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Layered Approach for Nonlinear Analysis of Hybrid Reinforced Concrete Beams Using Finite Elements Method

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Abstract

This paper is concerned with the development of computational model of nonlinear response of hybrid reinforced concrete beams using finite element method with layering technique. The degenerating shell element with material nonlinearity is used to analyze the beams. The area of steel reinforcement is represented by an equivalent thickness. Perfect bond between the concrete and reinforcement has been assumed. The elasto- plastic behavior of the steel is used. Both multilinear and elastic-perfect plastic models have been adopted to represent high-strength and normal concrete's stress-strain behavior, respectively. A comparison between the proposed model and experimental results that published by other researchers have shown a good agreement.

Keywords: layered approach, nonlinear analysis, hybrid, finite element

طريقة الطبقات للتحليل اللاخطي للعتبات الخرسانية المسلحة الهجينة باستخدام طريقة العناصر المحددة

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الخلاصة

هذه المقالة تعنى بتطوير نموذج رياضي للتصرف اللاخطي للعتبات الخرسانية المسلحة الهجينة باستخدام طريقة العناصر المحددة بأتباع اسلوب الطبقات. تم استخدام عناصر قشرية مندمجة مع لاخطية المادة لتحليل العتبات. مساحة التسليح تم تمثيلها بسمك مكافئ. تم فرض ان الترابط متكامل بين التسليح و الخرسانة المحيطة. تم استخدام تصرف اجهاد-انفعال مرن-لدن للتسليح الفولاذي بينما للخرسانة العادية و للخرسانة عالية المقاومة تم استخدام النموذج الخطي-المتعدد و مرن-لدن تام على التوالي. اظهرت المقارنة بين النموذج المقترح و النتائج المختبرية المنشورة من قبل باحثين اخرين تقارب جيد بينهما.

Introduction

High strength concrete with a compressive strength of up to 100 MPa can be now produced commercially. This is mainly because a large development in material technology and the availability of various types of minerals and chemical admixtures and very powerful superplasticizers [1].

High strength concrete does very well in compression members, such as columns and piles which lead to reduction in size and weight. In the design of flexural members, the decision to use either a relatively high or low strength concrete or steel depends on economics, the importance of special requirements, member size and the concerning mechanical behavior as deflection, ductility and crack width [2, 3].

Several ways were followed to enhance the behavior of reinforced concrete flexural members in order to satisfy different uses requirements, like the use of fiber of different materials and shapes, polymers and different types of admixtures, using two different strength concretes in one cross section to form a hybrid concrete strength cross section.

Beams of more than one homogenous material are commonly used. Wooden beams are often reinforced by metal straps. Reinforced concrete beams are the most familiar beams of two materials which consist of two different materials which coincide with each other in some properties, like coefficient of thermal expansion, and complete each other in other properties. Different types of hybrid beams were made by adding different types of fibers or polymers to concrete, partially or over all the depth, to enhance its structural behavior [4, 5].

Current codes of practice like BS, ACI and CEP-FIP [6, 7, 8], are used to find the load capacity of hybrid and homogenous high strength concrete beams. The Newmark's numerical procedure is used to predict more accurate results of deflections [9]. Some researchers have used nonlinear finite elements program provided by ANSYS software [10].

In present work a finite element method has been developed using the degenerated shell element and layered model for discretization and integration through the shell element thickness as given in Refs. [11, 12, 13].

Finite Element Formulation

In the degeneration process, four assumptions are taken into account:

1. The deflection of the shell mid-surface is small.
2. The shell mid-surface remains straight after deformation but the deflection is not necessarily normal to the shell mid-surface.
3. The strain energy associated with stresses perpendicular to the local surface is neglected, and
4. The normal stress component is constrained to zero.

Figure.(1) shows the quadratic degenerate shell element with the element geometry. Five degrees of freedom are specified at each total point, three translation displacements in the direction of the global axes and two rotations with respect to axes in the plane of the middle surface. Nine-node Lagrangian elements with total Lagrangian approach are employed in dealing with the material nonlinearity. In a total Lagrangian formulation, all quantities such as displacements, stresses, strains, cross-sectional properties, and material properties are referred to the initial configuration of the structure. Modified Newton-Raphson method with displacement convergence criterion is adopted in the present investigation and to reach more accurate results [11, 12]. The stiffness matrix is updated in the second iteration of each load increment. The reduced integration is used for the analysis of application.

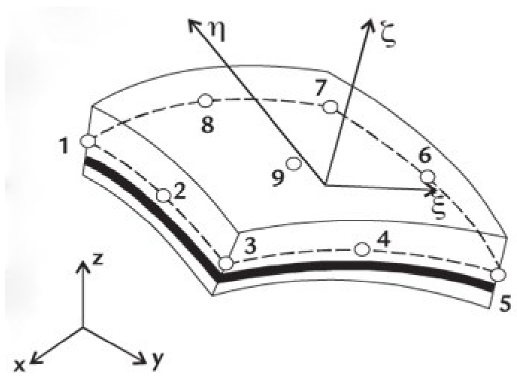


Fig .(1) Quadratic degenerate shell element.

Layered Model

A layering approach has been adopted for the nonlinear analysis of hybrid concrete beams. The nonlinear behaviors of concrete in compression, cracking and reinforcement have been represented across the cross-section. The cross-section may be divided into a number of high strength and normal concrete layers through thickness while the steel reinforcement is represented by a number of smeared layers with an equivalent thickness as shown in Fig. 2. Layers of different thicknesses can be employed, as well as different number of layers. Each layer contains Gauss points (stress points) on its mid-surface. The stress- strain and stress components of each layer are computed at those Gauss points and are assumed to be constant over the thickness of each layer as shown in **Figure.(2)** [11, 12].

