Sector Steel Plate Analysis Using Finite Difference Method

تحليل الصفائح المقوسة بإستخدام طريقة الفروقات المحددة

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ABSTRACT :

An investigation of validity of the Finite Difference method to simulate thin sectorial steel plate under the effect of uniform and concentrated loads with different types of boundary conditions was studied in this paper. The differential equation schemes of curved plate were solved by means of the Finite Difference method. Also bending moments Mr , Mϕ and Mrϕ schemes were derived. The effects of curvature, aspect ratio, load type and boundary conditions of the plate were studied. Avery good agreement between the results of deflections and moments obtained by the derived Finite Difference schemes and the result from the Finite Element method of (ABAQUS) computer program for all types of geometry, loading and support conditions.
Keywords: Curved Plate, Sector Plate, Thin Plate, Finite Deference Analysis Method (Numerical Analysis).

الخلاصة:

موضوع هذا البحث هو دراسة الصفائح الحديدية التي تكون بشكل قطاع دائري . يتناول البحث دراسة نظرية للصفائح النحيفة بإستخدام طريقة (الفروقات المحددة) ، حيث تم التحقق من مدى صحة دراسة هذا النوع من المشاكل الهندسية بطريقة (الفروقات المحددة). الأحمال المسلطة التي تم تناولها في البحث هي الحمل المنتشر و الحمل المركز ، مع الأخذ بنظر الإعتبار طبيعة أو نوع الإسناد.

حيث تم تناول أنماط مختلفة من إسناد أطراف الصفيحة كالإسناد البسيط والإسناد المحكم وأنماط مختلفة تجمع النوعين . وتم دراسة مدى تأثير انحناء الصفيحة وأبعاد الصفيحة على سلوك الصفائح .

المعادلة التفاضلية للصفائح الدائرية تم تمثيلها حسب طريقة الفروقات المحددة وتم إشتقاق الأنماط أو القوالب العددية التي يتم من خلالها إحتساب الهطول ، عزم الإنحناء القطري ، عزم الإنحناء الزاوي ، وعزم الالتفاف .

تم مقارنة النتائج المستحصلة من الدراسة الحالية بطريقة الفروقات المحددة مع النتائج المستحصلة بطريقة العناصر المحددة بإستخدام برنامج (ABAQUS) وقد وجد ان هناك توافق جيد بين النتائج وبذلك تكون الطريقة المقترحة في البحث الحالي صحيحة وممكن إستخدامها في تحليل الصفائح الدائرية .

1. Introduction :

Thin plate is a structural element of which its thickness is smaller than the other dimensions. The increase of using of rolled, machined, or thin-milled skins in aircraft and missile designs, makes the need of analyses of thin curved sectorial plates becomes more exigent. Analysis and design data for such plates are so limited.

The differential equation of plates were solved by several proposed approaches. Finite Element and Finite Difference methods are the well-known approaches to be usually used to find the solution of the differential equation of the current research. The advantage of the Finite Element method is suitibility of modeling the complex geometries of practical engineering problems. However, the main disadvantage of this method is the computational complexity involved in its constitutes, especially in the application of real-time.

In other view, the method is fast enough to analyze, relatively easy to program, and also it is more convenient for uniform members like plate system. The main big impediment of this method is the difficulty towards the problems with complex and irregular geometry. Moreover, since there is a difficulty in the Finite Difference method to vary the size of the difference in specific regions, it is not convient for cases of rapidly changing variables like cases of stress concentration. However, due to the uniformity of the geometry of the thin plates, Finite Difference method bocomes to be more applicable and quicker to calculate deformations, forces, stresses and strains.

In Finite Difference method, the plate region is divided into a mesh of divisions. Then, these divisions are written as terms of the derivatives of the governing partial deferential equations. Therefore, the Finite Difference is calculated to each point in grid division and thus the displacement at each node is related to displacements at the other nodes in the grid divisions connected directly to it. The boundary conditions of the problem then considered to obtain a unique solution for the overall system.

In 2014, ***Ghods and Mir*** [1], present a study to evaluate the modeling performance of rectangular thin plate by using Finite Difference method. The differential equations were discretized to apply Finite Difference analysis method to find functions of plates in-plane stress and derived sets of linear algebraic simultaneous equations. The authors found that the selection of the Finite Difference method is prefered to model thin plates problems. The authors concluded that the increasing of repeation steps number in Finite Difference method does not produce in increased attention to the problem and may caused the solution to be diverge.

Sucharda and ***Kubosek*** (2014) [2], presented a study to analyze slabs using Finite Difference method. Two types of square slabs were analyzed. The authors developed algorithms to calculate various systems of equations. They concerned deflections, specific bending moments and torsion moments for the analysis. The study aimed also to evaluate time needed for each method to solve a set of linear equations with algorithms developed in Matlab. A comparison for results of this method was made with those obtained by the Finite Element method. The authors concluded that the thin slabs can be analyzed in an algorithm created in the program Matlab which is based on the Finite Difference method.

