



## IMAGE TRANSMISSION OVER RAYLEIGH FADING CHANNELS USING TURBO CODES WITH MULTI-DIMENSIONAL CHAOTIC INTERLEAVERS

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**Abstract:** It is well known that the Turbo codes performance is considerably affected by its Interleaver design, which often has random permutation properties. In this paper we propose deterministic three types of 1-D Chaotic Interleavers based on chaotic dynamics that have a random like behavior to give the Interleaver the required degree of dispersion and correlation factors. Furthermore 2-D and 3-D Chaotic Interleavers are designed to have good separation parameters that have a major impact on the turbo codes performance especially over fading channels. Then, the designed chaotic Interleavers are compared with random one in terms of BER performance for random data and measuring the quality of the received image in terms of their PSNRs. The results show that, the Multi-dimensional Chaotic Interleavers outperform the random one, in terms of both: BER and the quality of the received image.

**Keywords:** Turbo codes, Chaos, Interleaver, Rayleigh channels, Estimation & Equalization.

### نقل الصور عبر قنوات الخفوت (رايلي) باستخدام الشيفرات النفاثة ذات مُبَدِّلات مواقع فوضوية متعددة الأبعاد

**الخلاصة:** من المعروف، ان اداء الشيفرات النفاثة يتأثر وبشكل كبير بتصميم مُبَدِّل المواقع الخاص به، والذي غالبا ما يكون له خصائص عشوائية. اقترحنا في هذا البحث ثلاث انواع من مُبَدِّلات المواقع الفوضوية احادية البعد المعرفة والتي تعتمد في عملها على انظمة من المعادلات الفوضوية والتي لها سلوك مشابه للعشوائي مما يعطي مُبَدِّل المواقع حد مقبول من معاملات التشتت والارتباط. بالاضافة لذلك قمنا بتصميم مُبَدِّل مواقع ثنائي واخر ثلاثي الأبعاد لضمان عامل إبعاد جيد لهن والذي له التأثير الكبير في اداء الشيفرات النفاثة، خصوصا في قنوات الخفوت. بعد ذلك قارنا اداء هذه المُبَدِّلات مع المُبَدِّل العشوائي من حيث معدل الرموز الخطأ (BER) عند نقل بيانات عشوائية، وكذلك في قياس جودة الصورة المستلمة من خلال اعلى نسبة إشارة الى الضوضاء في الصورة (PSNR). وبينت النتائج ان مبدل المواقع الفوضوي المتعدد الأبعاد (2D & 3D) تتفوق في الاداء على المُبَدِّل العشوائي، سواء من حيث ال BER او من حيث جودة الصورة المستلمة.

## 1. Introduction

In 1993 C. Berrou, et.al. proposed a new class of convolutional codes called Turbo Codes whose performance in terms of Bit Error Rate (BER) performance are close to the Shannon limit". For selected sub optimized parameters, Turbo Codes can operate within just 0.7dB away from the Shannon limit[1]. The potential performance offered by turbo codes has excited both academic and industrial researchers. This Turbo Code is not just a single code. It

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is, in fact, a combination of (at least) two codes that work together to achieve a synergy that would not be possible by merely using one code. In particular, a turbo code is formed by parallel concatenation of two constituent codes separated by an Interleaver. Each constituent code may be any type of FEC code used for conventional data communications. Although the two constituent encoders may be different, in practice they are nominally identical[2]. A generic structure for generating turbo codes is shown in Fig. 1, as can be seen that the turbo code consists of two constituent encoders, denoted as ENC-1 and ENC-2. The input data stream appears at the output as systematic bits ' $x$ ', ENC-1 deals with this frame of data in its original sequence to generate its own parity ' $y_1$ ', while ENC-2 generates its parity bits ' $y_2$ ' according to the Interleaved version of the data frame. Then parity outputs of the two parallel encoders are punctured to get the required total code rate. Finally they are serialized into a single turbo coded data by a multiplexer[3].

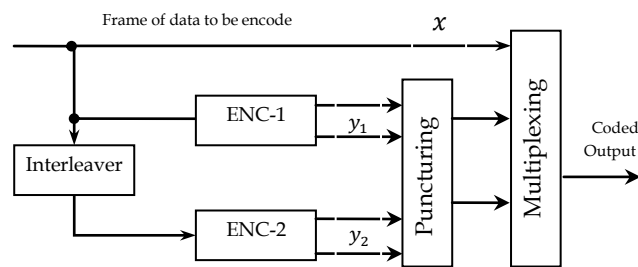


Figure 1. Turbo Codes Encoder

## 2. Turbo Decoders

Since convolutional codes have flexible code rate conversion as well as they have many efficient decoding algorithms like Soft Output Viterbi Decoding SOVA and Maximum A Posteriori 'MAP' decoding algorithm that can provide soft decision which is very important in suboptimal decoding. Referring to the encoder, which has two component codes, Turbo decoder must have two constituent decoders namely DEC-1 and DEC-2. Starting with DEC-1 it receives  $(x_k, y_{1k})$  and performs decoding in any algorithm (SOVA, MAP, ...) and produce its soft decision in terms of log likelihood ratio  $llr_1(d_k)$ . Then it sends its extrinsic information  $llr_1^e(d_k)$  to DEC-2 as an aprior information after interleaving, and the interleaved version of the systematic symbols  $x_k$  together with  $y_{k2}$  DEC-2 produces its decision  $llr_2(d_k)$ . But the key idea in the Turbo codes is the iterative decoding, where that the extrinsic information gained from the 2<sup>nd</sup> decoder  $llr_2^e(d_k)$  is used by the 1<sup>st</sup> decoder as an aprior information by means of a feedback loop after deinterleaving, and again DEC-1 recalculates the  $llr_1(d_k)$  and sends  $llr_1^{e2}(d_k)$  to the DEC-2 and again DEC-2 recalculates the  $llr_2(d_k)$  and sends  $llr_2^{e2}(d_k)$  to DEC-1. This exchanging of the information between the decoders is repeated until a satisfactory number of decoding iterations are done. At each iteration the estimated recovered binary data is taken as the sign of  $llr_2^{iter}(d_k)$ [3], as shown in Fig. 2.

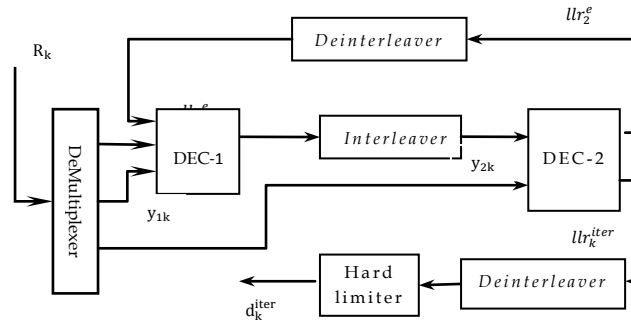


Figure 2. Iterative Decoder of Turbo Codes

### 3. The Interleaver in Turbo Codes

An Interleaver block normally takes set "χ" of symbols at the I/P and produces an identical set of symbols at the output but in a different order (indexes) from that in the input, typically it is denoted by :  $\chi \rightarrow \chi$ . Where π is the operation (Action) of the Interleaver[4], which represent the mapping every index 'i' into π(i) as shown in Fig. 3 below.

