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Cubic Bipolar Fuzzy Ideals with Thresholds (α, β), (ω , ϑ) of a Semigroup in KU-algebra

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Abstract

In this paper, we introduce the concept of cubic bipolar fuzzy ideals with thresholds (α,β),(ω,ϑ) of a semigroup in KU-algebra as a generalization of sets and in short **(CBF)**. Firstly, a **(CBF)** sub-KU-semigroup with a threshold (α,β),(ω,ϑ) and some results in this notion are achieved. Also, (cubic bipolar fuzzy ideals and cubic bipolar fuzzy *k*-ideals) with thresholds (α,β),(ω ,ϑ) are defined and some properties of these ideals are given. Relations between a **(CBF)** sub algebra and a **(CBF)** ideal are proved. A few characterizations of a **(CBF)** ***k***-ideal with thresholds (α, β), (ω,ϑ) are discussed. Finally, we proved that a **(CBF)** ***k***-ideal and a **(CBF)** ideal with thresholds (α, β), (ω,ϑ) of a KU-semi group are equivalent relations.

Keywords: A KU-semigroup, cubic ***k***-ideal, cubic bipolar fuzzy ***k***-ideal with thresholds (α, β), (ω,ϑ).

1. Introduction

The fuzzy sets were introduced by Zadeh [1] in 1956; after that, many authors applied this concept in different mathematics fields. Mostafa [2, 3] studied the notion of fuzzy KU-ideals of KU-algebras and Generalizations of Fuzzy sets, which are called bipolar- fuzzy n-fold KU-ideals. Jun [4- 6] studied the notion of a cubic set as a generalization of fuzzy set and interval-valued fuzzy set. Kareem and Hasan[7,8] defined the cubic ideals of a KU-semigroup and a homomorphism of a cubic set in this structure. Bipolar–valued fuzzy sets are extensions of fuzzy sets whose membership degree range is enlarged from the interval [0,1] to [-1,1]. Kareem and

Hassan[9] and Kareem and Awad [10] defined the concepts of bipolar fuzzy k-ideals and cubic bipolar ideals in KU-semigroup respectively, also Kareem and Abed [11] presented the idea of bipolar fuzzy k-ideals with a threshold of KU -semigroup.

The paper aims to introduce a cubic bipolar fuzzy ***k***-ideals with thresholds (α,β),(ω,ϑ) of KU-semi group and discuss some relations between a cubic bipolar fuzzy *k*-ideal with thresholds (α,β), (ω,ϑ) and a bipolar fuzzy ***k***-ideal.

2. Basic concepts

Definition(1)[12]. Algebra($\aleph, *, 0$) is a set $\aleph$,and a binary operation $*$ which is satisfies the following ,for all $\chi, \gamma, \tau \in \aleph$

$(ku_1) (\chi * \gamma) * [(\gamma * \tau) * (\chi * \tau)] = 0$

$(ku_2)\ \chi * 0 = 0$

$(ku_3)\ 0 * \chi = \chi$

$(ku_4)\ \chi * \gamma = \gamma * \chi = 0$ and $\gamma * \chi$ implies $\chi = \gamma$

$(ku_5)\ \chi * \chi = 0.$

We can define a binary operation $\leq$ on $\aleph$ is defined by $\chi \leq \gamma \Leftrightarrow \gamma * \chi = 0$. It follows that ($\aleph, \leq$)is a partially ordered set .

Theorem(2)[2]. In a KU-algebra ($\aleph, *, 0$) $\forall\ \chi, \gamma, \tau \in \aleph$, then the following holds

(1) $\chi \leq \gamma$ *imply* $\gamma * \tau \leq \chi * \tau$

(2) $\chi * (\gamma * \tau) = \gamma * (\chi * \tau)$

(3) $\gamma * \chi \leq \chi$,also $(\gamma * \chi) * \chi \leq \gamma$

Definition(3)[2]. A non-empty subset ***I*** of a KU-algebra $\aleph$ is named an ideal if for any $\chi, \gamma \in \aleph$, then

(1) $0 \in I$

(2) If $\chi * \gamma \in I$ implies that $\gamma \in I$.

Definition(4)[2]. A non-empty subset ***I*** of a KU-algebra $\aleph$ is named a KU-ideal if

(1) $0 \in I$

(2) If $\chi * (\gamma * \tau) \in I$, and $\gamma \in I$ imply that $\chi * \tau \in I$.

Definition(5)[13]. An algebra KU- semi group is a structure contains a nonempty set $\aleph$ with two binary operations $*, \circ$ and a constant 0 satisfying the following

(I) The set $\aleph$ with operation $*$ and constant 0 is KU-algebra

(II) The set $\aleph$ with operation $\circ$ is semigroup.

(III) $\chi \circ (\gamma * \tau) = (\chi \circ \gamma) * (\chi \circ \tau)$, and $(\chi * \gamma) \circ \tau = (\chi \circ \tau) * (\gamma \circ \tau)$, for all $\chi, \gamma, \tau \in \aleph$.

Definition(6)[13]. A non-empty subset A of $\aleph$ is called a sub-KU-semi group of $\aleph$ if $\chi * \gamma \in \boldsymbol{A}$, and $\chi \circ \gamma \in \boldsymbol{A}$, for all $\chi, \gamma \in \boldsymbol{A}$

Definition(7)[13]. In a KU-semi group $(\aleph, *, \circ, 0)$, the subset $\varphi \neq I$ of $\aleph$ is said to be S ideal , if

(i) It is an ideal in a KU-algebra

(ii) $\chi \circ a \in \boldsymbol{I}$, and $a \circ \chi \in \boldsymbol{I}$, $\forall \chi \in \aleph$, $a \in \boldsymbol{I}$

Definition(8)[13]. In KU-semigroup $(\aleph, *, \circ, 0)$, the subset $\varphi \neq A$ of $\aleph$ is named a $\boldsymbol{k}$ -ideal , if

(i) It is a KU-ideal of $\aleph$

(ii) $\chi \circ a \in \boldsymbol{I}$, and $a \circ \chi \in \boldsymbol{I}$, $\forall \chi \in \aleph$, $a \in \boldsymbol{I}$

In this part , we recall some concepts of fuzzy logic

A function $\mu: \aleph \longrightarrow [0,1]$ is said to be a fuzzy set of a set $\aleph$,and the set

is said to be a level set of μ, for t, where $1 \geq t \geq 0 U(\mu, t) = \{\chi \in \aleph : \mu(\chi) \geq t\}$

Now, an interval valued fuzzy set $\tilde{\mu}$ of $\aleph$ is defined as follows:

Remark(9)[7-8].A function $\tilde{\mu}: \aleph \longrightarrow D[0,1]$, where D[0,1] is a family of the closed sub - intervals of[0, 1]. The level subset of $\tilde{\mu}$ is denoted by $\tilde{\mu}_{\tilde{t}}$ and it is defined by

$\tilde{\mu}_{\tilde{t}} = \{\chi \in \aleph: \tilde{\mu}(\chi) \geq \tilde{t}\}$, for every $[0,0] \leq \tilde{t} \leq [1,1]$.

