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On Fuzzy SBA-Ideal of AB-Algebra

Lecture Showq Mohammed.E [1], Lecture Alaa Saleh Abed [2]

[1,2]*Faculty of Education for Girls, Department of Mathematics, University of kufa, Najaf, Iraq*

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Abstract:
In this paper, we introduce and study ideal in AB- Algebra, it is called SBA-ideal, we give some examples, properties and theorems about it .Also,we study the direct product of SBA-ideals finaly, we introduce and study fuzzy SBA –ideal of AB-Algebra.

Keywords: AB-algebra, fuzzy AB- ideal, the equivalence calss, level cut.

Introducing:
The notion of fuzzy subsets was defined by Zadeh in 1965 [7]. Then Y. Imai and K. Iseki introduced two classes of abstract algebras were BCK-algebras and BCI-algebras [5,6]. After that several papers have been published by mathematicians to defined the classical mathematical concepts and fuzzy mathematical concepts. In 2018 A.T. Hameed introduced a new notion, called a AB- algebra [1,2].
In this paper we itemized the ideas as we talk about in the abstract.

1-Preliminaries;

Definition (1.1) [7];
Let $\wp$ be a non- empty set a mapping $\mu : \wp \to [0,1]$ is named a fuzzy subset of $\wp$.

Definition (1.2) [7];
Let ∇ be a fuzzy subset of $\wp$. If ∇ (y) = 0 for every $y \in \wp$ then ∇ is named empty fuzzy set .

Definition (1.3) [3];
Let ∇, ∂ be two fuzzy sets of set AB-Algebra $(\wp; \bullet, 0)$ Then :
1- $(\nabla \bigcap \partial)(x) = \min\{\nabla(x), \partial(x)\}, \forall x \in \wp$ 2- $(\nabla \bigcup \partial)(x) = max\{\nabla(x), \partial(x)\}, \forall x \in \wp$.

Definition (1.4) [2];

An AB-algebra is a nonempty set $\wp$ with a constant 0 and a binary operation $\bullet$ satisfying three axioms:

$1-((x\bullet y)\bullet(z\bullet y))\bullet(x\bullet z)=0, \forall x,y,z\in\wp$

$2-0\bullet x=0 \quad , \forall x\in\wp$

$3-x\bullet 0=x$

Definition (1.5) [1];

A non-empty subset I of an AB-algebra $(\wp;\bullet,0)$ is named an AB-ideal of $\wp$ if the following two conditions are hold :

$1-0\in I$

$2-(x\bullet y)\bullet z\in I$ and $y\in I\rightarrow x\bullet z\in I, \forall x,y,z\in\wp.$

Proposition (1.6) [1];

Let $\{I_j\}_{j\in\hbar}$ be a family of AB-ideals of AB-algebra $(\wp;\bullet,0)$ then $\bigcap_{j\in\hbar} I_j$ is an AB-ideal of $\wp$.

Proposition (1.7) [2]:

Let $\{I_j\}_{j\in\hbar}$ be a family of AB-ideals of AB-algebra $(\wp;\bullet,0)$ where $I_j\subseteq I_{j+1}, \forall j\in\hbar$ then $\bigcup_{j\in\hbar} I_j$ is AB-ideal of $\wp$.

Definition (1.8) [2]:

Let $(\wp;\bullet,0)$ and $(G;\bullet',0')$ be two AB-algebras .A homomorphism from $\wp$ into G is a mapping $f:(\wp;\bullet,0)\rightarrow(G;\bullet',0')$ such that $f(x\bullet y)=f(x)\bullet' f(y)\ \forall x,y\in\wp$. The set ker (f)= $\{x\in X \mid f(x)=0'\}$ is called the kernel of f .

Definition (1.9) [1]:

Let I be an AB-ideal of AB-algebra $\wp$ .Given $x\in\wp$, the equivalence calss $[x]_t$ of $\wp$ is defined as the set of all element of $\wp$ that are quivalent to x that $[x]_t$ ={y∈ $\wp$: x~y},we define the set $\wp/I$ = { $[x]_t : x\in\wp\}$ and a binary operation ($\bullet$) on $\wp/I$ by $[x]_t\bullet[y]_t=[x\cdot y]_t$

Definition (1.10) [1]:

Let $f:(\wp;\bullet,0)\rightarrow(\wp/I\ ;\bullet',0')$ be an outo homomorphism, I be an AB-ideal of AB-algebra $\wp$. Then f is named the natural AB- homomorphism of $\wp$ onto $\wp/I$ if $f(x)=[x]_t, \forall x\in\wp$.