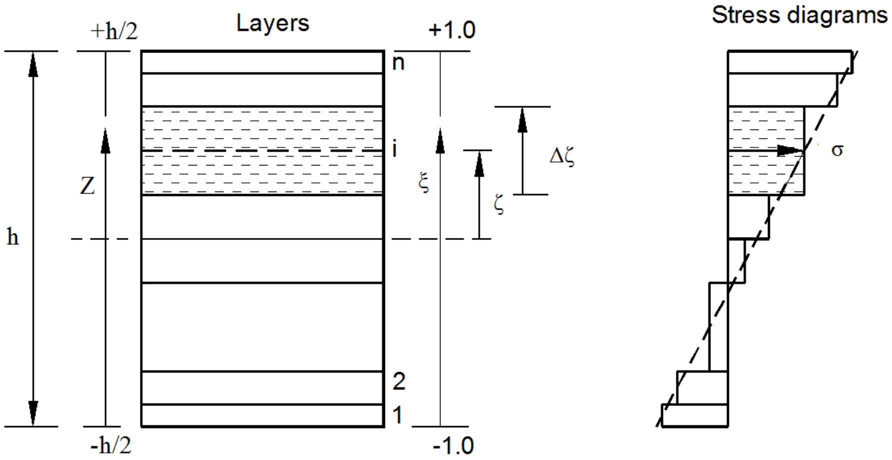


Fig .(2) Layered model and corresponding stress representation.

Reinforcement Representation

In the present investigation, concrete and reinforcement are represented with a single element. Perfect bond is assumed between the reinforcement and the surrounding concrete. The stiffness and internal forces associated with the reinforcement are integrated and added to those of the concrete to get the total stiffness and internal forces of the element. Each set of reinforcement is smeared and assumed to be distributed over the element with an equivalent thickness. Each steel layer deforms only in the direction of the bar and having strength and stiffness characteristics in the bar direction only. So, it exhibits a uniaxial response in the bar direction [11, 12].

Material Modeling and Failure Criterion

The uniaxial stress-strain relationship for the concrete in compression assumed the yield criterion is the basis for an elastic- perfectly plastic model for normal concrete as shown in **Figure.(3)** [11]. The model response is elastic until the effective stress reaches the value of f_c'. For high-strength concrete the multilinear model is adopted to idealize the stress-strain behavior as shown in Fig. 4. It can be defined by the following equations [10],

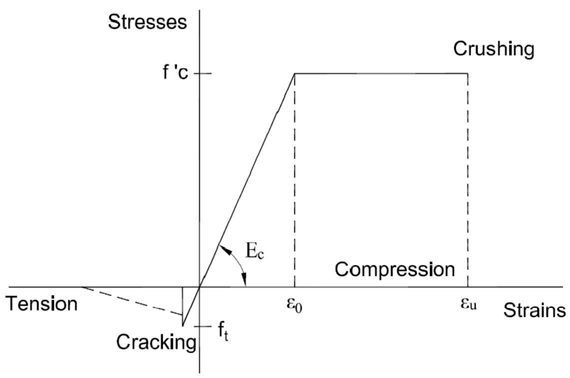


Fig .(3) One dimensional representation to the concrete constitutive model [11].

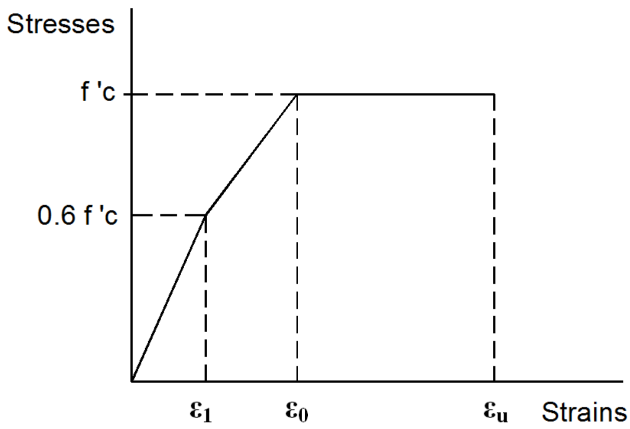


Fig .(4) Uniaxial stress strain model for high strength concrete [10].

$$f_c = \varepsilon E_c \quad \text{for} \quad \varepsilon \le \varepsilon_l \qquad \text{..................1}$$

$$f_c = 0.6\, f_c' + \frac{0.4\, f_c'}{(\varepsilon_o - \varepsilon_l)} (\varepsilon - \varepsilon_l) \quad \text{for} \quad \varepsilon_l \le \varepsilon \le \varepsilon_o \qquad \text{..................2}$$

$$f_c = f_c' \quad \text{for} \quad \varepsilon_o \le \varepsilon \le \varepsilon_u \qquad \text{..................3}$$

Where ε_l is the elastic limit strain and it equals to $\frac{0.6\, f_c'}{E_c}$ and ε_0 is the strain at peak stress and it equals to 0.002.

In tension, a smeared or distributed model for concrete cracking is assumed and an average shear modulus ρG is used for cracked zone as

$$\rho G = 0.5 G \qquad \text{..................4}$$

Tension stiffening effect in concrete in corporate to gradual release of the concrete stress component normal to the crack is shown in **Figure.(5)** [8].

