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Some Games in İ- PRE- g- separation axioms

Esmaeel R. B.

Department of mathematics,
College of Education for Pure Sciences,
Ibn Al-Haitham, University of Baghdad,
Baghdad, Iraq.
ranamumosa@yahoo.com

Ahmed A. Jassam

Department of mathematics,
College of Education for Pure Sciences,
Ibn Al-Haitham, University of Baghdad,
Baghdad, Iraq.
ahm7a7a@gmail.com

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Abstract

The primary purpose of this subject is to define new games in ideal spaces via İ- pre - g- open set. The relationships between games that provided and the winning and losing strategy for any player were elucidated.

Keywords. İ- pre- g- open set, İ- pre- g- open function, İ- pre- g- cotinuous function, İ- pre- g- separation axioms and game.

1.Introduction

Kuratowski [1] presented in 1933. A collection İ ⊂ Ƥ(X) is claims an ideal on a nonempty set X when the following two conditions are satisfied; (i) Ƀ ∈ İ whenever Ƀ ⊂ Ⱥ and Ⱥ ∈ İ (ii) Ⱥ ∪ Ƀ ∈ İ whenever Ⱥ and Ƀ ∈ İ. Vaidyanathaswamy [2]. Provides the concept of ideal spaces by giving the set operator $( )^*$: Ƥ(X) ⟶ Ƥ(X). Which is local function, so the topological spaces were circulated, claims ideal space and symbolize by (X, Ṭ, İ), [3-5].

Mashhour, Abd El- Monsef and El- Deeb, present the concept of "pre- open set", a set Ⱥ in (X, Ṭ) is a pre-open when Ⱥ ⊆ cl(int(Ⱥ)) [6]. Many researchers at that time used this concept in their studies [7-9].

Also, Ahmed and Esmaeel [10], use this concept to provide an İ - pre - g - closed set (symbolizes it, İpg - closed). If Ⱥ- Ų ∈ İ and Ų is a pre-open set, implies to cl(A) - Ų ∈ İ, so a set Ⱥ in (X, Ṭ, İ) is İpg - closed. And the set Ⱥ in X claims İ - pre - g - open set (symbolizes it, İpg - open), if X - Ⱥ is İpg - closed. The collection of all İpg - closed sets (respectively, İpg - open sets) in (X, Ṭ, İ) symbolizes it İpg - C(X) (respectively, İpg - O(X)). And İpg - O(X) is finer than Ṭ.

A space (X, Ṭ, İ) is namely İpg -$Ṭ_0$-space (respectively İpg -$Ṭ_1$-space, İpg -$Ṭ_2$-space), if for each element $r_1 \neq r_2$, there is an İpg-open set containing only one of them (respectively there is an

İpg-open sets $Џ_1$and $Џ_2$, satisfies $r_1 \in (Џ_1 - Џ_2)$ and $r_2 \in (Џ_2 - Џ_1)$, there is an İpg-open sets $Џ_1$and $Џ_2$, satisfies $r_1 \in Џ_1$ and $r_2 \in Џ_2$ such that $Џ_1 \cap Џ_2 = \emptyset$)[11].
The main point of this article is to provide new types of games in ideal spaces by using the concept of İpg - open set.

2. İ - Pre- g- openness on Game.

This portion is to provide new types of game by using the concept of İpg - openness, where the relationships between them are discussed. In the theory of game , there is always at least two participants called players $Ᵽ_1$ and $Ᵽ_2$. Denoted for player one by $Ᵽ_1$ and symbolizes for player two by $Ᵽ_2$ and Ģ be a game between two players $Ᵽ_1$and $Ᵽ_2$. The set of choices $I_1, I_2, I_3,\ldots\ldots, I_{ṃ}$ for each player. These choices are claims round, steps or options. In this research with games of type "Two- Zero- Sum Games". The games will be defined between two players and the payoff for any one of them equals to the loose of other player [11-13]

A function S is a strategy for $Ᵽ_1$ as follows $S = \{S_{ṃ}: Ⱥ_{ṃ-1} \times Ƀ_{ṃ-1} \rightarrow Ⱥ_{ṃ}$, such that $(Ⱥ_1, Ƀ_1, ------, Ⱥ_{ṃ-1}, Ƀ_{ṃ-1}) = Ⱥ_{ṃ}\}$ similarly a function T is a strategy for $Ᵽ_2$ as follows $T = \{T_{ṃ}\ ;\ Ⱥ_{ṃ} \times Ƀ_{ṃ-1} \rightarrow Ƀ_{ṃ}$, such that $(Ⱥ_1, Ƀ_1, ------, Ⱥ_{ṃ-1}, Ƀ_{ṃ-1}, Ⱥ_{ṃ}) = Ƀ_{ṃ}\}$. [15].
In this work, we provide the winning and losing strategy for any player Ᵽ in the game Ģ, if Ᵽ has a winning strategy in Ģ which symbolizes (Ᵽ $\hookrightarrow$ Ģ), if Ᵽ does not has a winning strategy symbolizes (Ᵽ $\not\hookrightarrow$ Ģ), if Ᵽ has a losing strategy symbolizes (Ᵽ $\hookleftarrow$ Ģ) and if Ᵽ does not has a losing strategy symbolizes (Ᵽ $\not\hookleftarrow$ Ģ).

Definition 2.1. Let (Ҳ, Ţ) be a topological space, define a game Ģ $(\dot{T}_0, Ҳ)$ (respectively, Ģ$(\dot{T}_0, İ)$)as follows: The two players $Ᵽ_1$and $Ᵽ_2$ play an inning for each natural numbers, in the ṃ- th inning, the first round, $Ᵽ_1$ will choose $ӿ_{ṃ} \neq ς_{ṃ}$. Next, $Ᵽ_2$ choose $Џ_{ṃ} \in Ţ$ (respectively $Џ_{ṃ} \in$ İpg- O(Ҳ)) such that $ӿ_{ṃ} \in Џ_{ṃ}$ and $ς_{ṃ} \notin Џ_{ṃ}$, $Ᵽ_2$ wins in the game where $Ƀ = \{Џ_1, Џ_2, Џ_3, \ldots, Џ_{ṃ}, \ldots\}$ satisfies that for all $ӿ_{ṃ} \neq ς_{ṃ}$ in Ҳ there exist $Џ_{ṃ} \in Ƀ$ such that $ӿ_{ṃ} \in Џ_{ṃ}$ and $ς_{ṃ} \notin Џ_{ṃ}$. Other hand $Ᵽ_1$wins.

Remark 2.2. For any ideal topological space(Ҳ, Ţ, İ):
1.if ($Ᵽ_2 \hookrightarrow$ Ģ$(\dot{T}_0$,Ҳ)) then ($Ᵽ_2 \hookrightarrow$ Ģ$(\dot{T}_0$, İ)).
2.if ($Ᵽ_2 \hookleftarrow$ Ģ$(\dot{T}_0$,Ҳ)) then ($Ᵽ_2 \hookleftarrow$ Ģ$(\dot{T}_0$, İ)).
3.if ($Ᵽ_1 \hookrightarrow$ Ģ$(\dot{T}_0$, İ)) then ($Ᵽ_1 \hookrightarrow$ Ģ$(\dot{T}_0$,Ҳ)).

Proposition 2.3. If (Ҳ, Ţ, İ)) is $\dot{T}_0$-space (respectively, İpg- $\dot{T}_0$- space) $\longleftrightarrow$ ($Ᵽ_2 \hookrightarrow$ Ģ $(\dot{T}_0$,Ҳ)). (respectively, ($Ᵽ_2 \hookrightarrow$ Ģ$(\dot{T}_0$, İ)).
Proof: since (Ҳ, Ţ, İ) is $\dot{T}_0$- space (respectively, İpg- $\dot{T}_0$- space), then, in the ṃ- th inning, any choice for the first player $Ᵽ_1$, $ӿ_{ṃ} \neq ς_{ṃ}$, the second player $Ᵽ_2$ can be found $Џ_{ṃ} \in Ţ$(respectively, $Џ_{ṃ} \in$ İpg- O(Ҳ)) $Џ_2 \in Ţ$ (respectively $Џ_{ṃ} \in$ İpg- O(Ҳ)). So $Ƀ = \{Џ_1, Џ_2, Џ_3, \ldots, Џ_{ṃ}, \ldots\}$ is the winning strategy for $Ᵽ_2$.
($\Leftarrow$) Clear.

