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# New Travelling Wave Solution of *Burgers Equations*

**Zainab John**
Department of Mathematical, College of Science, Al-Mustansiriyah University, Baghdad, Iraq.
zainabjohn22@uomustansiriyah.edu.iq

**Basim Akhudir Abbas**
Department of Mathematical, College of Science, Al-Mustansiriyah University, Baghdad, Iraq.
baasim_math@uomustansiriyah.edu.iq

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## Abstract

In this paper, we studied the travelling wave solving for some models of Burger's equations. We used sine-cosine method to solution nonlinear equation and we used a direct solution after getting travelling wave equation.

**Keywords:** travelling wave solution, cosine-sine Method, nonlinear differential equations, Wave equations.

## 1.Introduction

A large variety of physical, chemical, and biological phenomena are waves, such as sound waves, string waves, water waves, etc. Wave phenomena are very important in dispersion, diffusion, reaction, and convection, and such phenomena are represented by nonlinear partial differential equations.

Moving waves are visible in many linear equations and nonlinear wave modeling, such as heat waves, string waves, and the cycle life of some organisms one of the travelling waves, etc. One of the most important methods for solving wave equations is the Travelling Wave solution (TWS), which many researchers have discussed ( see [1,2], the tanh-coth method [3], the tanh [4,5] sine-cosine method [6], and see [7-12]).

## 2.Infinite Sires Method [5,9]

To clarify the travelling wave solutions method, we note the following:

Let the nonlinear partial differential equation has the form

$$g(u, u_t, u_x, u_{xx}, \dots) = 0, \qquad (1)$$

Where $u = (x, t)$ is a solution of equation (1), and $g$ is a polynomial with respect to $u$ and with its derivatives. To solve Eq. (1), we follow the following steps:

**Step 1:** First, we change the independent variables *x, t,* by using one independent variable $\xi$ that combines the two variables as shown

Let $u(x,t) = f(\xi) \quad where \;\; \xi = x - ct$ , (2)
Where $\xi$ is a wave variable and $c$ is constant.

**Step2:** derivative equation (2) with respect to $x\, and\, t$, we have the following ordinary differential equations:

$$u_x = u' = \frac{du}{d\,\xi}\,; u_{xx} = u'' = \frac{d^2u}{d\,\xi^2}\,;\;\; u_t = -cu' = -c\frac{du}{d\,\xi}\,;\;\; u_{tt} = c^2u'' = c^2\frac{d^2u}{d\,\xi^2};$$
$$u_{xt} = -cu'' = -c\frac{d^2u}{d\,\xi^2} \qquad (3)$$

When substitute (3) into (1), we find that equation (1) is transform to (linear or nonlinear) ordinary differential equation transformed into

$$G\left(f(\xi), \frac{df}{d\xi}, \frac{d^2f}{d\xi^2}, \dots\right) \qquad (4)$$

Where *G* is a polynomial in $f(\xi)$ and with its derivatives,

**Step 3:** integrate (4) with respect to $\xi$ , and let the constant of integration equal to zero.

**Step 4: (a)** we can solve equation (4) directly through many methods that solve ordinary differential equations.

or

**(b)** use sine-cosine method to solve equation (4).

## 3. Applications of the travelling wave solution
### 3.1: Some Models of Burgers Equations [13]

The viscid Burgers equation to be the nonlinear parabolic PDE has the form

$$v_t = v_{xx} + \in vv_x. \quad ,0 < \epsilon \leq 1. \qquad (5)$$

### 3.1.1. Travelling wave solution
The travelling wave solution of Eq.(5) is

$$\xi = x - ct \;\; \Rightarrow v(x,t) = v(\xi)\,,\;\; v_x = v' = \frac{du}{d\,\xi}\,; v_{xx} = v'' = \frac{d^2v}{d\,\xi^2}\,;\;\; v_t = -cu' = -c\frac{dv}{d\,\xi}.$$
(6)

Substituting Eq. (6) into Eq. (5), we have got

$$v'' + cv' + \in vv' = 0 \qquad (7)$$
$$v'' + cv' + \in \left(\frac{v^2}{2}\right)' = 0 \qquad (8)$$