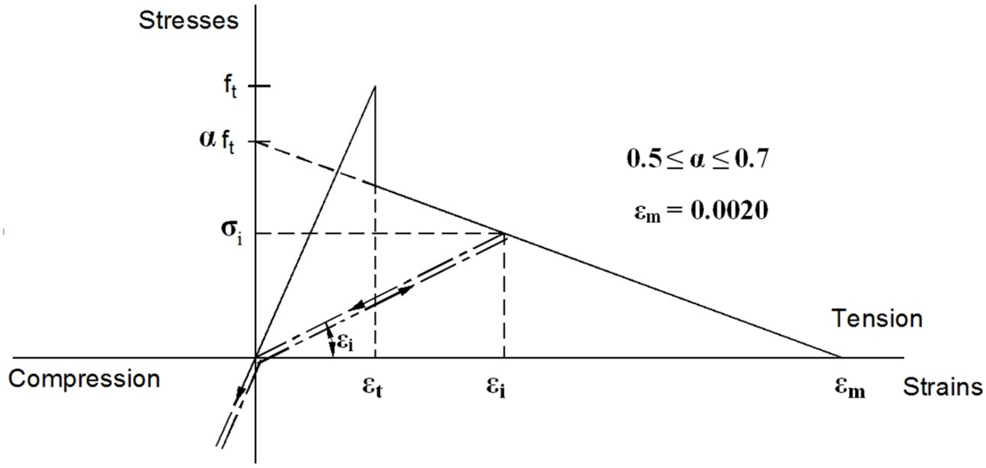


Fig .(5) Loading and unloading behavior of cracked concrete illustrating tension stiffening behavior [8].

Modeling of Steel Reinforcement

The reinforcement is modeled by smearing the steel bars into equivalent steel layers through the thickness of element which are assumed to be capable of transmitting axial forces only. The uniaxial elastic-plastic with hardening model for steel layers is shown in Figure(6) [11].

Computer Program

A computer program HYBRIDSHELL, capable of analyzing shell and beam structures based on a realistic analysis at the limit state of collapse is prepared here. A flowchart of the program is given in Appendix A. This program deals with nonlinear analysis of hybrid reinforced concrete beams. It is written in Fortran 77 language.

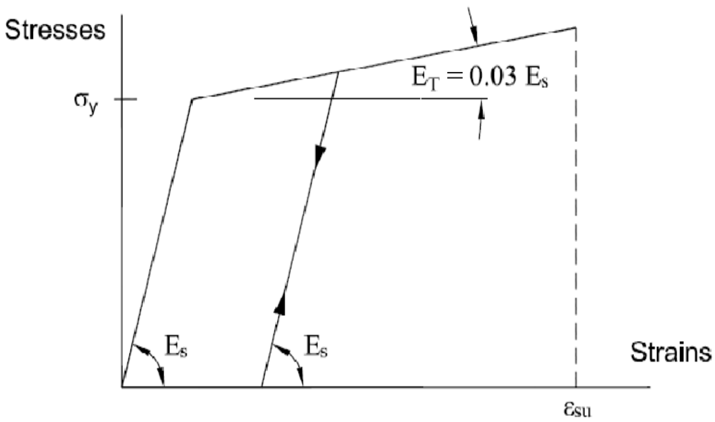
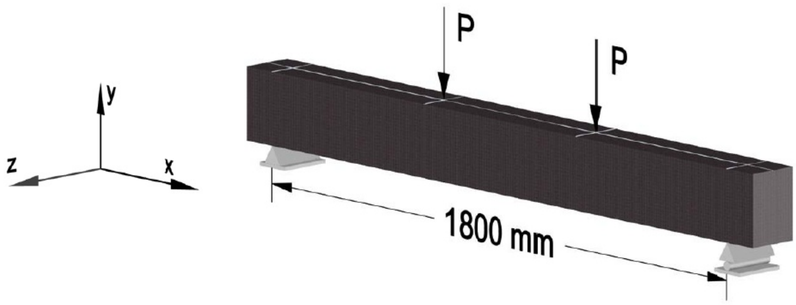


Fig .(6) Idealized elasto-plastic (with hardening) stress-strain relation for reinforcing [11].

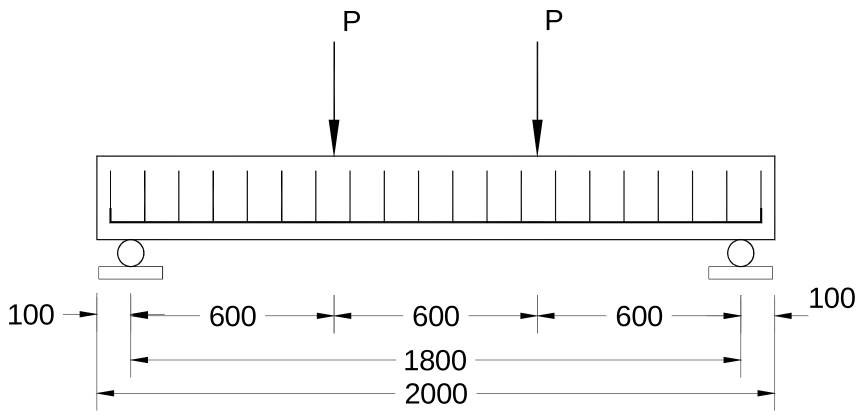
Test Results and Discussion

Simply supported hybrid reinforced concrete beams tested by Al-Shadidi [10] are analyzed in the present investigation. Two mixes were designed to obtain concrete of 25 and 70 MPa nominal compressive strengths. These beams were subjected to two equal third point loads. The dimensions of each beam are (2000x240x160 mm). Five beams with four different steel ratios were investigated. The geometry and details of beams are shown in (**Figure. 7. Figure. 8)** and **Table 1** shows the details of the beams which used in the present study. The meshing and modeling of the beam is shown in Fig.9. The material properties of the beams are presented in **Table 2.** Only beam labeled as B1 is not hybrid for comparison purpose.

