Ibn Al-Haitham Jour. for Pure & Appl. Sci. 34 (1) 2021

Ibn Al Haitham Journal for Pure and Applied Science

Journal homepage: http://jih.uobaghdad.edu.iq/index.php/j/index

$\alpha g_{Ị}$-open sets and $\alpha g_{Ị}$-functions

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Article history: Received 9,February,2020, Accepted 15,March,2020, Published in January 2021

Doi: 10.30526/34.1.2555

Abstract.

The objective of this paper is to show modern class of open sets which is an $\alpha g_{Ị}$-open. Some functions via this concept and the relationships among continuous function strongly $\alpha g_{Ị}$-continuous function $\alpha g_{Ị}$-irresolute function $\alpha g_{Ị}$-continuous function are studied.

Keywords. $\alpha g_{Ị}$-closed set, $\alpha g_{Ị}O$-functions, $\alpha g_{Ị}C$-functions, $\alpha g_{Ị}$-continuous function, Strongly $\alpha g_{Ị}$-continuous function, $\alpha g_{Ị}$-irresolute function, ideal.

1. Introduction.

An α-open was studied in 1965 by O. Njastad, a subset Ç is α-open set if Ç ⊆ $int(cl(int($Ç$)))$[1,2]. The notion of ideal was studied by Kuratowski[3,4],that Ị is an ideal on Ҳ, where Ị is a collection of all subsets of Ҳ an ideal have two properties (if Ç, Ð ∈ Ị, then Ç ∪ Ð ∈ Ị) and (if Ç ∈ Ị and Ð ⊆ Ç, then Ð ∈ Ị.

There are many types for the ideal[5-7]

i. $Ị_{\{\emptyset\}}$: the trivial ideal where Ị={∅}.

ii. $Ị_n$: the ideal of all nowhere dense sets

$Ị_n$ ={Ç ⊆ Ҳ: $int(cl($Ç$))$ = {∅}}.

iii. $Ị_f$: the ideal of all finite subsets of Ҳ

$Ị_f$ = { Ç ⊆ Ҳ: Ç is a finite set}.

The collection of all α-open sets is denoted by" $ƭ_{\alpha}$" and the collection of all α-closed is denoted by" $ɟ_{\alpha}$".

In this paper, we introduce $\alpha g_{Ị}$-closed set, and the complement of $\alpha g_{Ị}$-open set. More functions have been introduced via these concepts, such as $\alpha g_{Ị}$-open, $\alpha g_{Ị}^{*}$-open, $\alpha g_{Ị}^{**}$-open, $\alpha g_{Ị}$-continuous, $\alpha g_{Ị}$-irresolute and Strongly $\alpha g_{Ị}$-function.

2- On $\alpha g_{Ị}$-closed set

Definition 1: In ideal topological space (Ӽ, ĩ, Ị), Let Ç ⊆ Ӽ. Ç is said Ị-α-g-closed set denoted by "$\alpha g_{Ị}$-closed" ,if Ç-Ơ ∈ Ị then, cl(Ç)-Ơ ∈ Ị where Ơ ⊆ Ӽ and Ơ is an α-open sets.
Now, $Ç^{c}$ is Ị-α-g-open sets denoted by "$\alpha g_{Ị}$-open". The collection of all $\alpha g_{Ị}$-closed sets, where $Ç^{c}$ ∈ Ӽ, is denoted by "$\alpha g_{Ị}C$(Ӽ). The collection of all $\alpha g_{Ị}$-open sets "$\alpha g_{Ị}O$(Ӽ)".

Example 2: Consider the space (Ӽ, ĩ, Ị) where Ӽ={w,v}, ĩ={Ӽ,∅,{w}} and Ị={∅,{v}}.Then $ĩ_{\alpha}$={Ӽ,∅,{w}} and $ⱡ_{\alpha}$={Ӽ,∅,{v}}, so $\alpha g_{Ị}C$(Ӽ) = $\alpha g_{Ị}O$(Ӽ) = {Ӽ,∅,{w},{v}}.

Example 3: Consider the space (Ӽ, ĩ, Ị) where Ӽ={w,v,z}, ĩ={Ӽ,∅,{w}} and Ị={∅,{v}}. Then $ĩ_{\alpha}$={Ӽ,∅,{w},{w,v},{w,z}} $ⱡ_{\alpha}$={Ӽ,∅,{v,z},{z,},{v}}, so $\alpha g_{Ị}C$(Ӽ)={Ӽ,∅,{v,z},{z},{w,z}} $\alpha g_{Ị}O$(Ӽ) = {Ӽ,∅,{w},{w,v},{v}}.

Remark 4:

i. For each closed set in (Ӽ , ĩ) is an $\alpha g_{Ị}$-closed in (Ӽ, ĩ, Ị).
ii. For each open set in (Ӽ , ĩ) is an $\alpha g_{Ị}$-open in (Ӽ, ĩ, Ị).