In 2012, ***Abodi*** [3] investigated linear buckling case of of thin tapered steel plates under compression loading in-plane patch. The study concerned a case of in-plane patch loading subjected to steel plate of linearly tapered thickness. The researcher used Finite Difference method to treat stability problem and to study free vibration and buckling with various thickness variations of thin rectangular plates. The author compared results with those from previous works. The author concluded that the increasing of tapering ratio (keeping same volume of the plate) caused decreasing in values of buckling coefficients and so with increasing ratio of patch length.

Mosheer [4] present in 2011 a research to study the effect of plate thickness on the structural efficiency and whether it can be improved. The study used Finite Difference method to analyze steel plate with large displacement and variable thickness at x direction and to study the behavior of large deflection rectangular plates under the effects of tapering ratio, boundary condition, equation type of tapering and aspect ratio of the plate. The study concluded that thickness variation (tapering ratio) have a very sensitive effect on the large deflection behavior.

There are several other researches carried out earlier than those which are reviewed above, but, those which concerned the application of the Finite Difference on sectorial plates may be not carried out. However, one can say that the Finite Difference method is one of the good approaches to analyze the case of thin plates.

2. Governing Equations and Finite Difference Formulation

The governing equation of motion (deflection, ω) in polar coordinate for an isotropic plate of uniform thickness within plane load is:

$$\nabla^4 \omega = \frac{q}{D} \qquad (1)$$

Where (q) is the applied load on the plate, (D) is the flexural rigidity, and $\nabla^4 = \nabla^2 \nabla^2$ in which:

$$\nabla^2 = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2}{\partial \emptyset^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) \qquad (2)$$

$$D = \frac{E\, h^3}{12(1-\upsilon^2)} \qquad (3)$$

Where (E) is the modulus of elasticity, (h) is thickness of plate, and (υ) poisons ratio.

$$\nabla^4 \omega = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \emptyset^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \emptyset^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\omega = \frac{q}{D} \qquad (4)$$

The internal bending moments and twisting moment per unit length are expressed by the following formulas:

$$M_r = -D\left[\frac{\partial^2 \omega}{\partial r^2} + \upsilon\left(\frac{1}{r^2}\frac{\partial^2 \omega}{\partial \emptyset^2} + \frac{1}{r}\frac{\partial \omega}{\partial r}\right)\right] \quad (5)$$

$$M_\phi = -D\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \omega}{\partial \emptyset^2} + \upsilon\frac{\partial^2 \omega}{\partial r^2}\right) \qquad (6)$$

$$M_{r\phi} = -D(1-\upsilon)\left(\frac{1}{r}\frac{\partial^2 \omega}{\partial r \partial \emptyset} + \frac{1}{r^2}\frac{\partial \omega}{\partial \emptyset}\right) \quad (7)$$

Thus, the differential equations of normal, tangential and twisting stresses will be as followings:

$$\sigma_r = -z\left[\frac{\partial^2 \omega}{\partial r^2} + \upsilon\left(\frac{1}{r^2}\frac{\partial^2 \omega}{\partial \emptyset^2} + \frac{1}{r}\frac{\partial \omega}{\partial r}\right)\right] \quad (8)$$

$$\sigma_\phi = -z\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \omega}{\partial \emptyset^2} + \upsilon\frac{\partial^2 \omega}{\partial r^2}\right) \quad (9)$$

$$\sigma_{r\phi} = -z(1-\upsilon)\left(\frac{1}{r}\frac{\partial^2 \omega}{\partial r \partial \emptyset} + \frac{1}{r^2}\frac{\partial \omega}{\partial \emptyset}\right) \quad (10)$$

where:
z = distance from neutral axis to the extreme fiber (top or bottom) of the plate
Also, the situation of the boundary condition of the circular plate should be characterized to find the solution. Thus the boundary conditions of a circular plate may be defined as:

1. Fixed edge (clamped edge):
   $\omega = \frac{\partial w}{\partial r} = 0$
2. Simply supported edge:
   $\omega = M_r = 0$

To formulate the Finite Difference formula, consider a sector of circular plate with inner radius (Rin), outer radius (Rout) and angle (ϕ) as shown in Fig. (1) the circular plate divided by a polar mesh. The interval of mesh in the direction of the radius is (β), while (φi) is the angle of each interval of mesh in tangential direction.