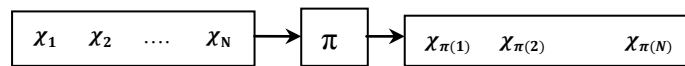


Figure 3. Action of the Interleaver

Deterministic Interleavers often described by some deterministic equations that specify its output sequence in terms of input sequence, or any Interleaver may be represented by a lookup table for the input and the output indexes. Whatever the Interleaver type, mathematically it can be substituted by a  $P_{NxN}$  matrix with single one in each  $i^{th}$  row lays in the  $\pi_i^{th}$  column. With the set input  $X_{Nx1}$  the output (Interleaved) set sequence  $Y_{1xN}$  is [5]:

$$Y_{Nx1} = P_{NxN} * X_{Nx1} \tag{1}$$

For every Interleaver, the Decoder must have its corresponding inverse operation named the "Deinterleaver" or  $\pi^{-1}$  where the interleaved (permuted) sequence recovers its original order and its permutation matrix simply got from  $P^T$  or  $P^{-1}$ , so the deinterleaved to its original sequence may be written as[5]:

$$X_{1xN} = P_{NxN}^T * Y_{Nx1} \tag{2}$$

#### 3.1 Main Objective of Turbo Codes Interleaver

##### 3.1.1 Correlation of the Interleaver:

If the input binary data sequence to the Interleaver is  $X = S^i = \{S_1^i, S_2^i \dots S_N^i\}$ , and Y is the output sequence from the Interleaver i. e.  $Y = S^o = \{S_1^o, S_2^o \dots S_N^o\}$ , then, the correlation of the Interleaver  $R_I$  is defined as [6]:

$$R_I = \frac{1}{N} \sum_{k=1}^N (2S_k^i - 1)(2S_k^o - 1) \tag{3}$$

As the Interleaver correlation approaches to zero, then, output data sequence approaches to be independent with the input sequence. An interleaver with low correlation is expected to have a good performance in concatenated codes where the extrinsic information exchange is optimal when the sequence of the processed data is fully independent. Since the data is normally a random variable with equal probability for both 0&1, so for different positions for data,  $R_I$  is almost zero except, only when data symbols that take the same indexes after Interleaving they give "+1" whatever  $S_k^i$ . Therefore the average correlation may be estimated by only the indexes that don't change (Immoveable points) after interleaving as follows [4],[6]:

$$R_I = \frac{\text{Number of Immoveable points of the Interleaver}}{\text{Interleaver size } N} \quad (4)$$

### 3.1.2 Dispersion of the Interleaver

Simply, dispersion is a set of distinct displacement vectors. It is used as a measure to the randomness of an Interleaver and it is defined by[4]:

$$\Gamma = \|\Delta x, \Delta y\| \quad (5)$$

Where  $\Delta x = j - i$  and  $\Delta y = \pi(j) - \pi(i) \forall 1 \leq i \leq j \leq N$ ,  $\|A, B\|$  here represents the cardinality, which is the number of non-repeated vectors in the set. But as the size of the Interleaver is increased, then  $\Gamma$  is increased, so that always Interleavers are compared in terms of *normalized dispersion* factor  $\gamma$  where  $0 \leq \gamma \leq 1$  and given by [7]:

$$\gamma = \frac{\|\Delta x, \Delta y\|}{(N(N-1)/2)} \quad (6)$$

A low dispersion value of an Interleaver indicates that the Interleaver is very regular. For example for identity Interleaver then  $\gamma = 2/N$ . For classical block Interleaver  $\gamma = 4/N(N+1)$  while for random Interleaver  $\gamma \approx 0.81$  [4].

### 3.1.3 S-parameter of the Interleaver (Depth of the Interleaver):

The depth of an Interleaver is defined as the minimum separation (in symbols) between any pair of symbols at the output of the Interleaver where these two symbols were adjacent or separated with distance less than  $t$ -symbols at the input of the interleaver. Then at the output of the Interleaver, there must be at least  $S$  symbols between these symbols. Let  $i$  and  $j$  be the two indexes for these symbols then at the output, this definition can be formulated as follows [4], [8]:

$$|i - j| < t \rightarrow |\pi(i) - \pi(j)| \geq S \quad (7)$$

The depth of an Interleaver has a significant importance for a burst of errors entering a deinterleaver at the receiver. If a burst of errors has duration less than the depth, then two symbols affected by the burst cannot be adjacent after de-interleaving, thereby can be corrected by a simple isolated error correcting codes[9].

#### 4. Proposed-1: 'Sorting' Chaotic Interleaver

By making use of chaotic signals which is a random like one. We can design a Chaotic Interleaver accomplishing job of random Interleaver in turbo codes as done with BICM-ID[6]. This design method assumes there is a known chaotic map with its initial condition and parameters unanimous between the Encoder and the Decoder, we will explain the design procedure steps as follows:-

**Step 0:** Prepare a Chaotic map with its control parameter(s) and negotiate it between the encoder and the decoder.

**Step1:** Using the defined chaotic map, generates a Chaotic Sequence with length equals to Frame length  $N$ .

**Step 2:** Sort this chaotic vector in ascending order according to the samples magnitude.

**Step3:** After sorting the chaotic vector according to ascending order, the indexes for the elements will change. Take these indexes as the interleaving look up table for both transmitter's encoder and the receiver's decoder. Fig.4 shows the scatter plot of the interleaved positions by using this algorithm for two Interleaver sizes 256, 512 bits and how does it looks a random like permutation.

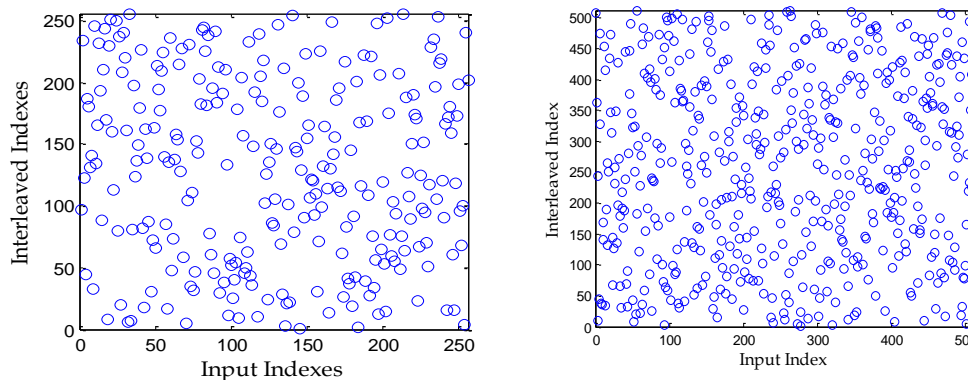


Figure 4.Scatter plot of interleaved indexes of Chaotic sorting Interleaver

The advantages of this design is the simplicity in its algorithm, in the same time it provides a high dispersion and low correlation Interleaver. But it does not ensure any S-parameter and the sorting process may be time consuming especially for large frames, since it requires  $N(N - 1)/2$  of comparison to sort  $N$  elements. By testing a 100 frame of random data the calculation of this Interleaver parameters gives *correlation*  $R_i \approx 1/N$  and *dispersion*  $\gamma \approx 0.82$  but without any guaranteed S-parameter for this algorithm.

#### 5. Proposed-2: 'Multiplying' Chaotic Interleaver

Another way of thinking, as in [10], we can design another chaotic Interleaver by direct multiplying the resulted chaotic value by a certain large number  $L$ , where  $L$  must be greater than the frame size/ $\max(x)$ , and it must be defined for both encoder and decoder in addition to the chaotic map with its initial condition, Then take the modulo with respect to the frame size to the nearest integer resulting from this multiplication. This procedure can be summarized as follows:

**Step 0:** Setup the chaotic map with its initial conditions, its control parameters and "L" .

**Step 1:** Multiply  $x_1$  by  $L$ , then take the module to the integer part, this results  $\pi(1)$ .

**Step 2:** For the every element index( $k=2$  to  $N$ ) iterate the chaotic map to generate new chaotic value.

**Step 3:** Multiply the last generated chaotic value i.e.  $x_k$  by  $L$  and take the module to the integer part this yields:

$$Z(i) = \text{Round}\{x_k * L \text{ mod } N\} + 1 \quad (8)$$

**Step 4:** Verifying: If the temporary variable  $Z(i)$  equals to any one of previously generated interleaved positions then go to step 2 otherwise assign this  $Z(i)$  to be current interleaved position  $\pi(k)$ .

