  –stable.

**Proof:** For any  $\varepsilon > 0$ , there is a  $\delta_M > 0$  such that for every  $\tau \in P - K$ ,

$$\inf_{z \in M} \|\pi(\tau, x) - z\| < \varepsilon \text{ whenever } \inf_{z \in M} \|x - z\| \leq \delta.$$

Therefore

$$x \in O(M) = \bigcup_{x \in M} B(x, \delta_x).$$

Since  $O(M)$  is open, so that,  $M$  is  $P$  –stable .

**Theorem (2.14):** If a set  $M \subset X$  is (uniformly)  $P$  – stable and  $P$  –weakly attracting, then it is  $P$  –attracting.

**Proof:** If the assertion is not true there exists at least one  $\{p_\lambda\}: p_\lambda \rightarrow \infty$ , such that

$$\inf_{z \in M} \|\pi(p_\lambda, x) - z\| \not\rightarrow 0.$$

Whereas there is a net  $\{\tau_\lambda\}, \tau_\lambda \rightarrow \infty$  such that

$$\inf_{z \in M} \|\pi(\tau_\lambda, x) - z\| \rightarrow 0.$$

Assume that  $\{p_\lambda\} \subseteq \{\tau_\lambda\}$ , then

$$\pi(p_\lambda, x) = \pi(p_\lambda - \tau_\lambda, \pi(\tau_\lambda, x))$$



This is a contradiction with Definition (A closed random set  $M$  in  $X$  is said to be  $P$  – stable if for every  $\varepsilon > 0$  and  $x \in M$ , there is a  $\delta \equiv \delta_{x,\varepsilon} > 0$  such that

$\gamma^P(S(x, \delta)) \subseteq S(x, \varepsilon)$ , for every  $t \in P - K$  . This implies that,

$M$  is  $P$  –attracting.

### 3-Conclusions:

The concept of  $P$  – Attractors of Closed Sets has been defined and explored In the context of replete semi-groups.

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