



Available online at: www.basra-science journal.org



ISSN –1817 –2695

Approximate Solution of Schrödinger Equation With Manning –Rosen Potential in Two Dimensions by using the shifted $1/N$ expansion method

R. H. Hammed

Physics Department , Science College , Basrah University , Basrah ,Iraq.

E-mail : Rashrash@yahoo.com

Received 9-10-2011, Accepted 21-12-2011

Abstract

We have used the shifted $1/N$ method to solve the Schrödinger equation with Manning – Rosen potential in two dimensions . This method is primarily tested on the 3D case and then detailed calculations carried out on the 2D case .The energy levels for both cases are listed as a function of the potential strengths α and A and the screening parameter b . A comparison with literature reveals that the $1/N$ method is reliable and powerful .

Key words : screening , expansion methods , centrifugal term .

1-Introduction

In non - relativistic quantum mechanics maximum information of a quantum system can be obtained when one knows the exact solution of the corresponding Schrödinger equation .However ,the exact analytical solution of the Schrödinger equation is only possible with the angular momentum $l = 0$ for some potential models like the nonharmonic oscillator [1], kratzer [1-3], pöschl-Teller [4,5], sacarf [6], Woods-Sax [7,8,9], Hulthen [10,11] , the Manning – Rosen [12,13,14] and Rosen-Morse potentials [15,16],etc. But when $l \neq 0$ the Schrödinger equation can only be solved approximately for different suitable approximation schemes such as the Factorization method [17], the super-symmetric quantum mechanics [18], Nikiforov-Uvarov method [19], the shifted $1/N$ expansion method [20],etc.

In the present paper, we attempt to solve approximately the Schrödinger equation in two dimensions with Manning –Rosen potential including the centrifugal term by using the shifted $1/N$ expansion .This approximation has been successfully used in connection with spherically symmetric potentials [21]. The method starts by that N is sufficiently large . In this way , a new effective potential can be defined and the kinetic energy term becomes negligible, resulting in a semiclassical approach[20]. For spherically symmetric potential one usually uses $x = \bar{k}^{1/2} (r - r_0)/r_0$ as a new variable instead of r , where $\bar{k} = N + 2l - a$, \bar{k} is the expansion parameter , l is the orbital quantum number and a is a suitable shift parameter [21] .If one expands the resulting equation of x ,an analytical structure similar to the one of the one –dimensional

unharmonic oscillator is found. Once the problem is collapsed to its actual dimension ($N=1,2,3,\dots$), it simply remains to relate the coefficients of both equations in order to get the energy corrections .

This paper is arranged as follows : In section 2, we outline the main steps the shifted $1/N$ approximations for the Manning

–Rosen potential . In section 3, we compare our numerical results for $N=3$ with the available accurate numerical results [24] , and our results for $N=2$ Which to our knowledge did not appear before in the literature , are presented thereafter. concluding remarks are given in section 4.

2- shifted $1/N$ expansion

The Manning –Rosen potential is given by [22]

$$V(r) = \frac{1}{Kb^2} \left[\frac{\alpha(\alpha-1)e^{-2r/b}}{(1-e^{-r/b})^2} - \frac{Ae^{-r/b}}{(1-e^{-r/b})} \right] \quad (1)$$

$$K = 2m/\hbar^2$$

$$\left[-\left\{ \frac{d^2}{dr^2} + \frac{N-1}{r} \frac{d}{dr} - \frac{l(l+N-2)}{r^2} \right\} + V(r) \right] \psi(r) = \psi(r) \quad (2)$$

where

$$r^2 = \sum_{i=1}^N X_i^2, \quad N = 1,2,3 \dots \dots \quad (3)$$

For $N = 2$,

$X_1 = r \cos \phi$, $X_2 = r \sin \phi$ and $l = |m|$, and

$N = 3$, $X_1 = r \sin \theta \cos \phi$, $X_2 =$

A and α are two dimensionless parameters, but the parameter b has the dimension of length .The radial Schrödinger equation in N dimensions is given by

$r \sin \theta \sin \phi$, $X_3 = r \cos \theta$ and $m = -l, \dots, 0, \dots, l$. The first derivative in equation (2) is removed by replacing ψ by ϕ from :

$$\psi(r) = \phi(r) \exp\left(-\frac{1}{2} \int \frac{N-1}{r} dr\right) = \phi(r) r^{\frac{(1-N)}{2}} \quad (4)$$