O. Hasan and F.Kareem **[7-8]** introduced the Cubic ideals of the KU-semigroup as follows:

Definition(10)[7-8]. In the KU-semigroup$(\aleph, *, \circ, 0)$, a cubic set Θ is the form $\Theta = \{\langle \chi, \tilde{\mu}_\Theta(\chi), \lambda_\Theta(\chi) \rangle : \chi \in \aleph\}$, such that $\lambda_\Theta(\chi)$ is a fuzzy set and $\tilde{\mu}_\Theta: \aleph \to D[0,1]$ is an interval-valued , briefly $\Theta = \langle \tilde{\mu}_\Theta, \lambda_\Theta \rangle$.

Definition(11)[7-8]. In the KU-semigroup $(\aleph, *, \circ, 0)$ a cubic set $\Theta = \langle \tilde{\mu}_\Theta, \lambda_\Theta \rangle$in $\aleph$ is named a cubic sub-KU-semigroup if: for all $\chi, \gamma \in \aleph$,

(1) $\tilde{\mu}_\Theta(\chi * \gamma) \geq rmin\{\tilde{\mu}_\Theta(\chi), \tilde{\mu}_\Theta(\gamma)\}$,$\lambda_\Theta(\chi * \gamma) \leq max\{\lambda_\Theta(\chi), \lambda_\Theta(\gamma)\}$

(2) $\tilde{\mu}_\Theta(\chi \circ \gamma) \geq rmin\{\tilde{\mu}_\Theta(\chi), \tilde{\mu}_\Theta(\gamma)\}$, $\lambda_\Theta(\chi \circ \gamma) \leq max\{\lambda_\Theta(\chi), \lambda_\Theta(\gamma)\}$.

Definition(12)[7-8]. The set Θ in $\aleph$ is named a cubic ideal of a KU-semigroup $(\aleph, *, \circ, 0)$ if, $\forall \chi, \gamma \in \aleph$

(CI₁) $\tilde{\mu}_\Theta(0) \geq \tilde{\mu}_\Theta(\chi)$ *and* $\lambda_\Theta(0) \leq \lambda_\Theta(\chi)$,

(CI₂) $\tilde{\mu}_\Theta(\gamma) \geq rmin\{\tilde{\mu}_\Theta(\chi * \gamma), \tilde{\mu}_\Theta(\chi)\}$, $\lambda_\Theta(\gamma) \leq max\{\lambda_\Theta(\chi * \gamma), \lambda_\Theta(\chi)\}$

(CI₃) $\tilde{\mu}_\Theta(\chi \circ \gamma) \geq rmin\{\tilde{\mu}_\Theta(\chi), \tilde{\mu}_\Theta(\gamma)\}$, $\lambda_\Theta(\chi \circ \gamma) \leq max\{\lambda_\Theta(\chi), \lambda_\Theta(\gamma)\}$.

Example(13)[7-8]. Let $\aleph = \{0,1,2\}$ be a set. Define the operations $*,\circ$ by the following tables.

$*$	0	1	2
0	0	1	2
1	0	0	1
2	0	1	0

$\circ$	0	1	2
0	0	0	0
1	0	1	0
2	0	0	2

Then the structure $(\aleph,*,\circ,0)$ is a KU-semi group. A cubic set $\Theta = \langle\tilde{\mu}_\Theta, \lambda_\Theta\rangle$ is defined by:

$$\tilde{\mu}_\Theta(x) = \begin{cases}[0.4,0.8] & if\ \chi \in \{0,2\} \\ [0.1,0.3] & if\quad \chi = 1\end{cases} \quad \text{and } \lambda_\Theta(x) = \begin{cases}0.1 & if\ \chi \in \{0,2\} \\ 0.3 & if\quad \chi = 1\end{cases}$$

Then $\Theta = \langle\tilde{\mu}_\Theta, \lambda_\Theta\rangle$ is a cubic ideal of $\aleph$.

Definition(14)[7-8]. In a KU-semigroup $(\aleph,*,\circ,0)$, a cubic set $\Theta = \langle\tilde{\mu}_\Theta, \lambda_\Theta\rangle$ in $\aleph$ is named a cubic $\boldsymbol{k}$-ideal if $\forall\, \chi, \gamma, \tau \in \aleph$

$(\boldsymbol{Ck_1})\tilde{\mu}_\Theta(0)) \geq \tilde{\mu}_\Theta(\chi)$, and $\lambda_\Theta(0) \leq \lambda_\Theta(x)$

$(\boldsymbol{Ck_2})\tilde{\mu}_\Theta(\chi * \tau) \geq rmin\{\tilde{\mu}_\Theta(\chi * (\gamma * \tau)), \tilde{\mu}_\Theta(\gamma)\}$,

$\lambda_\Theta(\chi * \tau) \leq max\{\lambda_\Theta(\chi * (\gamma * \tau)), \lambda_\Theta(\gamma)\}$

$(\boldsymbol{Ck_3})\tilde{\mu}_\Theta(\chi \circ \gamma) \geq rmin\{\tilde{\mu}_\Theta(\chi), \tilde{\mu}_\Theta(\gamma)\}$,$\lambda_\Theta(\chi \circ \gamma) \leq max\{\lambda_\Theta(\chi), \lambda_\Theta(\gamma)\}$.

In the following ,we recall some basic concepts of a bipolar fuzzy set.

Definition(15)[9]. A bipolar fuzzy set B in a set $\aleph$ is a form $\mathrm{B} = \{(\chi, \mu(\chi), \mu^+(\chi)) : \chi \in \aleph\}$, where $\mu^-(\chi) : \aleph \to [-1,0]$ and $\mu^+(\chi) : \aleph \to [0,1]$ are two fuzzy mappings. The two membership degrees $\mu^+(\chi)$ and $\mu^-(\chi)$ denote the fulfillment degree of $\aleph$ to the property corresponding of B and the fulfillment degree of $\aleph$ to some implicit counter-property of B , respectively.