Integral Eq. (8) with respect to $\xi$ we have got

$$v' + cv + \epsilon\frac{v^2}{2} = c_1 \qquad (9)$$

Let $c_1 = 0$

$$v' = -(cv + \epsilon \frac{v^2}{2}) \quad (10)$$

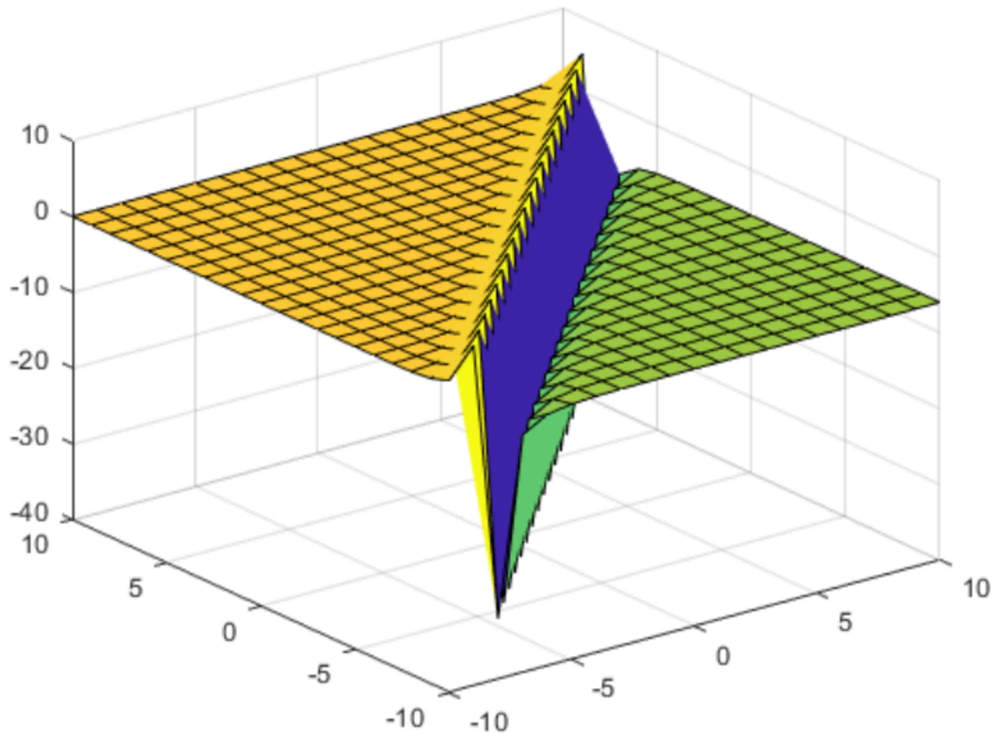
$$\frac{dv}{-(cv+\epsilon\frac{v^2}{2})} = d\xi$$

By solving this equation, we have got

$$v(\xi) = \frac{c}{\left(ce^{c(\xi+c_2)} - \epsilon/2\right)} \quad (11)$$

Where $= x - ct$ , substituted in (11), we have $v(x,t) = \frac{c}{\left(ce^{((x-ct)+c_2)} - \epsilon/2\right)}$, which is a general solution of Burgers equation (5).

By drawing the solution function *(v(x,t))*, we find the following figure




Singular kink of u

**Figure 1.** When c=1, $\delta$=1/3, x=[-10,10], t=[-10,10].

## 3.2. KdV-Burgers Equation:

### 3.2.1: Cosine Function Method:

In this part, we want to find (TWS) of KdV-Burgers equation, by using the cosine function method see [6,7]. KdV-Burgers equation [13,14] has the form

$$u_t + (u^2)_x - \gamma u_{xx} + \delta u_{xxx} = 0, \quad (12)$$

Where $\delta$ and $\Upsilon$>0 are constants and $u$ is a function of spatial variable $x$ and time variable $t$

let $u(x,t) = u(\xi)$ , $\quad \xi = x - ct \quad (13)$

Application (TWS) on Eq. (12) yields into the ordinary differential equation

$$-cu' + (u^2)' - \gamma u'' + \delta u''' = 0 \quad (14)$$

Integrating (14) we get

$$-cu + u^2 - \gamma u' + \delta u'' = 0 \tag{15}$$

Then we have the nonlinear travelling wave equation (15). Solution Eq.+(15) by cosine function method has the form.