Corollary 2.4. ($Ᵽ_2 \hookrightarrow$ Ģ$(\dot{T}_0$,Ҳ)) (respectively, ($Ᵽ_2 \hookrightarrow$ Ģ$(\dot{T}_0$, İ))$\longleftrightarrow \forall\, ӿ_1 \neq ӿ_2$ in Ҳ , $\exists\, \dot{F} \in Ƒ$ (respectively$\exists\, \dot{F} \in$ İpgC(Ҳ)) such that, $ӿ_1 \in \dot{F}$ and $ӿ_2 \notin \dot{F}$.

Corollary 2.5. If (Ҳ, Ţ, İ) is $\dot{T}_0$-space (respectively, İpg- $\dot{T}_0$-space)$\longleftrightarrow$ ($Ᵽ_1 \not\hookrightarrow$ Ģ$(\dot{T}_0$,Ҳ)). (respectively ($Ᵽ_1 \not\hookrightarrow$ Ģ$(\dot{T}_0$, İ)).

Proposition 2.6. If $(X, T, \acute{I})$ is not $\dot{T}_0$-space (respectively, not $\acute{I}$pg-$\dot{T}_0$-space) $\longleftrightarrow (P_1 \hookrightarrow G(\dot{T}_0, X))$ (respectively, $(P_1 \hookrightarrow G(\dot{T}_0, \acute{I}))$.

Corollary 2.7. If $(X, T, \acute{I})$ is not $\dot{T}_0$-space (respectively not $\acute{I}$pg-$\dot{T}_0$-space) $\longleftrightarrow (P_2 \not\hookrightarrow G(\dot{T}_0, X))$ (respectively$(P_2 \not\hookrightarrow G(\dot{T}_0, \acute{I}))$.

Definition 2.8. Let $(X, T, \acute{I})$ be a topological space, define a game $G(\dot{T}_1, X)$ (respectively $G(\dot{T}_1, \acute{I})$)as follows: The two players $P_1$and P_2 play an inning for each natural numbers, in the m-th inning, the first round, P_1 will choose $x_m \neq \varsigma_m$ where $x_m, \varsigma_m \in X$. Next, P_2 choose U_m, $V_m \in T$(respectively, U_m, $V_m \in \acute{I}pg\text{-}O(X)$) such that $x_m \in (U_m - V_m)$ and $\varsigma_m \in (V_m - U_m)$, P_2 wins in the game where $B = \{\{U_1, V_1\}, \{U_2, V_2\} \dots, \{U_m, V_m\} \dots\}$ satisfies that for all $x_m \neq \varsigma_m$ in X there exist $\{U_m, V_m\} \in B$ such that $x_m \in (U_m - V_m)$ and $\varsigma_m \in (V_m - U_m)$. Other hand P_1 wins.

Remark 2.9. For any ideal topological space$(X, T, \acute{I})$:
1. if $(P_2 \hookrightarrow G(\dot{T}_1, X))$ then $(P_2 \hookrightarrow G(\dot{T}_1, \acute{I}))$.
2. if $(P_2 \hookleftarrow G(\dot{T}_1, X))$ then $(P_2 \hookleftarrow G(\dot{T}_1, \acute{I}))$.
3. if $(P_1 \hookrightarrow G(\dot{T}_1, \acute{I}))$ then $(P_1 \hookrightarrow G(\dot{T}_1, X))$.

Proposition 2.10. If $(X, T, \acute{I})$ is $\dot{T}_1$-space (respectively $\acute{I}$pg-$\dot{T}_1$-space $\longleftrightarrow (P_2 \hookrightarrow G(\dot{T}_1, X))$. (respectively, $(P_2 \hookrightarrow G(\dot{T}_1, \acute{I}))$.

Proof: $(\Longrightarrow)$ Let $(X, T, \acute{I})$ be a topological space, in the first round, P_1 will choose $x_1 \neq \varsigma_1$. Next, since $(X, T, \acute{I})$ is $\dot{T}_1$-space (respectively $\acute{I}$pg-$\dot{T}_1$-space) P_2 can be found $U_1, V_1 \in T$ (respectively U_1, $V_1 \in \acute{I}pg\text{-}O(X)$) such that $x_1 \in (U_1 - V_1)$and $\varsigma_1 \in (V_1 - U_1)$, in the second round, P_1 will choose $x_2 \neq \varsigma_2$. Next, P_2 can be found U_2, $V_2 \in T$ (respectively U_2, $V_2 \in \acute{I}pg\text{-}O(X)$) such that $x_2 \in U_2 - V_2$ and $\varsigma_2 \in (V_2 - U_2)$, in the m-th round P_1 will choose $x_m \neq \varsigma_m$, Next, P_2 can be found U_m, $V_m \in T$(respectively, U_m, $V_m \in \acute{I}pg\text{-}O(X)$) such that $x_m \in (U_m - V_m)$ and $\varsigma_m \in (V_m - U_m)$. So $B = \{\{U_1, V_1\}, \{U_2, V_2\}, \dots, \{U_m, V_m\}, \dots\}$ is the winning strategy for P_2.
$(\Longleftarrow)$ Clear.

Corollary 2.11. $(P_2 \hookrightarrow G(\dot{T}_1, X))$ (respectively, $(P_2 \hookrightarrow G(\dot{T}_1, \acute{I}))$ $\longleftrightarrow \forall x_1 \neq x_2$ in X $\exists \dot{F}_1, \dot{F}_2 \in F$ (respectively $\exists \dot{F}_1, \dot{F}_2 \in \acute{I}pg\text{-}C(X)$) such that, $x_1 \in \dot{F}_1$ and $x_2 \notin \dot{F}_1$ and $x_1 \notin \dot{F}_2$ and $x_2 \in \dot{F}_2$.

Corollary 2.12. $(X, T, \acute{I})$ is $\dot{T}_1$-space (respectively, $\acute{I}$pg-$\dot{T}_1$-space) $\longleftrightarrow (P_1 \not\hookrightarrow G(\dot{T}_1, X))$. (respectively $(P_1 \not\hookrightarrow G(\dot{T}_1, \acute{I}))$.

Proposition 2.13. $(X, T, \acute{I})$ is not $\dot{T}_1$-space (respectively, not $\acute{I}$pg-$\dot{T}_1$-space $\longleftrightarrow (P_1 \hookrightarrow G(\dot{T}_1, X))$ (respectively$(P_1 \hookrightarrow G(\dot{T}_1, \acute{I}))$.

Corollary 2.14. $(X, T, \acute{I})$ is not $\dot{T}_1$-space (respectively, not $\acute{I}$pg-$\dot{T}_1$-space) $\longleftrightarrow (P_2 \not\hookrightarrow G(\dot{T}_1, X))$ (respectively$(P_2 \not\hookrightarrow G(\dot{T}_1, \acute{I}))$.

Definition 2.15. [10], [13] Let (X, T) be topological space, define a game $G(\dot{T}_2, X)$(respectively $G(\dot{T}_2, \acute{I})$) as follows: The two players $P_1$and P_2 play an inning for each natural numbers, in the m-th inning, the first round, P_1will choose $x_m \neq \varsigma_m$. Next, P_2 choose U_m, V_m are disjoint, U_m,

$V_m \in T$(respectively, U_m, $V_m \in \dot{I}pg\text{-}O(X)$) such that $x_m \in U_m$ and $\varsigma_m \in V_m$. P_2 wins in the game where $B = \{\{U_1, V_1\}, \{U_2, V_2\}, \dots, \{U_m, V_m\}, \dots\}$ satisfies that for all $x_m \neq \varsigma_m$ in X there exist $\{U_m, V_m\} \in B$, such that $x_m \in U_m$ and $\varsigma_m \in V_m$. Other hand P_1 wins.

Remark 2.16. For any ideal topological space$(X, T, \dot{I})$:

1. if $(P_2 \hookrightarrow G(\dot{T}_2, X))$ then $(P_2 \hookrightarrow G(\dot{T}_2, \dot{I}))$.
2. if $(P_2 \hookleftarrow G(\dot{T}_2, X))$ then $(P_2 \hookleftarrow G(\dot{T}_2, \dot{I}))$.
3. if $(P_1 \hookrightarrow G(\dot{T}_2, \dot{I}))$ then $(P_1 \hookrightarrow G(\dot{T}_2, X))$.

Proposition 2.17. If $(X, T, \dot{I})$ is $\dot{T}_2$-space (respectively, $\dot{I}pg\text{-}\dot{T}_2$-space) $\longleftrightarrow$ $(P_2 \hookrightarrow G(\dot{T}_2, X))$. (respectively, $(P_2 \hookrightarrow G(\dot{T}_2, \dot{I}))$.