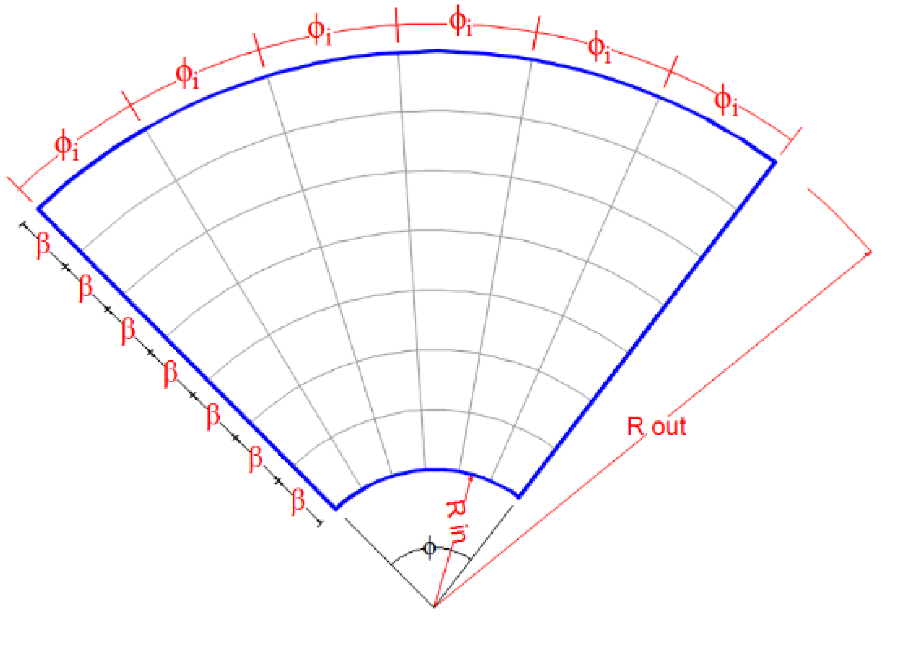


Figure (1) Plate mesh with division's symbol

In Fig.(2) the interval between the nodes in the perpendicular direction on the radius is denoted as (γ_{i-1} , γ_i , γ_{i+1}).

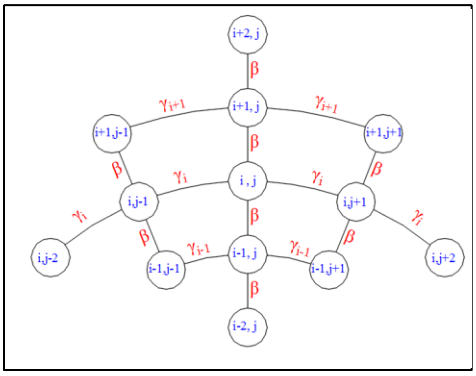
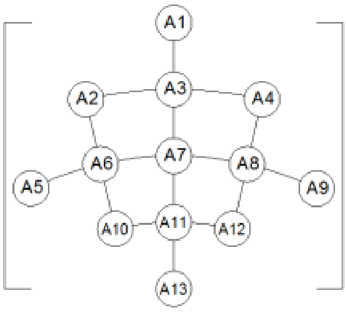


Figure (2) Finite difference intervals

According to that, the mentioned governing differential equations of a circular plate are formulated as follow:

w



$= q/D$ (11)

where:

$$A1 = \frac{1}{\beta^4} + \frac{1}{r\beta^3} + \frac{1}{4r^2\beta^2} \quad (12)$$

$$A2 = \frac{2}{r^2\varphi^2\beta^2} + \frac{1}{r^3\varphi^2\beta} \quad (13)$$

$$A3 = \frac{-4}{\beta^4} - \frac{2}{r\beta^3} - \frac{4}{r^2\varphi^2\beta^2} - \frac{2}{r^3\varphi^2\beta} \quad (14)$$

$$A4 = A2 \quad (15)$$

$$A5 = \frac{1}{r^4\varphi^4} \quad (16)$$

$$A6 = \frac{-4}{r^2\varphi^2\beta^2} - \frac{4}{r^4\varphi^4} \quad (17)$$

$$A7 = \frac{6}{\beta^4} + \frac{8}{r^2\varphi^2\beta^2} + \frac{6}{r^2\varphi^2} \quad (18)$$

$$A8 = A6 \quad (19)$$

$$A9 = A5 \quad (20)$$

$$A10 = \frac{2}{r^2\varphi^2\beta^2} - \frac{1}{r^3\varphi^2\beta} \quad (21)$$

$$A11 = \frac{-4}{\beta^4} + \frac{2}{r\beta^3} - \frac{4}{r^2\varphi^2\beta^2} + \frac{2}{r^3\varphi^2\beta} \quad (22)$$

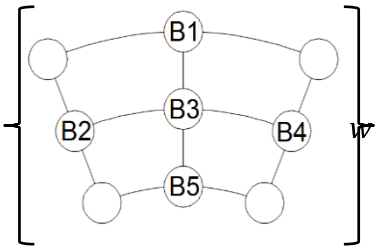
$$A12 = A10 \quad (23)$$

$$A13 = \frac{1}{\beta^4} - \frac{1}{r\beta^3} - \frac{1}{4r^2\beta^2} \quad (24)$$

