Journal University of Kerbala , Vol. 15 No.3 Scientific . 2017

Study the effect of aberrations on Point spread function and encircled energy of annular aperture

دراسة تأثير الزيوغ على دالة الانتشار النقطية و الطاقة التجميعية الدائرية لفتحة حلقية

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Abstract:

In this work, the encircled energy of point spread function (PSF)for annular aperture have been studied. This study is done for the case of diffraction limited system and for different kinds of first and third order aberrations were occurred, like tilt error, focus error, spherical aberration, and coma aberration. The study showed that the spherical aberration (third order aberration) can be balanced with focus error (first order aberration) to reduce the effect of these errors on encircled energy. The same thing occurs with coma aberration (third order aberration) which balanced with tilt error (first order aberration).

Keywords: Point spread function, encircled energy, annular aperture.

الخلاصة:

تم في هذا البحث دراسة الطاقة التجميعية الدائرية لدالة الانتشار النقطية (PSF) للفتحة الحلقية .حيث تم دراسة حالة النظام المحدد بالحيود، وكذلك أنواع مختلفة من زيوغ الرتبة الأولى والثالثة مثل الخطأ البؤري ،خطأ الإمالة ،الزيغ الكروي والزيغ المذنبي ، وقد تبين ان تأثير الزيغ الكروي والخطأ البؤري يمكن ان يقل على الطاقة التجميعية الدائرية اذا تواجدا سويا حيث يوازن احدهما الآخر. ونفس الشئ يحدث مع الزيغ المذنبي الذي يتم موازنته مع خطأ الإمالة.

الكلمات المفتاحية: دالة الانتشار النقطية، الطاقة التجميعية الدائرية، الفتحة الحلقة.

1. Introduction

The most important consequence of point spread function (PSF) is the encircled energy (ENCE), which measures the part of total energy in PSF that lies inside a specified radius in the plane of observation or detection. It regards as one of the significant parameters serves as an index of the performance of an optical system [1,2].

Many studies on encircled energy have been adopted.In 1961, Lansrauxand Boivin studied the effects of spherical aberration on encircled energy and they showed that aberration always tends to decrease the factor from the value obtained with an Airy pattern, and they showed that this factor may be increased by the use of an amplitude filter at the pupil of the optical system [2]. In 1988, J. Campos et.al.studied resolving power and encircled energy in aberrated optical systems with filters optimized for the strehl ratio [3].Then in1997, S.J. Park and C. S.Chung studied The influence of Bessel and Bessel-Gauss beams acting as a combined filter of amplitude and phase on the point spread function and the encircled energy for an optical system having spherical aberration,and they investigated it numerically [4]. In 2011, A. Srisailamet.al .studied encircled energy factor as a point-image quality-assessment parameter [1]. While in2013 A. Srisailam studied excluded energy and relative encircled energy that play important role in the optical systems as point-image quality-assessment parameters [5]. In 2013,C.Vijender, et.al.studied encircled energy factor in the PSF of an amplitude apodisedoptical system [6].

In this work, the ENCE were studied for the annular aperture, where many instruments, like the Cassegrain telescope, have a central opaque disk at the center of the entrance pupil. Then, the effective pupil is not a clear disk but an annular aperture [7].

2. Encircled energy for circular aperture

Encircled energy (ENCE) is the ratio of the energy in the PSF (or spot diagram) that is collected by a single circular detector element to the amount of the total energy that reaches the image plane from an object point. It can be computed for objects at different fields of view. This is a common metric for point images, especially for systems which need high signal to noise ratio.
The equation of ENCE for a circular aperture can be written as[1,8]

$$\mathbf{EE(w)} = \int_0^{2\pi}\int_0^{w} \mathbf{PSF(w,\psi)\, w\, dw\, d\psi} \qquad (1)$$

Where w and ψ are polar coordinates in the plane of observation, and PSF(w,ψ) is the point spread function in the point (w,ψ).
The above equation can be written in Cartesian coordinates as

$$\int_{-d}^{d}\int_{-\sqrt{d-v^2}}^{+\sqrt{d-v^2}} PSF(u,v)\, dv\, du \qquad (2)$$

Where u and v are Cartesian coordinates in observation plane, and if z=2πu and m=2πv, PSF for a circular aperture can be written as

$$\mathbf{PSF} = \mathbf{N}\left|\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{+\sqrt{1-x^2}} \mathbf{f(x,y)\, e^{i(zx+my)}\, dydx}\right|^2 \quad (3)$$

Where N is the normalization constant and f(x,y) represents pupil function which consists of two parts[9]:

$$\mathbf{f(x,y)} = \boldsymbol{\tau}(\mathbf{x,y})\boldsymbol{e^{ikw(x,y)}} \qquad (4)$$

τ(x,y) represents the real amplitude distribution in exit pupil coordinates (x,y), and it is called "pupil transparency" and $e^{ikw(x,y)}$ is the wave front of aberration function and W(x.y) is the aberration factor.