**Step 5:** The resulted interleaved positions  $\pi(1), \pi(2), \dots, \pi(N)$  forming the Interleaver permutation lookup table to be used at both transmitter and receiver.

**Note 1:** The multiplicand large number "L" for some chaotic maps have domains of 'U' interval extend from negative to positive axis, this L must be greater the or equal to frame size  $N/U$ . For example for with Lozi map given in eq(9) with  $a=1.7$  and  $b = 0.5$  that has U domain interval of  $[1.4 \leftrightarrow -1.4]$  and any frame size N, then  $L \geq \{N/(1.4 - (-1.4))\}$  or  $L \geq N/2.8$  .

$$y_{i+1} = bx_i \quad , \quad x_{i+1} = 1 - a|x_i| + y(i) \quad (9)$$

**Note 2:** There is no need for the multiplication by L and modulus operations for some integer defined chaotic maps, like Arnold cat map, discrete time baker map whose domain can be adjusted to match the Interleaver size. Fig. 5 shows the scatter plot for the input and the interleaved indexes for this algorithm accomplished using Lozi map for Interleaver size 256 and 512 symbols.

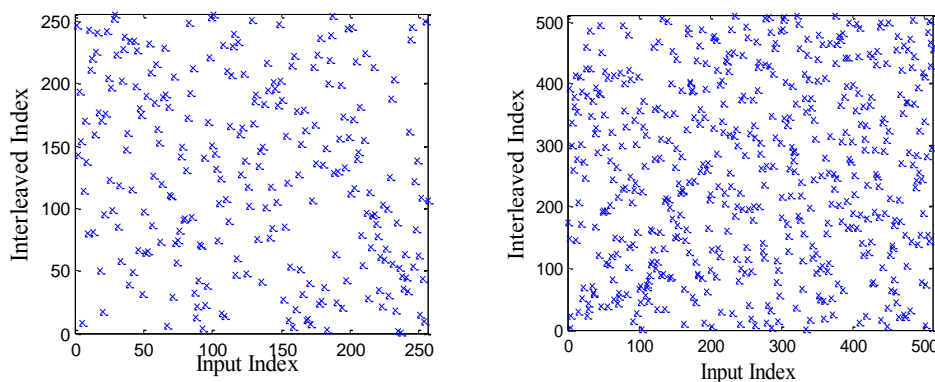


Figure 5. Scatter plot of interleaved indexes of multiplying Chaotic Interleaver

### 5. Proposed-3: 'Slicing' Chaotic Interleaver

Actually this approach was suggested by [11] to interleave few symbols for the purpose of PAPR reduction in OFDM systems, Here we propose to extend this algorithm for large number of symbols to be used as an Interleaver for Turbo codes. This design is mainly based on dividing the domain of the negotiated chaotic map into  $N$  sub domain (slices) and label the  $1, 2, \dots, N$ , then run the chaotic map. At every time instance the interleaved index is the index of the slice in which the chaotic value lays in. and if that position has already taken by previously interleaved positions leave it and iterate the chaotic map to try with next chaotic sample, and carry on by this procedure until all entries of Interleaver permutation table are filled. This can be simplified in the following steps:

**Step 0:** With the defined map which has a known domain(dynamic range), divide this domain into  $N$  slices, each one with Chaotic domain interval/ $N$  width.

**Step 1:** The first interleaved position  $\pi(1)$  is the index of the slice in which  $x_1$  lays. For the remaining indexes  $\pi(k)$  where  $k = 2, 3, \dots, N$ .

**Step 2:** Iterate the chaotic map to provide a new chaotic value and define  $Z(i)$  to be the index of the slice in which  $x_k$  lays.

**Step 3:** Compare  $Z(i)$  with all previously generated indexes  $\pi(1) \dots \pi(k-1)$ . If it equals to any of them go back to step 2 to get new  $x$  and  $Z$ , i.e.  $x_{i+1} \& Z(i+1)$  otherwise  $\pi(k) = Z(i)$ .

**Step 4:** Finally the interleaver lookup table is the indexes  $\pi(1), \pi(2) \dots \pi(N)$ .

Fig.6 shows the scatter plots for the input vs. interleaved indexes for this algorithm accomplished using logistic map for arbitrary Interleaver sizes 256 and 512 symbols.

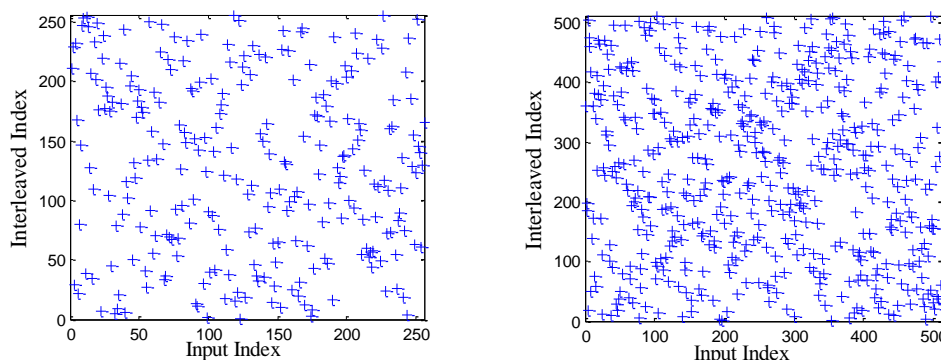


Figure 6. Scatter plot of interleaved indexes of Sliding Chaotic Interleaver

### 6. Proposed-4: Two Dimensional '2-D' Chaotic Interleaver

Since the chaotic system can't predict their long term results, thus all of previous (1-D) designs do not guarantee any separation ( $S$ -parameter) between symbols after Interleaving, so we have to design an other chaotic Interleaver that ensures that there will be at least " $S$ " symbol separation after interleaving between any two elements that were adjacent or separated by less than or " $t$ " symbols. Below we describe the design procedure of the "Two Dimensional Chaotic Interleaver".

**Step1** Write the frame of  $N$  data symbols inside a two dimensional array with  $R$ -rows,  $C$ -columns (*where*  $R \times C = N$ ) in row by row fashion.

**Step2** Using any one of previous one dimensional chaotic Interleaving generate  $R$ -sized Interleaver to permute element inside every column. This step is equivalent to rows permutation.

**Step3** Using the last generated chaotic value from step 2 as initial condition for the 1-D chaotic Interleaving to generate an Interleaver size equal to  $\lfloor C/2 \rfloor$  to permute the odd numbered columns again with the last generated chaotic sample evaluate another Interleaver size of  $\lfloor C/2 \rfloor$  to permute even numbered columns then combine these two Interleavers to permute columns. But if the firsts or the lasts elements form the odd and even indexes became neighbors then reject this chaotic value and try with next value and redo this step.

**Step4** Finally read the data from this matrix in a column by column manner.

Fig. 7 shows this interleaving strategy for Frame size  $N=30$  symbols we factorized the 30 into  $5 \times 6$  where we write just the indexes of the symbols.

**Note:** Step 2 and Step 3 are interchangeable, and step 3 gives a good separation between interleaved columns with semi random permutation, it can be replaced by designing a block Interleaver of size  $C$  with suitable choice for its dimensions. This guarantees neighbors columns no longer be so, but this may degrade the randomness degree of the total Interleaver, so that it may be compensated by assigning different interleaving permutation for each column element, by the same idea of using last chaotic value as initial condition for the successive columns Interleavers.