Moments

Also, moments (Mr, Mϕ and Mrϕ) Finite Difference schemes were derived from their differential equations as follows:

$M_r =$



w (25)

where:

$$B1 = -D\left(\frac{1}{\beta^2} + \frac{\upsilon}{2r\beta}\right) \quad (26)$$

$$B2 = -D\left(\frac{\upsilon}{r^2\varphi^2}\right) \quad (27)$$

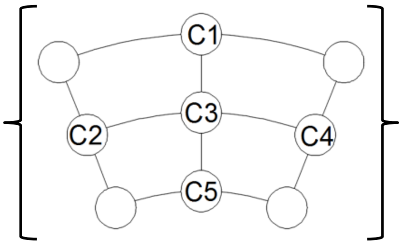
$$B3 = -D\left(\frac{-2}{\beta^2} + \frac{-2\upsilon}{r^2\varphi^2}\right) \quad (28)$$

$$B4 = -D\left(\frac{\upsilon}{r^2\varphi^2}\right) \quad (29)$$

$$B5 = -D\left(\frac{1}{\beta^2} + \frac{-\upsilon}{2r\beta}\right) \quad (30)$$

and

$M_{\phi} =$



w (31)

where:

$$C1 = -D\left(\frac{v}{\beta^2} + \frac{1}{2r\beta}\right) \quad (32)$$

$$C2 = -D\left(\frac{1}{r^2\varphi^2}\right) \quad (33)$$

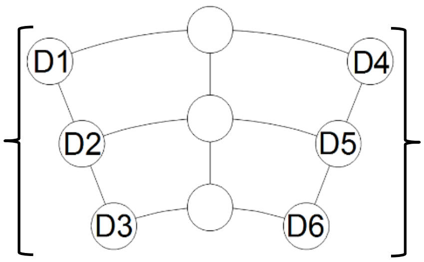
$$C3 = -D\left(\frac{-2v}{\beta^2} + \frac{-2}{r^2\varphi^2}\right) \quad (34)$$

$$C4 = -D\left(\frac{1}{r^2\varphi^2}\right) \quad (35)$$

$$C5 = -D\left(\frac{v}{\beta^2} + \frac{-1}{2r\beta}\right) \quad (36)$$

and

$M_{r\phi} =$



w (37)

where:

$$D1 = -D(1-v)\left(\frac{1}{4r\varphi\beta}\right) \quad (38)$$

$$D2 = -D(1-v)\left(\frac{-1}{2r^2\varphi}\right) \quad (39)$$

$$D3 = -D(1-v)\left(\frac{-1}{4r\varphi\beta}\right) \quad (40)$$

$$D4 = -D(1-v)\left(\frac{1}{4r\varphi\beta}\right) \quad (41)$$

$$D5 = -D(1-v)\left(\frac{1}{2r^2\varphi}\right) \quad (42)$$

$$D6 = -D(1-v)\left(\frac{1}{4r\varphi\beta}\right) \quad (43)$$

3. Numerical Results

The derived numerical schemes for deflection (w) and moments (Mr, Mϕ and Mrϕ) were programmed in order to analyze different cases for sectorial circular thin plate. The results were recorded for all the nodes of the generated mesh which generated as described before. Results of the

numerical analysis for the present investigation compared with those obtained from (ABAQUS) Finite Element analysis program. The 4-node doubly curved thin S4R shell was used to represent the plate in (ABAQUS) program. Also, S4R element is a robust, general-purpose element that is suitable for a wide range of applications. The element has several hourglass modes that may propagate over the mesh.

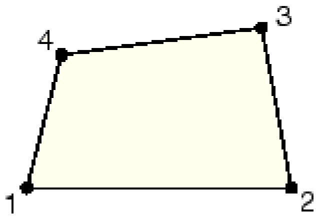


Figure (3) 4-Node S4R-element used in Abaqus Program

The present study was performed for a steel plate with modulus of elasticity (E = 200 GPa.), poisons ratio (υ = 0.3), thickness (h=10 mm) and a uniform distributed load of = 100 kN/m^2 or concentrated load of 100 kN.

Several parameters were concerned in this study in order to:

1. Verify the validity of the adopted approach of analysis.
2. Investigate the influence of each parameter on the adopted analysis approach.
3. Study the efficiency and adequacy of the adopted analysis approach through investigation of the state of internal stresses and ending moments.

The parameters concerned are:

(1) Aspect ratio (average width to height ratio)
(2) Supports type (clamped and pinned)
(3) Mesh size
(4) Loading type (uniform and concentrated loads)
(5) Internal moments and stresses.