The intensity distribution on the two axes z and m are symmetric; so, the last equation can be reduced to one axis only, therefore, equation (3)becomes

$$\mathrm{PSF} = \mathrm{N}\left|\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{+\sqrt{1-x^2}} f(x,y)\, e^{i(zx)}\, dydx\right|^2 \quad (5)$$

3. Encircled energy for annular aperture

The annular aperture is a circular aperture with an obscuration of smaller radius , as shown in **(Figure 1)**.The equation of encircled energy for obscured circular aperture is given by:

$$\mathbf{E\,NCE} = \mathbf{n.f}\int_{-d}^{d}\int_{-\sqrt{d^2-v^2}}^{\sqrt{d^2-v^2}}\left|\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{+\sqrt{1-x^2}} \mathbf{f(x,y)e^{i[(2\pi u)x+(2\pi v)y]}dydx} - \int_{-\varepsilon}^{\varepsilon}\int_{-\sqrt{\varepsilon^2-x^2}}^{+\sqrt{\varepsilon^2-x^2}} \mathbf{f(x,y)e^{i[(2\pi u)x+(2\pi v)y]}dydx}\right|^2 \mathbf{dudv} \quad (6)$$

Where n.f is the normalizing factor for ENCE.

4. Results and discussion

A MATLAB (version7) code were used in this work to solve the equations of ENCE, and, as it seen, these equations were too difficult to solve directly, and therefore, they were programmed using Simpson numerical method. The obscuration ratio is used here for annular aperture is equal to 0.5, and in the following we will illustrate the studied cases.

4.1 ENCE for annular aperture with diffraction limited system

Figures 2 and 3 (red curves) shows PSF and ENCE respectively for the system with no aberrations. In figure 3, the ENCE increased rapidly till it reach (u=0.4), which represents ($z=2\pi u\approx 2.5$), and this represents the region of central spot in PSF, then ENCE increased slowly till (u=1 or $z\approx 6$) which is the end of second maximum in PSF

After that the ENCE increased so slowly or became nearly constant, and in this case, the central spot has 86.8% of the total intensity, while the intensity became 98.6% at the end of the second peak.

4.2 ENCE for annular aperture with focus error and spherical aberration

The wave front aberration function can be expressed as a series of terms [10]

$$W = W(\sigma, r, \Phi) = \sum_{klm} \mathbf{w_{klm}\sigma^k r^l \cos^m} \Phi \qquad (7)$$

Where w_{klm} is the aberration factor, σ principal ray height from optical axis in exit pupil, r is the radial distance of wave in exit pupil, and Φ is the angle between r and x axis, while k, l, and m represent powers of σ, r, and Φ respectively.

The equations of focus error w_{20} or w_{020} and spherical aberration w_{40} or w_{040} can be respectively written as[11]

$$W = w_{020}(x^2+y^2) \quad \text{where } r^2=x^2+y^2 \qquad (8)$$

$$W = w_{040}(x^2+y^2)^2 \qquad (9)$$

Figures 2 and 3 represent PSF and ENCE respectively for the annular aperture with focus error and spherical aberration. They showed that the area under the curve of PSF and ENCE decreased as the values of w_{20} or w_{40} increased.

As w_{20} and w_{40} increased by (0.25, 0.5, 0.75, and 1), total intensity decreased by (9.4%, 32.8%, 59%, and 78.6%) and (14.6%, 45.6%, 70.5%, and 80.9%) respectively, while the percentage amount of the intensity of the central spot as w_{20} and w_{40} increased are (85.1%, 79.3%, 67.3%, and 61.8%) and (84.4%, 75.9%, 66.1%, and 62.3%

respectively. Also it can be seen that the first and second peak were displaced away from the central in w_{40} more than w_{20}. So, it's obvious that the spherical aberration affect PSF and ENCE more than focus error.

These two errors can be balanced with each other giving less effects on PSF and ENCE as illustrated in figures (4-a) and (5-a), where the color curves of different values of ($w_{20}= - w_{40}$) are close to each other unlike those of only w_{20} or only w_{40}, and the second peak displacement is decreased. The total intensity decreased by only (1.1%, 4.3%, 9.2%, and 15.5%), and the percentage amount of the intensity of the central spot are (86.7%, 86.4%, 86%, 85.4%) as $w_{20}=w_{40}$ increased.