Inserting equation (4) in equation (2), results in the following equation for ϕ :

$$\frac{d^2 \phi}{dr^2} + \left(E - \frac{4l(l+N-2) + (N-1)(N-3)}{4r^2} - V(r) \right) \phi(r) = 0 \quad (5)$$

Letting $\bar{k} = N + 2l - a$, where a is a suitable shifted parameter which has the

meaning of an additional degree of freedom , we get ([20])

$$\left[-\frac{d^2}{dr^2} + (\bar{k} + a - 1)(\bar{k} + a - 3)/4r^2 + V(r) \right] \phi(r) = E\phi(r). \quad (6)$$

Following the former procedure of the shifted $1/N$ expansion [20],we present here only the analytical expressions , which are required to obtain bound state energies $E_{n,l}$.Therefore ,

the relevant formulae up to the third –order correction in physical $N = 2,3$ spaces are obtained as follows:

$$E = E_0 + \frac{1}{r_0^2} [(1 - a) (3 - a)/4 + \alpha_1] + \frac{\alpha_2}{\bar{k} r_0^2} \quad (7)$$

Where

$$E_0 = V(r_0) + \bar{k}^2/4r_0^2 \quad (8)$$

and

$$a = 2 - 2(2n_r + 1)\omega \quad (9)$$

With

$$\omega = \sqrt{3 + r_0 V''(r_0)/V'(r_0)} \quad (10)$$

The parameter r_0 can be determined through the relation

$$\sqrt{2r_0^3 V'(r_0)} = N + 2l - a = Q^{\frac{1}{2}} \quad (11)$$

Where n_r is the radial quantum number and α_1 and α_2 are given in [23] and V' and V'' are the first and second derivatives of the potential with respect to r , respectively. The key quantity on which the shifted $1/N$ procedure is depending on is r_0 . To calculate r_0 from equation (11), ω should be first

evaluated by using equation (10) followed by the shifted parameter a from equation (9). This would require the calculation of the first and second derivatives of potential at $r = r_0$. Performing the last step and using equation (10), we get :

$$\omega = \left(\frac{r_0}{b} \times \frac{-A+4\alpha(\alpha-1)e^{-r_0/b}+[2\alpha(\alpha-1)+A]e^{-2r_0/b}}{A-2[\alpha(\alpha-1)+A]e^{-r_0/b}+[2\alpha(\alpha-1)+A]e^{-2r_0/b}} \right)^{1/2} \tag{12}$$

Using ω in equation (9) and then substituting into (11), we obtain (for $N = 3$) the following transcendental relation :

$$F = \sqrt{Q} \tag{13}$$

Where

$$F = \sqrt{2r_0^3 V(r_0)} = \sqrt{\frac{2r_0^3}{kb^3} \times \frac{-[2\alpha(\alpha-1)+A]e^{-2r_0/b} + Ae^{-r_0/b}}{(1-e^{-r_0/b})^3}} \tag{14}$$

and

$$\sqrt{Q} = 1 + 2l + (2n_r + 1) \omega \tag{15}$$

The relation (15) can be solved numerically for r_0 for a given set of A, α, b, l and n_r provided that the obtained root must satisfy the condition that

being minimum at $r = r_0$. Other quantities leading to the energy are followed sequentially (for more details see [23]).

$$V_{eff}(r) = \frac{1}{4} \frac{1}{r^2} + \frac{V(r)}{Q}$$

$$\varepsilon_3 = -1 + \frac{r_0^5}{6kQb^5} \frac{1}{(1-e^{-r_0/b})^5} [Ae^{-r_0/b} + (-8\alpha(\alpha-1) + 3A)e^{-2r_0/b} - (14\alpha(\alpha-1) + 3A)e^{-3r_0/b} - (2\alpha(\alpha-1) + A)e^{-4r_0/b}]$$

$$\varepsilon_4 = \frac{5}{4} + \frac{r_0^6}{24kQb^6} \frac{1}{(1-e^{-r_0/b})^6} [-Ae^{-r_0/b} + (16\alpha(\alpha-1) - 10A)e^{-2r_0/b} + 66\alpha(\alpha-1)e^{-3r_0/b} + (36\alpha(\alpha-1) + 10A)e^{-4r_0/b} + (2\alpha(\alpha-1) + A)e^{-5r_0/b}]$$