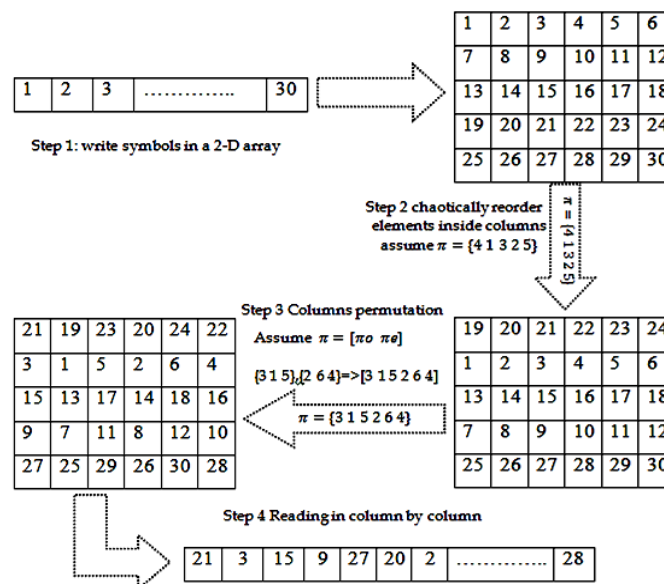


Figure 7. Example of '2-D' Chaotic Interleaver with  $N = 30, R = 5, C = 6$

Fig.8 shows scatter plots for the input and the interleaved indexes for two dimensional chaotic Interleaver with cubic map for reordering rows and columns for different Interleaver sizes: case1  $N = 256, R = 16, C = 16$ . Case2  $N = 512$  with  $R = 16, C = 32$ . These figures



indicate to existing degree of randomness provided by this technique for the Interleaved symbols, From these scatter plots we can see how the scattering area became more homogenous but it is evident that 2-D Chaotic interleaver has a limited degree of randomness since the scatter plots have some almost repeated patterns.

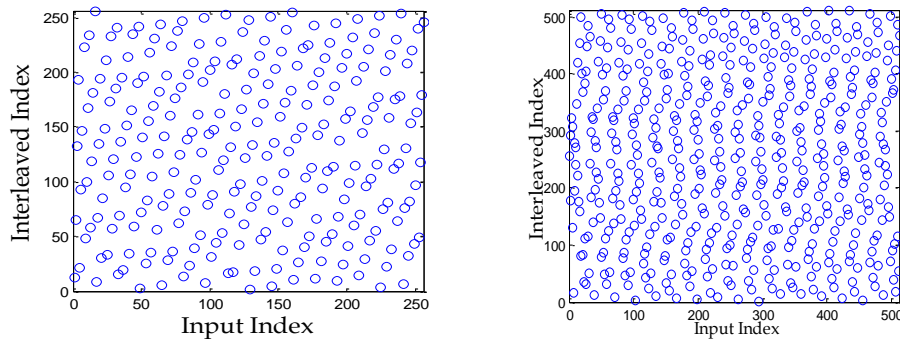


Figure 8. Scatter plot of interleaved positions of sliding Chaotic Interleaver for  $N=256, 512$

From Fig. 9 we can see that, there is no separation less than  $R - 1$  between the Interleaved symbols. Thereby, the 2-D chaotic Interleavers are expected to have a very good performance in burst error diffusion and awards a higher error correction capability to turbo code especially in Rayleigh Fading channels.

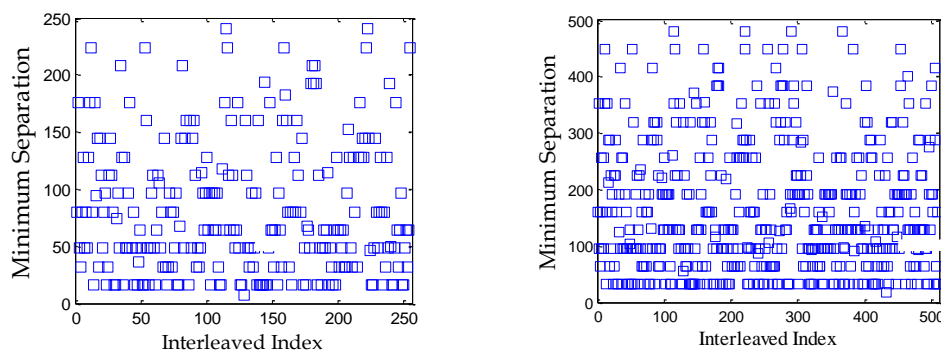


Figure 9. Minimum Separation between interleaved symbols in '2-D' Chaotic Interleaver.

## 7. Proposed-5: Three Dimensional '3-D' Chaotic Interleaver

2-D Chaotic Interleaver has a good S-parameter but it has many short periodic behavior in interleaving symbols appeared in its scatter plot which decreases its dispersion parameter thereby it mayn't achieve an excellent performance in Turbo Codes since its randomness is limited. In 3-D Chaotic Interleaver we aim to increase the randomness in the same time we attain S-parameter as larger as possible. Below we describe design procedure of the "Three Dimensional 3-D Chaotic Interleaver":

**Step 0** Setup the chaotic map with its initial conditions

**Step1** Write the frame of  $N$  symbols to be interleaved inside a three dimensional array having  $R$ -rows,  $C$ -columns,  $A$ -aisles in row by row for every aisle then aisle by aisle for the third dimension. Where:  $(R \times C \times A=N)$ .

**Step1** Inside every aisle which is a(2-D)  $R \times C$  array, using the known chaotic map, chaotically reorder every column entry in same procedure performed in 2-D Chaotic Interleaver, but for every aisle the last chaotic value used as an initial condition for the next aisle to obtain different configuration 2-D arrays.

**Step2** As in 2-D Chaotic Interleaver: sequentially, inside every aisle using the latest generated chaotic sample from step 2 as an initial condition to generate an Interleaver of size equals to  $\lfloor C/2 \rfloor$  to permute the odd numbered columns, again with the last generated chaotic evaluate another Interleaver size equals to  $\lfloor C/2 \rfloor$  to permute even numbered columns. Then combine these two Interleavers to permute columns but firsts or lasts elements mustn't be neighbor in the total Interleaver.

**Step3** For every row index and column index chaotically reorder elements in the 3<sup>rd</sup> dimension (within different aisles), in a variable manner, where the final chaotic element obtained is used as an initial condition for the new  $r, c$  indexes and so on. Thereby we get a different ordering Sequence for all row and columns. This continuous different reordering elements in side aisles gives the 3-D chaotic Interleaver a strong randomness property especially these elements originally separated by  $(R \times C)$  elements.

**Step4** : Now read element from this matrix aisle by aisle then column by column for all rows.

Fig. 10 shows an example for 3-D Chaotic interleaving for frame size=120 symbols, where just the index for the symbols are written inside the 3-D matrix, and here we must adjust the dimension of the matrix to be equal to frame size. These factors of the 120 selected to be: 4, 5, 6, so that the three dimensional array will has: 4 Rows \* 5 Columns \* 6 Aisles. Fig. 11 shows the scatter plot for the interleaved positions by using '3-D' chaotic Interleaver to  $N=256,1000$ , where it is clear from these two plots that the 3-D Interleaver has a good randomness property in the same time it has a more homogenous diffusion in interleaved symbols in comparison with all previous 1-D chaotic Interleavers.