Definition (1.11) [2]:

A fuzzy subset ∇ of AB-algebra $\wp$ is known fuzzy AB- ideal of $\wp$ if satifies the following:

$1-\nabla(0)\geq\nabla(x), \quad \forall x\in\wp$

$2-\nabla(x\bullet z)\geq\min\{\nabla((x\bullet y)\bullet z),\nabla(y)\}, \forall x,y,z\in\wp.$

Theorem (1.12) [2]:

Let ∇ be a fuzzy subset of AB-algebra $\wp$. Then ∇ is a fuzzy AB- ideal of $\wp$ if and only if , $\forall t \in [0,1], \nabla_t$ then either empty or an AB-ideal of $\wp$.

Definition (1.13) [4]:

Let ∇ be a fuzzy subset of a set $\wp$. For any t$\in$[0,1], the set

$\nabla_t = U(\nabla, t) = \{ x \in \wp : \nabla(x) \geq t \}$ is called a level set (upper level cut) of ∇.

Theorem (1.14) [2]:

Let $(\wp;\bullet,0)$ and $(G;\bullet',0')$ be two AB-algebras and $\varpi:(\wp;\bullet,0)\rightarrow(G;\bullet',0')$ be an onto homomorphism . Then if ∇ is a fuzzy AB - ideal of $\wp$, then $\varpi(\nabla)$ is a fuzzy AB - ideal of G.

Definition (1.15) [9]:

Let ∇ be a fuzzy ideal of $\wp$ and $f:(\wp;\bullet,0)\rightarrow(G;\bullet',0')$ then we called ∇ is f-invariant if and only if for all $z, y \in \wp$, $f(z) = f(y)$ implies ∇ (z) = ∇ (y).

Definition (1.16) [8]:

Let $\{\nabla_\varepsilon,\ \varepsilon \in \varpi\}$ be a family of fuzzy subsets of a set $\wp$. Define the fuzzy subset of $\wp$ (intersection) by: $\bigcap_{\varepsilon\in\varpi}\nabla_\varepsilon(x) = \inf_{\varepsilon\in\varpi}\{\nabla_\varepsilon(x)\}, \forall\, x \in \wp$,define the fuzzy subset of $\wp$ (union) by $\bigcup_{\varepsilon\in\varpi}\nabla_\varepsilon(x) = \sup_{\varepsilon\in\varpi}\{\nabla_\varepsilon(x), \forall x \in \wp.$

2-Mean Results:

In this section we introduce the notion SBA-ideal of AB- algebra $\wp$. We will discuse proposition about the image of it under onto homomorphism.

Definition (2.1):

An AB- ideal S of AB- algebra $\wp$ is named SBA-ideal if it satisfies two conditions : for all $a, m \in \wp$:

$1-0 \in S$,

$2-a \in S \wedge a \bullet m \in S \rightarrow a \bullet (m \bullet a) \in S$.

Example (2.2):

Consider AB-algebra $\wp$={0,1,2,3,4,5} that is defined by following table:

$\bullet$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	0	0	0	0	1
2	2	2	0	0	1	1
3	3	2	1	0	1	1
4	4	4	4	4	0	1
5	5	5	5	5	5	0

Let S={0,1,2}, then S is SBA-ideal of $\wp$.

Theorem (2.3):

Let$\{S_j : j \in \hbar\}$be a family of SBA-ideals of AB-algebra $\wp$, then $\bigcap_{i\in\hbar} S_j$ is an SBA-ideal of AB-algebra $\wp$.

Proof

Since S_j $\forall j \in \hbar$ is SBA-ideal $\rightarrow S_j$ $\forall j \in \hbar$ is an ideal that means $\bigcap_{j\in\hbar} S_j$ is an ideal by using Proposition (1.7) and $0 \in \bigcap_{j\in\hbar} S_j$.

Let $a, m \in \wp$ such that $a \in \bigcap_{j\in\hbar} S_j$, $a \bullet m \in \bigcap_{j\in\hbar} S_j$ this implies $a, m \bullet a \in S_j$, $\forall j \in \hbar$ .amd we have S_j $\forall j \in \hbar$ is SBA-ideal of $\wp$ then $a \bullet (m \bullet a) \in S_j$ $\forall j \in \hbar$,we get $a \bullet (m \bullet a) \in \bigcap_{j\in\hbar} S_j$. Thus $\bigcap_{j\in\hbar} S_j$ is SBA-ideal.