3.1 Aspect Ratio

The ratio between the average width of the plate (b) to its height (a) defines the geometry of each case, Fig. (4). For all cases of the study, the inner radius is taken to be (2.00 m) and the outer radius is (3.00 m). Seven cases were studied in this case by changing the length of the middle arc (b) in order to change the aspect ratio.

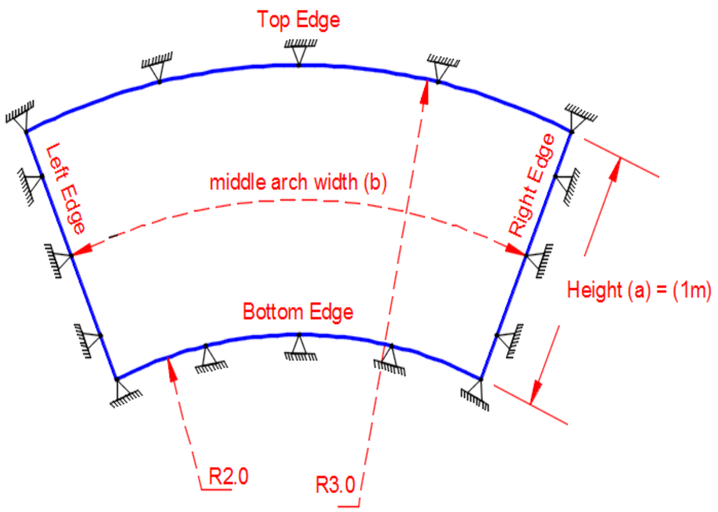


Figure (4) Plate geometry and dimensions for case 1

The concerned values of the middle arc width were (0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00) m. All edges in this group were simply supported and the plate is under a uniform distributed load of intensity (100 kN/m^2). Table (1) shows the analysis results this case for both of the current Finite Difference and (ABAQUS) Finite Element methods.

Table (1) Effect of Aspect Ratio

Middle arc width(m)	Length (m)	W_{max} (mm)		Difference ratio %
		F.D.M	F.E.M.	
0.50	1	3.48	3.60	3.45
0.75	1	11.15	11.44	2.60
1.00	1	22.08	21.86	1.00
1.25	1	32.51	32.57	0.18
1.50	1	42.88	41.68	2.80
1.75	1	49.63	49.08	1.11
2.00	1	55.20	54.58	1.12

The presented results in Table (1) shows that when the middle arc width equals to the height of the sectorial plate the difference between the two approaches of analysis. Generally when the middle arc width is greater than the height the results of the two approaches indicate smaller difference than the cases when the middle arc width is smaller than the height.

3.2 Supports Type

The four edges of the circular sectorial plate were taken in different manner to study the effect of supporting type. The aspect ratio for this case was concerned to be equal to 1 (with middle width = 1.00m), Fig.(4), and the load intensity was (100 kN/m^2). Five cases were studied in this case depending on boundary condition type. Table (2) shows the analysis results of this case for both of the current Finite Difference and (ABAQUS) Finite Element methods.

Table (2) Effect of Supporting Type

Case No.	Fixed Edges	Simply Supported Edges	W_{max} (mm)		Difference ratio %
			F.D.M	F.E.M.	
1	None	all	22.08	21.85	1.05
2	All	none	7.06	6.822	3.49
3	B & T *	R & L *	10.64	10.38	2.50
4	R & L	B & T	10.72	10.41	2.98
5	T & L	B & R	11.34	11.13	1.89

* T=Top, B=Bottom, L=Left, R=Right,

The results show that there is a clear effect of supports type on the behavior of the model. The model indicates maximum deflection when the edges were simply supported and minimum deflection when the edges were fixed. In addition to that the difference between the results of (F.D.M) and (F.E.M) indicate minimum value when the model was simply supported and maximum value when the supporting was fixed.

3.3 Mesh Size

The mesh was selected to produce five cases with different number of divisions (i.e. different mesh size). The aspect ratio for this case was concerned to be equal to1 and all edges in this group were simply supported under a load intensity of (100 kN/m^2).

Table (3) shows the analysis results of this case, where the maximum deflection was recorded for each. The recorded deflection for each case was compared with that obtained for the mesh (3) as a percentage ratio. The control case (Mesh No.3) was selected since there is a regularity in it's dimensions (middle arc = height).