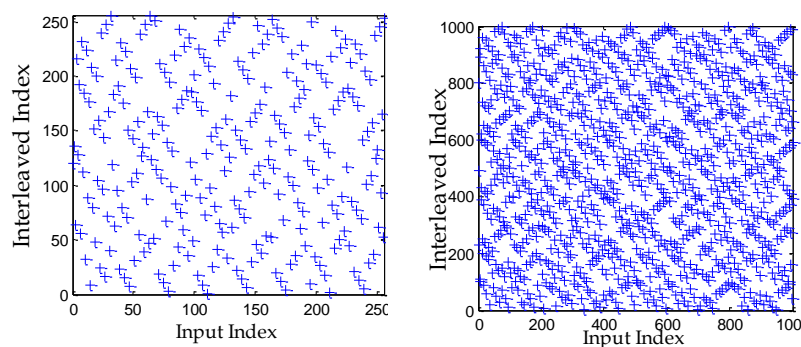


Figure 12. Minimum Separation between interleaved symbols in '3-D'Chaotic Interleaver

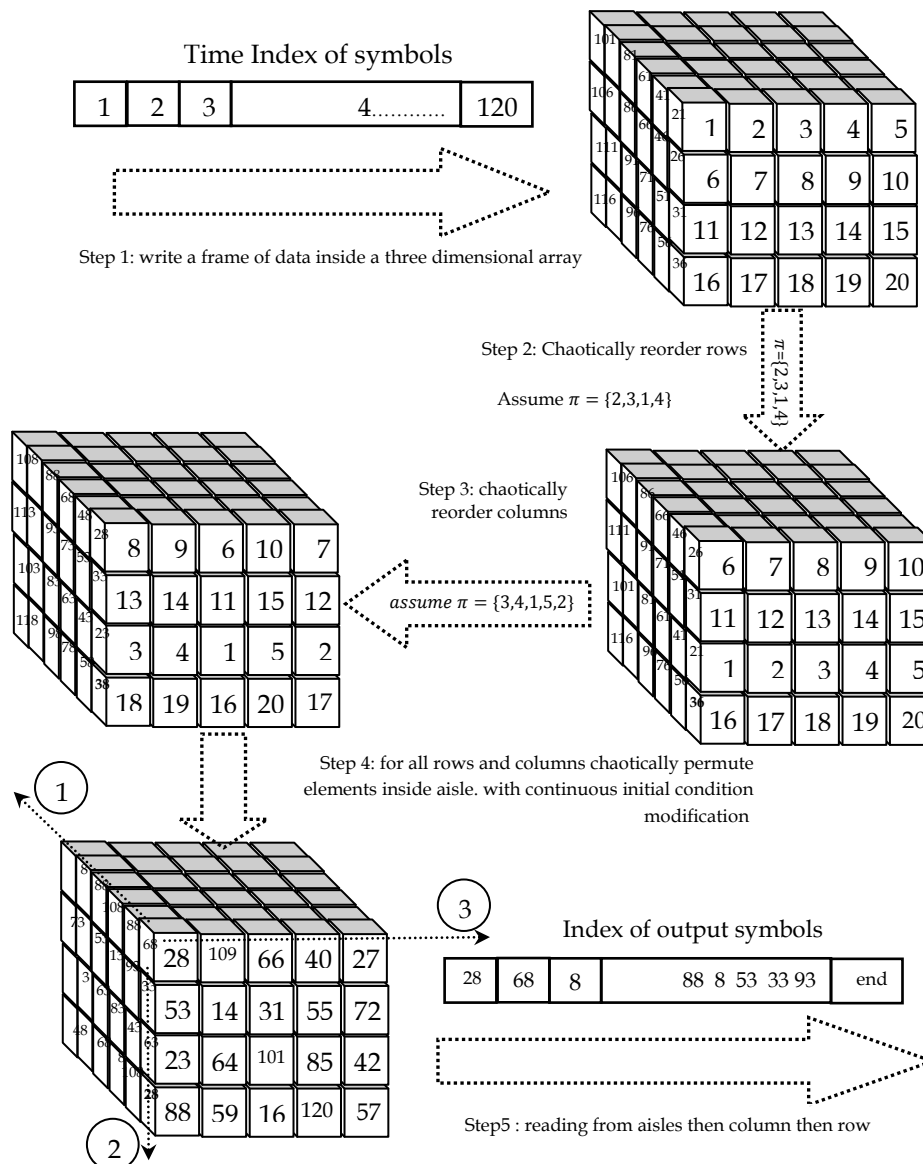


Figure.10 Example of '3-D' Chaotic Interleaver where N=120 with R=4, C=5, A=6

## 8. Simulation Results

To prove whether the designed chaotic Interleavers are capable of competing others Interleavers design for Turbo, below we relayed on the simulation results accomplished using Matlab platform with parameters set for the Turbo Codes are chosen as close as possible for that used in Long Term Evolution "LTE" systems over different wireless channels chosen from ITU standard channels for the purpose of testing systems in simulations[12].

### 8.1:Chaotic Interleavers Performance in Turbo code over Rayleigh Flat Fading Channel

Although Turbo codes performance is considerably enhanced as the Interleaver size is increased. It was proven in many previous studies that the Interleaver type has a major influence on its performance, particularly at the higher SNRs[4]. Fading channel normally

results in bursts of errors[13], so it may be regarded as a real test for the performance of multidimensional Interleavers that have the capability of diffusion the bursts. So to study the effect of the Interleaver type, all parameters of Turbo Code are fixed as follows: six decoding iterations with MAP algorithm, 1/3 global code rate, generating polynomials were  $G = (1,15/13)_{oct}$  for both convolutional codes, the Interleaver size is 4096 bits, it is a medium size and divisible into two factors (for Block, Helical and 2-D Chaotic) and into three factors for 3-D chaotic Interleaver. Figure-13 shows the BER performance comparison among Block<sub>64x64</sub>, Helical<sub>64x64</sub> unity slope, Random<sub>1x4096</sub>, (Sorting<sub>1x4096</sub>, Multiplying<sub>1x4096</sub>, sliding<sub>1x4096</sub>) Chaotic Interleavers, 2-D Chaotic<sub>64x64</sub> Interleaver, and 3-D Chaotic<sub>16x16x16</sub> Interleaver.

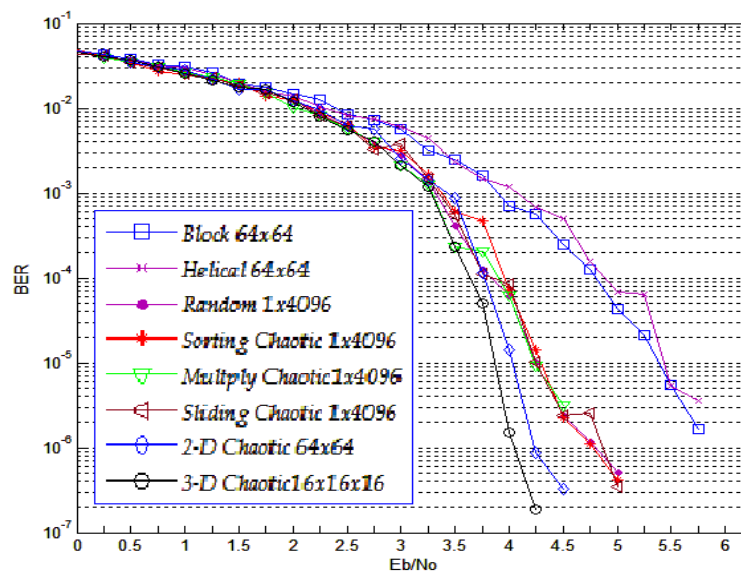


Figure 13. Chaotic Interleavers BER Performance over Rayleigh flat fading channel.

Fig.13 shows the performance of the chaotic Interleavers in comparison with previously mentioned Interleavers. The channel is chosen from UMTS standard (ITU-A) for indoor communications ( $f_d=30$  Hz,  $T_s=1\mu\text{sec}$ ), the channel state information 'CSI' is estimated by Pilot Symbol Assisted Modulation PSAM technique [14] with cyclic prefix of 1bit sent every 32-bits. From this figure, we can conclude that all 1-D Chaotic Interleavers are equivalent in performance to the Random Interleaver, but the 2-D Chaotic one has a gain of about 0.75 dB over the random Interleaver measures at  $BER = 10^{-6}$  and the 3-D Chaotic Interleaver has a gain of about 1dB over the Random one. This superiority of the last two Interleavers is due to that, they have a good degree of randomness and guaranteed separation between interleaved symbols (S-Parameter) to diffuse burst errors caused by time varying fading channel and make bursts as isolated errors that can be corrected in between the two RSC decoders.