Theorem (2.4):

Let $\{ S_j \}_{j\in\hbar}$ be a chian of SBA-ideals of $\wp$ where $S_j \subseteq S_{j+1}$, $\forall j \in \hbar$,then $\bigcup_{j\in\hbar} S_j$ is SBA-ideal $\wp$

Proof

Let $\{ S_j \}_{j\in\hbar}$ be a chain of SBA-ideal of $\wp \rightarrow \bigcup_{i\in\hbar} S_j$ is an ideal of $\wp$ by using Proposition (1.7)and

$0 \in \bigcup_{j\in\hbar} S_j$.

Let $a, m \in \wp, a \in \bigcup_{j\in\hbar} S_j \wedge a \bullet m \in \bigcup_{j\in\hbar} S_j$, then there exist $S_k \in \{ S_j \}_{j\in\hbar}$ such that

$a \in S_k \wedge a \bullet m \in S_k \Rightarrow a \bullet (m \bullet a) \in S_k$. Since $\wp$ ideal of -is SBA S_k

$\rightarrow a \bullet (m \bullet a) \in \bigcup_{i\in\hbar} S_j \Rightarrow \bigcup_{i\in\hbar} S_j$ is SBA-ideal of $\wp$

Theorem (2.5):

Let $\zeta : (\wp_1, \bullet, 0) \rightarrow (\wp_2, \bullet', 0')$ be an AB- onto homomorphism , S be SBA-ideal of $\wp_1$ then is $\zeta(S)$ SBA –ideal of $\wp_2$.

Proof

Let S be a SBA-ideal of $\wp_1$ we have $\zeta(S) = \{\zeta(i) : i \in S\}$ is an ideal of $\wp_2$. To prove –is SBA $\zeta(S)$ ideal .

let $0' \in \zeta(S)$ $,\zeta(a) \in \zeta(S)$ $,\zeta(a) \bullet \zeta(m) \in \zeta(S)$ then $\zeta(a) \in \zeta(S) \wedge \zeta(a \bullet m) \in \zeta(S) \Rightarrow$

$a \in S$ and $a \bullet m \in S \rightarrow$ $a \bullet (m \bullet a) \in S$ since S is SBA-ideal of $\wp_1$

thus $\zeta(a \bullet (m \bullet a)) \in \zeta(S)$

$\zeta(a) \bullet' (\zeta(m) \bullet' \zeta(a)) \in \zeta(S)$

Then $\zeta(S)$ is SBA –ideal of $\wp_2$.

Proposition (2.6):

Let $\zeta:(\wp_1,\bullet,0)\to(\wp_2,\bullet',0')$ be an AB- outo homomorphism, $\angle$ be SBA-ideal of $\wp_2$,then $\zeta^{-1}(\angle)$. $\ker\zeta\subseteq\zeta^{-1}(\angle)$,where $\wp_1$ ideal of –is SBA

Proof

Let $\angle$ is a SBA –ideal of $\wp_2'$ and $\zeta^{-1}(\angle)=\{a\in\wp_1:\zeta(a)\in\angle\}$ is an ideal of $\wp_1$, since $0'\in\angle$, we have

$\zeta^{-1}(0')=0\in\zeta^{-1}(\angle)$.

Let $a\in\zeta^{-1}(\angle)\wedge a\bullet m\in\zeta^{-1}(\angle)$

$\to\zeta(a),\zeta(a\bullet m)\in\angle$.

Since $\angle$ is SBA-ideal of $\wp_2$

$\zeta(a)\bullet'\zeta(m)\bullet'\zeta(a)\in\angle$.

$\zeta(a\bullet(m\bullet a))\in\angle\to\zeta^{-1}(\zeta(a\bullet(m\bullet a)))\in\zeta^{-1}(\angle)$

$\to a\bullet(m\bullet a)\in\zeta^{-1}(\angle)$

$\zeta^{-1}(\angle)$ is SBA –ideal of $\wp_1$.

Proposition (2.7):

Let $\{\wp_j\}_{j\in\hbar}$ a family of AB- algebras and S_j be a SBA –ideal of $\wp_j$ $\forall j\in\hbar$,then $\prod_{j\in\hbar}S_j$ be SBA –ideal of direct product $\prod_{j\in\hbar}\wp_j$ .Where $\prod_{j\in\hbar}\wp_j=\{(x_j):x_j\in\wp_j,\forall j\in\hbar\}$.