Table (3) Effect of Mesh Size (Divisions Numbers)

Mesh No.	Divisions	W_{max} (mm)	Difference (%)
1	6 × 6	22.04	0.18
2	7 × 7	21.45	2.85
3	10 × 10	22.08	0
4	12 × 12	22.08	0
5	15 × 15	22.05	0.13

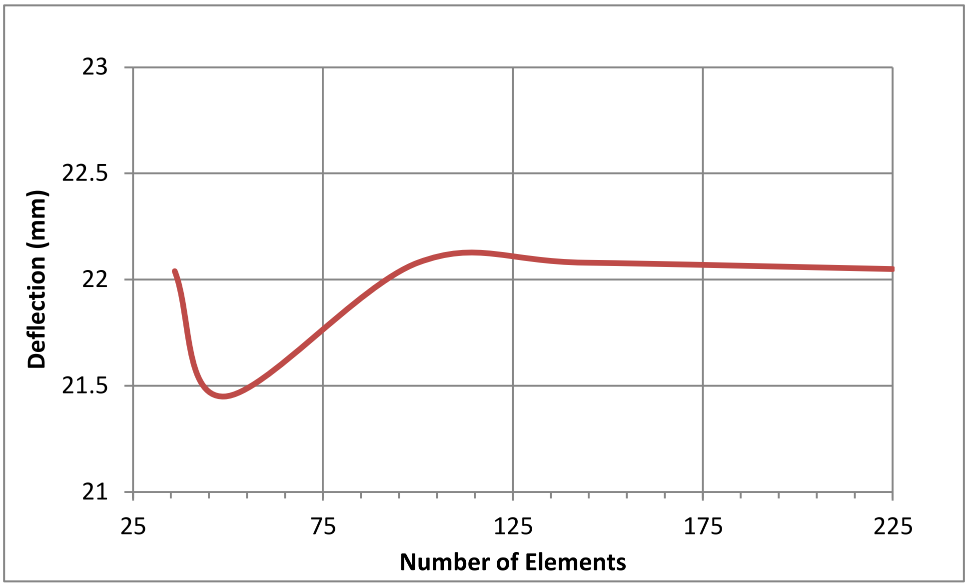


Figure (5) The Effect of the Number of Elements on the Deflection

Fig.(5) shows the relation between the maximum deflection and the number of divisions of the plate. The presented results in Table(3) and Fig.(5) show that the results of deflection was converged when the number of divisions between (100-225).

3.4 Loading Type

The load was changed from distributed load of intensity (100kN/m^2) to concentrated load of (100 kN) applied at central node of the plate. The results were compared with those obtained from the Finite Element analysis program (ABAQUS) in order to exam the efficiency of the derived Finite Difference method on the behavior of the circular sectorial thin plate with different load types. Results of this case is shown in Table (4).

Table (4) Effect of Loading Type

Case No.	Load type	Support type				W_{max} (mm)		Difference (%)
		Bottom	Right	Top	Left	F.D.M	F.E.M.	
1	Point Load	S. S. *	S. S.	S. S.	S. S.	66.05	63.48	4.05
2	Point Load	C **	C	C	C	33.89	30.54	10.97
3	U. D. L.	S. S.	S. S.	S. S.	S. S.	22.08	22.16	0.36
4	U. D. L.	C	C	C	C	7.43	6.89	7.84
5	U. D. L.	C	S. S.	C	S. S.	11.10	10.42	6.53
6	U. D. L.	S. S.	C	S. S.	C	11.06	10.35	6.86

*S.S.=simply supported, C** = Clamped

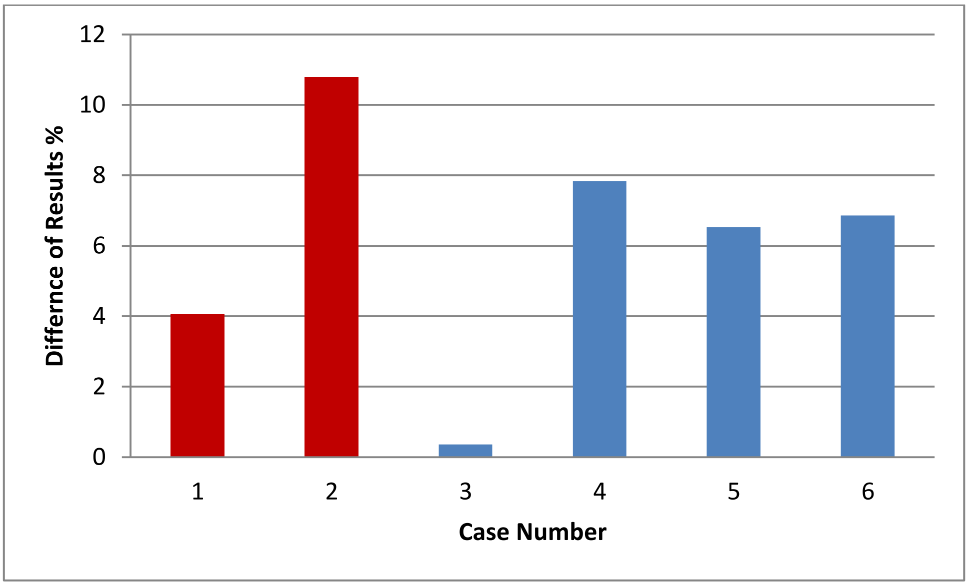


Figure (6) Difference between FDM and FEM for different types of loading

The results of the load parameter are presented and compared in Table(4) and Fig.(6). It is noticed that tha maximum percentage of the difference in the results is about 10%, also, the cases of the samples of simply supported edges generally indicate lower percentage than that of fixed supported edges.