The ultimate loads the beams can carry according to a nonlinear solution carried out using both present model and that used by Al-Shadidi [10] employing ANSYS program, are shown in **Table 3.** The results are compared with corresponding experimental values as given by [10], which seems to be acceptable.



(a) Test set-up.



(b) Loading and longitudinal section in the beam (dimensions in mm).

Fig .(7)

Homogenous beam cross section, B1
f'_c = 70 MPa
As = 2Φ12
Stirrups:Φ6@100

Hybrid beam cross section, B2
f'_c = 25, 70 MPa
As = 2Φ12
Stirrups:Φ6@100

Hybrid beam cross section, B3
f'_c = 25, 70 MPa
As = 4Φ12
Stirrups:Φ6@100

Hybrid beam cross section, B4
f'_c = 25, 70 MPa
As = 4Φ16
Stirrups:Φ10@100

Hybrid beam cross section, B5
f'_c = 25, 70 MPa
As = 6Φ16
Stirrups:Φ10@100

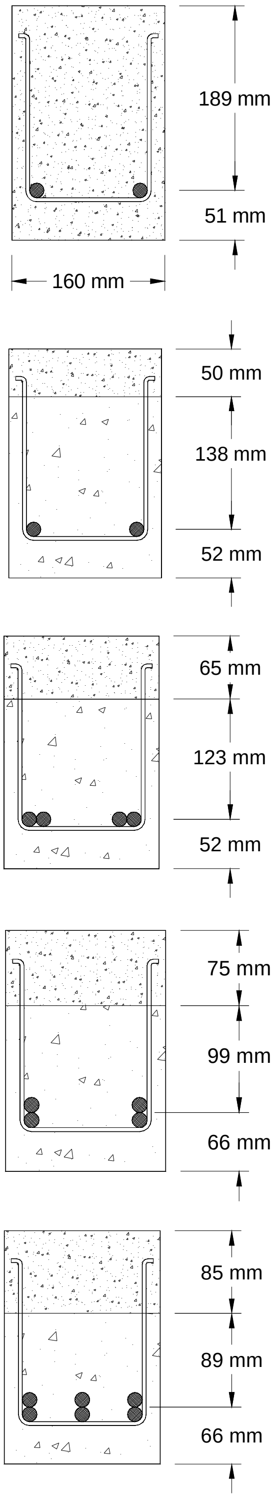
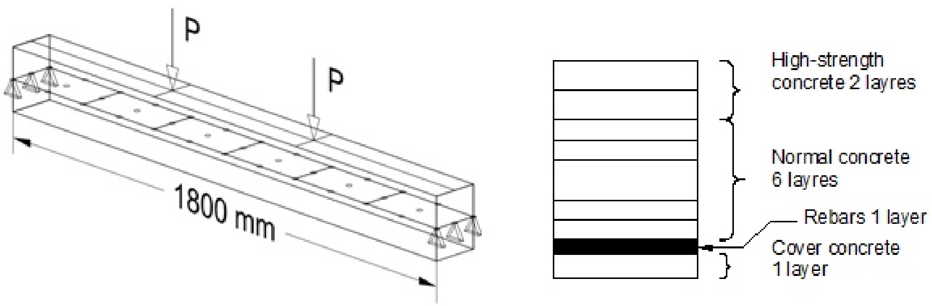


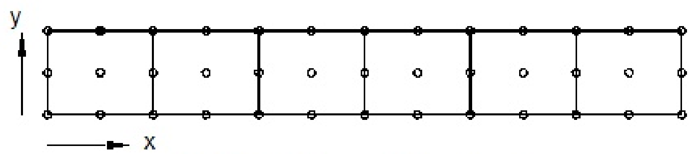
Fig .(8) Beams Cross-sections details.

Table .(1) Notations and reinforcing details of flexural and shear beams.

Beam no.	Thickness of high-strength concrete	Concrete f'_c (MPa)	Main reinforcement	Stirrups
B1	240	70	2ϕ12 mm	ϕ6 @ 100mm
B2	50	25, 70	2ϕ12 mm	ϕ6 @ 100mm
B3	65	25, 70	4ϕ12 mm	ϕ6 @ 100mm
B4	75	25, 70	4ϕ16 mm	ϕ10 @ 100mm
B5	85	25, 70	6ϕ16 mm	ϕ10 @ 100mm



(a) Degenerated shell element mesh and cross section layering.



(b) Shell elements for the beam.

Fig .(9) Meshing and modeling of the beams.

Table .(2) Material properties and parameters of beams.

Beam No.	Materials Properties*			
	f'_c (MPa)	f_r (MPa)	E_c (GPa)	f_y (MPa)
B1	69.5	6.9	36.7	560
B2	26, 72	2.8	24.5, 39	560
B3	24.5, 72	2.3	25, 36.3	560
B4	25.5, 73	2.8	24.7, 38	596
B5	26, 71	2.9	27.1, 38.2	596
Parameters for all beams				
Tension stiffening parameters			$\alpha = 0.6$	
			$\varepsilon_m = 0.002$	

*Assume $\nu = 0.18$ for concrete

Table .(3) Comparison the load capacity of beams with Ref. [10].