Proof

Let $a_j,m_j\in\prod_{j\in\hbar}\wp_j$

If $a_j\in\prod_{j\in\hbar}S_j$,$a_j\bullet m_j\in\prod_{j\in\hbar}S_j$

Then $a_j\in S_j$,$a_j\bullet m_j\in S_j$

S_j is SBA-ideal of $\wp_j$ $\forall j\in\hbar$

$\to a_j\bullet(m_j\bullet a_j)\in S_j$

$a_j\bullet(m_j\bullet a_j)\in\prod_{j\in\hbar}S_j$

.$\prod_{j\in\hbar}\wp_j$ ideal of -is SBA thus $\prod_{j\in\hbar}S_j$

Proposition (2.8):

Assume $\Im$ be a normal subalgebra of AB –algebra $\wp$. If S is a SBA –ideal of $\Im$, then $S/\Im$ is SBA-ideal of $\wp/\Im$.

Proof

Let S is a SBA –ideal, that means S is an ideal of $\wp\Rightarrow S/\Im$ is an ideal of $\wp/\Im$.

Then $[0]_{\Im} \in S/_{\Im}$, since $0 \in S$

Let $[a]_{\Im},[m]_{\Im} \in \wp/_{\Im}$, So $[a]_{\Im},[a]_{\Im} \bullet [m]_{\Im} \in S/_{\Im}$

Then $[a]_{\Im},[a \bullet m]_{\Im} \in S/_{\Im}$

Thus $[a]_{\Im} \in S/_{\Im} \wedge [m \bullet a]_{\Im} \in S/_{\Im} \Rightarrow a \in S \wedge a \bullet m \in S$, but S is SBA - ideal

then $a \bullet (m \bullet a) \in S$. It followes $[a \bullet (m \bullet a)]_{\Im} = [a]_{\Im} \bullet ([m]_{\Im} \bullet [a]_{\Im}) \in S/_{\Im}$

Hence $S/_{\Im}$ is SBA - ideal of

Theorem (2.9):

If $\zeta : (\wp_1, \bullet, 0) \to (\wp_2, \bullet', 0')$ be a hommorphism from commutative AB-algebra $\wp_1$ into AB-algebra $\wp_2$,then ker (ζ) is a SBA-ideal of $\wp_1$.

Proof

$\zeta(0) = 0'$

Let $a \in \ker(\zeta) \wedge a \bullet m \in \ker(\zeta)$, $\forall a, m \in \wp_1$

then $\zeta(a) = 0' \wedge \zeta(a \bullet m) = 0'$

$\zeta(a) \bullet \zeta(m \bullet a) = 0' \bullet \zeta(m \bullet a) = 0'$ by using def AB - Algebra (2)

So $\zeta(a \bullet (m \bullet a)) = 0'$

$a \bullet (m \bullet a) \in \ker(\zeta)$

Thus ker (ζ) is a SBA-ideal of $\wp_1$.

3-Fuzzy SBA-Ideal:

In this section, we introduce the concept of a fuzzy SBA-ideal of AB- algebra $\wp$. We will discuse proposition about its the image of it under onto homomorphism.

Definition (3.1):

A fuzzy ideal ∇ of AB-algebra $\wp$ is named a fuzzy SBA –ideal and denoted it by F-SBA -ideal of $\wp$ if $\forall a, m \in \wp$, $\nabla(a \bullet (m \bullet a)) \geq \min\{\nabla(a), \nabla(a \bullet m)\}$

Example (3.2):

Let $\wp = \{0, \varepsilon, \tau, \partial\}$ be a set with the accompanying table:

$\bullet$	0	ε	τ	∂
0	0	0	0	0
ε	ε	0	0	0
τ	τ	ε	0	0
∂	∂	τ	τ	0

Then $(\wp, \bullet, 0)$ is an AB-algebra and defined fuzzy set $\nabla : \wp \to [0,1]$,when

$$\nabla = \begin{cases} 1 & , x = 0 \\ 0.5 & , x = \{\varepsilon, \tau, \partial\} \end{cases}$$ is F-SBA -ideal of $\wp$

Theorem (3.3):