Proof: $(\Rightarrow)$ Let $(X, T, \dot{I})$ be a topological space, in the first round, P_1 will choose $x_1 \neq \varsigma_1$. Next, since $(X, T, \dot{I})$ is $\dot{T}_2$- space (respectively, $\dot{I}pg\text{-}\dot{T}_2$- space), P_2 can be found $U_1$and $V_1 \in T$ (respectively $U_1$and $V_1 \in \dot{I}pg\text{-}O(X)$) such that $x_1 \in U_1$ and $\varsigma_1 \in V_1$, $U_1 \cap V_1 = \emptyset$, in the second round, P_1 will choose $x_2 \neq \varsigma_2$. Next, P_2 choose $U_2$and $V_2 \in T$(respectively $U_1$and $V_2 \in \dot{I}pg\text{-}O(X)$) such that $x_2 \in U_2$ and $\varsigma_2 \in V_2$, $U_2 \cap V_2 = \emptyset$, in the m- th round, P_1 will choose $x_m \neq \varsigma_m$. Next, P_2 choose U_m and $V_m \in T$(respectively, $U_m$and $V_m \in \dot{I}pg\text{-}O(X)$) such that $x_m \in U_m$ and $\varsigma_m \in V_m$, $U_m \cap V_m = \emptyset$.

So $B = \{\{U_1, V_1\}, \{U_2, V_2\}, \dots, \{U_m, V_m\} \dots\}$ is the winning strategy for P_2.

$(\Leftarrow)$ Clear.

Corollary 2.18. If $(X, T, \dot{I})$ is $\dot{T}_2$-space (respectively, $\dot{I}pg\text{-}\dot{T}_2$-space) $\longleftrightarrow$ $(P_1 \nrightarrow G(\dot{T}_2, X))$. (respectively, $(P_1 \nrightarrow G(\dot{T}_2, \dot{I}))$.

Proposition 2.19. $(X, T, \dot{I})$ is not $\dot{T}_2$-space(respectively not $\dot{I}pg\text{-}\dot{T}_2$-space) $\longleftrightarrow$ $(P_1 \hookrightarrow G(\dot{T}_2, X))$ (respectively $(P_1 \hookrightarrow G(\dot{T}_2, \dot{I}))$.

Corollary 2.20. $(X, T, \dot{I})$ is not $\dot{T}_2$-space (respectively not $\dot{I}pg\text{-}\dot{T}_0$-space) $\longleftrightarrow$$(P_2 \nrightarrow G(\dot{T}_2, X))$ (respectively$(P_2 \nrightarrow G(\dot{T}_2, \dot{I}))$.

Remark 2.21. For any space $(T, X, \dot{I})$

1. $(P_2 \hookrightarrow G(\dot{T}_{i+1}, X))$(respectively $G(\dot{T}_{i+1}, \dot{I})$); $i = \{0,1\}$ then $(P_2 \hookrightarrow G(\dot{T}_i, X))$(respectively $G(\dot{T}_i, \dot{I})$).
2. $(P_2 \nrightarrow G(\dot{T}_{i+1}, X))$(respectively $G(\dot{T}_{i+1}, \dot{I})$); $i = \{0,1\}$ then $(P_2 \nrightarrow G(\dot{T}_i, X))$(respectively $G(\dot{T}_i, \dot{I})$).

The following (fig) illustrates the relationships given in Remark 2.2

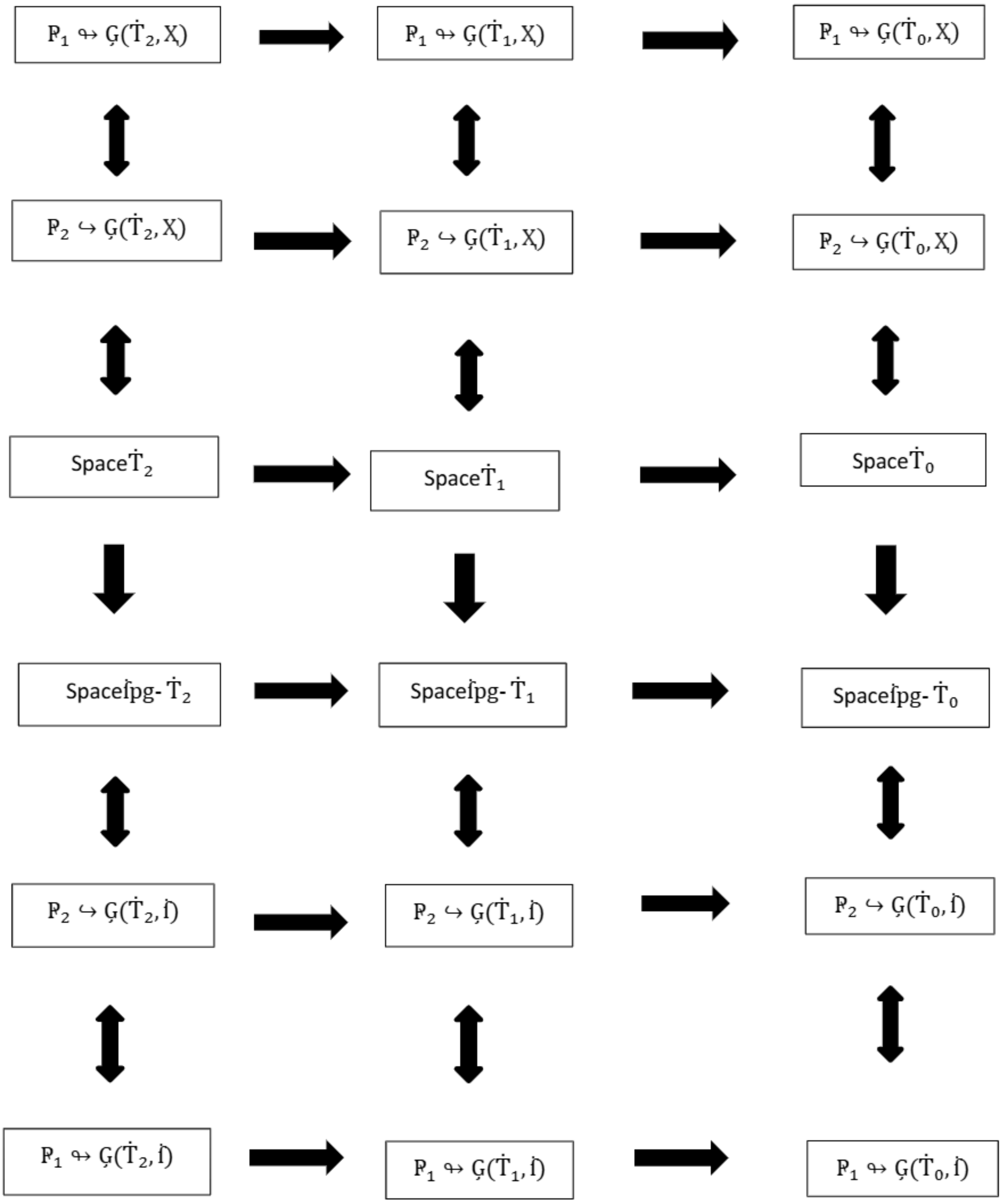

Figure 1. The winning strategy for $Ᵽ_2$ in $Ģ(\dot{T}_i, X)$, i= {0, 1, 2}

Remark 2.22. For any space (Ţ, X, İ)

1. ($Ᵽ_1 \hookrightarrow Ģ(\dot{T}_i, X)$)(respectively $Ģ(\dot{T}_i, İ)$); i = {0,1} then ($Ᵽ_1 \hookrightarrow Ģ(\dot{T}_{i+1}, X)$)(respectively $Ģ(\dot{T}_{i+1}, İ)$).
2. ($Ᵽ_2 \not\hookrightarrow Ģ(\dot{T}_i, X)$)(respectively $Ģ(\dot{T}_i, İ)$); i = {0,1} then ($Ᵽ_2 \not\hookrightarrow Ģ(\dot{T}_{i+1}, X)$)(respectively $Ģ(\dot{T}_{i+1}, İ)$).