$$u(\xi) = \lambda cos^{\beta}(\mu\xi) \tag{16}$$

Where **λ**, **β** and **μ** are unknown parameters to find its, we derivatives (16) we get

$$u' = -\lambda\mu\beta cos^{\beta-1}(\mu\xi)\sin(\mu\xi) \tag{17}$$

$$u'' = -\lambda\beta\mu^2 cos^{\beta}(\mu\xi) + \lambda\mu^2\beta(\beta-1)cos^{\beta-2}(\mu\xi). \tag{18}$$

Substituting (16-18) into equation (15) gives

$$-c\lambda cos^{\beta}(\mu\xi) + \left(\lambda cos^{\beta}(\mu\xi)\right)^2 + \gamma\lambda\mu\beta cos^{\beta-1}(\mu\xi)\sin(\mu\xi) + \delta\left(-\lambda\beta\mu^2 cos^{\beta}(\mu\xi) + \lambda\mu^2\beta(\beta-1)cos^{\beta-2}(\mu\xi)\right) = 0$$

The following algebraic equation system is obtained by equating the exponents and the coefficient of each pair of the cosine functions.

$$-c\lambda - \delta\lambda\beta\mu^2 = 0 \tag{19}$$

$$\lambda^2 + \delta\lambda\mu^2\beta(\beta-1) = 0 \tag{20}$$

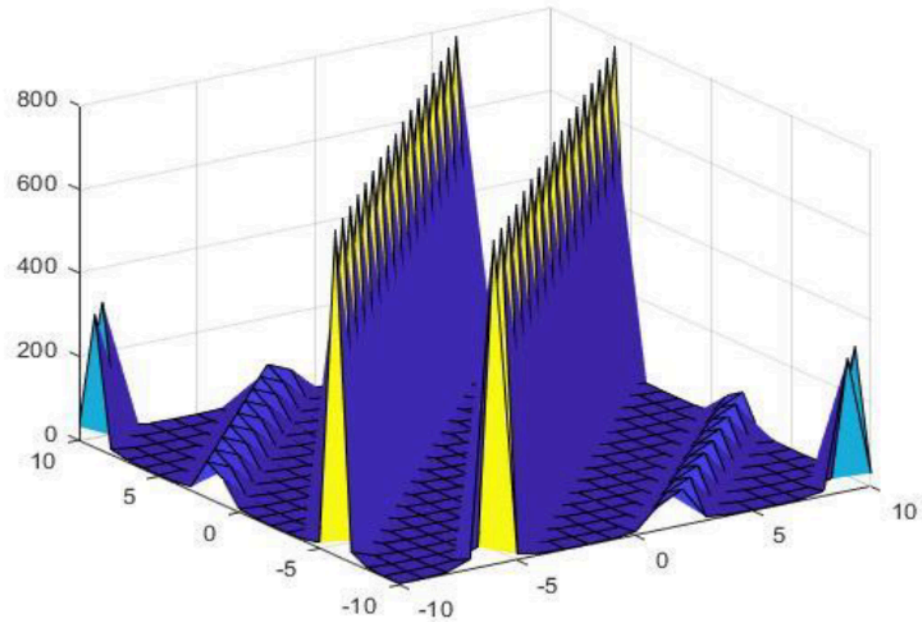
$2\beta = \beta - 2 \Rightarrow \beta = -2$ , substitute in (19-20) we have

$\mu = \mp\sqrt{\frac{c}{2\delta}}$ , $\lambda = 3c$, substitute in (16) we get the solution

$u(\xi) = 3c\ cos^{-2}(\sqrt{\frac{c}{2\delta}}\ \xi)$ or $u(\xi) = -3c\ cos^{-2}(\sqrt{\frac{c}{2\delta}}\ \xi)$

$$u(x,t) = 3c\ sec^2(\sqrt{\frac{c}{2\delta}(x-ct)}\ ) \tag{21}$$

By drawing the solution function *u(x,t)*, we find the following figure




The periodic solution of u

**Figure 2.** When c=1, δ=1/3, x=[-10,10], t=[-10,10].

### 3.2.2.Sine Function Method:

Solve equation (15) by use (sine-function method), we have the form,

$$u = \lambda sin^{\beta}(\mu\xi) \tag{22}$$

Where **λ**, **β** and **μ** are unknown parameters, to find its, we Derivatives (22) we get.

