SEIF-ADJIONT OPERATOR OF PERIDYNAMIC MODEL

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ABSTRACT
In this paper, we study a functional analytical framework for a linear peridynamic model of spring system in one dimension,Various properties of the peridynamic operators are examining for general micromodulus,one this properties of peridynamic operator are self-adjont.

KEYWORD: *self-adjoin , peridynamic ,mathematical analysis* **.**

1-INTRODUCTION

The peridynamics,a continuum theory that employs a nonlocal model of force interaction. Specifically, the stress/strain relationship of classical elasticity is replaced by an integral operator that sums internal forces separated by a finite distance [1] .for a review of the recent applications of the peridynamic (PD) framework The peridynamic theory is alternative based on integral[3].The relation between general linear peridynamic model and the classical Navier equation[4].It explained in [8] how the general state-based PD material model converges to the continuum elasticity model as the ratio of the PD horizon to effective length scale decreases, assuming that the underlying deformation is sufficiently smooth.

Rather differential equation the purpose of peridynamic theory is provide more generalizes or other framework than the classical theory for problems involving discontinuities or other singularities in the deformation the integral equation express nonlocal force model that describes long-rang material interaction the convergence peridynamic model to classical elasticity theory by the limit small the horzin, i.e $\delta \to 0$ [9]. Such properties make Peridynamic theory a powerful tool for modeling problems involving cracks, interfaces or defects.

In this paper ,by mathematical analysis we discuss some propertics of perdynamic model. The properties of the models depend crucially on the particular micromodulus functions used to specify the spring systems. we discussed the self –adjont of peridynamic operator.Since the operator is one-to-one we conclude to the peridynamic operator is isomatry.By mathematical analysis method which indication the model is elasticity if the operator is self-adjoin .

2-**The peridynamic model**[2] :

The peridynamic is the second-order partial integro-differential equation in time variable[4],,[5],[7]:

$$\rho\ddot{u} = \int_{R^\circ} f(x,x`,u(x,t),u(x`,t))dx`+b(x,t)...............(1)$$

where ρ denotes the mass density , u the displacement field of the body , f the pairwise force function that describes the internal forces, and b an in homogeneity that collects all external forces per unit volume. By t >0, the time under consideration is denoted. and Rº denote the open ball of radius δ where δ>0 is the so-called peridynamic horizon of interaction such that :

$R^\circ = \{x \in \Omega : |x`-x| \leq \delta\}$

is sub region of R(the set of real number) Ω Where

The assumption of no explicit time dependence, and Newton's third law (For every action, there is an equal and opposite reaction)lead to:

$$f(x,x`,u,u`) = f(x`-x,u`-u)$$

with

$$f(x`,x,-\eta) = -f(x,x`,\eta), \forall x,x`,\eta = u`-u........(3)$$

It is typical for the peridynamic model to require

$$f(x,x`,\eta) = 0, if |x`-x| \geq \delta........(4)$$

A first-order approximation justifies for small relative displacements

$$f(x,x`,\eta) = f_0(x,x`) + C(x,x`)\eta.........(5)$$

with the stiffness tensor (or micromodulus function[12]) C =C(x, x`) and denoting forces in the reference configuration $f_0$ without loss of generality, we may assume $f \equiv 0$ since otherwise $f_0$ can be incorporated into the right-hand side b in general, the stiffness tensor C is neither definite nor depending on only $|x`-x|$ the length However, C has to be symmetric with respect to its arguments as well as with respect to its tensor structure such that

$$C(x`,x) = C(x,x`)..............(6)$$

$$C(x`,x)^T = C(x`,x).........(7)$$

$C(x`,x) = 0 if |x`-x| \geq \delta$ view of (4)

The stiffness tensor can be shown to read as:

$$C(x,x`) = \lambda_\delta(|x`-x)|(x`-x) \otimes (x`-x).........(8)$$

For the special case of proportional materials the equation (8)take the form :

$$C(x,x`) = \frac{c_\delta}{|x`-x|^3}$$

The linear peridynamic equation of motion (1) now reads as

$$\rho \ddot{u} = c_{\delta} \int_{R^{\circ}} \frac{(x`-x)\otimes(x`-x)}{|x`-x|^{3}} u(x`,t) - u(x,t) dx` \ldots\ldots\ldots\ldots(9)$$

In this paper we discuss along with boundary condition(case steady –state ,one dimensional , homogenous and linear model) , the eq (9) and eq(1) reduces to:

$$\rho \ddot{u} = \frac{1}{\delta^{2}} \int_{R^{\circ}} \frac{u(x`,t) - u(x,t)}{|x`-x|} dx` + b(x,t) \ldots\ldots\ldots\ldots(10)$$

Where $L_{\delta} = \frac{1}{\delta^{2}} \int_{R^{\circ}} \frac{u(x`,t) - u(x,t)}{|x`-x|} dx` \ldots\ldots(11)$

We called $\sigma(|x' - x|) = \frac{1}{|x' - x|}$ is kernel function of the peridynamic integral operator which also determines the micromodulus function.