The following **Figure** illustrates the relationships given in Remark 2.22:

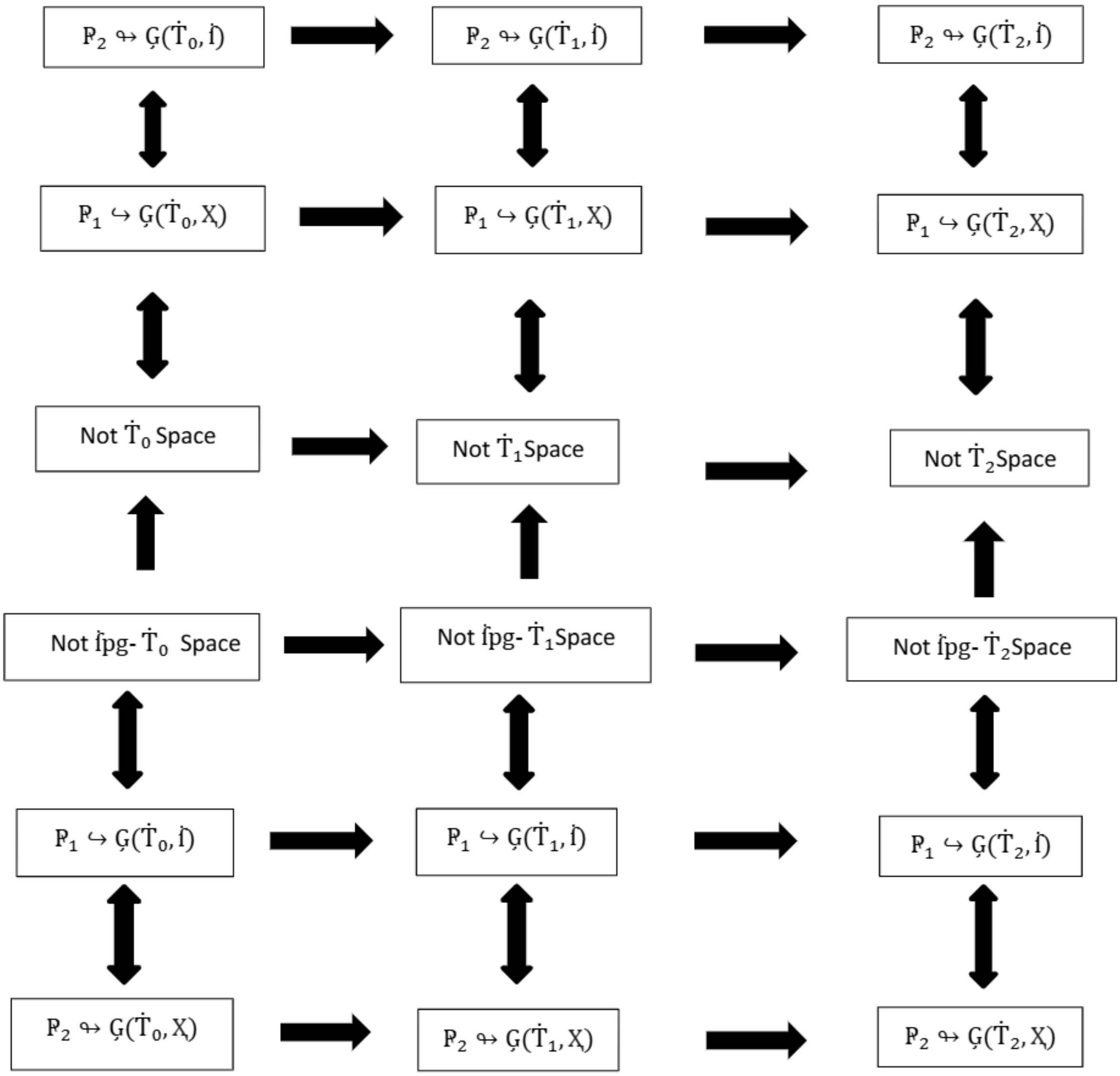

Figure 2. The winning strategy for $Ᵽ_1$ in $Ģ(Ṫ_i, Ҳ)$, i= {0, 1, 2}

3. The games with open functions via Ịpg-open sets.

By using open function via Ịpg-open sets; you can determine the winning strategy for any players in $Ģ(Ṫ_i, Ҳ)$); and $Ģ(Ṫ_i, Ị)$) where i={0,1,2}.

Definition 3.1. (1) A function $ƒ : (Ҳ, Ţ, Ị) \to (Ƴ, Ƭ, ɉ)$ is

1. Ị-pre-g-open function, symbolizes Ịpgo-function if $ƒ(Ų)$ is a ɉpg-open set in Ƴ whenever Ų is an Ịpg-open set in Ҳ.
2. $Ị^*$-pre-g-open function, symbolizes $Ị^*$pgo-function if $ƒ(Ų)$ is a ɉpg-open set in Ƴ whenever Ų is an open set in Ҳ.
3. $Ị^{**}$-pre-g-open function, symbolizes $Ị^{**}$pgo-function if $ƒ(Ų)$ is an open in Ƴ whenever Ų is an Ịpg-open set in Ҳ.

Proposition 3.1. If the function ƒ : (X, Ț, İ) → (Ύ, Ŧ, ĵ) is surjective open (respectively İ-pre-g-open function) and (Ᵽ$_2$ ↪ Ģ(Ṫ$_i$, X)) (respectively(Ᵽ$_2$ ↪ Ģ(Ṫ$_i$, İ)) then (Ᵽ$_2$ ↪ Ģ(Ṫ$_i$, Ύ) (respectively (Ᵽ$_2$ ↪ Ģ(Ṫ$_i$, ĵ)), where (i=0,1and 2 respectively).

*Proof***(1).** In the game Ģ(Ṫ$_i$, Ύ) (respectively, Ģ(Ṫ$_i$, ĵ)) where(i=0), in the first round, Ᵽ$_1$will choose ς$_1$ ≠ z$_1$ such that ς$_1$, z$_1$ ∈ Ύ. Next, Ᵽ$_2$ in Ģ(Ṫ$_0$, Ύ) (respectively Ᵽ$_2$ in Ģ(Ṫ$_0$, ĵ) will hold account ƒ$^{-1}$(ς$_1$), ƒ$^{-1}$(z$_1$) ∈ X, ƒ$^{-1}$(ς$_1$) ≠ ƒ$^{-1}$(z$_1$), but (Ᵽ$_2$ ↪ Ģ(Ṫ$_0$, X)) (respectively (Ᵽ$_2$ ↪ Ģ(Ṫ$_0$, İ)), ∃Џ$_1$ ∈ Ț (respectively ∃Џ$_1$ ∈ İpg-O(X)), ƒ$^{-1}$(ς$_1$) ∈ Џ$_1$ and ƒ$^{-1}$(z$_1$) ∉ Џ$_1$ since ƒ is an open respectively İ-pre-g-open function then ς$_1$ ∈ ƒ(Џ$_1$) and z$_1$ ∉ ƒ(Џ$_1$) this implies Ᵽ$_2$ in Ģ(Ṫ$_0$, Ύ) (respectively Ᵽ$_2$ in Ģ(Ṫ$_0$, ĵ)) choose ƒ(Џ$_1$) is open (respectively ĵpg-open sets), in the second round, Ᵽ$_1$ in Ģ(Ṫ$_0$, Ύ) (respectively Ᵽ$_1$ in Ģ(Ṫ$_0$, ĵ) choose ς$_2$ ≠ ẓ$_2$ such that ς$_2$, ẓ$_2$ ∈ Ύ. Next, Ᵽ$_2$ in Ģ(Ṫ$_0$, Ύ) (respectively Ᵽ$_2$ in Ģ(Ṫ$_0$, ĵ)) will hold account ƒ$^{-1}$(ς$_2$), ƒ$^{-1}$(ẓ$_2$) ∈ X, ƒ$^{-1}$(ς$_2$) ≠ ƒ$^{-1}$(ẓ$_2$), but (Ᵽ$_2$ ↪ Ģ(Ṫ$_0$, X)), (respectively (Ᵽ$_2$ ↪ Ģ(Ṫ$_0$, İ)), ∃Џ$_2$ ∈ Ț(respectively ∃Џ$_2$ ∈ İpg-O(X)), ƒ$^{-1}$(ς$_ṃ$) ∈ Џ$_2$ and ƒ$^{-1}$(ẓ$_2$) ∉ Џ$_2$, then ς$_2$ ∈ ƒ(Џ$_2$) and z$_2$ ∉ ƒ(Џ$_2$) this implies Ᵽ$_2$ in Ģ(Ṫ$_0$, Ύ) (respectively Ᵽ$_2$in Ģ(Ṫ$_0$, Ύ) will choose ƒ(Џ$_2$) is open (respectively ĵpg-open sets) and in the ṃ-th round, Ᵽ$_1$ in Ģ(Ṫ$_0$, Ύ) (respectively Ᵽ$_1$ in Ģ(Ṫ$_0$, ĵ) choose ς$_ṃ$ ≠ ẓ$_ṃ$ such that ς$_ṃ$, ẓ$_ṃ$ ∈ Ύ. Next, Ᵽ$_2$ in Ģ(Ṫ$_0$, Ύ) (respectively Ᵽ$_1$ in Ģ(Ṫ$_0$, ĵ) will hold account ƒ$^{-1}$(ς$_ṃ$), ƒ$^{-1}$(ẓ$_ṃ$) ∈ X, ƒ$^{-1}$(ς$_ṃ$) ≠ ƒ$^{-1}$(ẓ$_ṃ$), but (Ᵽ$_2$ ↪ Ģ(Ṫ$_0$, X)), (respectively (Ᵽ$_2$ ↪ Ģ(Ṫ$_0$, İ)), so,∃Џ$_ṃ$ ∈ Ț(respectively ∃Џ$_ṃ$ ∈ İpg-O(X)); ƒ$^{-1}$(ς$_ṃ$) ∈ Џ$_ṃ$ and ƒ$^{-1}$(ẓ$_ṃ$) ∉ Џ$_ṃ$, then ς$_ṃ$ ∈ ƒ(Џ$_ṃ$) and z$_ṃ$ ∉ ƒ(Џ$_ṃ$); this implies Ᵽ$_2$ in Ģ(Ṫ$_0$, Ύ) (respectively Ᵽ$_2$ in Ģ(Ṫ$_0$, ĵ)will choose ƒ(Џ$_ṃ$) is open (respectively ĵpg-open sets); thus Ɓ = {ƒ{ Џ$_1$}, ƒ{Џ$_2$}, … , ƒ{Џ$_ṃ$}…} is the winning strategy for Ᵽ$_2$ in Ģ(Ṫ$_0$, Ύ) (respectively, Ᵽ$_2$ in Ģ(Ṫ$_0$, ĵ)).