Proof:

i. Let Ç is any closed set in (Ӽ, ĩ, Ị) and Ơ be an α-open set such that Ç-Ơ ∈ Ị since cl(Ç) = Ç this implies that Ç is an $\alpha g_{Ị}$-closed set.
ii. Let Ơ ∈ Ӽ, then $Ơ^{c}$ is a closed set this implies that $Ơ^{c}$ is an $\alpha g_{Ị}$-closed set, so Ơ is an $\alpha g_{Ị}$-open set.

The reverse way of Remark 2.4 is wrong in general see Example 2.2.

Remark 5: A space (Ӽ, ĩ, Ị):

i. If Ị = Ᵽ(Ӽ) then $\alpha g_{Ị}C$(Ӽ) = $\alpha g_{Ị}O$(Ӽ) = Ᵽ(Ӽ).
ii. If ĩ = D then $\alpha g_{Ị}C$(Ӽ) = $\alpha g_{Ị}O$(Ӽ) = Ᵽ(Ӽ).

Remark 6: For any space (Ӽ, ĩ, Ị), then the two idea $\alpha g_{Ị}$-closed set and αg^{*}-closed set are the same, if Ị={∅}.
The following example display that the two notion $\alpha g_{Ị}$-closed set and αg^{*}-closed set are separate, in general.

Example 7:

i. The set {w} in Example 2.2 is an $\alpha g_{Ị}$-closed set but not αg^{*}-closed set, and {v} is an $\alpha g_{Ị}$-open set, but not αg^{*}-open set.
ii. For a space (Ӽ, ĩ, Ị), where Ӽ={ṟ,ș,w,v}, ĩ = {Ӽ,∅,{ṟ,ș},{w,v}} and Ị={∅,ṟ}. Then $ĩ_{\alpha}$= ĩ, leads to $\alpha g^{*}C$(Ӽ) = Ᵽ(Ӽ) and $\alpha g^{*}O$(Ӽ) = Ᵽ(Ӽ). It seems obvious that the set {ṟ} is αg^{*}-closed set but not $\alpha g_{Ị}$-closed.

Remark 8: For any set Ӽ, let ӽ ∈ Ӽ and ĩ = {Ӽ,∅,{ӽ}}, Ị=$Ị_{n}$={Ç ⊆ Ӽ: $int(cl($Ç$))$={∅}} then $\alpha g_{Ị}C$(Ӽ) = Ᵽ(Ӽ).
Proof:
Let $Ị_{n}$ = { Ç ⊆ Ӽ: $int(cl($Ç$))$ = {∅}}, Ӽ be any set and ĩ = {Ӽ, ∅, {ӽ}} such that ӽ ∈ Ӽ, $ĩ_{\alpha}$={Ơ ⊆ Ӽ; ӽ ∈ Ơ} ∪ {∅}, for any set Ç ⊆ Ӽ, and Ơ is α-open set, if Ç-Ơ ∈ $Ị_{n}$ this implies ӽ ∉ (Ç-Ơ), so

cl(Ç-Ơ) = Ӽ/{ӽ}, then $int(cl$(Ç-Ơ)) = ∅, then ӽ ∉ Ç and ӽ ∈ Ơ, since ӽ ∉ Ç this implies cl(Ç) = Ӽ/{ӽ}, thus (Ӽ/{ӽ}-Ơ)∈ $Ị_n$,if ӽ ∈ Ç and ӽ ∈ Ơ then ӽ ∉ (cl(Ç)-Ơ), so cl(Ç)-Ơ ∈ $Ị_n$, hence $\alpha g_{Ị}C$(Ӽ) = Ᵽ(Ӽ).

Theorem 9: Let Ç and Đ are two $\alpha g_{Ị}$-closed sets then Ç ∪ Đ is an $\alpha g_{Ị}$-closed.

Proof: Let Ç and Đ are two $\alpha g_{Ị}$-closed set in (Ӽ, ĩ, Ị) and Ơ ∈ $ĩ_\alpha$ subset of Ӽ, where (Ç ∪ Đ)-Ơ ∈ Ị, then Đ-Ơ ∈ Ị and Ç-Ơ ∈ Ị, so cl(Đ)-Ơ ∈ Ị and cl(Ç)-Ơ ∈ Ị therefore, (cl(Ç)-Ơ) ∪ (cl(Đ)-Ơ) ∈ Ị, *so* cl(Ç ∪ Đ)-Ơ ∈ Ị.Hence Ç ∪ Đ is an $\alpha g_{Ị}$-closed sets.

Corollary 10: Let Ç and Đ are two $\alpha g_{Ị}$-open sets then Ç ∩ Đ is an $\alpha g_{Ị}$-open.

Proof: Let Ç and Đ are two $\alpha g_{Ị}$-open sets in Ӽ then $Ç^c$, $Đ^c$ are two $\alpha g_{Ị}$-closed sets therefore, $Ç^c$ ∪ $Đ^c$ is an $\alpha g_{Ị}$-closed set by theorem 2.9. Hence$(Ç ∩ Đ)^c$ is an $\alpha g_{Ị}$-closed set so Ç ∩ Đ is an $\alpha g_{Ị}$-open set.