3-Mathematical analysis for the peridynamic model[6][10]

To set up a suitable functional setting to discuss convergence properties of peridynamic model equations, we first make some definition on the kernel function,[1]:

$$\ell_{k,d,\delta} = \int_{0}^{\delta} \lambda_{d,\delta}(r) r^{k+d-1} dr \text{ ,for all } r = |x`-x|, k = 2,4,6,\ldots\ldots 2n \ldots\ldots\ldots(12)$$

$$\ell_{k,1,\delta} = \int_{0}^{\delta} \lambda_{1,\delta}(r) r^{k} dr.$$

(in one dimensional)

$$L^{k} u(x) = \sum_{k} \frac{2}{k!} \ell_{k,1,\delta} u^{k}(x), \forall k = 2,4,6,\ldots\ldots, 2n \ldots\ldots\ldots\ldots(13)$$

$$\ell_{2,1,\delta} = \frac{1}{\delta^{2}} \int_{0}^{\delta} \frac{1}{r} r^{2} dr = \frac{1}{2}$$

$$\ell_{4,1,\delta} = \frac{1}{\delta^{2}} \int_{0}^{\delta} \frac{1}{r} r^{4} dr = \frac{1}{48} \delta^{2}$$

$\vdots$

If we assume that u(x) is sufficiently smooth ,by performing the Taylor extension, we can introduce an equivalent definition of peridynamic operator[6] ,the equation (12) take the form:

$$L_{\delta} = \sum_{k} L_{\delta}^{k} u(x) = \frac{\sum_{k} \frac{2}{k!} \ell_{k,1,\delta} u^{k}(x)}{}, \forall k = 2,4,6,\ldots\ldots 2n \ldots\ldots\ldots\ldots(14)$$

It follows $L_{\delta}^{k} u(x) = 0$, $\forall k$ odd , since the integrand is an odd function in $x`-x$

The eq(14) become:

$$-L_\delta = \frac{1}{2}u``(x) + \frac{1}{48}\delta^2 u````(x) + \cdots .... \quad (15)$$

For all $k = 2,4,6,...,2n$.

We denote $-L_\delta$ be form:

$$-L_\delta = \sum_k \varphi_\delta u^*(x) ....(16)$$

where $\varphi_\delta = \frac{2}{\delta^{k+2}} \int_0^\delta \lambda(r) r^k dr$

and $u^*(x) = \frac{u^k(x)}{k!}\delta^k$, and $u^k(x)$ is the derivative of order k

the φ_δ a real-valued and symmetric positive vector which we can use to determine the right-hand side of (15) for polynomial exact solutions $u(x)$.

Definition (3.1):

The space $s_\sigma(\Omega)$ dependent on the kernel function[4], consists of all the functions $u(x) \in L^2(\Omega)$ such that

Norm $s_\sigma(\Omega)$ for which the $s_\sigma(\Omega)$

$$\|u\|_{S_\sigma} = \{\sum_k \int_\Omega u^*(x).(\underline{1} + \varphi_\delta) u^*(x) dx\}^{1/2}, \forall k = 2,4,6,....2n.$$

We also define the corresponding inner product associated with the $s_\sigma(\Omega)$ Norm:

$$(u,v)_{S\sigma} = \{\sum_k \int_\Omega v^*(x).(\underline{1} + \varphi_\delta) u^*(x) dx\}^{1/2} \qquad \forall u,v \in s_\sigma(\Omega)$$

we use $s^-{}_\sigma(\Omega)$ to denote the dual space of $s_\sigma(\Omega)$ and $\underline{1}$ is vector.

Remark (3.2):

The norm is well defined since $\underline{1} + \varphi_\delta$ is real-valued symmetric positive definite vector and it is uniformly bounded below by $\underline{1}$.

Lemma (3.3):

The space $s_\sigma(\Omega)$ is Hilbert space[11] corresponding to the inner product $(.,.)_{S^2_\sigma(R)}$

Proof:

Let $\{u_n\}$ be coushy sequence in $s_\sigma(\Omega)$, by definition, it is equivalent to say

$$\{(\underline{1}+\varphi_\delta)^{1/2}u_n^*(x)\}$$

is coushy sequence in $L^2(\Omega)$

So by the completeness of , there exists an element $v\in L^2(\Omega)$, such that

$$\sum_k \ \| \ (\underline{1}+\varphi_\delta)u^*(x)-v(x)\ \| L^2 \to o \ , \ \text{as } n\to\infty, k=2,4,6,....,2n$$