(2). In the game Ģ(Ṫ$_i$, Ύ) (respectively, Ģ(Ṫ$_i$, ĵ)) where (i=1), in the ṃ-th inning, Ᵽ$_1$will choose ς$_ṃ$ ≠ z$_ṃ$ such that ς$_ṃ$, z$_ṃ$ ∈ Ύ. Next, Ᵽ$_2$ in Ģ(Ŧ$_1$, Ύ) (respectively, Ᵽ$_2$ in Ģ(Ṫ$_1$, ĵ) will hold account ƒ$^{-1}$(ς$_ṃ$), ƒ$^{-1}$(z$_ṃ$) ∈ X, ƒ$^{-1}$(ς$_ṃ$) ≠ ƒ$^{-1}$(z$_ṃ$), but (Ᵽ$_2$ ↪ Ģ(Ṫ$_1$, X)) (respectively, (Ᵽ$_2$ ↪ Ģ(Ṫ$_1$, İ)), ∃Џ$_ṃ$, ѵ$_ṃ$ ∈ Ț (respectively ∃Џ$_ṃ$, ѵ$_ṃ$ ∈ İpg-O(X)), ƒ$^{-1}$(ς$_1$) ∈ (Џ$_ṃ$-ѵ$_ṃ$) and ƒ$^{-1}$(z$_ṃ$) ∈ (ѵ$_ṃ$ -Џ$_ṃ$) and since ƒ is an open ,respectively İ-pre-g-open function; this implies Ᵽ$_2$ in Ģ(Ṫ$_1$, Ύ) (respectively Ᵽ$_2$ in Ģ(Ṫ$_1$, ĵ)) choose ƒ(Џ$_ṃ$) , ƒ(ѵ$_ṃ$) are open (respectively ĵpg-open sets), thus Ɓ = {{ƒ(Џ$_1$), ƒ(ѵ$_1$)}, {ƒ(Џ$_2$), ƒ(ѵ$_2$)}, … , {ƒ(Џ$_ṃ$), ƒ(ѵ$_ṃ$)}…} is the winning strategy for Ᵽ$_2$ in Ģ(Ṫ$_1$, Ύ)) (respectively Ᵽ$_2$ in Ģ(Ṫ$_1$, ĵ)).In the same way, we can proof (Ᵽ$_2$ ↪ Ģ(Ṫ$_2$, Ύ) (respectively(Ᵽ$_2$ ↪ Ģ(Ṫ$_1$, ĵ)) but ƒ(Џ$_ṃ$) ∩ ƒ(ѵ$_ṃ$) = ∅.Thus, Ɓ = {{ƒ(Џ$_1$), ƒ(ѵ$_1$)}, {ƒ(Џ$_2$), ƒ(ѵ$_2$)}, … , { ƒ(Џ$_ṃ$), ƒ(ѵ$_ṃ$)}…} is the winning strategy for Ᵽ$_2$ in Ģ(Ṫ$_2$, Ύ)(respectively Ᵽ$_2$ in Ģ(Ṫ$_2$, ĵ)).

Proposition 3.3. If the function ƒ : (X, Ț, İ) → (Ύ, Ŧ, ĵ) is surjective İ*pgo-function and (Ᵽ$_2$ ↪ Ģ(Ṫ$_i$, X)), then, (Ᵽ$_2$ ↪ Ģ(Ṫ$_i$, ĵ)) , where (i=0,1and 2 respectively).

Proof **(1).** In the game Ģ(Ṫ$_i$, ĵ), where (i=0), in the first round, Ᵽ$_1$ will choose ς$_1$ ≠ ẓ$_1$ such that ς$_1$, ẓ$_1$ ∈ Ύ. Next, Ᵽ$_2$ in Ģ(Ț$_0$, ĵ) will hold account ƒ$^{-1}$(ς$_1$), ƒ$^{-1}$(ẓ$_1$) ∈ X, ƒ$^{-1}$(ς$_1$) ≠ ƒ$^{-1}$(z$_1$), but (Ᵽ$_2$ ↪ Ģ(Ṫ$_0$, X)), ∃Џ$_1$ ∈ Ț, ƒ$^{-1}$(ς$_1$) ∈ Џ$_1$ and ƒ$^{-1}$(z$_1$) ∉ Џ$_1$, and since ƒ is İ*pgo-function this implies Ᵽ$_2$ in Ģ(Ṫ$_0$, X) will choose ƒ(Џ$_1$) is a ĵpg-open set, in the second round, Ᵽ$_1$ in Ģ(Ṫ$_0$, ĵ) choose ς$_2$ ≠ ẓ$_2$, ς$_2$, ẓ$_2$ ∈ Ύ. Next, Ᵽ$_2$ in Ģ(Ṫ$_0$, ĵ) will hold account ƒ$^{-1}$(ς$_2$), ƒ$^{-1}$(ẓ$_2$) ∈ X, ƒ$^{-1}$(ς$_2$) ≠ ƒ$^{-1}$(ẓ$_2$), but (Ᵽ$_2$ ↪ Ģ(Ṫ$_0$, X)), ∃Џ$_2$ ∈ Ț, ƒ$^{-1}$(ς$_2$) ∈ Џ$_2$ and ƒ$^{-1}$(ẓ$_2$) ∉ Џ$_2$, this implies Ᵽ$_2$ in Ģ(Ṫ$_0$, X) will choose ƒ(Џ$_2$) is a ĵpg-open set and in ṃ-th round Ᵽ$_1$ in Ģ(Ṫ$_0$, ĵ) choose ς$_ṃ$ ≠ ẓ$_ṃ$, ς$_ṃ$, ẓ$_ṃ$ ∈ Ύ, Next, Ᵽ$_2$ in Ģ(Ṫ$_0$, ĵ) will hold account ƒ$^{-1}$(ς$_ṃ$), ƒ$^{-1}$(ẓ$_ṃ$) ∈ X, ƒ$^{-1}$(ς$_ṃ$) ≠ ƒ$^{-1}$(ẓ$_ṃ$), but (Ᵽ$_2$ ↪ Ģ(Ṫ$_0$, X)), ∃Џ$_ṃ$ ∈ Ț, ƒ$^{-1}$(ς$_ṃ$) ∈ Џ$_ṃ$ and ƒ$^{-1}$(ẓ$_ṃ$) ∉ Џ$_ṃ$, this implies Ᵽ$_2$ in Ģ(Ṫ$_0$, X) will choose ƒ(Џ$_ṃ$) is a ĵpg-open set, thus Ɓ{ƒ{ Џ$_1$}, ƒ{Џ$_2$} … , ƒ{Џ$_ṃ$}…} is the winning strategy for Ᵽ$_2$ in Ģ(Ṫ$_0$, X)).