Remark 11:

i. The union of any collection of $\alpha g_{Ị}$-closed sets is not necessarily $\alpha g_{Ị}$-closed.

ii. The intersection of collection of $\alpha g_{Ị}$-open sets is not necessarily $\alpha g_{Ị}$-open.

For example: Consider a space (Ӽ, ĩ, Ị), when Ӽ = Ɲ, the set of all natural numbers, ĩ = ĩ cof, is a topology of all sets that complement is a finite set and Ị=$Ị_{Ị_f}$={Ơ ⊆ Ɲ, Ơ is a finite set}, $ĩ_\alpha$={Ơ ⊆ Ɲ, Ơ is an infinite set}∪{∅}.

Clearly, {ŋ} is an $\alpha g_{Ị}$-closed set, ∀ ŋ ∈ $Ȩ^+$, where $Ȩ^+$ is the positive even numbers, but ∪{{ŋ}:ŋ∈ $Ȩ^+$}=$Ȩ^+$ which is not $\alpha g_{Ị}$-closed set. Similarly; $Ç_n$=Ɲ-{ŋ} is an of $\alpha g_{Ị}$-open set, ∀ ŋ ∈ $Ȩ^+$ but ∩{$Ç_n$: ŋ ∈ $Ȩ^+$}=$Ộ^+$, where $Ộ^+$ is the positive odd number ,$Ộ^+$ is not $\alpha g_{Ị}$-closed set.

Theorem 12: In (Ӽ, ĩ, Ị), let Ç ⊆ Ӽ. Ç is an $\alpha g_{Ị}$-open set if and only if (Ꞙ − int(Ç)) ∈ Ị, whenever (Ꞙ-Ç) ∈ Ị, ∀ Ꞙ ∈ $ɟ_\alpha$.

Proof: (→)Let Ç ⊆ Ӽ, where Ç be an $\alpha g_{Ị}$-open sets and (Ꞙ-Ç) ∈ Ị, Ꞙ ∈ $ɟ_\alpha$, since (Ӽ-Ç) is an $\alpha g_{Ị}$-closed set and (Ӽ-Ç)-Ơ ∈ Ị, Ơ ∈ $ĩ_\alpha$implies cl(Ӽ-Ç)-Ơ ∈ Ị, whenever (Ӽ-Ç)-Ơ ∈ Ị, for each Ơ ∈ $ĩ_\alpha$, cl(Ӽ-Ç)-Ơ = (Ӽ-Ơ)-(Ӽ-cl(Ӽ-Ç) since Ç-Đ = (Ӽ-Đ)-(Ӽ-Ç), thus (Ӽ-Ơ)-(Ӽ-(Ӽ-int(Ӽ-Ӽ-Ç))) = (Ӽ-Ơ)-int(Ç) = Ꞙ-int(Ç) ∈ Ị.

(←) Let Ꞙ-int(Ç) ∈ Ị, whenever Ꞙ-Ç ∈ Ị, for each Ꞙ ∈ $ɟ_\alpha$. Let (Ӽ-Ç)-Ơ ∈ Ị; Ơ ∈ $ĩ_\alpha$, (Ӽ-Ç)-Ơ = (Ӽ-Ơ)-Ç ∈ Ị, let Ӽ-Ơ = Ꞙ ∈ $ɟ_\alpha$ and Ꞙ-Ç ∈ Ị this implies Ꞙ-int(Ç) ∈ Ị, now Ꞙ-int(Ç) = cl(Ӽ-Ç)-(Ӽ-Ꞙ) = cl(Ӽ-Ç)-Ơ ∈ Ị, thus (Ӽ-Ç) is an $\alpha g_{Ị}$-closed set, hence Ç is an $\alpha g_{Ị}$-open set.

3-Open function

Definition 1: The function ʃ: (Ӽ, ĩ, Ị) → (Y, ɟ, ʝ) is called;

i. $\alpha g_{Ị}$-open function, denoted by "$\alpha g_{Ị}$o-function" if ʃ(Ơ) is an $\alpha g_{ʝ}$-open set in Y. Whenever Ơ is an $\alpha g_{Ị}$-open in Ӽ.

ii. $\alpha g_{Ị}^*$-open function, denoted by "$\alpha g_{Ị}^*$o-function" if ʃ(Ơ) is an $\alpha g_{ʝ}$-open set in Y. Whenever Ơ ∈ ĩ.

iii. αg_{I}^{**}-open function, denoted by "αg_{I}^{**}o-function" if ƒ(Ơ) is an open set in Ƴ. Whenever Ơ is an αg_{I}-open set in Ӽ.