4.3 ENCE for annular aperture with tilt error and comma aberration

The equations of tilt error w_{11} and comma aberration w_{31} can be respectively written as[11]

$$W = w_{011}\, x \qquad (10)$$

$$W = w_{031}(x^2+y^2)\, x \qquad (11)$$

Figures 2 and 3 represent the PSF and ENCE respectively for the annular aperture with tilt error (c) and comma aberration (d), and also the area under the curve of PSF and ENCE decreased as the values of w_{11} or w_{31} increased.

When w_{11} and w_{31} increased by (0.25, 0.5, 0.75, and 1), total intensity decreased rapidly by (51.4%, 84.1%, 80.4%, and 95.4%) and (31.2%, 75%, 93.1%, and 97.4%) respectively, while the percentage amount of the intensity of the central spot with respect to total intensity as w_{11} and w_{31} increased are (72.7%, 35.8%,82.2 and 72.2%) and (82.2%, 65.5%, 45.1%, and 46.2%) respectively. So, it's obvious that the tilt error and comma aberration affect the intensity more than focus error and spherical aberration.

Comma aberration can be balanced with tilt error giving less effect on PSF and ENCE by choosing appropriate value of their ratio. Figures 4-b and 4-c for PSF and Figures 5-b and 5-c for ENCE, showed two kinds of balancing $w_{31}=w_{11}$ and $w_{31}=0.5w_{11}$ respectively. To compare between the two cases it found that for $w_{31}=w_{11}$, the intensity of total PSF is decreased as the value of errors increased (0.25-1) by (9.3%, 31.3%, 53,5%, and 66.5%), while for $w_{31}=0.5w_{11}$ the percentage decreases are (5.89%, 20.6%, 38.2%, and 53.6%). And the percentage intensity of the central spot are (84.6%, 76.5%,59.4%, and 37.6%) for $w_{31}=w_{11}$, while for $w_{31}=0.5w_{11}$ (86.1%, 84.1%,81.2%, and78.4%), and it's obvious that the second type, $w_{31}=0.5w_{11}$, is better than $w_{31}=w_{11}$ in making the effect of these errors minimum, i.e. it must be looking for best ratio of the balanced error.

4.4 ENCE for annular aperture with different values of obscuration ratio

Figure 6represents PSF and ENCE for annular aperture with different values of obscuration ratio from $\varepsilon=0$ of no obscuration to $\varepsilon=0.6$. This figure shows that the area under the curve of PSF and ENCE decreases as ε increases.

References

[1] A. Srisailam, V. Dharmaiah, M.V Ramanamurthy.and P.Kumar Mondal, ,Adv. Appl. Sci. Res, 2011, 2, 6, 145-154.

[2] G. Lansraux and G., Can. J. Phys., 1961, 39, 158-189.

[3] J. Campos, F. Calvo, and M. J. Yzuel,J . Optics, 1988, 19, 3, 135-144.

[4] S.J. Park and C. S.Chung , J. K. P. S.,1997,30, 2, 194-201.

[5] A. Srisailam, , Ultra Scientist, 2013, 25, 2, A 333-338.

[6] C.Vijender, A.Srisailam, M.V.Ramana Murthy, I. J. E. R. A., 2013, 3, 3, 1-3.

[7] D. Malacara and Z. Malacara, Handbook of Optical Design, 2nded., Marcel Dekker, America, (2004).

[8] J. E. Greivenkamp, Field Guide to Geometrical Optics, SPIE Press, Bellingham, WA (2004).

[9] J.W. Goodman , " Introduction to Fourier Optics ", 2nd edition, McGraw-Hill Companies, Inc., America, (1996).

[10] R.D.Guenther, Modern Optics, Ch. 5, John Wiley& Sons, Inc., Canada , (2005).

[11] Robert D.Guenther, " MODERN OPTICS", Chapter four, John Wiley & Sons, Inc., Canada, (2005).