Beam No.	Experimental $P_{u,1}$ (kN)	Present Analysis $P_{u,2}$ (kN)	F.E. Solution [10] $P_{u,3}$ (kN)	$P_{u,1} / P_{u,2}$	$P_{u,1} / P_{u,3}$ [10]
B1	90	83	87.5	1.08	1.03
B2	90	82.8	82.5	1.09	1.09
B3	170	158	181.5	1.08	0.93
B4	281	260	272	1.08	1.03
B5	376	356.4	351	1.05	1.07

Figure.(10) shows that the present method is giving more stable results and it always on the conservative side. While the results given by [10] using ANSYS program may give underestimate of ultimate load capacity of the beams.

The beams are modeled mathematically and subjected to the same loading arrangements in the experimental program. **Figures (11 to 15**) show load-deflection curves for the beams.

Similar to the typical behavior of reinforced concrete beams, the curves show that beams fail by yielding of tension reinforcement. Three different stages are obvious in load-deflection behavior of beams B1 to B3. The precracking linear part is the first stage where both concrete and steel are still in their elastic response due to small strains and where the uncracked moment of inertia is the control factor. The range of this stage depends on the modulus of rupture of concrete.

Figure (11) is of nonhybrid beam that its strength in the tension zone is nominally 70 MPa. So it is clear that the cracking load P_{cr} is greater than that of the remaining hybrid sections of other beams. This is because that the uncracked flexural rigidity of hybrid beams is less than that of nonhybrid beam.

Immediately after cracking, another almost linear portion will begin from a distinct point on the curve corresponding to P_{cr}. The postcracking portion has a less slope than that of the first stage because of the successive immigration of moment of inertia from uncracked to cracked starting from the highest stressed zone.

The point of steel yielding initiation will terminate the postcracking stage to get in the postyielding one where the deflection increases at a fast rate to reach a secondary crushing failure stage. **Figure (14 and 15**) show the load deflection curves for beams B4 and B5 which fail in compression where just the first two stages clearly appear.

The presence of the steel in the concrete cross section increases its gross moment of inertia, but its effect is higher on the cracked moment of inertia as can be seen in **Figure (16).** It can be seen that beams B4 and B5 where the steel reinforcements are the highest are less ductile in failure manner.

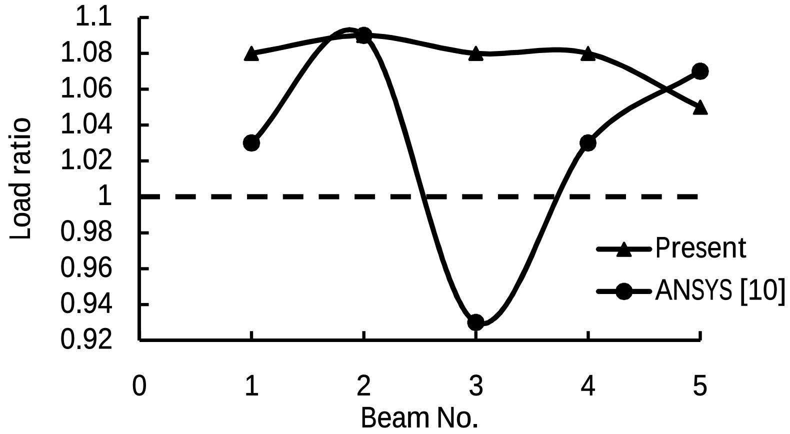


Fig .(10) Ultimate load capacity of beams.

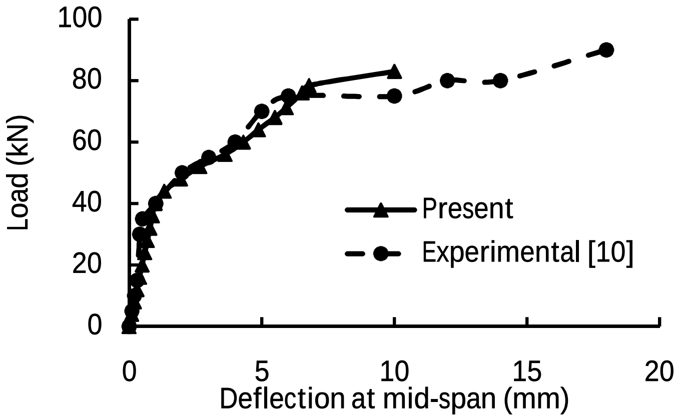


Fig .(11): Load deflection curve for B1.

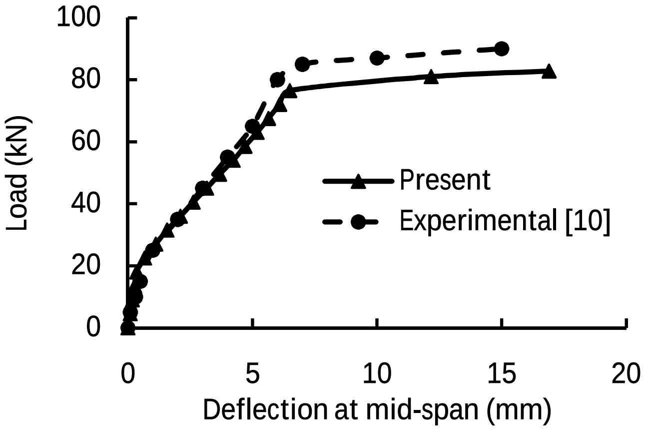


Fig .(12): Load deflection curve for B2.