## 8.2 Performance of Chaotic Interleavers in Turbo Codes over Frequency selective Rayleigh fading Channel

Such a channel type has inherent ISI because of the many paths signals arrive to the receiver with delays greater than the symbol duration, and because of continuous time varying in paths gain in addition to AWGN that makes the estimator infer wrong or not exact CSI that

yields clusters of errors[13]. And we have hypothesized that both 2-D, 3-D can diffuse such clusters in addition to their irregular behavior that increasing their dispersion and decreasing correlation factors, so that they are expected to outperform others Interleavers. Fig. 14 shows a comparison of Chaotic Interleavers with others Interleavers in Turbo code performance over frequency selective Rayleigh fading channel chosen from the ITU-A standard for outdoor mobile simulations[12]. Where this channel assumes the Doppler frequency is 30 Hz, relative path delays(0,1,2)  $\mu$ sec, average path gains of (0,3,9)dB and the symbol duration is assumed to be 1  $\mu$ sec. And the channel fading is estimated using PSAM with cyclic prefix greater than the channel ISI and equalized using simple linear equalizer to compensate the channel effects.

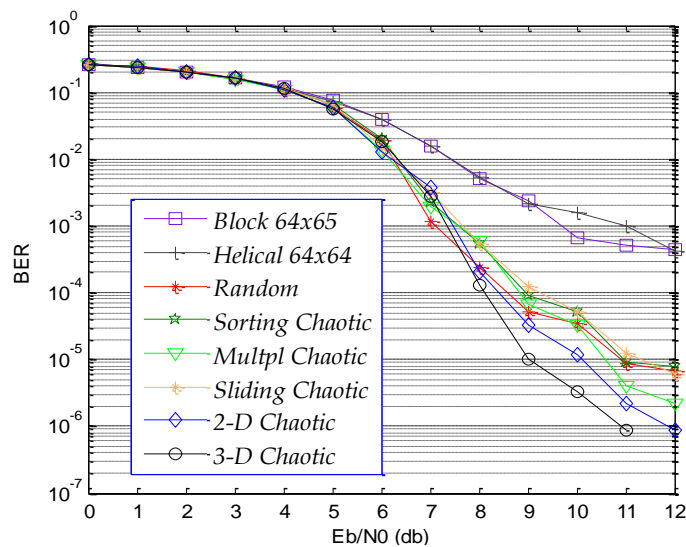


Figure 14. Chaotic Interleavers BER Performance over frequency selective Rayleigh fading channel

From Fig. 14 we can conclude same result concerning 1-D Chaotic Interleavers, they are equivalent to the Random Interleaver. But the 2-D chaotic one has a gain of about 1dB while the 3-D chaotic Interleaver has a gain of approximately 2dB measured at BER  $10^{-5}$

## 9. Image Transmission

The importance of Image transmission comes from the fact that it is one important type of digital data forms and the digital videos are basically composed of groups of Images pass through our eyes in swift manner, so that our eyes imagine and see a continuous motion. Hence if we succeed in image transmission then we can say our system can convey digital video like DVB signal with sincerity [15],[16]. It is well-known that, images may be written in raw manner as in RGB image(.Bmp) or semi compressed like 8-bits colored Indexed images such as Portable Network Graphic(.PNG Images) or compressed images such as the JPEG image. But which way or format is the best for transmission over various channels type by using the Turbo Codes?! Simulation will answer us.

### 9.1 Turbo code for image transmission over flat Rayleigh fading channel

Table.1 shows an example of reconstructed images after transmission the same image "C. Berrue" image over Rayleigh flat fading Channel in these three formats, but keeping in mind

that the total number of bits in compressed file is less than these in the uncompressed one. Then to make a fair comparison among the three formats, the same amount of power must be given to each file. In this example the JPEG technique compresses this image with a rate of 6:1 then the energy per bit given to the JPEG file is 6 times that given to that of the RGB file and the Indexed Image about three times that of RGB image. Using the same specifications in the Turbo codes for the three formats where the generating polynomial  $g = (1,15/13)$ , and Random permutation Interleaver with size 4096 bits, code rate 1/3, since the comparisons is normally performed in the waterfall region of the BER curves, in Table-1 the JPEG image transmission has done at  $E_b/N_o = 6 \text{ dB}$  and since the JPEG compression ratio in this Image is 6:1 then the energy per bit used for the RGB Image is one sixth of that in JPEG image so that the RGB image transmission carried out at  $E_b/N_o$  in dB of  $[6 - 10 \log_{10}(6)] \cong -1.8 \text{ dB}$ . For the Indexed image the energy per bit is approximately three times that of RGB image, so that its transmission has been performed at  $E_b/N_o$ (dB) of  $[-1.8 + 10 \log_{10}(3)] \cong 2.9 \text{ dB}$ . After channel decoding and corresponding source decoding it is clear from that table that the most of JPEG image is clear from the fifth iteration, and the Indexed Image is enhanced very slowly with iterations, while the raw RGB image has no noticeable improvement with iterative decoding .

Table 1. Different image formats transmission over Rayleigh flat Fading channel










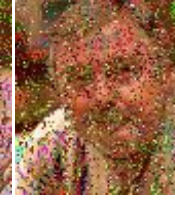


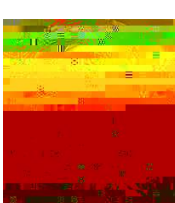





|               | 1 <sup>st</sup> iter  | 2 <sup>nd</sup> iter  | 3 <sup>rd</sup> iter  | 4 <sup>th</sup> iter  | 5 <sup>th</sup> iter   | 6 <sup>th</sup> iter  |
|---------------|---|---|---|---|--|---|
| RGB Image     |  |  |  |  |  |  |
| Indexed Image |  |  |  |  |  |  |
| JPEG Image    |  |  |  |  |  |  |

Table2 shows the reconstructed Images quality in terms of PSNR given by [17]:

$$PSNR = 10 \log_{10} \left\{ \frac{255^2}{MSE} \right\} \tag{10}$$

Where 
$$MSE = \frac{1}{M \times N} \sum_{i=1}^M \sum_{j=1}^N [P_{T_x}(i, j) - P_{R_x}(i, j)]^2 \tag{11}$$

With the notations  $P_{Tx}(i, j)$ ,  $P_{Rx}(i, j)$  are transmitted and decoded pixel values respectively that laying in  $i^{th}$  row,  $j^{th}$  column of the image having  $M$  rows and  $N$  columns.

Table 2: received RGB, Indexed and JPEG "Berrue" Image transmitted over flat Rayleigh fading channel

| Image Type    | PSNR                 |                      |                      |                      |                      |                      |
|---------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
|               | 1 <sup>st</sup> iter | 2 <sup>nd</sup> iter | 3 <sup>rd</sup> iter | 4 <sup>th</sup> iter | 5 <sup>th</sup> iter | 6 <sup>th</sup> iter |
| RGB Image     | 10.25                | 10.54                | 10.55                | 10.59                | 10.62                | 10.60                |
| Indexed Image | 12.99                | 13.81                | 13.85                | 13.72                | 13.80                | 15.49                |
| JPEG Image    | 6.27                 | 7.18                 | 8.61                 | 8.31                 | 36.75                | 36.75                |

From this table we can conclude that, the compression is useful in transmission in flat fading channel, since at the 5<sup>th</sup> and the 6<sup>th</sup> iteration the JPEG image has been reconstructed perfectly, while the Indexed image quality is very slowly enhanced with iterations. The RGB bad quality is almost unchanged with iterations. Thereby we can conclude from this table after just six decoding iterations that, Indexed image outperforms the RGB image by 4.5 dB, and the JPEG decoded image outperforms the RGB by about 16 dB. Note that the 36.75 dB in the PSNR of the intact JPEG is simply due to the fact that the JPEG compression is a lossy compression technique.