Kareem and Awad**[10]** introduced the cubic bipolar ideals of a KU- semigroup in KU-algebra as follows:

Definition(16)[10]. Let $\aleph$ be a non-empty set. A cubic bipolar set in a set $\aleph$ is the structure $\Theta = \{\langle\chi, \tilde{\mu}^+_\Theta(\chi), \tilde{\mu}^-_\Theta(\chi), \lambda^+_\Theta(\chi), \lambda^-_\Theta(\chi) : \chi \in \aleph\rangle\}$ is denoted as

$\Theta = \langle N, K\rangle$, where $N(\chi) = \{\tilde{\mu}^+_\Theta(\chi), \tilde{\mu}^-_\Theta(\chi)\}$ is called interval-valued bipolar fuzzy set and $K(\chi) = \{\lambda^+_\Theta(\chi), \lambda^-_\Theta(\chi)\}$ is a bipolar fuzzy set. Consider $\tilde{\mu}^+_\Theta : \aleph \to D[0,1]$ such that $\tilde{\mu}^+_\Theta(\chi) = [\xi^+_{\Theta_L}(\chi), \xi^+_{\Theta_U}(\chi)]$ and

$\tilde{\mu}^-_\Theta : \aleph \to D[-1,0]$ such that $\tilde{\mu}^-_\Theta(\chi) = [\xi^-_{\Theta_L}(\chi), \xi^-_{\Theta_U}(\chi)]$, also $\lambda^+_\Theta : \aleph \to [0,1]$ and $\lambda^-_\Theta : \aleph \to [-1,0]$ it follows that

$\Theta = \{< \chi, \{[\xi^{+}_{\Theta_L}(\chi), \xi^{+}_{\Theta_U}(\chi)], [\xi^{-}_{\Theta_L}(\chi), \xi^{-}_{\Theta_U}(\chi)]\}, \quad \lambda^{+}_{\Theta}(\chi), \lambda^{-}_{\Theta}(\chi)\} >: \chi \in \aleph\}$

Definition(17)[10]. A (CB) $\Theta = \langle N, K\rangle$ in $\aleph$ is named a (CB) sub-KU-semigroup if: $\forall \chi, \gamma \in \aleph$,

(1) $\tilde{\mu}^{+}_{\Theta}(\chi * \gamma) \geq rmin\{\tilde{\mu}^{+}_{\Theta}(\chi), \tilde{\mu}^{+}_{\Theta}(\gamma)\}, \tilde{\mu}^{-}_{\Theta}(\chi * \gamma) \leq rmax\{\tilde{\mu}^{-}_{\Theta}(\chi), \tilde{\mu}^{-}_{\Theta}(\gamma)\}$

$\lambda^{+}_{\Theta}(\chi * \gamma) \geq min\{\lambda^{+}_{\Theta}(\chi), \lambda^{+}_{\Theta}(\gamma)\}, \lambda^{-}_{\Theta}(\chi * \gamma) \leq max\{\lambda^{-}_{\Theta}(\chi), \lambda^{-}_{\Theta}(\gamma)\}$,

(2) $\tilde{\mu}^{+}_{\Theta}(\chi \circ \gamma) \geq rmin\{\tilde{\mu}^{+}_{\Theta}(\chi), \tilde{\mu}^{+}_{\Theta}(\gamma)\}, \tilde{\mu}^{-}_{\Theta}(\chi \circ \gamma) \leq rmax\{\tilde{\mu}^{-}_{\Theta}(\chi), \tilde{\mu}^{-}_{\Theta}(\gamma)\}$

$\lambda^{+}_{\Theta}(\chi \circ \gamma) \geq min\{\lambda^{+}_{\Theta}(\chi), \lambda^{+}_{\Theta}(\gamma)\}, \lambda^{-}_{\Theta}(\chi \circ \gamma) \leq max\{\lambda^{-}_{\Theta}(\chi), \lambda^{-}_{\Theta}(\gamma)\}$,

Example(18)[10]: The following table is Illustrates that the set $\aleph = \{0,1,2,3\}$ with binary operations $*$ and $\circ$

*	0	1	2	3
0	0	1	2	3
1	0	0	0	2
2	0	2	0	1
3	0	0	0	0

∘	0	1	2	3
0	0	0	0	0
1	0	1	0	1
2	0	0	2	2
3	0	1	2	3

Then$(\aleph, *, \circ, 0)$ is a KU-semigroup. Define $\Theta = \langle N, K\rangle$ as follows

$$M(x) = \begin{cases} \{[-0.2, -0.5], [0.1, 0.9]\} & if \quad \chi = \{0,1\} \\ \{[-0.6, -0.2], [0.2, 0.5]\} & if \quad otherwise \end{cases},$$

$$\lambda^{+}_{\Theta}(x) = \begin{cases} 0.5 & if \quad \chi = \{0,1\} \\ 0.3 & if \quad otherwise \end{cases} \quad \lambda^{-}_{\Theta}(x) = \begin{cases} -0.6 & if \quad \chi = \{0,1\} \\ -0.3 & if \quad otherwise \end{cases}$$

And by applying definition **2.17**, we can easily prove that $\Theta = \langle N, K\rangle$ is a cubic bipolar sub KU-semigroup of $\aleph$.

3. Cubic bipolar ideals of a KU-semi group with thresholds *(α, β), (ω ,ϑ)*

In this part, the notion of cubic bipolar ***k***-ideals with thresholds $(\alpha, \beta), (\omega, \vartheta)$ of a KU-semi group and some properties are defined. In the following, we denote a cubic bipolar fuzzy set by (CBF) ,and let $\alpha, \beta \in D[0,1]$, *and*, $\omega, \vartheta \in [0,1]$, *such that*

$[0,0] < \alpha < \beta < [1,1] \quad , 0 < \omega < \vartheta < 1$**,** where ω, ϑ are arbitrary values, and

α, β**,** are arbitrary closed sub-intervals

Definition(19). A ***(CBF)*** set $\Theta = \langle M, L\rangle$ is named a ***(CBF)*** sub-KU-semi group with thresholds $(\alpha, \beta), (\omega, \vartheta)$ if $\forall \chi, \gamma \in \aleph$

(1)$min\{\tilde{\mu}^{-}_{\Theta}(\chi * \gamma), -\alpha\} \leq rmax\{\tilde{\mu}^{-}_{\Theta}(\chi), \tilde{\mu}^{-}_{\Theta}(\gamma), -\beta\}$

$rmax\{\tilde{\mu}^{+}_{\Theta}(\chi * \gamma), \alpha\} \geq rmin\{\tilde{\mu}^{+}_{\Theta}(\chi), \tilde{\mu}^{+}_{\Theta}(\gamma), \beta\}$