There exist $u^*(x)=(\underline{1}+\varphi_\delta)^{-1/2}v(x)$

Such that

$$\sum_k \ \|(\underline{1}+\varphi_\delta)^{1/2}\ u_n^*(x) \quad -(\underline{1}+\varphi_\delta)^{1/2}\ u^*(x)\ \|_{L^2}\to 0 \quad \text{when } n\to\infty, k=2,4,6,...,2n$$

$$\sum_k \ \left\|(\underline{1}+\varphi_\delta)^{1/2}(u_n^*(x)-u(x))\right\| \to 0 \quad \text{when } n\to\infty, k=2,4,6,...,2n$$

space is complete, and it is thus a Hilbert space $S_\sigma(\Omega)$ then the

Lemma (3.4):

The peridynamic operator $-L_\delta$ is self-adjont operator on $S_\sigma(\Omega)$. $-L_\delta+\underline{1}$ isomatry from $S_\sigma(\Omega) to S^-{}_\sigma(\Omega)$.

Proof:

By relation

$$(-L_\delta u,v)=\frac{1}{2\delta^2}\int_\Omega\int_{x-\delta}^{x+\delta}(u(x`)-u(x)).(v(x`)-v(x))dx`dx$$

Since the kernel function is symmetric i.e. $|x`-x|=|x-x`|$

$$\Rightarrow \frac{1}{2\delta^2}\int_{x-\delta}^{x+\delta}\int_\Omega \ (v(x`)-v(x).u(x`)-u(x))dxdx`$$

then

$$=(-L_\delta v,u)$$

$$(-L_\delta u,v)=(-L_\delta v,u)$$

$$(-L_\delta u,v)-(-L_\delta v,u)=0$$

$$-L_\delta u-(-L_\delta v)=0$$

$$-L_\delta u=-L_\delta v$$

then $-L_\delta$ is self-adjoint

to prove $-L_\delta + 1$ isometry from $S_\sigma^2(\Omega)$ to $S_\sigma^{-2}(\Omega)$

we prove -$L_\delta + 1$ is one-to-one

$$\ker(-L_\delta + 1) = \{0\}$$
$$u \in \ker(-L_\delta + 1)$$
$$(-L_\delta + 1)(u) = 0$$
$$-L_\delta u + u = 0$$
$$-L_\delta u = -u$$
$$(-L_\delta u, u) = (-u, u)$$
$$= -(u, u)$$
$$= -\|u\|^2$$
$$\sin ce(-L_\delta u, u) \geq 0$$
$$\Rightarrow -\|u\|^2 \geq 0$$
$$\|u\|^2 \leq 0$$
$$but \|u\|^2 \geq 0$$
$$\Rightarrow \|u\|^2 = 0$$
$$\|u\| = 0 \Rightarrow u = o$$

$-L_\delta + 1$ is one to one

then $-L_\delta + 1$ isometry from $S_\sigma(\Omega)$ to $S_\sigma^{-}(\Omega)$

REFRENCES

[1] E. Askari, F. Bobaru, R.B. Lehoucq, M.L. Parks, S.A. Silling and O. Weckner, ``Peridynamics for multiscale materials modeling``, J. Phys. Conf. Ser. 125 (2008) 012078.

[2] Xi Chen1and Max Gunzburger1,`` Continuous and Discontinuous Finite Element Methods for a Peridynamics Model of Mechanics ``,Florida State University(2005).

[3] K. Dayal and K. Bhattacharya,``Kinetics of phase transformations in the peridynamic formulation of continuum mechanics``J. Mech. Phys. Solids 54 (2006) 1811–184

[4] Q. Du and K. Zhou, Mathematical analysis for the peridynamic nonlocal continuum theory,Mathematical Modeling and Numerical Analysis, 2010, doi:10.1051/m2an/2010040

[5] E. Emmrich and O. Weckner, ``The peridynamic equation of motion in nonlocal elasticity theory, in III European Conference on Computational Mechanics – Solids, Structures and Coupled Problems in Engineering, Springer (2006).

[6] E. Emmrich and O. Weckner, ``On the well-posedness of the linear peridynamic model and its convergence towards the Navier equation of linear elasticity``, Commun. Math. Sci. 5 (2007) 851–864.

[7] S.A. Silling, ``Linearized theory of peridynamic states. Sandia National Laboratories``, SAND (2009) 2009–2458.

[8] S.A. Silling and R.B. Lehoucq,`` Convergence of peridynamics to classical elasticity theory``, J. Elasticity 93 (2008) 13–37.

[9] S.A. Silling, O. Weckner, E. Askari and F. Bobaru,``Crack nucleation in a peridynamic solid``, Preprint (2009).

[10] S.A. Silling,`` Reformulation of elasticity theory for discontinuities and long-range forces``,Sandia National Laboratories,USA(1999).

[11]A.H.Siddiqi ,``Functional Analysis With Applications``,aligarh

[12] S. Torquat,`` Effective Stiffness Tensor of Composite Media-I. Exact Series Expansions``. NJ, U.S.A.1997.