3.5 Effect of Number of Element on Internal Moments and Stresses

Several cases were analyzed by using both methods (current Finite Difference and ABAQUS Finite Element program) to study the effect of number of elements on the results of internal bending moments and stresses. Tables (5.a) to (5.f) show the analysis results of internal bending moments and stresses for both methods. The analyses were carried out for a plate of inner radius 2.0 m and outer radius 3.0 m (height = 1m), average width = 1m and thickness = 10 mm, Fig(4).

Table (5.a) Maximum Internal Bending Moments Mr

Supporting				Mr (kN.m)				Difference (%)	
Bot.	Right	Top	Left	F.D.M		F.E.M.			
				Mid.	Edge	Mid.	Edge	Mid.	Edge
S. S.	S. S.	S. S.	S. S.	4.439	0	4.733	0.00	6.21	0.00
C	C	C	C	2.177	4.476	2.283	4.483	4.64	0.16
S. S.	C	S. S.	C	3.067	6.142	3.317	6.60	7.54	6.94
C	S. S.	C	S. S.	2.413	1.960	2.700	2.033	10.63	3.59

The results presented in Table (5.a) show that there is a difference in the values of the maximum internal bending moment (Mr) between the two approaches (F.D. & F.E.) is about (0-4)% for edges and about (6-10) for mid region.

Table (5.b) Maximum Internal Bending Moments Mϕ

Supporting				Mϕ (kN.m)				Difference (%)	
Bot.	Right	Top	Left	F.D.M		F.E.M.			
				Mid.	Edge	Mid.	Edge	Mid.	Edge
S. S.	S. S.	S. S.	S. S.	4.908	0	4.667	0	5.16	0.00
C	C	C	C	2.295	4.945	2.267	4.383	1.24	12.82
S. S.	C	S. S.	C	2.867	1.842	2.433	2.000	17.84	7.90
C	S. S.	C	S. S.	3.315	6.535	3.267	6.067	1.47	7.71

In Table (5.b), the percentage of difference of the obtained results by the two approaches is about (0-8)% for the edges and about (1-18)% for mid region.

Table (5.c) Maximum Internal Bending Moments Mrϕ

Supporting				Mrϕ (kN.m)		Difference (%)
Bot.	Right	Top	Left	F.D.M	F.E.M.	
				Max.	Max.	
S. S.	S. S.	S. S.	S. S.	3.227	3.100	4.10
C	C	C	C	0.747	0.783	4.60
S. S.	C	S. S.	C	1.352	1.250	8.16
C	S. S.	C	S. S.	1.561	1.417	10.16

The percentage difference of the results of the maximum internal bending moment (Mrϕ) as shown in Table (5.c) is about (4-10)%

Table (5.d) Maximum Internal Stress σr

Supporting				σr (kN/m^2)				Difference (%)	
Bot.	Right	Top	Left	F.D.M		F.E.M.			
				Mid.	Edge	Mid.	Edge	Mid.	Edge
S. S.	S. S.	S. S.	S. S.	296	0	284	0	4.23	0.00
C	C	C	C	138	279	137	269	0.73	3.72
S. S.	C	S. S.	C	198	417	199	396	0.50	5.30
C	S. S.	C	S. S.	174	109	162	122	7.41	10.66

Table (5.e) Maximum Internal Tangential Stress σϕ

Supporting				σϕ (kN/m^2)				Difference (%)	
Bot.	Right	Top	Left	F.D.M		F.E.M.			
				Mid.	Edge	Mid.	Edge	Mid.	Edge
S. S.	S. S.	S. S.	S. S.	266	0	280	0	5.00	0.00
C	C	C	C	130	267	136	263	4.41	1.52
S. S.	C	S. S.	C	144	125	146	120	1.37	4.17
C	S. S.	C	S. S.	184	365	196	364	6.12	0.27

Table(5.d) shows that the percentage difference between the results of the two approaches is about (4-7)% for mid region. The edges indicate greater percentage which is about (0-11)%. While the percentage difference in Table (5.e) for maximum internal tangential stress (σϕ) for edges is smaller than mid region. The percentage in mid region is about (1-6)% and in edges is about (0-4)%.

Table (5.f) Maximum Internal Twisting Stress σrϕ

Supporting				σrϕ (kN/m^2)		Difference (%)
Bot.	Right	Top	Left	F.D.M	F.E.M.	
				Max.	Max.	
S. S.	S. S.	S. S.	S. S.	186	186	0.00
C	C	C	C	47	47	0.00
S. S.	C	S. S.	C	78	75	4.00
C	S. S.	C	S. S.	75	85	11.76

Table (5.f) shows the results of the maximum internal twisting stress (σrϕ) and the percentage difference between the two approaches which is about (0-12)%.