$min\{\lambda^{-}_{\Theta}(\chi * \gamma), -\omega\} \leq max\{\lambda^{-}_{\Theta}(\chi), \lambda^{-}_{\Theta}(\gamma), -\vartheta\}$

$max\{\lambda^+_\Theta(\chi * \gamma), \omega\} \geq min\{\lambda^+_\Theta(\chi), \lambda^+_\Theta(\gamma), \vartheta\}$

(2)$rmin\{\tilde{\mu}^-_\Theta(\chi \circ \gamma), -\alpha\} \leq rmax\{\tilde{\mu}^-_\Theta(\chi), \tilde{\mu}^-_\Theta(\gamma), -\beta\}$

$rmax\{\tilde{\mu}^+_\Theta(\chi \circ \gamma), \alpha\} \geq rmin\{\tilde{\mu}^+_\Theta(\chi), \tilde{\mu}^+_\Theta(\gamma), \beta\}$

$min\{\lambda^-_\Theta(\chi \circ \gamma), -\omega\} \leq max\{\lambda^-_\Theta(\chi), \lambda^-_\Theta(\gamma), -\vartheta\}$

$max\{\lambda^+_\Theta(\chi \circ \gamma), \omega\} \geq min\{\lambda^+_\Theta(\chi), \lambda^+_\Theta(\gamma), \vartheta\}$

Remark(20). Every ***(CBF)*** sub-KU-semi group of ℵ is a ***(CBF)*** sub-KU-semigroup with thresholds ***(α, β), (ω ,ϑ)*** , but not converse as it is shown in the following example

Example(21). Let $\aleph = \{0,1,2,3\}$ be a set with two operations $*$ and $\circ$ which are defined by the following tables.

*	0	1	2	3
0	0	1	2	3
1	0	0	0	2
2	0	2	0	1
3	0	0	0	0

∘	0	1	2	3
0	0	0	0	0
1	0	1	0	1
2	0	0	2	2
3	0	1	2	3

Then$(\aleph, *, \circ, 0)$ is a KU-semi group.Now, we define $\Theta = \langle M, L \rangle$ by the next

$$M(x) = \begin{cases} [-0.9, -0.8], [0.8, 0.9] & if \quad \chi = 0 \\ [-0.8, -0.7], [0.7, 0.8] & if \quad \chi = 1 \\ [-0.6, -0.5], [0.5, 0.6] & if \quad \chi = 3 \\ [-0.3, -0.2], [0.2, 0.3] & if \quad \chi = 2 \end{cases}$$

$$L(x) = \begin{cases} -0.9, & 0.9 \quad if \quad \chi = 0 \\ -0.5, & 0.6 \quad if \quad \chi = 1 \\ -0.4, & 0.5 \quad if \quad \chi = 3 \\ -0.2, & 0.2 \quad if \quad \chi = 2 \end{cases}$$

And by applying definition **(19**), we can easily prove that $\Theta = \langle M, L \rangle$ is a***(CBF)***sub KU-semi group with thresholds $(\alpha, \beta) = ([0.1, 0.2], [0.2, 0.2])$, and $(\omega, \vartheta) = (0.1, 0.2)$, but not a ***(CBF)***sub KU-semi group since

$\tilde{\mu}^+_\Theta(1 * 3) \geq rmin\{\tilde{\mu}^+_\Theta(1), \tilde{\mu}^+_\Theta(3)\}$

$\{\tilde{\mu}^+_\Theta(2)\} \geq rmin\{\tilde{\mu}^+_\Theta(1), \tilde{\mu}^+_\Theta(3)\}$

$[0.2, 0.3] \geq rmin\{[0.7, 0.8], [0.5, 0.6]\}$

$[0.2, 0.1] \geq [0.5, 0.6]$,which is incorrect phrase

$\tilde{\mu}^-_\Theta(1 * 3) \leq rmax\{\tilde{\mu}^-_\Theta(1), \tilde{\mu}^-_\Theta(3)\}$

$\tilde{\mu}_{\Theta}^{-}(2) \leq rmax\{[-0.8\,,-0.7],[-0.6\,,-0.5]\}$

$[-0.3, -0.2\leq$ $[-0.6\,,-0.5]$, which is the incorrect phrase, and

$\lambda_{\Theta}^{+}(1*3) \geq min\{\lambda_{\Theta}^{+}(1)\,,\lambda_{\Theta}^{+}(3)\}$

$\lambda_{\Theta}^{+}(2) \geq min\{0.6\,,0.5\}$

$0.2 \geq 0.5$, it is wrong

$\lambda_{\Theta}^{-}(1*3) \leq max\{\lambda_{\Theta}^{-}(1)\,,\lambda_{\Theta}^{-}(3)\}$

$\lambda_{\Theta}^{-}(2) \leq max\{-0.5\,,-0.4\}$

$-0.2 \leq -0.4$, which is also wrong.

Remark(22). If $\Theta = \langle M, L\rangle$ is a ***(CBF)*** sub KU-semi group with thresholds (α, β), (ω , ϑ) such that α = [0,0], β = [1,1,] , ω = 0, and ϑ = 1 ,then $\Theta = \langle M, L\rangle$ is a ***(CBF)*** sub-KU-semi group of ℵ.

Proposition(23). If $\Theta = \langle M, L\rangle$ is a cubic bipolar sub-KU-semi group with thresholds (α, β), $(\omega\,,\vartheta)$ of ℵ ,then for all $\chi \in$ ℵ

(1) $rmax\{\tilde{\mu}_{\Theta}^{+}(0),\ \alpha\} \geq rmin\{\tilde{\mu}_{\Theta}^{+}(\chi),\ \beta\}$

(2) $rmin\{\tilde{\mu}_{\Theta}^{-}(0), -\alpha\} \leq rmax\{\tilde{\mu}_{\Theta}^{-}(\chi), -\beta\}$

(3) $max\{\lambda_{\Theta}^{+}(0),\ \omega\} \geq min\{\lambda_{\Theta}^{+}(\chi),\ \vartheta\}$

(4)$min\{\lambda_{\Theta}^{-}(0), -\omega\} \leq max\{\lambda_{\Theta}^{-}(\chi), -\vartheta\}$

Proof: by **(ku5)** $\chi * \chi = 0$, and since $\Theta = \langle M, L\rangle$ is a cubic bipolar sub-KU-semi group with thresholds (α, β), $(\omega\,,\vartheta)$ of ℵ ,

$rmax\{\tilde{\mu}_{\Theta}^{+}(0),\ \alpha\} = rmax\{\tilde{\mu}_{\Theta}^{+}(\chi*\chi),\ \alpha\} \geq rmin\{\tilde{\mu}_{\Theta}^{+}(\chi), \tilde{\mu}_{\Theta}^{+}(\chi),\ \beta\}$