Proposition 2: Let ƒ: (Ӽ, ĩ, Ị) → (Ƴ, ɟ, ʝ) is a function;

i. If ƒ is an open function then ƒ is αg_{I}^{*}o-function

Proof: Let Ơ ∈ ĩ, since ƒ is an open function then ƒ(Ơ)∈ ɟ and since for each open sets is an αg_{I}-open set then ƒ(Ơ) is an αg_{J}-open set in Ƴ, then ƒ is an αg_{I}^{*}o-function.

ii. If ƒ is an αg_{I}^{**}o-function then ƒ is an αg_{I}-open function.

Proof: Let Ơ be an αg_{I}-open set in Ӽ, since ƒ is an αg_{I}^{**}o-function, then ƒ(Ơ)∈ ɟ, since for each open set is an αg_{I}-open set, this implies that ƒ(Ơ) is an αg_{J}-open set in Ƴ, then ƒ is an αg_{I}-open function.

iii. If ƒ is an αg_{I}o-function then ƒ is an αg_{I}^{*}o-function.

Proof: Let Ơ ∈ ĩ, since for each open set is an αg_{I}-open set, then ƒ(Ơ) is an αg_{J}-open set in Ƴ, thus ƒ is an αg_{I}^{*}o-function.

iv. If ƒ is an αg_{I}^{**}o-function then ƒ is an open function.

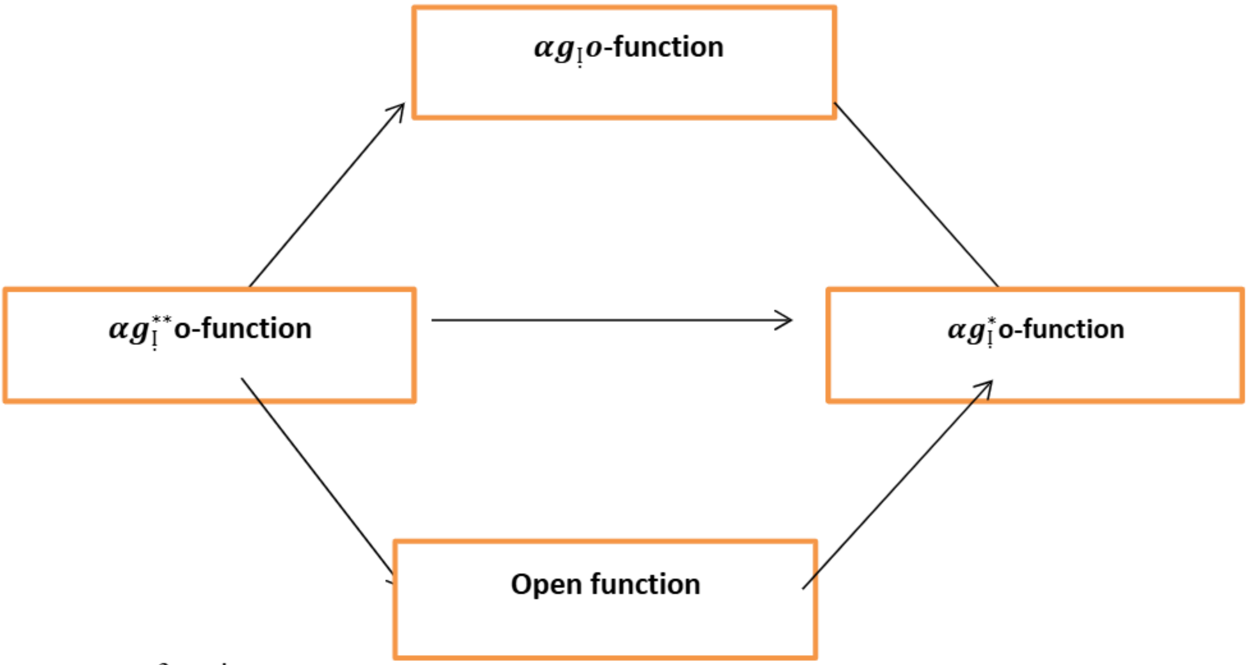
Proof: Let Ơ ∈ ĩ, since for each open set is an αg_{I}-open set, then Ơ be an αg_{I}-open set in Ӽ, since ƒ is an αg_{I}^{**}o-function thus ƒ(Ơ) is an open set in Ƴ, then ƒ is an open function.

v. If ƒ is an αg_{I}^{**}o-function then ƒ is an αg_{I}^{*}o-function.

Proof: By proposition 3.2-ii and proposition 3.2-iii, prove is over.

The following scheme explains the relationship between the various concepts presented in Definition 3.1.

Arrow chart
(3.1)



αg_{I}-open function

The following are some examples showing that the opposite direction of the above schema is incorrect.