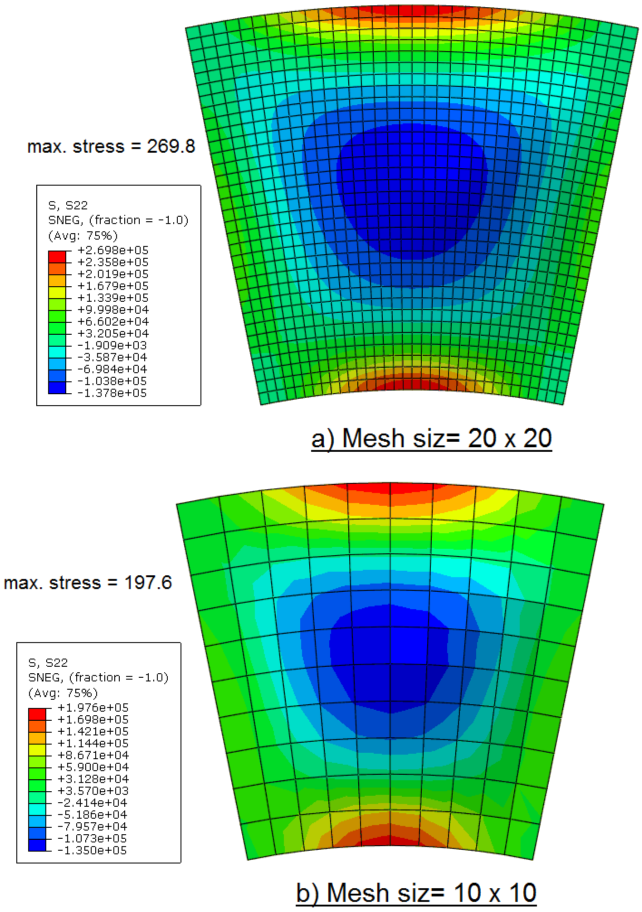


Figure (7) Maximum Negative Stress due to Mϕ for Different Mesh Size

The Finite Element analyses were carried out to a sectorial circular thin plate by using (ABAQUS) with mesh size (Nr×Nϕ) varied from (10×10) to (20×20). These F.E. results show that there is a considerable variation in results values for the maximum negative stress reaches about (26%) when the mesh size changed from (10×10) to (20×20). It could be noticed from Fig.(7) that maximum negative stress for (10×10) mesh is 197.6 kN/m^2, and for (20×20) mesh is 269.8 kN/m^2, while the variation of stress for the same plate was about (1.8%) when same change of mesh was applied by the adopted Finite Difference method.

4. Conclusions

The behavior of thin sectorial circular plate was investigated in this paper. A Finite Difference approach was presented to analyze the plate. The validity of the adopted approach was compared with Finite Element approach using software package (ABAQUS). The investigation was carried out to simulate thin sectorial plate under the effect of uniform and concentrated loads with different types of boundary conditions, aspect ratios and mesh size. The following points were concluded from the present study:

1. The results obtained from the present study shows the validity of the derived Finite Difference schemes with a very small difference and for any suitable mesh size, loading type and boundary conditions if compared with Finite Element analysis program package results.
2. However, in F.E.M the number of elements (mesh size) should be chosen after carrying out a convergence study to avoid the differences in stress calculations which could be reached up to 26%, while no need for mesh optimization in the adopted F.D.M, because the results showed that the tolerance in stress values was very small (about 1.8%).
3. The two approaches of the analyses (F.E.M) and (F.D.M) give approximately same results if the middle arc width equals to the height of the sectorial plate.
4. Generally the number of the mesh size (number of division) in the F.E. scheme has a little effect on the results of the analysis.
5. The supporting condition and the type of loading have a clear effect on the results and the percentage defference of the results between F.E.M and F.D.M. The cases of uniformly distributed load indicate smaller percentage of difference between the two approaches. The cases of simply supporting cases also indicate the same response. It is concluded that if the sectorial plate is simply supported and under uniformly distributed load, then the two approaches gives very close results.

List of symbols

ω = deflection
$\emptyset$ = angle of rotation of the plate
υ= Poisson ratio
σ_r= normal stresses
σ_ϕ= tangential stresses
$\sigma_{r\phi}$= twisting stresses
D = flexural rigidity
F.D.M = Finite Difference method
F.E.M = finite element method
h= plate thickness
M_r= bending moment about r direction
M_ϕ= bending moment about $\emptyset$ direction
$M_{r\phi}$= twisting moment
Nr = number of division in r-direction
Nϕ = number of division in ϕ-direction
q = applied load
r = radius of plate
z = distance from neutral axis

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