### 9.2 Interleaver Type Effect in JPEG Image Transmission over Flat Rayleigh Fading Channel

JPEG Image proved its superiority in transmission over flat fading channel upon other formats with Turbo codes. Table.3 shows a comparison of the reconstructed the same JPEG (C. Berue) image with different interleaver designs for Turbo codes, all have the size of 4096 bits over the same (ITU-A) channel with  $E_b/N_0 = 6$  dB. From this table we can see that both Block and Helical Interleavers have too much residual errors, though after the sixth iteration even though they are optimized in their dimension. And all of '1-D' (sorting, multiplying and sliding) Chaotic Interleaves are adequate to the Random one since they almost restore the image after the fifth decoding iteration, on other hand the 2-D chaotic Interleaver shows its profit upon them, since it restores the JPEG image intact from the forth iteration. while '3-D' Chaotic Interleaver proves its worthiness to be the best one, since it recovers the JPEG Image perfectly before them all, from the third iteration. Table.4 lists the MSE and PSNR for the reconstructed images shown in Table-3 in comparison with the original JPEG image, for the mid iteration (the third one): all 1-D Chaotic Interleavers roughly have the same quality to the random one, but there is about 3 dB PSNR enhancement for the 2-D Chaotic Interleaver over the average of all 1-D Interleavers. While the 3-D Chaotic proves its superiority over the all, since it recovers the JPEG image exact after this iteration. And for the 6<sup>th</sup> iteration, the chaotic Interleavers the random one recover the image intact and only Block and helical Interleaver cannot do that, this is simply due to their regular interleaving strategies that lead to low dispersion and high correlation coefficients, which degrade the Turbo codes performance in its iterative decoder.

Table.3 Restored Images using Different Interleavers with Turbo Codes for JPEG Image Transmitted over Rayleigh Flat Fading Channel

|        |  | 1 <sup>st</sup> iter | 2 <sup>nd</sup> iter | 3 <sup>rd</sup> iter | 4 <sup>th</sup> iter | 5 <sup>th</sup> iter | 6 <sup>th</sup> iter |
|--------|--|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| 1x4096 | Block <sub>64x64</sub>                       |                      |                      |                      |                      |                      |                      |
|        | Helical <sub>64x64</sub>                     |                      |                      |                      |                      |                      |                      |
|        | Random                                       |                      |                      |                      |                      |                      |                      |
|        | Sort Chaotic                                 |                      |                      |                      |                      |                      |                      |
|        | Slide Chaotic                                |                      |                      |                      |                      |                      |                      |
|        | Mult Chaotic                                 |                      |                      |                      |                      |                      |                      |
|        | '2-D' Chaotic <sub>C<sub>64x46</sub></sub>   |                      |                      |                      |                      |                      |                      |
|        | '3-D' Chaotic <sub>C<sub>16x16x1</sub></sub> |                      |                      |                      |                      |                      |                      |



Table.4 MSE &amp; PSNR of JPEG "C. Berrue" Image transmitted over flat fading channel

| Interleaver Type                       | $3^{rd}$ Iteration |          | $4^{th}$ Iteration |          |
|--|--------------------|----------|--------------------|----------|
|  | MSE                | PSNR     | MSE                | PSNR     |
| Block $_{64 \times 64}$                | 11323              | 7.4711   | 8800               | 8.68     |
| Helical $_{64 \times 64}$              | 14458              | 6.5296   | 17721              | 5.64     |
| Random $_{1 \times 4096}$              | 4331               | 11.764   | 0                  | $\infty$ |
| Sort Chaotic                           | 39453              | 8.4207   | 0                  | $\infty$ |
| Mult Chaotic                           | 19131              | 8.9161   | 0                  | $\infty$ |
| Sliding Chaotic                        | 14664              | 7.7316   | 0                  | $\infty$ |
| 2D Chaotic $_{64 \times 64}$           | 3964               | 12.148   | 0                  | $\infty$ |
| 3D Chaotic $_{16 \times 16 \times 16}$ | 0                  | $\infty$ | 0                  | $\infty$ |

### 9.3 Image Transmission over Frequency Selective Rayleigh Fading Channel

Using the same principle of energy per bit management between the compressed image and the raw image the "Monaliza" image is used for testing and transmitting the JPEG image at  $E_b/N_o = 12$  dB. Because the JPEG achieves a compression ratio about 7:1 for this image, we have to transmit the RGB image at  $E_b/N_o = 12 - 10 \log_{10}(7) \cong 3.6$  dB and the indexed image transmission at  $E_b/N_o = 3.6 + 10 \log_{10}(3) \cong 8.1$  dB. With N=4096-bits using random interleaving strategy, Table.5 shows that, the raw RGB image has no improvement with iterative decoding, while the Indexed image has a wonderful enhancement with iterations, and the compressed (JPEG) image is still in error though after the 6<sup>th</sup> iteration. Table.6 lists the PSNRs for the six iterative decoded images in their corresponding formats.

Table.5 Different image formats transmission over Frequency selective Rayleigh Fading channel.



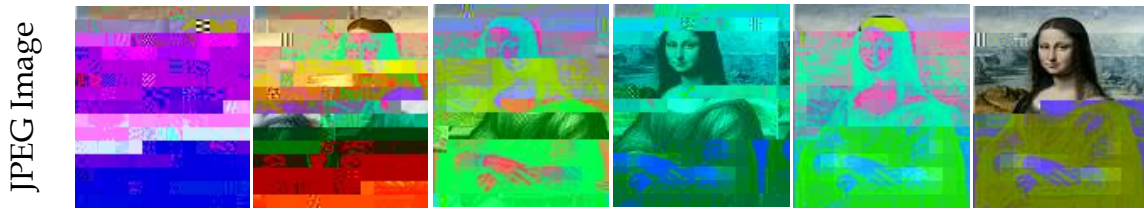


Table6 PSNR for Received RGB, Indexed and JPEG "Monaliza"Image transmitted over frequency selective Rayleigh fading channel

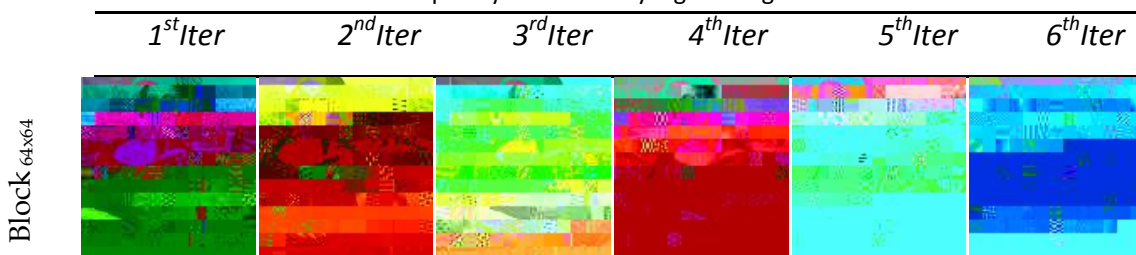
| Image Type    | PSNR                 |                      |                     |                     |                      |                      |
|---------------|----------------------|----------------------|---------------------|---------------------|----------------------|----------------------|
|               | 1 <sup>st</sup> iter | 2 <sup>nd</sup> iter | 3 <sup>rd</sup> ite | 4 <sup>th</sup> ite | 5 <sup>th</sup> iter | 6 <sup>th</sup> iter |
| RGB Image     | 11.35                | 11.83                | 11.91               | 11.91               | 12.03                | 12.05                |
| Indexed Image | 13.65                | 16.23                | 17.37               | 17.92               | 18.00                | 18.47                |
| JPEG Image    | 6.27                 | 7.18                 | 8.61                | 8.31                | 8.01                 | 13.30                |