$= rmin\{\tilde{\mu}_{\Theta}^{+}(\chi), \beta\}$, that is ***(1)***

$rmin\{\tilde{\mu}_{\Theta}^{-}(0), -\alpha\} = rmin\{\tilde{\mu}_{\Theta}^{-}(\chi*\chi), -\alpha\} \leq rmax\{\tilde{\mu}_{\Theta}^{-}(\chi), \tilde{\mu}_{\Theta}^{-}(\chi), -\beta\}$

$= rmax\{\tilde{\mu}_{\Theta}^{-}(\chi), -\beta\}$,that is ***(2)***

$max\{\lambda_{\Theta}^{+}(0),\ \omega\} = max\{\lambda_{\Theta}^{+}(\chi*\chi), \omega\} \geq min\{\lambda_{\Theta}^{+}(\chi), \lambda_{\Theta}^{+}(\chi),\ \vartheta\}$

$= min\{\lambda_{\Theta}^{+}(\chi)\,, \vartheta\}$,that is ***(3)***

$min\{\lambda_{\Theta}^{-}(0), -\omega\} = min\{\lambda_{\Theta}^{-}(\chi*\chi), -\omega\} \leq max\{\lambda_{\Theta}^{-}(\chi), \lambda_{\Theta}^{-}(\chi), -\vartheta\}$

$= max\{\lambda_{\Theta}^{-}(\chi), -\vartheta\}$,that is ***(4)***

Proposition(24). If $\Theta = \langle M, L\rangle$ is a ***(CBF)*** sub-KU-semi group with thresholds (α, β), $(\omega\,,\vartheta)$ of ℵ ,then for all $\chi \in$ ℵ

(1) $rmax\{\tilde{\mu}_{\Theta}^{+}(0\circ\chi),\ \alpha\} \geq rmin\{\tilde{\mu}_{\Theta}^{+}(\chi),\ \beta\}$

(2) $rmin\{\tilde{\mu}_{\Theta}^{-}(0\circ\chi), -\alpha\} \leq rmax\{\tilde{\mu}_{\Theta}^{-}(\chi), -\beta\}$

(3) $max\{\lambda_{\Theta}^{+}(0\circ\chi),\ \omega\} \geq min\{\lambda_{\Theta}^{+}(\chi),\ \vartheta\}$

(4)$min\{\lambda_{\Theta}^{-}(0\circ\chi), -\omega\} \leq max\{\lambda_{\Theta}^{-}(\chi), -\vartheta\}$

Proof: Since $\Theta = \langle M, L \rangle$ is a ***(CBF)*** sub-KU-semi group with thresholds (α, β), (ω, ϑ) of $\aleph$, we have

$rmax\{\tilde{\mu}_{\Theta}^{+}(0 \circ \chi),\ \alpha\} \geq rmin\{\tilde{\mu}_{\Theta}^{+}(0),\ \tilde{\mu}_{\Theta}^{+}(\chi),\ \beta\} = rmin\{\tilde{\mu}_{\Theta}^{+}(\chi),\ \beta\}$, which is ***(1)***

$rmin\{\tilde{\mu}_{\Theta}^{-}(0 \circ \chi), -\alpha\} \leq rmax\{\tilde{\mu}_{\Theta}^{-}(0),\ \tilde{\mu}_{\Theta}^{-}(\chi) - \beta\} = rmax\{\tilde{\mu}_{\Theta}^{-}(\chi), -\beta\}$, which is ***(2)***

$max\{\lambda_{\Theta}^{+}(0 \circ \chi), \omega\} \geq min\{\lambda_{\Theta}^{+}(0), \lambda_{\Theta}^{+}(\chi),\ \vartheta\} = min\{\lambda_{\Theta}^{+}(\chi),\ \vartheta\}$, which is ***(3)***

$min\{\lambda_{\Theta}^{-}(0 \circ \chi), -\omega\} \leq max\{\lambda_{\Theta}^{-}(0), \lambda_{\Theta}^{-}(\chi), -\vartheta\} = max\{\lambda_{\Theta}^{-}(\chi), -\vartheta\}$,which is ***(4)***

Definition(25). A ***(CBF)*** set $\Theta = \langle M, L \rangle$ is named a ***(CBF)*** ideal of the KU-semi group with thresholds (α, β), (ω, ϑ) if $\forall\ \chi, \gamma \in \aleph$

(CBT₁) $rmin\{\tilde{\mu}_{\Theta}^{-}(0), -\alpha\} \leq rmax\{\tilde{\mu}_{\Theta}^{-}(\chi), -\beta\}$

$rmax\{\tilde{\mu}_{\Theta}^{+}(0),\ \alpha\} \geq rmin\{\tilde{\mu}_{\Theta}^{+}(\chi),\ \beta\}$,and

$min\{\lambda_{\Theta}^{-}(0), -\omega\} \leq max\{\lambda_{\Theta}^{-}(\chi), -\vartheta\}$

$max\{\lambda_{\Theta}^{+}(0),\ \omega\} \geq min\{\lambda_{\Theta}^{+}(\chi),\ \vartheta\}$

(CBT₂) $rmin\{\tilde{\mu}_{\Theta}^{-}(\gamma), -\alpha\} \leq rmax\{\tilde{\mu}_{\Theta}^{-}(\chi * \gamma), \tilde{\mu}_{\Theta}^{-}(\chi), -\beta\}$

$rmax\{\tilde{\mu}_{\Theta}^{+}(\gamma),\ \alpha\} \geq rmin\{\tilde{\mu}_{\Theta}^{+}(\chi * \gamma), \tilde{\mu}_{\Theta}^{+}(\chi),\ \beta\}$

$min\{\lambda_{\Theta}^{-}(\gamma), -\omega\} \leq max\{\lambda_{\Theta}^{-}(\chi * \gamma), \lambda_{\Theta}^{-}(\chi), -\vartheta\}$

$max\{\lambda_{\Theta}^{+}(\gamma),\ \omega\} \geq min\{\lambda_{\Theta}^{+}(\chi * \gamma), \lambda_{\Theta}^{+}(\chi),\ \vartheta\}$

(CBT₃) $rmin\{\tilde{\mu}_{\Theta}^{-}(\chi \circ \gamma), -\alpha\} \leq rmax\{\tilde{\mu}_{\Theta}^{-}(\chi), \tilde{\mu}_{\Theta}^{-}(\gamma), -\beta\}$