Example 3: A function ƒ: (Ӽ, ĩ, Ị) → (Ӽ, ĩ, ȷ̧), where Ӽ = {$ẻ_1$, $ẻ_2$, $ẻ_3$} such that ƒ($ẻ_1$) = ($ẻ_2$), ƒ($ẻ_2$) = ($ẻ_1$), ƒ($ẻ_3$) = ($ẻ_3$), ĩ={Ӽ,∅,{$ẻ_1$}}, Ị={∅} and ȷ̧={∅,{$ẻ_2$},{$ẻ_3$},{$ẻ_2$,$ẻ_3$}} then $ĩ_\alpha$={Ӽ,∅,{$ẻ_1$},{$ẻ_1$, $ẻ_2$},{$ẻ_1$, $ẻ_3$}} then $\alpha g_{Ị}C$(Ӽ) = {Ӽ,∅,{$ẻ_2$,$ẻ_3$}} and $\alpha g_{Ị}O$(Ӽ) = {Ӽ,∅,{$ẻ_1$}}. So $\alpha g_{ȷ̧}C$(Ӽ) = Ᵽ(Ӽ) and $\alpha g_{ȷ̧}O$(Ӽ) = Ᵽ(Ӽ).
Then ƒ is $\alpha g_{Ị}$o-function and $\alpha g_{Ị}^{*}$o-function which is not $\alpha g_{Ị}^{**}$o-function and not an open function, since {$ẻ_1$} is an open set in Ӽ and $\alpha g_{Ị}$-open set, but ƒ($ẻ_1$)=($ẻ_2$) which is not open.

Example 4: The function ƒ: (Ӽ, ĩ, Ị) → (Ӽ, ĩ, Ị); where Ӽ={$ẻ_1$, $ẻ_2$, $ẻ_3$} such that ƒ(ẻ) = (ẻ), ∀ ẻ∈Ӽ, ĩ={Ӽ,∅,{$ẻ_1$}}, Ị={∅,{$ẻ_2$},{$ẻ_3$},{$ẻ_2$, $ẻ_3$}} and ȷ̧={∅}. Then $ĩ_\alpha$={Ӽ,∅,{$ẻ_1$},{$ẻ_1$, $ẻ_2$},{$ẻ_1$, $ẻ_3$}} then $\alpha g_{Ị}C$(Ӽ) = Ᵽ(Ӽ) and $\alpha g_{Ị}O$(Ӽ) = Ᵽ(Ӽ). So $\alpha g_{ȷ̧}C$(Ӽ) = {Ӽ,∅,{$ẻ_2$,$ẻ_3$}} and $\alpha g_{ȷ̧}O$(Ӽ) = {Ӽ,∅,{$ẻ_1$}}.
It is easy to see that ƒ is an open function and $\alpha g_{Ị}^{*}$o-function but it is not $\alpha g_{Ị}$o-function and not $\alpha g_{Ị}^{**}$c-function, since {$ẻ_2$}∈ $\alpha g_{Ị}O$(Ӽ) but ƒ($ẻ_2$) = ($ẻ_1$) which is not open and not $\alpha g_{ȷ̧}$-open set.

Definition 5: The function ƒ: (Ӽ, ĩ, Ị) → (Ƴ, ɟ, ȷ̧) is said,

i. $\alpha g_{Ị}$-closed function, denoted by "$\alpha g_{Ị}$c-function" if ƒ(Ơ) is $\alpha g_{ȷ̧}$-closed in Ƴ whenever Ơ is an $\alpha g_{Ị}$-closed in Ӽ.
ii. $\alpha g_{Ị}^{*}$-closed function, denoted by "$\alpha g_{Ị}^{*}$c-function", if ƒ(Ơ) is $\alpha g_{ȷ̧}$-closed in Ƴ whenever Ơ is an closed in Ӽ.
iii. $\alpha g_{Ị}^{**}$-closed function, denoted by "$\alpha g_{Ị}^{**}$c-function", if ƒ(Ơ) is closed in Ƴ whenever Ơ is an $\alpha g_{Ị}$-closed in Ӽ.