$$\delta_5 = -\frac{3}{2} + \frac{r_0^7}{120kQb^7} \frac{1}{(1-e^{-r_0/b})^7} [Ae^{-r_0/b} + (-32\alpha(\alpha-1) + 25A)e^{-2r_0/b} + (-262\alpha(\alpha-1) + 40A)e^{-3r_0/b} - (342\alpha(\alpha-1) + 40A)e^{-4r_0/b} - (82\alpha(\alpha-1) + 25A)e^{-5r_0/b} - (2\alpha(\alpha-1) + A)e^{-6r_0/b}]$$

$$\delta_6 = \frac{7}{4} + \frac{r_0^8}{720kQb^8} \frac{1}{(1 - e^{-r_0/b})^8} \left[-Ae^{-r_0/b} + (64 \alpha(\alpha - 1) - 56A)e^{-2r_0/b} \right. \\ \left. + (946 \alpha(\alpha - 1) - 245A)e^{-3r_0/b} + (2416 \alpha(\alpha - 1))e^{-4r_0/b} \right. \\ \left. + (1436 \alpha(\alpha - 1) + 245A)e^{-5r_0/b} + (176 \alpha(\alpha - 1) + 56A)e^{-6r_0/b} \right. \\ \left. + (2 \alpha(\alpha - 1) + A)e^{-7r_0/b} \right]$$

Other quantities such as $e_i (i = 0, 1, \dots, 4)$ and $d_j (j = 1, 2, \dots, 6)$ are obtained directly by the substitution of the corresponding ε_i and δ_j , respectively.

Table 1. Eigenvalues for the states 1s-6g for 3D and for $\alpha = 0.75$ and $\alpha = 1.5$
 $A = 2b$ ($m = h = 1$).

States	1/b	$\alpha = 0.75$		$\alpha = 1.5$	
		Present	Lucha et al[24]	Present	Lucha et al[24]
1S	0.025	-0.778 6517	-----	-0.214 2915	-----
	0.050	-0.762 4770	-----	-0.206 1857	-----
	0.075	-0.746 6566	-----	-0.198 2301	-----
2S	0.025	-0.130 7914	-----	-0.073 4304	-----
	0.050	-0.118 5602	-----	-0.063 4813	-----
	0.075	-0.106 9576	-----	-0.054 1877	-----
2P	0.025	-0.120 4882	-0.120 5271	-0.090 0173	-0.089 9708
	0.050	-0.108 1768	-0.108 2151	-0.080 0854	-0.080 0400
	0.075	-0.096 4098	-0.096 4469	-0.070 6134	-0.070 5701
3S	0.025	-0.047 3453	-----	-0.033 3553	-----
	0.050	-0.036 6831	-----	-0.023 8450	-----
	0.075	-0.027 4435	-----	-0.015 8277	-----
3P	0.025	-0.045 7105	-0.045 8779	-0.037 2129	-0.036 9134
	0.050	-0.034 9100	-0.035 0633	-0.027 5397	-0.027 2696
	0.075	-0.025 4350	-0.025 5654	-0.019 1714	-0.018 9474
3d	0.025	-0.044 7732	-0.044 7743	-0.039 4817	-0.039 4789
	0.050	-0.033 6920	-0.033 6930	-0.029 4520	-0.029 4496
	0.075	-0.023 7612	-0.023 7621	-0.020 4682	-0.020 4663
4S	0.025	-0.021 1153	-----	-0.016 3555	-----
	0.050	-0.012 2565	-----	-0.008 2296	-----
	0.075	-0.006 0316	-----	-0.002 9437	-----
4P	0.025	-0.020 6536	-0.020 8097	-0.017 5201	-0.017 1740
	0.050	-0.011 6268	-0.011 7365	-0.009 1809	-0.008 9134
	0.075	-0.005 0928	-0.005 0945	-0.003 3961	-0.003 1884
4d	0.025	-0.020 2905	-0.020 3017	-0.018 2424	-0.018 2115
	0.050	-0.010 9839	-0.010 9904	-0.009 5392	-0.009 5167
	0.075	-0.004 0373	-0.004 0331	-0.003 1577	-0.003 1399
4f	0.025	-0.019 9796	-0.019 9797	-0.018 6131	-0.018 6137
	0.050	-0.010 2392	-0.010 2393	-0.009 4016	-0.009 4015
	0.075	-0.002 6449	-0.002 6443	-0.002 2315	-0.002 2307
5P	0.025	-0.009 7010	-0.009 8079	-0.008 3607	-0.008 0816
5d	0.025	-0.009 4995	-0.009 5141	-0.008 5893	-0.008 5415
5f	0.025	-0.009 2813	-0.009 2825	-0.008 6664	-0.008 6619
5g	0.025	-0.009 0330	-0.009 0330	-0.008 151	-0.008 6150
6P	0.025	-0.004 3103	-0.004 3583	-0.003 6914	-0.003 4876
6d	0.025	-0.004 1601	-0.004 1650	-0.003 7311	-0.003 6813
6f	0.025	-0.003 9803	-0.003 9803	-0.003 6871	-0.003 6774
6g	0.025	-0.003 5633	-0.003 7611	-0.003 5633	-0.003 5623