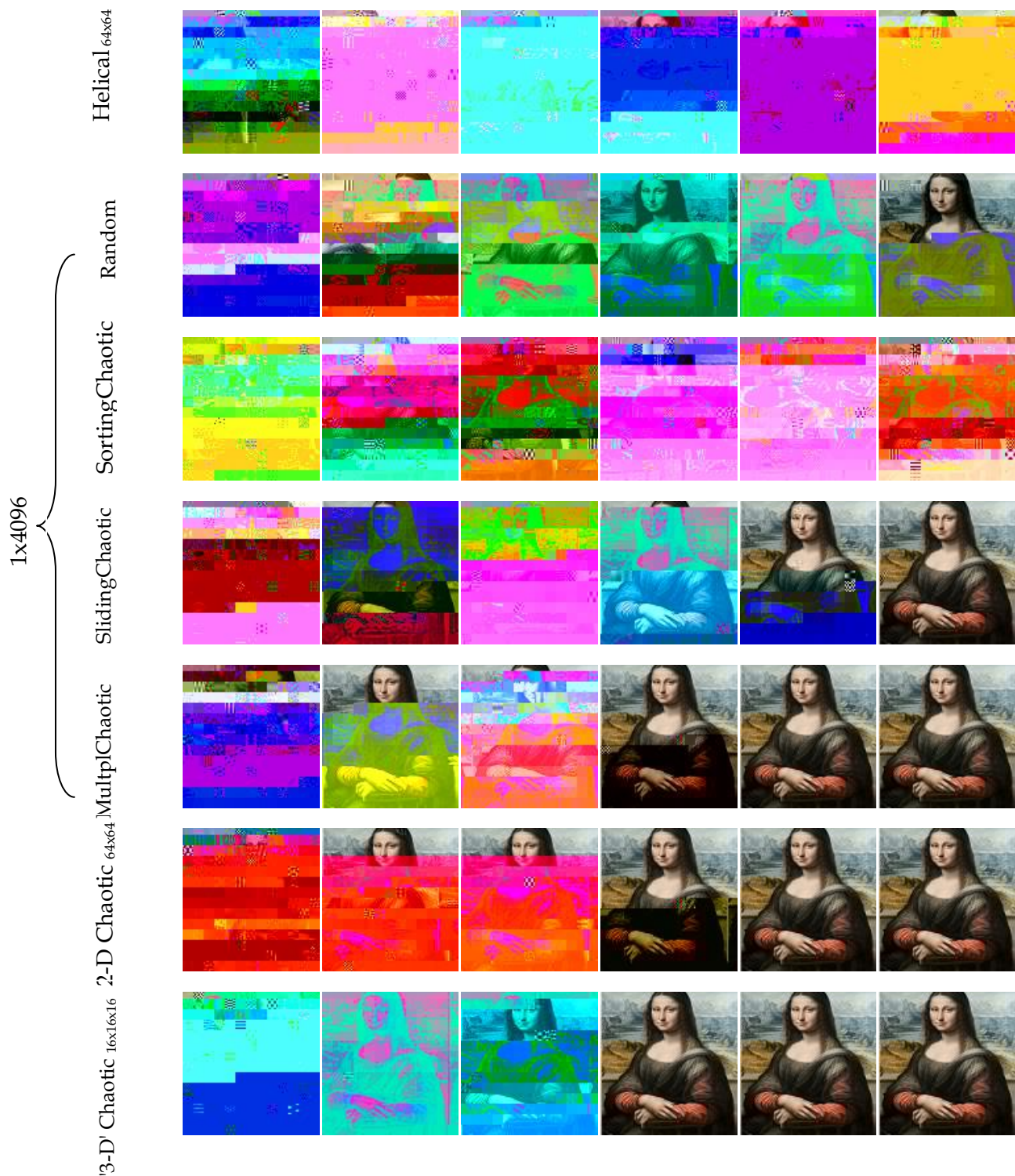
Because of the nature of this channel, it has an inherent ISI, so that it is a risk to send the JPEG compressed file though such a channel. For this reason we noticed that the JPEG image after the 6<sup>th</sup> iteration is not recovered correctly. On other hand, the Indexed image has a remarkable enhancement in its quality with iteration and it has PSNR gain of 5dB over the JPEG image and about 6.4 dB over the RGB Image.

#### 9.4 Chaotic Interleavers Gain in JPEG Image transmission over Frequency Selective Rayleigh Fading Channel

Multi-Dimensional Chaotic Interleavers proved their superiority over all tested Interleavers in terms of BER. So they may be capable to restore the JPEG image intact since the compression has many benefits in data storage and bandwidth exploitation. Here we investigate the advantages of the designed Interleaver in JPEG image transmission. Over frequency selective (ITU-A) standard channel for outdoor mobile communications. Table-A6 shows the decoded images using different Interleavers. From this table we can see that: the Block and Helical Interleavers have too much errors such that we can't see even just the image features, and the 1-D Chaotic Interleavers and the Random one have many residual errors even though after six decoding iterations and they are almost equipotent, But the 2-D Chaotic Interleaver recovered completely the JPEG image intact after the 5<sup>th</sup> iterations. While the 3-D Chaotic one recovered the image completely at the 4<sup>th</sup> iteration.

Table7. Restored Image using Different Interleavers with Turbo Codes for JPEG Image transmission over frequency selective Rayleigh fading channel





This fast data recovery in less number of iterations fertilized by the multi dimensional Chaotic Interleaver has significant importance in real time or short latency communication since it minimizes the global decoding time of Turbo Codes which may be regarded the main drawback of this powerful error correcting code. Table-8 shows the MSE & PSNR of decoded Image using different Interleavers, One can notice that the 2-D and 3-D Chaotic Interleavers have about 1.5 dB, 22 dB PSNR gain respectively over Random one after the 4<sup>th</sup> iteration and handout the image intact while the traditional Random Interleaver can't do that even though after the six decoding Iterations.

Table8 MSE &amp; PSNR for reconstructed JPEG "Monaliza" Image

| Interleaver Type                | 4 <sup>th</sup> Iteration |       | 6 <sup>th</sup> Iteration |       |
|---------------------------------|---------------------------|-------|---------------------------|-------|
|                                 | MSE                       | PSNR  | MSE                       | PSNR  |
| Block <sub>64x64</sub>          | 14395                     | 6.54  | 80032                     | 8.68  |
| Helical <sub>64x64</sub>        | 15787                     | 6.57  | 17723                     | 5.64  |
| Random <sub>1x4096</sub>        | 17262                     | 8.76  | 12093                     | 13.31 |
| Sort Chaotic                    | 21074                     | 4.89  | 14229                     | 6.59  |
| Multiply Chaotic                | 197                       | 26.16 | 157                       | 28.17 |
| Sliding Chaotic                 | 77723                     | 9.22  | 160                       | 26.08 |
| 2-D Chaotic <sub>64x64</sub>    | 12226                     | 9.32  | 0                         | ∞     |
| 3-D Chaotic <sub>16x16x16</sub> | 0                         | ∞     | 0                         | ∞     |

## 10. Conclusions

Although Chaotic Interleavers belong to deterministic Interleavers family that make both the encoder and decoder build them by themselves, that means there is no need to transmit all Interleaver lookup table entry as in case of random Interleaver. All three types of '1-D' Chaotic Interleavers (Sorting, Multiplying, Sliding) have the same performance to the Random one over all channels whether in terms of BER or PSNR of the decoded images. '2-D' Chaotic Interleaver is better than the random one at higher SNRs since it has less error floor, and in flat fading channel '2-D' Interleaver outperforms the Random Interleaver with power gain of about 0.6dB measured at  $BER = 10^{-6}$ , and in Frequency selective channel, the '2-D' outperforms the Random one with power gain of about 1dB, measured at  $BER = 10^{-5}$ . while the '3-D' Chaotic Interleaver proves its superiority over tested Interleavers, since in flat fading Rayleigh channel it outperforms the Random Interleaver with power gain of about 0.75dB measured at  $BER = 10^{-6}$ , and in Frequency selective Rayleigh fading channel it has a power gain of about 2dB measured at  $BER = 10^{-5}$ . And with respect to the image transmission the best candidate to transmission over Rayleigh flat fading channel is the JPEG image format, and in frequency selective channel the best was the 8-bit indexed image. The advantage of the chaotic Interleaver was: the 2-D Chaotic Interleaver was able to restore the JPEG image intact with about one decoding iteration gain, while the 3-D one has a gain of about two decoding iterations in comparison with random interleaver during transmission over fading channels.

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