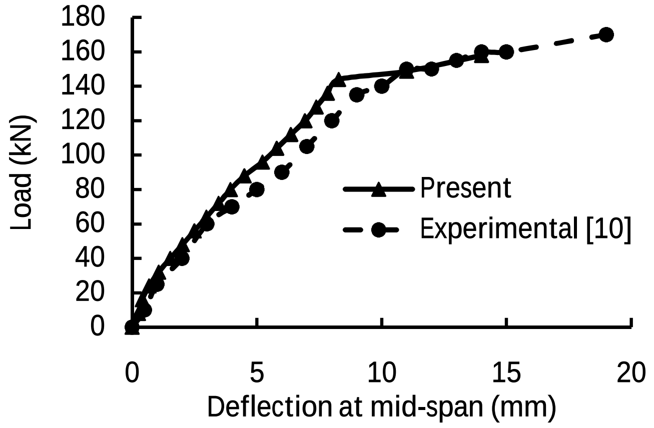


Fig .(13) Load deflection curve for B3.

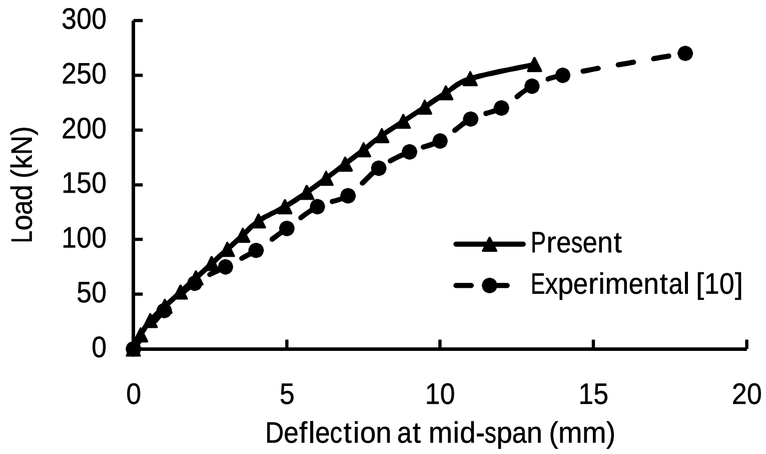


Fig .(14): Load deflection curve for B4.

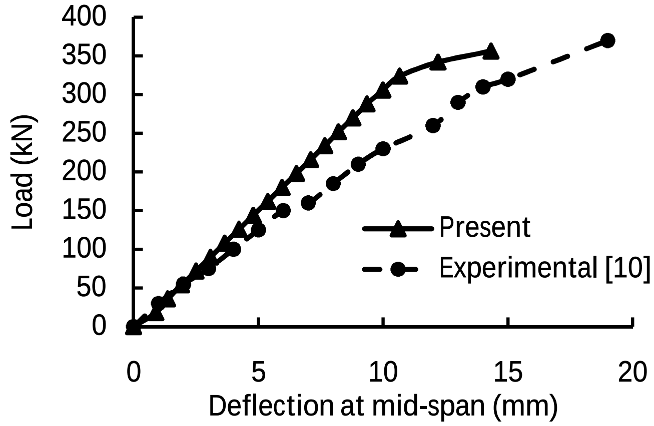


Fig .(15): Load deflection curve for B5.

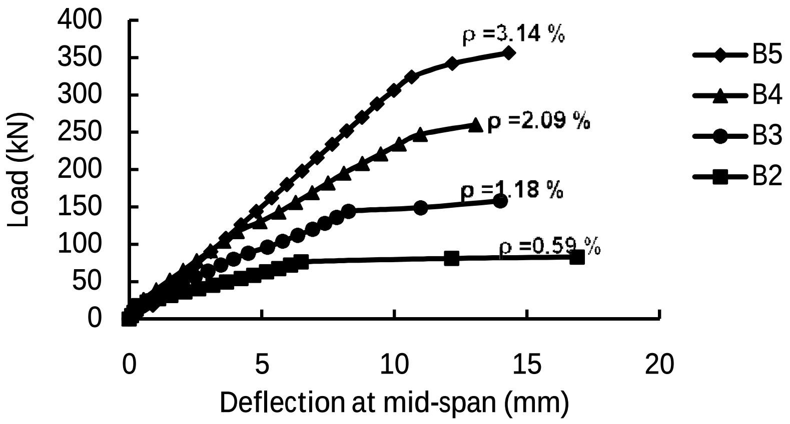


Fig .(16) Load-deflection curves for hybrid beams with different steel reinforcement ratios.

Conclusions

The application of the layered approach for finite element method to analyze hybrid reinforced concrete beams has been described in this paper. Based on the results reached here, the following conclusions can be drawn.

1. A comparison of the results given by the method with those measured experimentally has established the accuracy of the computational model adopted to predict the behavior of beams at different stages of loading up to failure.
2. While the results using ANSYS program gave underestimate of ultimate load capacity of the beams, the present method is of more stable results and it always on the conservative side. This may due to that the layering is more convenient to deal with beam section composed of different properties.
3. It can be noted from **Figures.(11-15)** that the deviation between experimental and analytical results increases with increasing the depth of the high strength concrete zone layers and increasing steel reinforcement ratios. This may due to high stresses induced on the elements representing the cross section and one may increase layers to get more accurate results.
4. No big difference in ultimate load capacity of beams B1 and B2. Both beams B1 and B2 are identical except that beam B1 is nonhybrid of high strength concrete while B2 is hybrid of 20% of high strength concrete only.
5. The effect of increasing percentages of steel reinforcement is as expected to be giving more strength but the ductile manner will be reduced in similar to the typical reinforced concrete beams.