Table 2. Eigenvalues for the states 1S-6g for 2D and for $\alpha = 0.75$ and $\alpha = 1.5$, $A = 2b$ ($m = \hbar = 1$).

States	1/b	$\alpha = 0.75$	$\alpha = 1.5$
1S	0.025	-----	-0.236 3920
	0.050	-----	-0.228 6190
	0.075	-----	-0.220 9431
2S	0.025	-0.318 4997	-0.087 5315
	0.050	-0.305 0352	-0.077 5116
	0.075	-0.291 9843	-0.067 9264
2P	0.025	-0.240 4777	-0.141 1529
	0.050	-0.227 2783	-0.131 8815
	0.075	-0.214 4151	-0.122 8812
3S	0.025	-0.078 4777	-0.043 2362
	0.050	-0.067 0632	-0.033 2931
	0.075	-0.056 6607	-0.024 4207
3P	0.025	-0.073 2084	-0.053 0067
	0.050	-0.061 7208	-0.043 1070
	0.075	-0.051 1787	-0.034 1524
3d	0.025	-0.070 6319	-0.058 8111
	0.050	-0.058 9514	-0.048 6775
	0.075	-0.048 0851	-0.039 2595
4S	0.025	-0.031 7458	-0.021 8887
	0.050	-0.021 9759	-0.012 9788
	0.075	-0.014 2329	-0.006 2754
4P	0.025	-0.030 7476	-0.024 4191
	0.050	-0.020 8724	-0.015 4320
	0.075	-0.012 9103	-0.008 4253
4d	0.025	-0.030 0741	-0.026 0631
	0.050	-0.019 9884	-0.016 7950
	0.075	-0.011 6879	-0.009 2777
4f	0.025	-0.029 6071	-0.027 0031
	0.050	-0.019 1614	-0.017 2951
	0.075	-0.010 2908	-0.009 0554
5P	0.025	-0.014 3107	-0.011 8462
5d	0.025	-0.014 0397	-0.012 3868
5f	0.025	-0.013 7858	-0.012 6826
5g	0.025	-0.013 5265	-0.012 7710
6P	0.025	-0.006 6285	-0.005 5422
6d	0.025	-0.006 4693	-0.005 7168
6f	0.025	-0.006 2889	-0.005 7746
6g	0.025	-0.006 0766	-0.005 727

3-Numerical results

We find by comparing the results in tables (1) and (2) that the energy electron correlation

kernel in the case of two dimensions is larger than that in the case of three dimensions as a

result of restricting its movement within a plane in the former. Also the degree of degeneracy changes ; in three dimensions it is $n_g = n^2$ (excluding spin) while in the case of two dimensions all values $(|m_l| + n_\rho + 1)$ are possible , So the degree of degeneracy of the first case is more than the second and we can attribute that to the falling down in the symmetry in the case of the two dimensions compared with the three dimensions as shown

in figure (1) . At specific value for n we find this energy is less when increasing the number of quantitative orbital l in both cases as shown in figure (2) ,but for the quantum numbers l, n_r the energy in the case of 3D depends on both terms while , in the case of 2D the energy depends on the magnetic quantum number m_l instead and the radial quantum number n_ρ .