Let S be a SBA –ideal on $\wp$, ∇ be a fuzzy subset of AB-algebra $\wp$ .For $\iota \in (0,1)$, there exists a F-SBA -ideal of $\wp$ such that $\nabla_\iota = S$

Proof

Let $\iota \in (0,1)$, defined $\nabla : \wp \to [0,1]$ by $\nabla(a) = \iota$ if $a \in S$ and $\nabla(a) = 0$ when $a \notin S$,$\nabla_\iota = \{a \in \wp : \nabla(a) \geq \iota\}$ $\Rightarrow \nabla_\iota = \{a \in \wp : \nabla(a) = \iota\} = S$,suppose ∇ is not F-SBA -ideal of $\wp$ ideal–Since S is SBA , $\nabla(a \bullet (m \bullet a)) \leq \min\{\nabla(a), \nabla(a \bullet m)$

$a \in S, a \bullet m \in S \to a \bullet (m \bullet a) \in S$

$\to \nabla(a) = \iota$ and ,$\nabla(a \bullet m) = \iota$ then we have

$\nabla(a \bullet (m \bullet a)) \leq \min\{\nabla(a), \nabla(a \bullet m)$

$\to \iota \leq \min\{\iota, \iota\}$

$\to \iota \leq \iota$

This is contradiction ∇ is F-SBA -ideal of $\wp$

Theorem (3.4):

Let ∇ be a fuzzy subset of an AB-algebra $\wp$,and ∇ is a F-SBA -ideal of $\wp$. Then ∇_* is SBA -ideal of $\wp$ .where $\nabla_* = \{x \in \wp | \nabla(x) = \nabla(0)\}$.

Proof

Let $a, m \in \wp$ such that $a, a \bullet m \in \nabla_*, \nabla(a) = \nabla(0), \nabla(a \bullet m) = \nabla(0)$

since ∇ is F-SBA -ideal of $\wp$

$\nabla(a \bullet (m \bullet a)) \geq \min\{\nabla(a), \nabla(a \bullet m)\}$

$\nabla(a \bullet (m \bullet a)) = \nabla(0) \Rightarrow a \bullet (m \bullet a) \in \nabla_*$

Then ∇_* SBA -ideal of $\wp$.

Proposition (3.5):

Let ∇ be F-SBA -ideal of AB- algebra $\wp$, then ∇_ι is SBA -ideal for $\iota \in [0, \nabla(0)]$

Proof

By using definition ∇_ι we have $\nabla_\iota = \{\tau \in \wp : \nabla(\tau) \geq \iota\}, \forall a, m \in \wp, a \in \nabla_\iota, a \bullet m \in \nabla_\iota$

$\nabla(a) \geq \iota, \nabla(a \bullet m) \geq \iota$ since ∇ is F-SBA -ideal of $\wp$ that mean

$\nabla(a \bullet (m \bullet a)) \geq \min\{\nabla(a), \nabla(a \bullet m)\} \geq t$

then $a \bullet (m \bullet a) \in \nabla_\iota$

∇_ι is SBA -ideal of $\wp$.

proposition (3.6):

Let $\zeta : (\wp_1, \bullet, 0) \to (\wp_2, \bullet', 0')$ be an onto homomorphism,let ∇ be a fuzzy ideal of a $\wp_1$. For $\iota \in [0, \nabla(0)]$if ∇_ι is SBA -ideal of $\wp_1$,then $\zeta(\nabla_\iota)$ is SBA -ideal of $\wp_2$.

Proof

By using Theorem (2.5) we can prove that $\zeta(\nabla_\iota)$ is SBA -ideal of $\wp_2$ obviously .

Proposition (3.7):

Let $\not{\zeta}:(\wp_1,\bullet,0)\rightarrow(\wp_2,\bullet',0')$ be an onto homomorphism, it is f-invaiant , then ∇ is F- SBA -ideal of $\wp_2$ if and only if $\not{\zeta}^{-1}(\nabla)$ is F- SBA -ideal of $\wp_1$.