$$u' = \lambda\mu\beta sin^{\beta-1}(\mu\xi)\cos(\mu\xi) \tag{23}$$

$$u'' = -\lambda\beta\mu^2 sin^{\beta}(\mu\xi) + \lambda\mu^2\beta(\beta-1)sin^{\beta-2}(\mu\xi) \tag{24}$$

Substituting (22-24) into equation (15) yield in

$$-c\lambda sin^{\beta}(\mu\xi) + \left(\lambda sin^{\beta}(\mu\xi)\right)^2 - \gamma\lambda\mu\beta sin^{\beta-1}(\mu\xi)\cos(\mu\xi) + \delta(-\lambda\beta\mu^2 sin^{\beta}(\mu\xi) + \lambda\mu^2\beta(\beta-1)sin^{\beta-2}(\mu\xi)) = 0$$

We find the following system of algebraic and we obtain to parameters by equating the exponents and the coefficient of each pair of the (cosine function).

$$-c\lambda - \delta\lambda\beta\mu^2 = 0 \tag{25}$$

$$\lambda^2 + \lambda\mu^2\beta(\beta-1) = 0\ , \tag{26}$$

$$2\beta = \beta - 2 \Rightarrow \beta = -2 \tag{27}$$

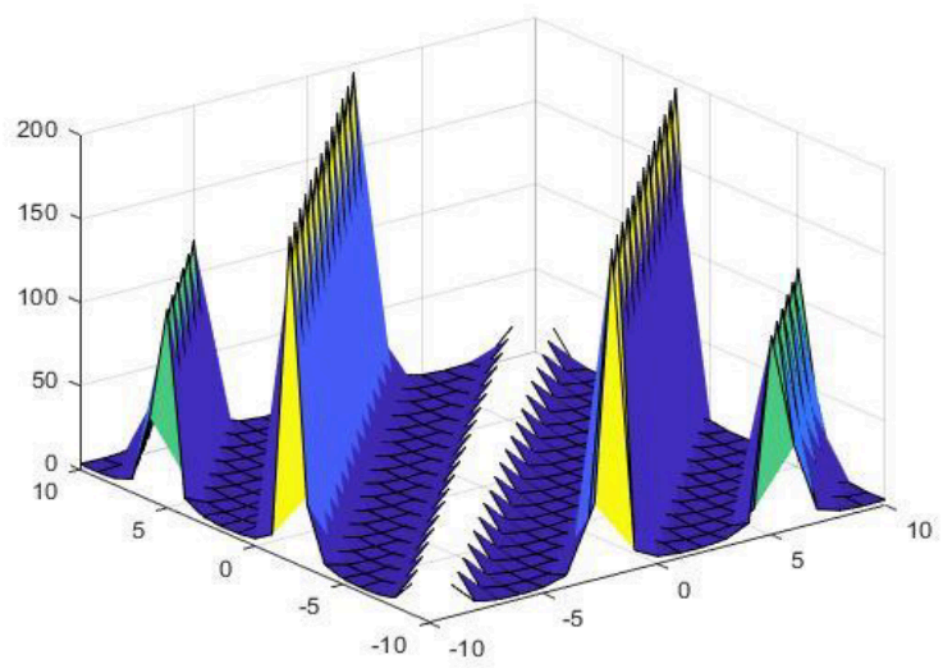
Substitute (27) in (25,26) we have

$$\mu = \mp\sqrt{\frac{c}{2\delta}},\ \lambda = 3\text{c}$$

Then, we have the solution

$$u = -3c\ sin^{-2}(\sqrt{\frac{c}{2\delta}}\xi)\ \text{ or } u = 3c\ sin^{-2}(\sqrt{\frac{c}{2\delta}}\xi)$$

$$u = \mp 3c\ csc^2(\sqrt{\frac{c}{2\delta}}(x-ct))$$




The periodic solutions of u

**Figure 3**. When c=1, $\delta$=1/3, x=[-10,10], t=[-10,10].

## Conclusion

In this work, we conclude that the method of sine and cosine function gives the exact solution to the nonlinear partial differential equation, which is difficult to find in other ways.

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