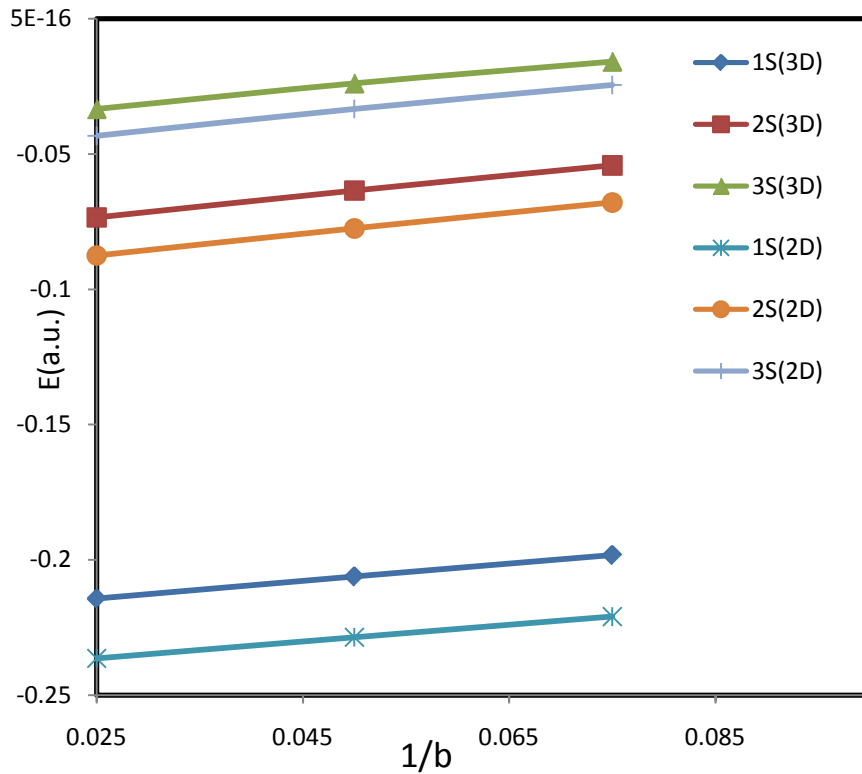


Figure 1. Energy eigenvalues for $n = 1, 2, 3$ of s states for 2D and 3D cases when $\alpha = 1.5$

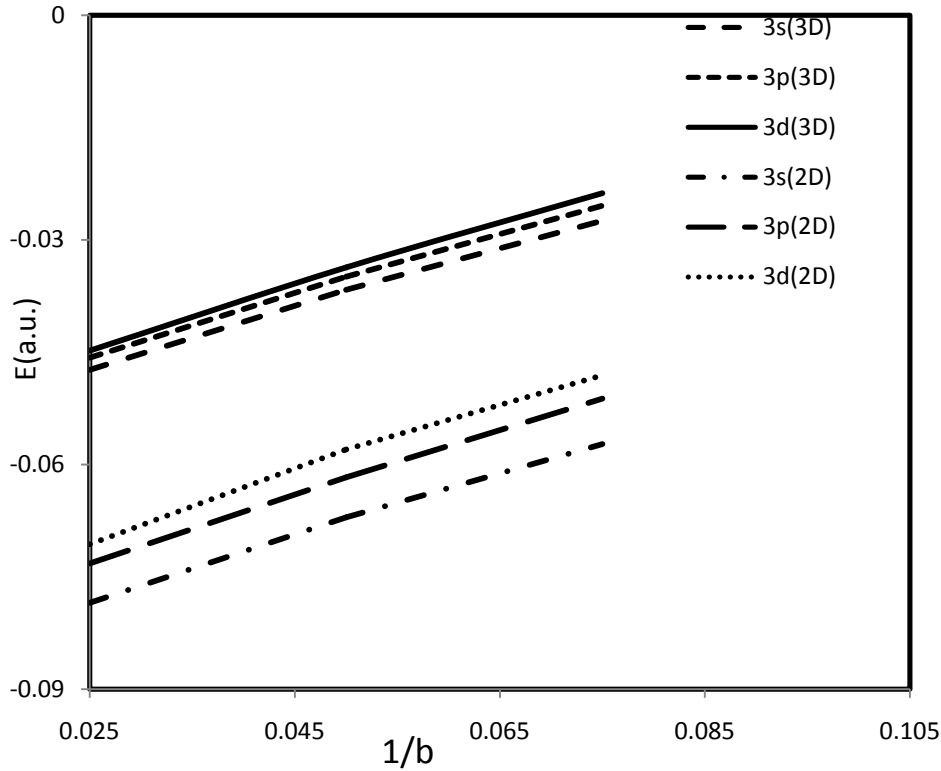


Figure 2. Energy eigenvalues of the orbital s,p and d state for 2D and 3D cases when $\alpha = 0.75$