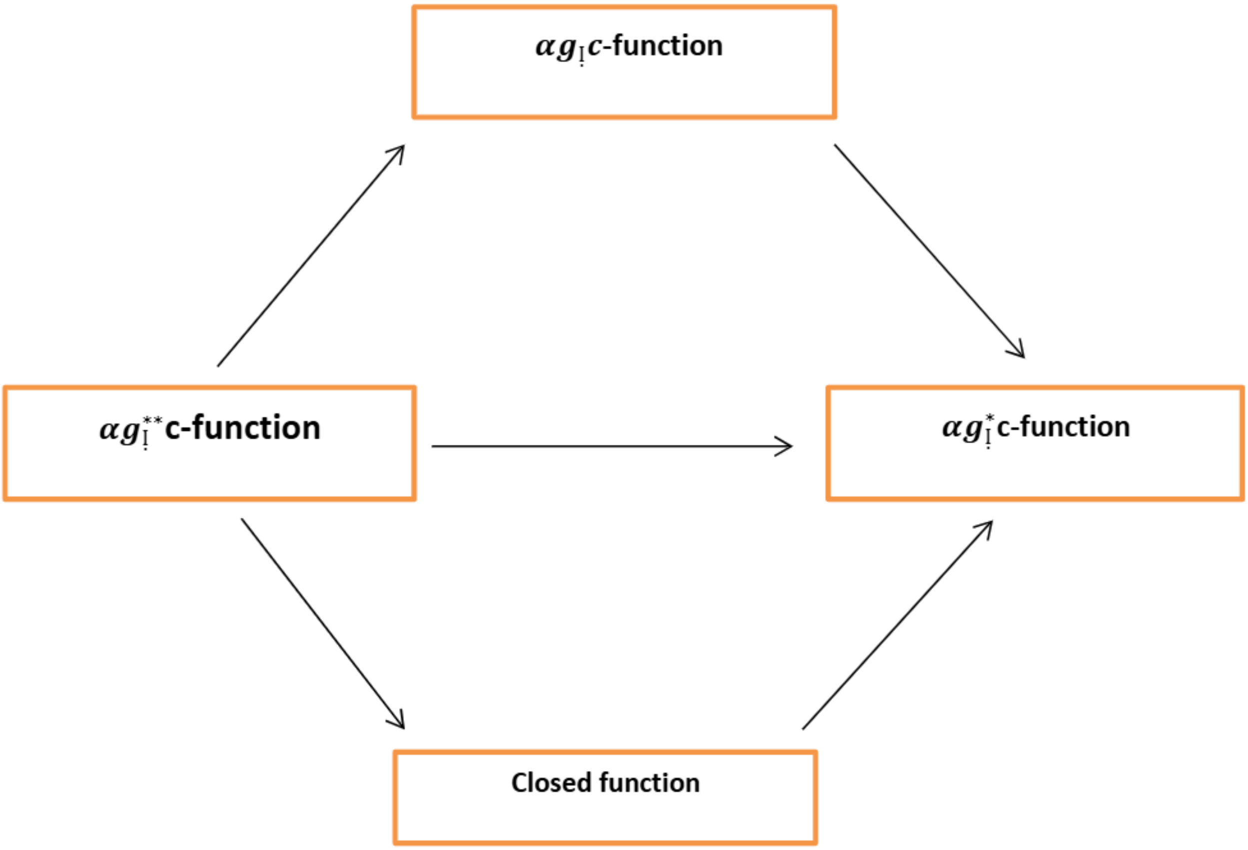
Proposition 6: Let ƒ: (Ӽ, ĩ, Ị) → (Ƴ, ɟ, ȷ̧) is function,

i. If ƒ is a closed function then ƒ is an $\alpha g_{Ị}^{*}$c-function.
ii. If ƒ is an $\alpha g_{Ị}^{**}$c-function then ƒ is an $\alpha g_{Ị}$c-function.
iii. If ƒ is an $\alpha g_{Ị}^{**}$c-function then ƒ is a closed function.
iv. If ƒ is an $\alpha g_{Ị}$c-function then ƒ is an $\alpha g_{Ị}^{*}$c-function.
v. If ƒ is an $\alpha g_{Ị}^{**}$c-function then ƒ is an $\alpha g_{Ị}^{*}$c-function.

Proof: By Remark 2.4 and Definition 3.5.

The follow Diagram shows the relationships between the different concepts that are inserted in Definition 3.5

Arrow chart
(3.2)



$\alpha g_{Ị}$-closed function

Example 3.3 and 3.4 show that the opposite direction of the above chart is incorrect.

Remark 7: If ʄ is onto function then:

i. $\alpha g_{Ị}$o-function and $\alpha g_{Ị}$c-function are the same.
ii. $\alpha g_{Ị}^{*}$o-function and $\alpha g_{Ị}^{*}$c-function are the same.
iii. $\alpha g_{Ị}^{**}$o-function and $\alpha g_{Ị}^{**}$c-function are the same.

Proof: since ʄ is an onto function then the prove is easy by using Definition 3.1 and Definition 3.5

4- Near continuous function

Definition 1: A function ʄ: (Ҳ, ĩ, Ị) → (Ƴ, ɟ, ʝ) is called;

i. Ị-α-g-continuous function, denoted by "$\alpha g_{Ị}$-continuous function", if $ʄ^{-1}$(Ơ) is an $\alpha g_{Ị}$-open set in Ҳ, where Ơ ∈ ɟ.
ii. Strongly Ị-α-g-continuous function, denoted by "Strongly $\alpha g_{Ị}$-continuous function" if $ʄ^{-1}$(Ơ) ∈ ĩ, whenever Ơ is an $\alpha g_{ʝ}$-open set in Ƴ.
iii. Ị-α-g-irresolute function, denoted by "$\alpha g_{Ị}$-irresolute function", if $ʄ^{-1}$(Ơ) is an $\alpha g_{Ị}$-open set in Ҳ, where Ơ is an $\alpha g_{ʝ}$-open set in Ƴ.