(2). In the game $\c{G}(\dot{T}_i, \j)$ where$(i = 1)$, in the m-th round P_1 in $\c{G}(\dot{T}_1, \j)$ choose $\varsigma_m \neq z_m$, $\varsigma_m, z_m \in Y$. Next, P_2 in $\c{G}(\dot{T}_1, \j)$ will hold account $f^{-1}(\varsigma_m), f^{-1}(z_m) \in X$, $f^{-1}(\varsigma_m) \neq f^{-1}(z_m)$, but $(P_2 \hookrightarrow \c{G}(\dot{T}_1, X))$, $\exists U_m, V_m \in T$, $f^{-1}(\varsigma_m) \in (U_m\text{-}V_m)$ and $f^{-1}(z_m) \in (V_m\text{-}U_m)$, this implies P_2 in $\c{G}(\dot{T}_1, X)$ will choose $f(U_m)$ and $f(V_m)$ are jpg-open sets, thus $B = \{\{f(U_1), f(V_1)\}, \{f(U_2), f(V_2)\}, \dots, \{f(U_m), f(V_m)\}\dots\}$ is the winning strategy for P_2 in $\c{G}(\dot{T}_1, X))$. By the same way we can proof$(P_2 \hookrightarrow \c{G}(\dot{T}_2, X))$ but, $f(U_m) \cap f(V_m) = \emptyset$. Thus $B = f(U_m) \cap f(V_m) = \emptyset$ is the winning strategy for P_2 in $\c{G}(\dot{T}_2, X))$.

Corollary. If the function $f : (X, T) \to (Y, T)$ is a surjective open function and $P_2 \hookrightarrow \c{G}(\dot{T}_i, X)$, then $P_2 \hookrightarrow \c{G}(\dot{T}_i, \j)$, where i= {0, 1, 2}.

Proposition 3.4. If the function $f : (X, T, \dot{I}) \to (Y, T, \j)$ is a surjective $\dot{I}$ **pgo-function and $(P_2 \hookrightarrow \c{G}(\dot{T}_0, \dot{I})$ then, $(P_2 \hookrightarrow \c{G}(\dot{T}_0, Y)$, where (i = 0,1and 2 respectively).

Proof(**1**). In the game $\c{G}(\dot{T}_i, Y)$ where$(i = 0)$, in the first round, P_1in $\c{G}(\dot{T}_0, Y)$ will choose $\varsigma_1 \neq z_1$ such that $\varsigma_1, z_1 \in Y$. Next, P_2 in $\c{G}(\dot{T}_0, Y)$ will hold account $f^{-1}(\varsigma_1), f^{-1}(z_1) \in X$, $f^{-1}(\varsigma_1) \neq f^{-1}(z_1)$, but $(P_2 \hookrightarrow \c{G}(\dot{T}_0, \dot{I}))$, $\exists U_1 \in \dot{I}pgO(X)$, $f^{-1}(\varsigma_1) \in U_1$and $f^{-1}(z_1) \notin U_1$, $\varsigma_1 \in f(U_1)$ and $z_1 \notin f(V_1)$ and since f is $\dot{I}$ **pgo-function this implies P_2 in $\c{G}(\dot{T}_0, \dot{I})$ will choose $f(U_1)$ such that $\varsigma_1 \in f(U_1)$, $z_1 \notin f(U_1)$ open, in the second round, P_1 in $\c{G}(T_0, Y)$ choose $\varsigma_2 \neq z_2$, $\varsigma_2, z_2 \in Y$. Next, P_2 in $\c{G}(\dot{T}_0, Y)$ will hold account $f^{-1}(\varsigma_2), f^{-1}(z_2) \in X$, $f^{-1}(\varsigma_2) \neq f^{-1}(z_2)$, but $(P_2 \hookrightarrow \c{G}(\dot{T}_0, \dot{I}))$, $\exists U_2 \in \dot{I}pgO(X)$, $f^{-1}(\varsigma_2) \in U_2$ and$f^{-1}(z_2) \notin U_2$, $\varsigma_2 \in f(U_2)$ and $z_2 \notin f(V_2)$ this implies P_2 in $\c{G}(\dot{T}_0, \dot{I})$ will choose $f(U_2)$ and in m-th round P_1 choose $\varsigma_m \neq z_m$, $\varsigma_m, z_m \in Y$. Next, P_2 in $\c{G}(\dot{T}_0, Y)$ will hold account $f^{-1}(\varsigma_m), f^{-1}(z_m) \in X$, $f^{-1}(\varsigma_m) \neq f^{-1}(z_m)$, but $(P_2 \hookrightarrow \c{G}(\dot{T}_0, \dot{I}))$, $\exists U_m \in \dot{I}pgO(X)$, $f^{-1}(\varsigma_m) \in U_m$ and $f^{-1}(z_m) \notin U_m$, $\varsigma_m \in f(U_m)$ and $z_m \notin f(V_m)$ this implies P_2 in $\c{G}(\dot{T}_0, \dot{I})$ will choose $f(U_m)$; thus $B = \{f(U_1), f(U_2), \dots, f(U_m)\dots\}$ is the winning strategy for P_2 in $\c{G}(T_0, Y))$.

(2). In the game $\c{G}(\dot{T}_i, Y)$where$(i = 1)$, in the m-th round P_1choose $\varsigma_m \neq z_m$, $\varsigma_m, z_m \in Y$. Next, P_2 in $\c{G}(\dot{T}_1, Y)$will hold account $f^{-1}(\varsigma_m), f^{-1}(z_m) \in X$, $f^{-1}(\varsigma_m) \neq f^{-1}(z_m)$, but $(P_2 \hookrightarrow \c{G}(\dot{T}_1, \dot{I}))$, $\exists U_m, V_m \in \dot{I}pg\text{-}O(X)$, $f^{-1}(\varsigma_m) \in (U_m\text{-}V_m)$ and $f^{-1}(z_m) \in (V_m\text{-}U_m)$, so P_2 in $\c{G}(T_1, \dot{I})$ will choose $f(U_m)$, $f(V_m)$;thus $B = \{\{f(U_1), f(V_1)\}, \{f(U_2), f(V_2)\}, \dots, \{f(U_m), f(V_m)\}..\}$ is the winning strategy for P_2 in $\c{G}(T_1, Y))$.
In the same way, we can proof $(P_2 \hookrightarrow (\dot{T}_2, Y))$, but $f(U_m) \cap f(V_m) = \emptyset$.
Thus $B = \{\{f(U_1), f(V_1)\}, \{f(U_2), f(V_2)\}, \dots, \{f(U_m), f(V_m)\}\dots\}$ is the winning strategy for P_2 in $\c{G}(\dot{T}_2, Y))$.

4. The games with a continuous function via $\dot{I}$pg-open sets.

In this part, we will using *continuous* function via $\dot{I}$pg- open set to explain a winning strategy for P_1 and P_2 in $\c{G}(\dot{T}_i, X)$ and $\c{G}(\dot{T}_i, \dot{I})$ where I ={0,1,2}.

Definition 3.6. (1) A function $f : (X, T, \dot{I}) \to (Y, T, \j)$ is;

1. $\dot{I}$-pre-g-continuous function, symbolizes $\dot{I}$pg-continuous, if $f^{-1}(V) \in \dot{I}pgO(X)$ for all $V \in T$.
2. Strongly-$\dot{I}$-pre-g-continuous function, Symbolizes strongly-$\dot{I}$pg-continuous, if $f^{-1}(V) \in T$, for all $V \in \j pgO(Y)$.
3. $\dot{I}$-pre-g-irresolute function, symbolizes $\dot{I}$pg-irresolute, if $f^{-1}(V) \in \dot{I}pgO(X)$ for all $V \in \j pgO(Y)$.

Proposition 4.6. If the function $f : (X, T, \dot{I}) \to (Y, T, \j)$ is an injective $\dot{I}$-pre-g-continuous function and $(P_2 \hookrightarrow \c{G}(\dot{T}_i, Y)$ then $(P_2 \hookrightarrow \c{G}(\dot{T}_i, \dot{I}))$, where (i=0,1and 2 respectively).