$rmax\{\tilde{\mu}_{\Theta}^{+}(\chi \circ \gamma),\ \alpha\} \geq rmin\{\tilde{\mu}_{\Theta}^{+}(\chi), \tilde{\mu}_{\Theta}^{+}(\gamma),\ \beta\}$

$min\{\lambda_{\Theta}^{-}(\chi \circ \gamma), -\omega\} \leq max\{\lambda_{\Theta}^{-}(\chi), \lambda_{\Theta}^{-}(\gamma), -\vartheta\}$

$max\{\lambda_{\Theta}^{+}(\chi \circ \gamma),\ \omega\} \geq min\{\lambda_{\Theta}^{+}(\chi), \lambda_{\Theta}^{+}(\gamma),\ \vartheta\}$

Example(26). The following table Illustrates the set $\aleph = \{0,1,2\}$ with binary operations $*$ and $\circ$

$*$	0	1	2
0	0	1	2
1	0	0	1
2	0	1	0

$\circ$	0	1	2
0	0	0	0
1	0	1	0
2	0	0	2

Then$(\aleph, *, \circ, 0)$ is a KU-semigroup. Define $\Theta = \langle M, L \rangle$ as follows:

$$M(\chi) = \begin{cases} [-0.8, -0.7]\,, [0.6, 0.8] & if\ \ \chi = 0 \\ [-0.6, -0.5]\,, [0.4, 0.6] & if\ \ \chi = 1 \\ [-0.4, -0.3]\,, [0.3, 0.2] & if\ \ \chi = 2 \end{cases}$$

$$L(\chi)\begin{cases}-0.6\,, & 0.8 \qquad if\ \chi=0\\ -0.5\,, & 0.6 \qquad if\ \chi=1\\ -0.3\,, & 0.3 \qquad if\ \chi=2\end{cases}$$

We can show that $\Theta=\langle M,L\rangle$ is a (***CBF***) ideal with thresholds ([0.1, 0.1], [0.3,0.2]) and (0.4, 0.2) of ℵ

Definition(27). A (***CBF***)set $\Theta=\langle M,L\rangle$ is named a (***CBF***)***k***-ideal of KU-semigroup with thresholds $(\alpha,\beta),(\omega,\vartheta)$ if $\forall\ \chi,\gamma,\tau\in\aleph$

(CBK₁) $rmin\{\tilde{\mu}_\Theta^-(0),-\alpha\}\le rmax\{\tilde{\mu}_\Theta^-(\chi),-\beta\}$

$rmax\{\tilde{\mu}_\Theta^+(0),\ \alpha\}\ge rmin\{\tilde{\mu}_\Theta^+(\chi),\ \beta\}$

$min\{\lambda_\Theta^-(0),-\omega\}\le max\{\lambda_\Theta^-(\chi),-\vartheta\}$

$max\{\lambda_\Theta^+(0),\ \omega\}\ge min\{\lambda_\Theta^+(\chi),\ \vartheta\}$

(CBK₂) $rmin\{\tilde{\mu}_\Theta^-(\chi*\tau),-\alpha\}\le rmax\{\tilde{\mu}_\Theta^-\big(\chi*(\gamma*\tau)\big),\tilde{\mu}_\Theta^-(\gamma),-\beta\}$

$rmax\{\tilde{\mu}_\Theta^+(\chi*\tau),\ \alpha\}\ge rmin\{\tilde{\mu}_\Theta^+(\chi*(\gamma*\tau),\tilde{\mu}_\Theta^+(\gamma),\ \beta\}$

$min\{\lambda_\Theta^-(\chi*\tau),-\omega\}\le max\{\lambda_\Theta^-\big(\chi*(\gamma*\tau)\big),\lambda_\Theta^-(\gamma),-\vartheta\}$

$max\{\lambda_\Theta^+(\chi*\tau),\ \omega\}\ge min\{\lambda_\Theta^+\big(\chi*(\gamma*\tau)\big),\lambda_\Theta^+(\gamma),\ \vartheta\}$

(CBK₃) $rmin\{\tilde{\mu}_\Theta^-(\chi\circ\gamma),-\alpha\}\le rmax\{\tilde{\mu}_\Theta^-(\chi),\tilde{\mu}_\Theta^-(\gamma),-\beta\}$

$rmax\{\tilde{\mu}_\Theta^+(\chi\circ\gamma),\ \alpha\}\ge rmin\{\tilde{\mu}_\Theta^+(\chi),\tilde{\mu}_\Theta^+(\gamma),\ \beta\}$

$min\{\lambda_\Theta^-(\chi\circ\gamma),-\omega\}\le max\{\lambda_\Theta^-(\chi),\lambda_\Theta^-(\gamma),-\vartheta\}$

$max\{\lambda_\Theta^+(\chi\circ\gamma),\ \omega\}\ge min\{\lambda_\Theta^+(\chi),\lambda_\Theta^+(\gamma),\ \vartheta\}$

Lemma(28). Every ***(CBF) k***-ideal of ℵ is a ***(CBF) k***-ideal with thresholds $(\alpha,\beta),(\omega,\vartheta)$ of ℵ

Proof: Suppose that $\Theta=\langle M,L\rangle$ is a ***(CBF) k***-ideal of ℵ , then let

$rmax\{\tilde{\mu}_\Theta^+(0),\alpha\}<rmin\{\tilde{\mu}_\Theta^+(\chi),\beta\}$,and $\alpha<\beta$ it follows that $\tilde{\mu}_\Theta^+(0)<\tilde{\mu}_\Theta^+(\chi)$. But that is a contradiction, since Θ is a***(CBF) k***-ideal of ℵ ,

$rmax\{\tilde{\mu}_\Theta^+(0),\alpha\}\ge rmin\{\tilde{\mu}_\Theta^+(\chi),\beta\}$,

also let $min\{\lambda_\Theta^-(0),-\omega\}>max\{\lambda_\Theta^-(\chi),-\vartheta\}$, and $\omega<\vartheta$, it follows that $\lambda_\Theta^-(0)>\lambda_\Theta^-(\chi)$; this is a contradiction since Θ is a***(CBF) k***-ideal of ℵ . this means that $min\{\lambda_\Theta^-(0),-\omega\}\le max\{\lambda_\Theta^-(\chi),-\vartheta\}$, in the same way, we can prove $rmin\{\tilde{\mu}_\Theta^-(0),-\alpha\}\le rmax\{\tilde{\mu}_\Theta^-(\chi),-\beta\}$, and $max\{\lambda_\Theta^+(0),\omega\}\ge min\{\lambda_\Theta^+(\chi),\vartheta\}$