Proposition 2: Let ƒ: (X̧, ĩ, Ị) → (Ƴ, ɟ, ȷ̧) is a function;

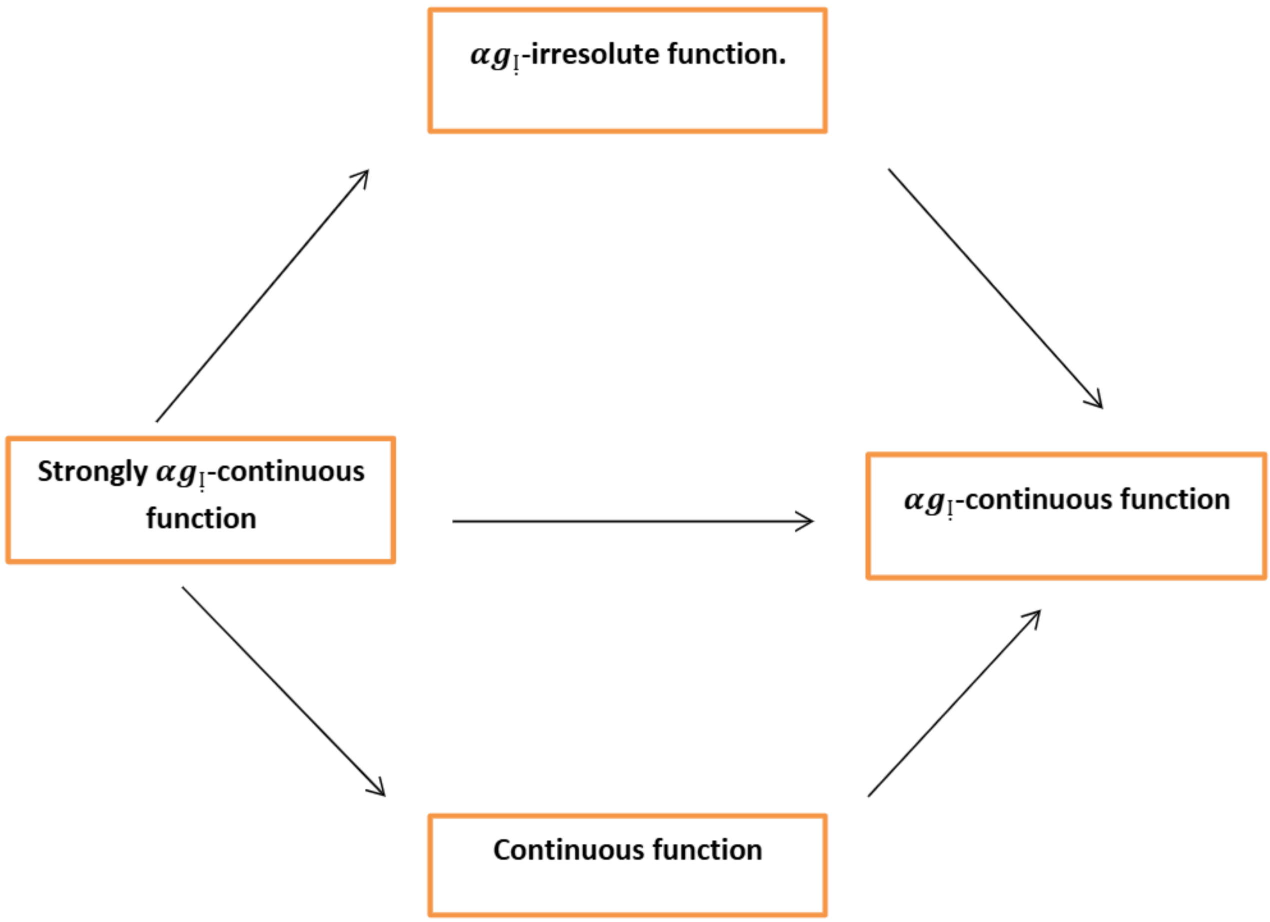
i. If ƒ is a continuous function, then ƒ is an αg_{I}-continuous function.
ii. If ƒ is Strongly αg_{I}-continuous function, then ƒ is a continuous function.
iii. If ƒ is an αg_{I}-irresolute function, then ƒ is an αg_{I}-continuous function.
iv. If ƒ is Strongly αg_{I}-continuous function, then ƒ is an αg_{I}-irresolute function.
v. If ƒ is Strongly αg_{I}-continuous function, then ƒ is an αg_{I}-continuous function.