Proof $\rightarrow$

Suppose that ∇ is a F- SBA -ideal of $\wp_2$

$\not{\zeta}^{-1}(\nabla)(a)=\nabla(\not{\zeta}(a))$ and

$\not{\zeta}^{-1}(\nabla)(a\bullet m)=\nabla(\not{\zeta}\ (a\bullet m))$

$=\nabla(\not{\zeta}(a\bullet(m\bullet a)))\geq\min\{\nabla(\not{\zeta}(a)),\nabla(\not{\zeta}(a\bullet m))\}$

then $\not{\zeta}^{-1}(\nabla)(a\bullet(m\bullet a))\geq\min\{\not{\zeta}^{-1}(\nabla)(a),\not{\zeta}^{-1}(\nabla)(a\bullet m)\}$

So $\wp_1$ ideal of -SBA -is F $\not{\zeta}^{-1}$

Proof $\leftarrow$

Assume that $\not{\zeta}^{-1}(\nabla)$ is F- SBA -ideal of $\wp_1$,let

$\not{\zeta}(a),\not{\zeta}(a\bullet m)\in\wp_2,\ \forall a,\ \ a,m\in\wp_1$

$\nabla(\not{\zeta}(a)\bullet'(\not{\zeta}(m)\bullet'\not{\zeta}(a)))=\nabla(\not{\zeta}(a\bullet(m\bullet a)))$

$=\not{\zeta}^{-1}(\nabla)(a\bullet(m\bullet a))\geq\min\{\not{\zeta}^{-1}(\nabla)(a),\not{\zeta}^{-1}(\nabla)(a\bullet m)\}$

Since $\not{\zeta}^{-1}(\nabla)$ is F- SBA -ideal of $\wp_1$

$\nabla(\not{\zeta}(a)\bullet'(\not{\zeta}(m)\bullet'\not{\zeta}(a)))\geq\min\{\nabla(\not{\zeta}(a)),\nabla(\not{\zeta}(a\bullet m))\}$

So $\not{\zeta}$ is SBA -ideal of $\wp_2$.

Theorem (3.8):

Let $\{\nabla_\varepsilon\}_{\varepsilon\in\varpi}$ be a family of F- SBA –ideals of $\wp$,then $\bigcap_{\varepsilon\in\varpi}\nabla_\varepsilon$ is F-SBA –ideal $\wp$.

Proof

Let $a,m\in\wp$,

$\bigcap_{\varepsilon\in\varpi}\nabla_\varepsilon(a\bullet(m\bullet a)))=\inf_{\varepsilon\in\varpi}\{\nabla_\varepsilon(a\bullet(m\bullet a))\}$ by Definition (1.16)

$\inf_{\varepsilon\in\varpi}\{\nabla_\varepsilon(a\bullet(m\bullet a))\}\geq\inf_{\varepsilon\in\varpi}\{\min\{\nabla_\varepsilon(a),\nabla_\varepsilon(a\bullet m)\}\}$, since ∇_ε is F - SBA - ideal

$=\min\{\inf_{\varepsilon\in\varpi}\{\nabla_\varepsilon(a)\},\inf_{\varepsilon\in\varpi}\{\nabla_\varepsilon(a\bullet m)\}\}=\min\{\bigcap_{\varepsilon\in\varpi}\nabla_\varepsilon(a),\bigcap_{\varepsilon\in\varpi}\nabla_\varepsilon(a\bullet m)\}$

then $\bigcap_{\varepsilon\in\varpi}\nabla_\varepsilon(a\bullet(m\bullet a))\geq\min\{\bigcap_{\varepsilon\in\varpi}\nabla_\varepsilon(a),\bigcap_{\varepsilon\in\varpi}\nabla_\varepsilon(a\bullet m)\}$

Hence $\bigcap_{\varepsilon\in\varpi}\nabla_\varepsilon$ F-SBA –ideal of $\wp$.

Theorem (3.9):

Let $\{\nabla_\varepsilon\}_{\varepsilon\in\varpi}$ be a chain of F- SBA –ideal $\wp$,then $\bigcup_{\varepsilon\in\varpi}\nabla_\varepsilon$ is F- SBA –ideal $\wp$.

Proof

Let $a,m\in\wp$, such that

$\bigcup_{\varepsilon\in\varpi}\nabla_{\varepsilon}(a\bullet(m\bullet a)) = \sup_{\varepsilon\in\varpi}\{\nabla_{\varepsilon}(a\bullet(m\bullet a))\text{by Definition}(1.16)$

$\sup_{\varepsilon\in\varpi}\{\nabla_{\varepsilon}(a\bullet(m\bullet a)) \geq \sup_{\varepsilon\in\varpi}\{\min\{\nabla_{\varepsilon}(a),\nabla_{\varepsilon}(a\bullet m)\}\},\text{since}\,\nabla_{\varepsilon}\text{ is F-SBA-ideal}$