Proof **(1).** In the game $Ģ(\dot{T}_i, \dot{ꞁ})$ where $(i = 0)$, in the first round, P_1 will choose $x_1 \neq r_1$ such that, x_1, $r_1 \in X$. Next, P_2 in $Ģ(\dot{T}_0, \dot{ꞁ})$ will hold account $f(x_1)$, $f(r_1) \in Y$, $f(x_1) \neq f(r_1)$, but ($P_2 \hookrightarrow Ģ(\dot{T}_0, Y)$, $\exists v_1 \in T$, $f(x_1) \in v_1$ and $f(r_1) \notin v_1$, but f is $\dot{ꞁ}$pg-continuous function, so $f^{-1}(v) \in \dot{ꞁ}pgO(X)$, this implies P_2 in $Ģ(\dot{T}_0, \dot{ꞁ})$ choose $f^{-1}(v_1)$ is an $\dot{ꞁ}pgO(X)$, in the second round, P_1 in $Ģ(\dot{T}_0, \dot{ꞁ})$ will choose $x_2 \neq r_2$ such that x_2, $r_2 \in X$. Next, P_2 in $Ģ(\dot{T}_0, X)$ will hold account $f(x_2)$, $f(r_2) \in Y$, $f(x_2) \neq f(r_2)$, but ($P_2 \hookrightarrow Ģ(\dot{T}_0, Y)$, $\exists v_2 \in T$, $f(x_2) \in v_2$ and $f(r_2) \notin v_2$, this implies P_2 in $Ģ(\dot{T}_0, \dot{ꞁ})$ choose $f^{-1}(v_2)$ is an $\dot{ꞁ}pgO(X)$ and in m-th round P_1 in $Ģ(\dot{T}_0, \dot{ꞁ})$ will choose $x_m \neq r_m$ such that x_m, $r_m \in X$. Next, P_2 in $Ģ(\dot{T}_0, X)$ choose $f(x_m)$, $f(r_m) \in Y$, $f(x_m) \neq f(r_m)$, but ($P_2 \hookrightarrow Ģ(\dot{T}_0, Y)$, $\exists v_m \in T$, $f(x_m) \in v_m$ and $f(r_m) \notin v_m$, this implies P_2 in $Ģ(\dot{T}_0, \dot{ꞁ})$ choose $f^{-1}(v_m)$ is an $\dot{ꞁ}pgO(X)$ thus $B = \{ f^{-1}(v_1), f^{-1}(v_2), \dots, f^{-1}(v_m)\} \dots \}$ is winning strategy for P_2 in $Ģ(\dot{T}_0, \dot{ꞁ})$).

(2) In the game $Ģ(\dot{T}_i, \dot{ꞁ})$ where $(i = 1)$, in m-th round P_1 in $Ģ(\dot{T}_1, \dot{ꞁ})$ will choose $x_m \neq r_m$ such that x_m, $r_m \in X$. Next, P_2 in $Ģ(\dot{T}_1, X)$ will hold account $f(x_m)$, $f(r_m) \in Y$, $f(x_m) \neq f(r_m)$, but ($P_2 \hookrightarrow Ģ(\dot{T}_1, Y)$, $\exists U_m, v_m \in T$, $f(x_m) \in (U_m - v_m)$ and $f(r_m) \in (v_m - U_m)$, this implies P_2 in $Ģ(\dot{T}_1, \dot{ꞁ})$ choose $f^{-1}(U_m)$, $f^{-1}(v_m)$, are $\dot{ꞁ}pgO(X)$, thus $B = \left\{\{f^{-1}(U_1), f^{-1}(v_1)\}, \{f^{-1}(U_2), f^{-1}(v_2)\}, \dots, \{ f^{-1}(U_m), f^{-1}(v_m)\} \dots \right\}$ is winning strategy for P_2 in $Ģ(\dot{T}_1, \dot{ꞁ})$). By the same way we can prove $P_2 \hookrightarrow Ģ(\dot{T}_2, \dot{ꞁ})$.but $f^{-1}(U_m) \cap f^{-1}(v_m) = \emptyset$, thus $B = \left\{\{f^{-1}(U_1), f^{-1}(v_1)\}, \{f^{-1}(U_2), f^{-1}(v_2)\}, \dots, \{ f^{-1}(U_m), f^{-1}(v_m)\} \dots \right\}$ is winning strategy for P_2 in $Ģ(\dot{T}_2, \dot{ꞁ})$).

Proposition 4.7. If the function $f : (X, T, \dot{ꞁ}) \to (Y, T, \dot{j})$ is an injective strongly-$\dot{ꞁ}$pg- continuous and ($P_2 \hookrightarrow Ģ(\dot{T}_i, \dot{j})$) then ($P_2 \hookrightarrow Ģ(\dot{T}_i, X)$) where (i=0,1and 2 respectively).

Proof **(1).** In the game $Ģ(\dot{T}_i, X)$ where $(i = 0)$, in the first round, P_1 will choose $x_1 \neq r_1$ such that $x_1, r_1 \in X$. Next, P_2 in $Ģ(\dot{T}_0, X)$) will hold account $f(x_1)$, $f(r_1) \in Y$, $f(x_1) \neq f(r_1)$, but ($P_2 \hookrightarrow Ģ(\dot{T}_0, \dot{j})$, so $\exists v_1 \in \dot{j}pgO(Y)$), $f(x_1) \in v_1$ and $f(r_1) \notin v_1$ but f is strongly-$\dot{ꞁ}$pg- continuous then, $f^{-1}(v_1) \in T$ this implies P_2 in $Ģ(\dot{T}_0, X)$ choose $f^{-1}(v_1)$, in the second round, P_1 in $Ģ(\dot{T}_0, X)$ choose $x_2 \neq r_2$ such that $x_2, r_2 \in X$. Next, P_2 in $Ģ(\dot{T}_0, X)$ will hold account $f(x_2)$, $f(r_2) \in Y$, $f(x_2) \neq f(r_2)$, but ($P_2 \hookrightarrow Ģ(\dot{T}_0, \dot{j})$, $\exists v_2 \in \dot{j}pgO(Y)$), $f(x_2) \in v_2$ and $f(r_2) \notin v_2$, this implies P_2 in $Ģ(\dot{T}_0, X)$ choose $f^{-1}(v_2)$ and in m - th round, P_1 in $Ģ(\dot{T}_0, X)$ choose $x_m \neq r_m$, $x_m, r_m \in X$. Next, P_2 in $Ģ(\dot{T}_0, X)$ will hold account $f(x_m)$, $f(r_m) \in Y$, $f(x_m) \neq f(r_m)$, but ($P_2 \hookrightarrow Ģ(\dot{T}_0, \dot{j})$, $\exists v_m \in \dot{j}pgO(Y)$, $f(x_m) \in v_m$ and $f(r_m) \notin v_m$, this implies P_2 in $Ģ(\dot{T}_0, X)$ choose $f^{-1}(v_m) \in T$,thus $B = \{f^{-1}\{ v_1\}, f^{-1}\{v_2\} \dots, f^{-1}\{v_m\} . . \}$ is winning strategy for P_2 in $Ģ(\dot{T}_0, X)$.

(2). In the game $Ģ(\dot{T}_i, X)$, where $(i = 1)$, in the m- th round P_1 in $Ģ(\dot{T}_1, X)$ choose $x_m \neq r_m$ such that $x_m, r_m \in X$, P_2 in $Ģ(\dot{T}_1, X)$ will hold account $f(x_m)$, $f(r_m) \in Y$, $f(x_m) \neq f(r_m)$, but ($P_2 \hookrightarrow Ģ(\dot{T}_1, \dot{j})$, $\exists U_m, v_m \in \dot{j}pgO(Y)$, $f(x_m) \in (U_m - v_m)$ and $f(r_m) \in (v_m - U_m)$, this implies P_2 in $Ģ(\dot{T}_1, X)$ choose $f^{-1}(U_m)$, $f^{-1}(v_m) \in T$. Thus $B = \left\{\{f^{-1}(U_1), f^{-1}(v_1)\}, \{f^{-1}(U_2), f^{-1}(v_2)\}, \dots, \{ f^{-1}(U_m), f^{-1}(v_m)\} \dots \right\}$ is winning strategy for P_2 in $Ģ(\dot{T}_1, X)$. In the same way, we can prove $P_2 \hookrightarrow Ģ(\dot{T}_2, X)$, but $f^{-1}(U_m) \cap f^{-1}(v_m) = \emptyset$. Thus $B = \left\{\{f^{-1}(U_1), f^{-1}(v_1)\}, \{f^{-1}(U_2), f^{-1}(v_2)\}, \dots, \{ f^{-1}(U_m), f^{-1}(v_m)\} \dots \right\}$ is winning strategy for P_2 in $Ģ(\dot{T}_2, \dot{ꞁ})$).