Again , assume that

$rmax\{\tilde{\mu}_\Theta^+(\chi*\tau),\alpha\}<rmin\{\tilde{\mu}_\Theta^+(\chi*(\gamma*\tau),\tilde{\mu}_\Theta^+(\gamma),\beta\}$, and $\alpha<\beta$ it follows that $\tilde{\mu}_\Theta^+(\chi*\tau)<rmin\{\tilde{\mu}_\Theta^+(\chi*(\gamma*\tau),\tilde{\mu}_\Theta^+(\gamma)\}$, which is a contradiction, so

$rmax\{\tilde{\mu}_{\Theta}^{+}(\chi * \tau), \alpha\} \geq rmin\{\tilde{\mu}_{\Theta}^{+}(\chi * (\gamma * \tau), \tilde{\mu}_{\Theta}^{+}(\gamma), \beta\}$,

Also let

$min\{\lambda_{\Theta}^{-}(\chi * \tau), -\omega\} > max\{\lambda_{\Theta}^{-}(\chi * (\gamma * \tau)), \lambda_{\Theta}^{-}(\gamma), -\vartheta\}$, and $\omega < \vartheta$, so

$\lambda_{\Theta}^{-}(\chi * \tau) > max\{\lambda_{\Theta}^{-}(\chi * (\gamma * \tau)), \lambda_{\Theta}^{-}(\gamma)\}$, which is a contradiction. That is $min\{\lambda_{\Theta}^{-}(\chi * \tau), -\omega\} \leq max\{\lambda_{\Theta}^{-}(\chi * (\gamma * \tau)), \lambda_{\Theta}^{-}(\gamma), -\vartheta\}$

In the same way, we get

$rmin\{\tilde{\mu}_{\Theta}^{-}(\chi * \tau), -\alpha\} \leq rmax\{\tilde{\mu}_{\Theta}^{-}(\chi * (\gamma * \tau)), \tilde{\mu}_{\Theta}^{-}(\gamma), -\beta\}$

$max\{\lambda_{\Theta}^{+}(\chi * \tau), \omega\} \geq min\{\lambda_{\Theta}^{+}(\chi * (\gamma * \tau)), \lambda_{\Theta}^{+}(\gamma), \vartheta\}$,and the condition ***(CBK₃)***

Then, $\Theta = \langle M, L\rangle$is a (CBF) ***k***-ideal with thresholds (α, β),(ω, ϑ) of ℵ.

Proposition(29). Let $\Theta = \langle M, L\rangle$ be a cubic bipolar ***k***-ideal with thresholds (α, β), (ω, ϑ) of ℵ if $\chi \leq \gamma$, then

(a) $rmin\{\tilde{\mu}_{\Theta}^{-}(\chi), -\alpha\} \leq rmax\{\tilde{\mu}_{\Theta}^{-}(\gamma), -\beta\}$, $rmax\{\tilde{\mu}_{\Theta}^{+}(\chi), \alpha\} \geq rmin\{\tilde{\mu}_{\Theta}^{+}(\gamma), \beta\}$

(b) $min\{\lambda_{\Theta}^{-}(\chi), -\omega\} \leq max\{\lambda_{\Theta}^{-}(\gamma), -\vartheta\}$, $max\{\lambda_{\Theta}^{+}(\chi), \omega\} \geq min\{\lambda_{\Theta}^{+}(\gamma), \vartheta\}$

Proof: Since $\chi \leq \gamma$,then $\gamma * \chi = 0$,and by ***(ku₃)*** $0 * \chi = \chi$

Since $\Theta = \langle M, L\rangle$ is a (CB) ***k***-ideal with thresholds (α, β), (ω, ϑ) of ℵ ,we get

$$rmin\{\tilde{\mu}_{\Theta}^{-}(\chi), -\alpha\} = rmin\{\tilde{\mu}_{\Theta}^{-}(0 * \chi), -\alpha\} \leq rmax\{\tilde{\mu}_{\Theta}^{-}(0 * (\gamma * \chi)), \tilde{\mu}_{\Theta}^{-}(\gamma), -\beta\}$$
$$= rmax\{\tilde{\mu}_{\Theta}^{-}(0 * 0), \tilde{\mu}_{\Theta}^{-}(\gamma), -\beta\}$$
$$= rmax\{\tilde{\mu}_{\Theta}^{-}(0), \tilde{\mu}_{\Theta}^{-}(\gamma), -\beta\}$$
$$= rmax\{\tilde{\mu}_{\Theta}^{-}(\gamma), -\beta\}$$

$$rmax\{\tilde{\mu}_{\Theta}^{+}(\chi), \alpha\} = rmax\{\tilde{\mu}_{\Theta}^{+}(0 * \chi), \alpha\} \geq rmin\{\tilde{\mu}_{\Theta}^{+}(0 * (\gamma * \chi)), \tilde{\mu}_{\Theta}^{+}(\gamma), \beta\}$$
$$= rmin\{\tilde{\mu}_{\Theta}^{+}(0 * 0), \tilde{\mu}_{\Theta}^{+}(\gamma), \beta\}$$
$$= rmin\{\tilde{\mu}_{\Theta}^{+}(0), \tilde{\mu}_{\Theta}^{+}(\gamma), \beta\}$$

$= rmin\{\tilde{\mu}_{\Theta}^{+}(\gamma), \beta\}$,which is ***(a)*** , And

$$min\{\lambda_{\Theta}^{-}(\chi), -\omega\} = min\{\lambda_{\Theta}^{-}(0 * \chi), -\omega\}$$
$$\leq max\{\lambda_{\Theta}^{-}(0 * (\gamma * \chi)), \lambda_{\Theta}^{-}(\gamma), -\vartheta\}$$
$$= max\{\lambda_{\Theta}^{-}(0 * 0), \lambda_{\Theta}^{-}(\gamma), -\vartheta\}$$
$$= max\{\lambda_{\Theta}^{-}(0), \lambda_{\Theta}^{-}(\gamma), -\vartheta\}$$
$$= max\{\lambda_{\Theta}^{-}(\gamma), -\vartheta\}$$

$$max\{\lambda_{\Theta}^{+}(\chi), \omega\} = max\{\lambda_{\Theta}^{+}(0 * \chi), \omega\} \geq min\{\lambda_{\Theta}^{+}(0 * (\gamma * \chi)), \lambda_{\Theta}^{+}(\gamma), \vartheta\}$$
$$= min\{\lambda_{\Theta}^{+}(0 * 0), \lambda_{\Theta}^{+}(\gamma), \vartheta\}$$
$$= min\{\lambda_{\Theta}^{+}(0), \lambda_{\Theta}^{+}(\gamma), \vartheta\}$$

$= min\{\lambda_{\Theta}^{+}(\gamma),\ \vartheta\}$,which is ***(b)***

Theorem(30). Let $\Theta = \langle M, L\rangle$ be a cubic bipolar fuzzy set of a KUsemigroup$(\aleph, *, \circ, 0)$ then, Θ is a ***(CBF) k***-ideal with thresholds (α, β), (ω, ϑ) of $\aleph$ if and only if it is a ***(CBF)***-ideal with thresholds (α, β), (ω, ϑ) of $\aleph$.