$= \min\{\sup_{\varepsilon\in\varpi}\{\nabla_{\varepsilon}(a)\},\sup_{\varepsilon\in\varpi}\{\nabla_{\varepsilon}(a\bullet m)\}\} = \min\{\bigcup_{\varepsilon\in\varpi}\nabla_{\varepsilon}(a),\bigcup_{\varepsilon\in\varpi}\nabla_{\varepsilon}(a\bullet m)\}$

$\Rightarrow \bigcup_{\varepsilon\in\varpi}\nabla_{\varepsilon}(a\bullet(m\bullet a)) \geq \min\{\bigcup_{\varepsilon\in\varpi}\nabla_{\varepsilon}(a),\bigcup_{\varepsilon\in\varpi}\nabla_{\varepsilon}(a\bullet m)\}.$

So $\bigcup_{\varepsilon\in\varpi}\nabla_{\varepsilon}$ is F-SBA-ideal.

Theorem (3.10):

Let ∇ be a fuzzy subset of AB-ideal of $\wp$,then ∇ is a F-SBA-ideal iff ∇^{*} is a F-SBA-ideal of $\wp$.

Proof

Let $a,m\in\wp$,

$\rightarrow$

$\nabla^{*}(a\bullet(m\bullet a)) = \nabla(a\bullet(m\bullet a)) - \nabla(0) + 1$

since ∇ is F-SBA-ideal we have

$= \nabla(a\bullet(m\bullet a)) \geq \min\{\nabla(a),\nabla(a\bullet m)\}$

$= \nabla^{*}(a\bullet(m\bullet a)) \geq \min\{\nabla(a),\nabla(a\bullet m)\} - \nabla(0) + 1$

$= \min\{\nabla(a) - \nabla(0) + 1, \nabla(a\bullet m) - \nabla(0) + 1\}$

$= \min\{\nabla^{*}(a),\nabla^{*}(a\bullet m)\}$

∇^{*} is F-SBA-ideal.

$\leftarrow$ by using deafination ∇^{*}

$\nabla^{*}(a\bullet(m\bullet a)) = \nabla(a\bullet(m\bullet a)) - \nabla(0) + 1$

since ∇^{*} is F-SBA-ideal we have

$\nabla^{*}(a\bullet(m\bullet a)) \geq \min\{\nabla^{*}(a),\nabla^{*}(a\bullet m)\}$

$\nabla(a\bullet(m\bullet a)) - \nabla(0) + 1 \geq \min\{\nabla(a) - \nabla(0) + 1, \nabla(a\bullet m) - \nabla(0) + 1\}$

$\nabla(a\bullet(m\bullet a)) - \nabla(0) + 1 \geq \min\{\nabla(a),\nabla(a\bullet m)\} - \nabla(0) + 1$

$\nabla(a\bullet(m\bullet a)) \geq \min\{\nabla(a),\nabla(a\bullet m)\}$

$= \min\{\nabla^{*}(a),\nabla^{*}(a\bullet m)\}$

∇ is F-SBA- ideal.

References:

[1] A.T. Hameed and B. N. Abbas, "Derivation of AB-Ideals of and Fuzzy AB-Ideals of AB- Algebras", a theses Submitted to council of the Faculty of Education for Girls, kufa university,2018.

[2] A.T. Hameed, "AB-Ideals of AB- Algebras ", Applied Mathmatical, Vol,11, no.35,1715-1723,2017.

[3] D. Dubis and H, prade, "Fuzzy Set and Systems", academic press, INC, (London)LTD., Academic press, INC, fifth Avenue, New York, 1980.

[4] Das P.S.," Fuzzy Groups and Level Subgroups", Journal of Mathematical 230. Analysis and Applications, vol.84, pp.264-269, 1981.

[5] K. Iseki and S. Tanaka,"An introduction to the theory of BCK-algebras, Math, Japonica 23, no.1,1-26,1978.
[6] K. Iseki,"On BCI-algebras, Mathemayics Seminer Notes 8,125-130,1980.
[7] L.A. Zadeh, "Fuzzy Set", Information and control vol. 8, pp 338-358,1965.
[8] V. Kumbhojkar. and Bapat M.S., "Notso-fuzzy ideals", Fuzzy Set and Systems,
vol.37 (1991), pp.237-243.
[9] W.B. vasantha andasamy," Samarandache Fuzzy Algebra", United States of America, pp.9-10, pp.47-58,292,2003.