Corollary 4.8. Let $\mathfrak{f}:(X,T,\dot{I})\rightarrow(Y,\mathcal{T},\mathfrak{j})$ is injective Strongly-$\acute{I}$pg-continuous function and $(P_2 \hookrightarrow G(\dot{T}_i,\mathfrak{j})$, then$(P_2 \hookrightarrow G(\dot{T}_i,\dot{I})$, where (i = 0,1and2 respectively).

Proposition 4.9. If the function $\mathfrak{f}:(X,T,\dot{I})\rightarrow(Y,\mathcal{T},\mathfrak{j})$ is an injective open continuous (respectively $\acute{I}$-pre-g-irresolute function) and $(P_2 \hookrightarrow G(\dot{T}_0,Y)$ respectively $(P_2 \hookrightarrow G(\dot{T}_0,\mathfrak{j}))$ then $(P_2 \hookrightarrow G(\dot{T}_0,X)$ (respectively $(P_2 \hookrightarrow G(\dot{T}_0,\dot{I}))$.

Proof(1): In the game $G(\dot{T}_0,X)$(respectively in $G(\dot{T}_0,\dot{I})$), in the first round, P_1 will choose $x_1 \neq r_1$, $x_1,r_1 \in X$, Next P_2 in $G(\dot{T}_0,X)$(respectively P_2 in $G(\dot{T}_0,\dot{I})$)choose $\mathfrak{f}(x_1)$, $\mathfrak{f}(r_1)\in Y$, $\mathfrak{f}(x_1)\neq \mathfrak{f}(r_1)$, but $(P_2 \hookrightarrow G(\dot{T}_0,Y)$(respectively$(P_2 \hookrightarrow G(\dot{T}_0,\mathfrak{j}))$, $\exists v_1\ \mathcal{T}$ (respectively $\exists v_1\ \mathfrak{j}pgO(Y)$), $\mathfrak{f}(x_1)\in v_1$ and $\mathfrak{f}(r_1)\notin v_1$ and since $\mathfrak{f}$ is open continuous (respectively $\acute{I}$-pre-g-irresolute function)this implies P_2 in $G(\dot{T}_0,X)$ (respectively in $G(\dot{T}_0,\dot{I})$) choose $\mathfrak{f}^{-1}(v_1)$, in the second round, P_1 in $G(\dot{T}_0,X)$ (respectively in $G(\dot{T}_0,\dot{I})$) choose $x_2 \neq r_2$ such that x_2, $r_2 \in X$. Next, P_2 in $G(\dot{T}_0,X)$ (respectively P_2 in $G(\dot{T}_0,\dot{I})$) choose $\mathfrak{f}(x_2)$, $\mathfrak{f}(r_2)\in Y$, $\mathfrak{f}(x_2)\neq\mathfrak{f}(r_2)$, but $(P_2 \hookrightarrow G(\dot{T}_0,Y)$(respectively$(P_2 \hookrightarrow G(\dot{T}_0,\mathfrak{j}))$, $\exists v_2 \in \mathcal{T}$ (respectively $\exists v_2 \in \mathfrak{j}pgO(Y)$), $\mathfrak{f}(x_2)\in v_2$ and $\mathfrak{f}(r_2)\notin v_2$, this implies P_2 in $G(T_0,X)$ (respectively P_2 in $G(\dot{T}_0,\dot{I})$) choose $\mathfrak{f}^{-1}(v_2)$and in m-th step P_1 in $G(\dot{T}_0,X)$ (respectively in $G(\dot{T}_0,\dot{I})$) choose $x_m \neq r_m$, x_m, $r_m \in X$. Next, P_2 in $G(\dot{T}_0,X)$ (respectively P_2 in $G(\dot{T}_0,\dot{I})$) choose $\mathfrak{f}(x_m)$, $\mathfrak{f}(r_m)\in Y)$, $\mathfrak{f}(x_m)\neq\mathfrak{f}(r_m)$, but$(P_2 \hookrightarrow G(\dot{T}_0,Y))$(respectively $(P_2 \hookrightarrow G(\dot{T}_0,\mathfrak{j}))$,$\exists v_m \in \mathcal{T}$(respectively $\exists v_m \in \mathfrak{j}pgO(Y)$, $\mathfrak{f}(x_m)\in v_m$ and $\mathfrak{f}(r_m)\notin v_m$ this implies P_2 in $G(\dot{T}_0,X)$ respectively P_2 in $G(\dot{T}_0,\dot{I})$) choose $\mathfrak{f}^{-1}(v_m)$, thus $B=\{\mathfrak{f}^{-1}\{v_1\},\mathfrak{f}^{-1}\{v_2\}\dots,\mathfrak{f}^{-1}\{v_m\}\dots\}$ is winning strategy for P_2 in $G(\dot{T}_0,X)$) (respectively P_2 in $G(\dot{T}_0,\dot{I})$).

(2). In the game $G(\dot{T}_1,X)$, (respectively $G(\dot{T}_1,\dot{I})$), in the m-th round, P_1 in $G(\dot{T}_1,X)$ (respectively in $G(\dot{T}_1,\dot{I})$) choose $x_m \neq r_m$ such that x_m, $r_m \in X$. Next, P_2 in $G(\dot{T}_1,X)$ (respectively P_2 in $G(\dot{T}_1,\dot{I})$) choose $\mathfrak{f}(x_m)$, $\mathfrak{f}(r_m)\in Y)$, $\mathfrak{f}(x_m)\neq\mathfrak{f}(r_m)$, but $(P_2 \hookrightarrow G(\dot{T}_1,Y))$, $\exists u_m, v_m \in \mathcal{T}$ (respectively $\exists u_m, v_m \in \mathfrak{j}pgO(Y)$); $\mathfrak{f}(x_m)\in(u_m - v_m)$ and $\mathfrak{f}(r_m)\in(v_m - u_m)$, this implies P_2 in $G(\dot{T}_1,X)$(respectively P_2in $G(\dot{T}_1,\dot{I})$) choose $\mathfrak{f}^{-1}(u_m),\mathfrak{f}^{-1}(v_m)$ thus $B=\left\{\{\mathfrak{f}^{-1}(u_1),\mathfrak{f}^{-1}(v_1)\},\{\mathfrak{f}^{-1}(u_2),\mathfrak{f}^{-1}(v_2)\},\dots,\{\mathfrak{f}^{-1}(u_m),\mathfrak{f}^{-1}(v_m)\}\dots\right\}$ is winning strategy for P_2 in $G(\dot{T}_1,X)$(respectively P_2 in $G(\dot{T}_1,\dot{I})$). By the same way we can prove $P_2 \hookrightarrow G(\dot{T}_2,X)$ respectively, P_2in $G(\dot{T}_2,\dot{I})$, but $\mathfrak{f}^{-1}(u_m)\cap\mathfrak{f}^{-1}(v_m)=\emptyset$ thus $B=\left\{\{\mathfrak{f}^{-1}(u_1),\mathfrak{f}^{-1}(v_1)\},\{\mathfrak{f}^{-1}(u_2),\mathfrak{f}^{-1}(v_2)\},\dots,\{\mathfrak{f}^{-1}(u_m),\mathfrak{f}^{-1}(v_m)\}\dots\right\}$ is winning strategy for P_2 in $G(\dot{T}_2,X)$)(respectively P_2 in $G(\dot{T}_2,\dot{I})$).

Corollary 4.10. If $\mathfrak{f}:(X,T)\rightarrow(Y,\mathcal{T})$ is homeo then $(P_2 \hookrightarrow G(\dot{T}_i,X)) \longleftrightarrow (P_2 \hookrightarrow G(\dot{T}_i,Y))$ such that (i=0,1and 2 respectively).

5.Conclusion

The main aim of this work is to submit new near open sets which are called $\dot{I}$-pre-g-closed sets and it is complement $\dot{I}$-pre-g-open set, and interested also in studying new species of the games by application separation axioms via $\dot{I}$-pre-g-open sets and gives the strategy of winning and losing to any one of the two players in $G(\dot{T}_i,X)$, i= {0, 1, 2} .

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