Proof: $\Rightarrow$ Let $\Theta = \langle M, L\rangle$ be a cubic bipolar ***k***-ideal with thresholds (α, β), (ω, ϑ) of $\aleph$,if we put $\chi = 0$ in ***(CBK₂)*** , we get

$rmin\{\tilde{\mu}_{\Theta}^{-}(0 * \tau), -\alpha\} \leq rmax\{\tilde{\mu}_{\Theta}^{-}(0 * (\gamma * \tau)), \tilde{\mu}_{\Theta}^{-}(\gamma), -\beta\}$ is

$rmin\{\tilde{\mu}_{\Theta}^{-}(\tau), -\alpha\} \leq rmax\{\tilde{\mu}_{\Theta}^{-}(\gamma * \tau), \tilde{\mu}_{\Theta}^{-}(\gamma), -\beta\}$

,also $rmax\{\tilde{\mu}_{\Theta}^{+}(0 * \tau), \alpha\} \geq rmin\{\tilde{\mu}_{\Theta}^{+}(0 * (\gamma * \tau), \tilde{\mu}_{\Theta}^{-}(\gamma),\ \beta\}$ is

$rmax\{\tilde{\mu}_{\Theta}^{+}(\tau),\ \alpha\} \geq rmin\{\tilde{\mu}_{\Theta}^{+}(\gamma * \tau), \tilde{\mu}_{\Theta}^{-}(\gamma),\ \beta\}$,and

$min\{\lambda_{\Theta}^{-}(0 * \tau), -\omega\} \leq max\{\lambda_{\Theta}^{-}(0 * (\gamma * \tau)), \lambda_{\Theta}^{-}(\gamma), -\vartheta\}$ is

$min\{\lambda_{\Theta}^{-}(\tau), -\omega\} \leq max\{\lambda_{\Theta}^{-}(\gamma * \tau)), \lambda_{\Theta}^{-}(\gamma), -\vartheta\}$,

Also $max\{\lambda_{\Theta}^{+}(0 * \tau),\ \omega\} \geq min\{\lambda_{\Theta}^{+}(0 * (\gamma * \tau)), \lambda_{\Theta}^{+}(\gamma),\ \vartheta\}$

$max\{\lambda_{\Theta}^{+}(\tau),\ \omega\} \geq min\{\lambda_{\Theta}^{+}(\gamma * \tau), \lambda_{\Theta}^{+}(\gamma),\ \vartheta\}$,the other conditions ***(CBT₁)***, ***(CBT₃)*** are holds from the definition of ***(CBF)k***-ideal; therefore $\Theta = \langle M, L\rangle$is a (CB)-ideal with thresholds (α, β), (ω, ϑ) of $\aleph$

$\Leftarrow$ Let $\Theta = \langle M, L\rangle$ be a cubic bipolar ideal with thresholds (α, β), (ω, ϑ) of $\aleph$,

By ***(CBT₂)*** $rmin\{\tilde{\mu}_{\Theta}^{-}(\chi * \tau), -\alpha\} \leq rmax\{\tilde{\mu}_{\Theta}^{-}(\gamma * (\chi * \tau), \tilde{\mu}_{\Theta}^{-}(\gamma), -\beta\}$, also

$rmax\{\tilde{\mu}_{\Theta}^{+}(\chi * \tau),\ \alpha\} \leq rmin\{\tilde{\mu}_{\Theta}^{+}(\gamma * (\chi * \tau), \tilde{\mu}_{\Theta}^{+}(\gamma),\ \beta\}$,

$min\{\lambda_{\Theta}^{-}(\chi * \tau), -\omega\} \leq max\{\lambda_{\Theta}^{-}(\gamma * (\chi * \tau), \lambda_{\Theta}^{-}(\gamma), -\vartheta\}$, also

$max\{\lambda_{\Theta}^{+}(\chi * \tau), \omega\} \geq min\{\lambda_{\Theta}^{+}(\gamma * (\chi * \tau), \lambda_{\Theta}^{+}(\gamma), \vartheta\}$

Applying theorem **2 (2)** to the previous four steps ,we obtain

$rmin\{\tilde{\mu}_{\Theta}^{-}(\chi * \tau), -\alpha\} \leq rmax\{\tilde{\mu}_{\Theta}^{-}(\chi * (\gamma * \tau), \tilde{\mu}_{\Theta}^{-}(\gamma), -\beta\}$,

$rmax\{\tilde{\mu}_{\Theta}^{+}(\chi * \tau), \alpha\} \leq rmin\{\tilde{\mu}_{\Theta}^{+}(\chi * (\gamma * \tau), \tilde{\mu}_{\Theta}^{+}(\gamma),\ \beta\}$, and

$min\{\lambda_{\Theta}^{-}(\chi * \tau), -\omega\} \leq max\{\lambda_{\Theta}^{-}(\chi * (\gamma * \tau), \lambda_{\Theta}^{-}(\gamma), -\vartheta\}$,

$max\{\lambda_{\Theta}^{+}(\chi * \tau), \omega\} \geq min\{\lambda_{\Theta}^{+}(\chi * (\gamma * \tau), \lambda_{\Theta}^{+}(\gamma),\ \vartheta\}$, which is ***a (CBF) k***-ideal,

The remaining two conditions ***(CBK₁)***,***(CBK₃)*** are holds from the definition of ***(CBF)***-ideal .

4.Conclusion

During this work, we present the definitions of the cubic bipolar sub-KU-semigroup with thresholds (α, β), (ω, ϑ) and cubic bipolar ***k***-ideal with thresholds (α, β), (ω, ϑ) of $\aleph$. The relationship among these types of ideals and some properties are studied, We obtained the following result: every (CBF) sub-KU-semi group of $\aleph$ is a (CBF) sub-KU-semi group with thresholds (α, β), (ω, ϑ) of $\aleph$,but the converse is not true. Finally, we proved that a cubic bipolar fuzzy k-ideal with thresholds (α, β), (ω, ϑ) and a cubic bipolar fuzzy ideal with thresholds (α, β), (ω, ϑ) of a KU-semi group are equivalents.

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