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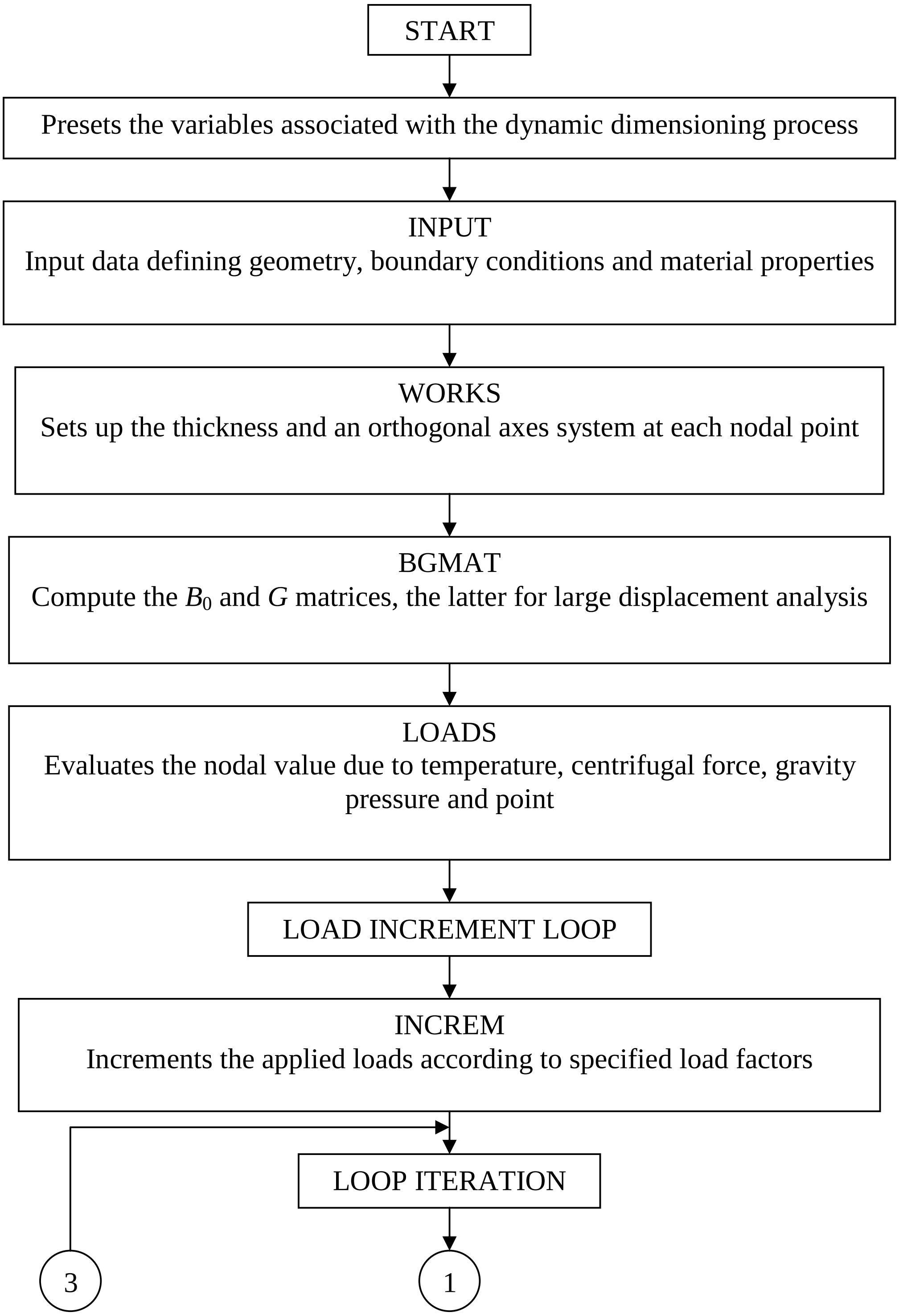
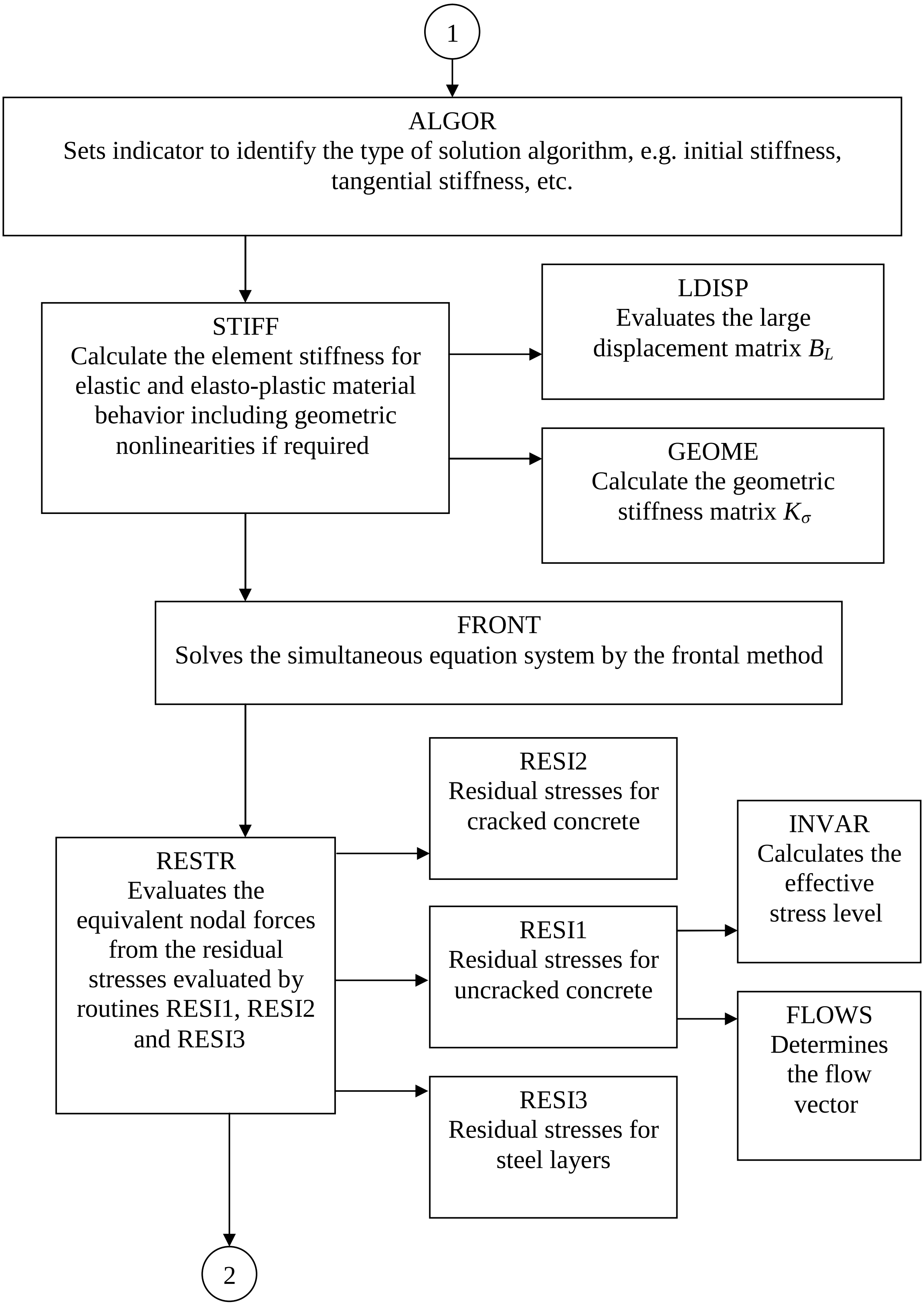
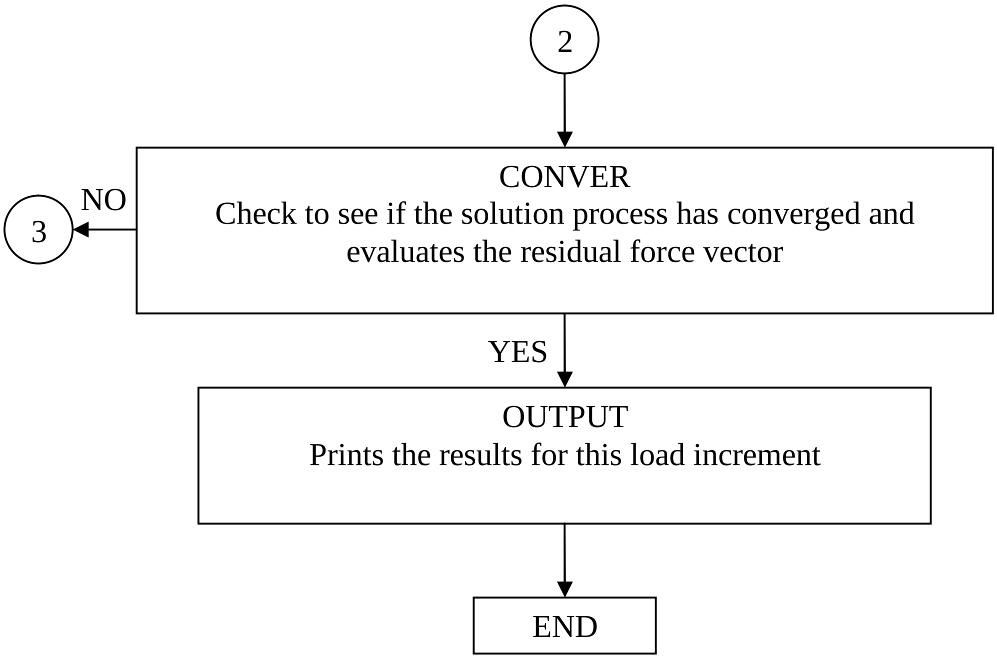
Appendix A: Computer Program HYBRIDSHELL Flowchart




1
ALGOR
Sets indicator to identify the type of solution algorithm, e.g. initial stiffness, tangential stiffness, etc.
STIFF
Calculate the element stiffness for elastic and elasto-plastic material behavior including geometric nonlinearities if required
LDISP
Evaluates the large displacement matrix B_L
GEOME
Calculate the geometric stiffness matrix K_σ
FRONT
Solves the simultaneous equation system by the frontal method
RESTR
Evaluates the equivalent nodal forces from the residual stresses evaluated by routines RESI1, RESI2 and RESI3
RESI2
Residual stresses for cracked concrete
RESI1
Residual stresses for uncracked concrete
RESI3
Residual stresses for steel layers
INVAR
Calculates the effective stress level
FLOWS
Determines the flow vector
2


2
CONVER
Check to see if the solution process has converged and evaluates the residual force vector
NO
3
YES
OUTPUT
Prints the results for this load increment
END