Proof:

i. Let Ơ ∈ ɟ. Since ƒ is a continuous function, then $ƒ^{-1}$(Ơ) ∈ ĩ. $ƒ^{-1}$(Ơ) is an αg_{I}-open set in X̧ By Remark 2.4. Hence ƒ is an αg_{I}-continuous function.
ii. Let Ơ ∈ ɟ. By Remark 2.4, Ơ is an αg_{j}-open set in Ƴ. Since ƒ is Strongly αg_{I}-continuous function, then $ƒ^{-1}$(Ơ) ∈ ĩ. Hence ƒ is a continuous function.
iii. Let Ơ ∈ ɟ, this implies to Ơ is αg_{j}-open set in Ƴ. Since ƒ is an αg_{I}-irresolute function then $ƒ^{-1}$(Ơ) is an αg_{I}-open set in X̧. Then ƒ is an αg_{I}-continuous function
iv. Let Ơ is an αg_{j}-open set in X̧. Since ƒ is a Strongly αg_{I}-continuous function, then $ƒ^{-1}$(Ơ) ∈ ĩ. By Remark 2.4, ƒ(Ơ) is αg_{I}-open set in X̧. This implies ƒ is an αg_{I}-irresolute function.
v. Let Ơ ∈ ɟ this implies Ơ is an αg_{j}-open set and since ƒ is a Strongly αg_{I}-continuous function, thus $ƒ^{-1}$(Ơ) is open set in X̧ by Remark 2.4 $ƒ^{-1}$(Ơ) is an αg_{I}-open set, so ƒ is an αg_{I}-continuous function.

The follow scheme shows the relation between the variant notions were presented in Definition 4.1.

Arrow chart
(4.1)



Ị-α-g-continuous function

The following are some examples showing that the opposite direction of the above schema is incorrect.

Example 3: The function ƒ: (Ҳ, ĩ, Ị) → (Ҳ, ĩ, ʝ), where Ҳ={ė$_1$, ė$_2$, ė$_3$} such that ƒ(ė$_1$)=(ė$_1$), ƒ(ė$_2$)=(ė$_2$), ƒ(ė$_3$)=(ė$_3$), ĩ = {Ҳ,∅,{ė$_1$}}, Ị={∅} and ʝ={∅,{ė$_2$},{ė$_3$},{ė$_2$,ė$_3$}} then ĩ$_\alpha$={Ҳ,∅,{ė$_1$},{ė$_1$, ė$_2$},{ė$_1$, ė$_3$}} then $\alpha g_{Ị}C$(Ҳ) = {Ҳ,∅,{ė$_2$,ė$_3$}} and $\alpha g_{Ị}O$(Ҳ) = {Ҳ,∅,{ė$_1$}}. So $\alpha g_{ʝ}C$(Ҳ) = Ᵽ(Ҳ) and $\alpha g_{ʝ}O$(Ҳ) = Ᵽ(Ҳ).
It is possible to see clearly that ƒ is continuous and $\alpha g_{Ị}$-continuous function but not $\alpha g_{Ị}$-irresolute function since {ė$_3$} is an $\alpha g_{ʝ}$-open set in Ƴ but ƒ$^{-1}$(ė$_3$) = ė$_3$ is not an $\alpha g_{Ị}$-open set in Ҳ.
Example 4: The function ƒ: (Ҳ, ĩ, Ị) → (Ҳ, ĩ, ʝ), where Ҳ={ė$_1$, ė$_2$, ė$_3$} such that ƒ(ė$_1$)=(ė$_1$), ƒ(ė$_2$)=(ė$_2$), ƒ(ė$_3$)=(ė$_3$), ĩ = {Ҳ,∅,{ė$_1$}}, ʝ={∅} and Ị={∅,{ė$_2$},{ė$_3$},{ė$_2$,ė$_3$}} then ĩ$_\alpha$={Ҳ,∅,{ė$_1$},{ė$_1$, ė$_2$},{ė$_1$, ė$_3$}} then $\alpha g_{ʝ}C$(Ҳ) = {Ҳ,∅,{ė$_2$,ė$_3$}} and $\alpha g_{ʝ}O$(Ҳ) = {Ҳ,∅,{ė$_1$}}. So $\alpha g_{Ị}C$(Ҳ) = Ᵽ(Ҳ) and $\alpha g_{Ị}O$(Ҳ) = Ᵽ(Ҳ).

It is possible to see clearly that ƒ is $\alpha g_{Ị}$-continuous function but not continuous function since {ė$_1$}∈ ĩ but ƒ$^{-1}$(ė$_1$) = ė$_2$ is not open in Ҳ, and not Strongly $\alpha g_{Ị}$-continuous function since {ė$_1$} ∈ $\alpha g_{ʝ}O$(Ҳ) but ƒ$^{-1}$(ė$_1$) = ė$_2$ is not open in Ҳ.

5- Conclusion

The concept of closed and open sets was used with the ideal concept to introduce new notions from these categories; $\alpha g_{Ị}$-closed set, $\alpha g_{Ị}$-open set. And we introduce a new functions like: $\alpha g_{Ị}$-open function, $\alpha g_{Ị}^{*}$-open function, $\alpha g_{Ị}^{**}$-open function, $\alpha g_{Ị}$-closed function, $\alpha g_{Ị}^{*}$-closed function and $\alpha g_{Ị}^{**}$-closed function with near continuous functions.

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