4-Concluding remarks

The Manning Rosen potential with centrifugal term is a rather involved problem and most works tend to model such term by fitting with a parameter that approximates analytical solution with known confluent type functions. The above procedure has its own shortcomings which render the above method unstable. The shifted $1/N$ method on the other hand is a ready recipe to deal with analytical potentials regardless of singularities near the origin. This is especially true with 3D case as shown in table (1). In table (2) we list the energy terms up to

$l = 5$ with remarkable numerical stability. The accuracy of these results unfortunately cannot be evaluated at the present due to the lack of parallel numerical ones in the literature. However, in view of the almost complete agreement with corresponding 3D results, we are confident that our results are accurate. Further instigation in this regard is certainly required for firm evaluation of the $1/N$ method for reduced dimensional MR potentials.

Acknowledgements

Thanks are due to Dr.H.A.Kassim for his interest and useful comments regarding this

work and also to Dr.N. AL-Maliky for his encouragement .

References

- [1] Ahmed Z. (2002) *phys.Lett. A* 294 , 287-291 .
- [2] Bayrak O. and Boztosum I. (2007) *Int. J. of Quant .Chem.* 107,540-544.
- [3] Setare M. R. and Karimi E. (2007) *phys. Scr.* 75 , 90-93 .
- [4] Dong S. H. (2008) *Ann. Of phys .* 323, 1136-1149 .
- [5] Yahiaoui S. A. , Hattou S. and Bentaiba M. (2007) *Ann. Of phys .* 322 ,2733-2744 .
- [6] Begchi B. and Quesne C. (2002) *phys.Lett. A* 300, 18-26 .
- [7] Berkdemir C. , Berkdemir A. and Sever R. (2008) *J. Math. Chem.* 43, 944-954.
- [8] Ikhdair S. M. and Sever R. (2007) *ANNALEN DER PHYSIK* ,16 ,218-232.
- [9] Meyur S. and Debnath S. (2010) *Lat. Am. J. phys. Educ.* Vol. 4, No.3 Sept.
- [10] Meyur S. and Debnath S. (2008) *Mod. phys. Lett. A* 23, 2077-2084 .
- [11] Gu X. Y. and Sun J. Q. (2010) *J. Math. phys.* 51 , 022106-022111.
- [12] Meyur S. and Debnath S. (2009) *Lat. Am. J. phys. Educ.* 3 , 300-306.
- [13] Ahmed S. and Buragohain L. (2010) *EJTP* 7, 23 .
- [14] Dong S. H. and Garcia-Ravelo J. (2007) *phys. Scr.* 75, 307 .
- [15] Akbarieh A. R. and Motavalli H. (2008) *Mod. phys. Lett. A* 23, 3005-3013 .
- [16] Akaninyene D. ,Akpan N. and Louis E. (2010) *ISSN 1450-216X Vol .46 No.1* 107-118.
- [17] Dong S. H.(2007) *Factorization method in Quantum Mechanics* (The Netherlands: Springer)
- [18] Nikiforov A. F. and Uvarov V. B. (1988) (Birkhauser, Basel).
- [19] Bender C. M. and Boettcher S. (1998) *Phys. Rev. Lett.* 80, 5243-5246.
- [20] Chrisyiansen H. , Epele L. N. , Fanchiotti H. and Garcia C. A. (1989) *Phys. Rev A* Vol. 40 ,No. 4 ,1760-1764.
- [21] Imbo T. , Pagnamenta A. and Sukhatme U. (1983) *Phys. Rev D* , Vol. 29 ,No.8 1669-1681.
- [22] Manning M. F. and Rosen N. (1933) *phys. Rev.* 44 953.
- [23] Kassim H. A. and Al-Maliky N. S. (2006) *J.Phys.B:At.Mol. Opt.Phys.* 39 3057-3071.
- [24] Qiang W. C. and Dong S. H. (2009) *phys .Scr* 79 045004 (5pp).