The Figure

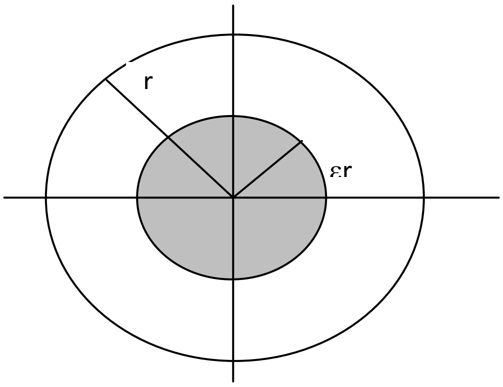


Figure 1: The Obscured Circular Aperture

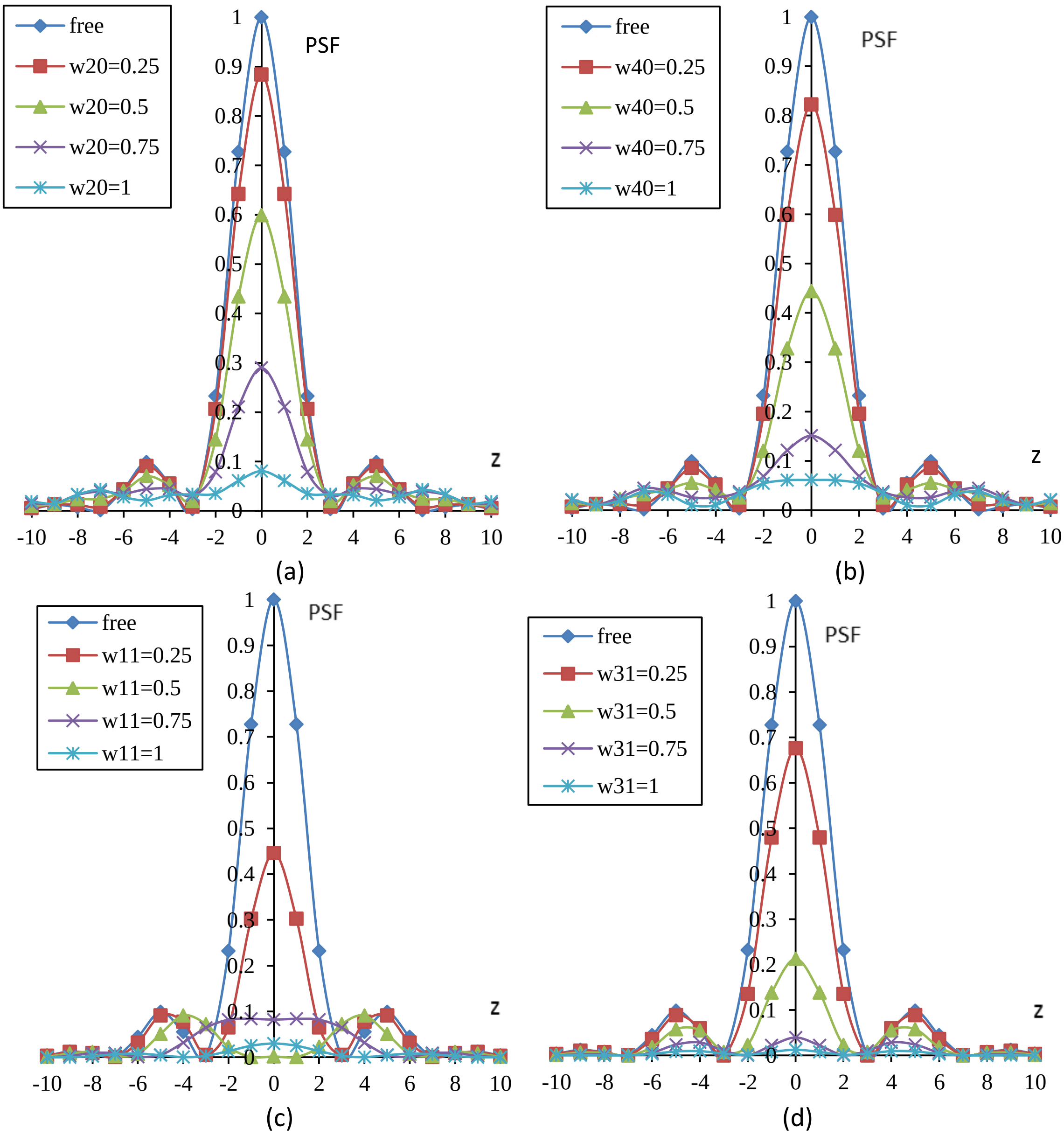


Figure 2 :PSF for annular aperture of obscuration ratio (ε =0.5) with.
a) different values of W_{20} (focus error) b) different values of W_{40} (spherical aberration)
c) different values of W_{11} (tilt error) d) different values of W_{31} (comma aberration).

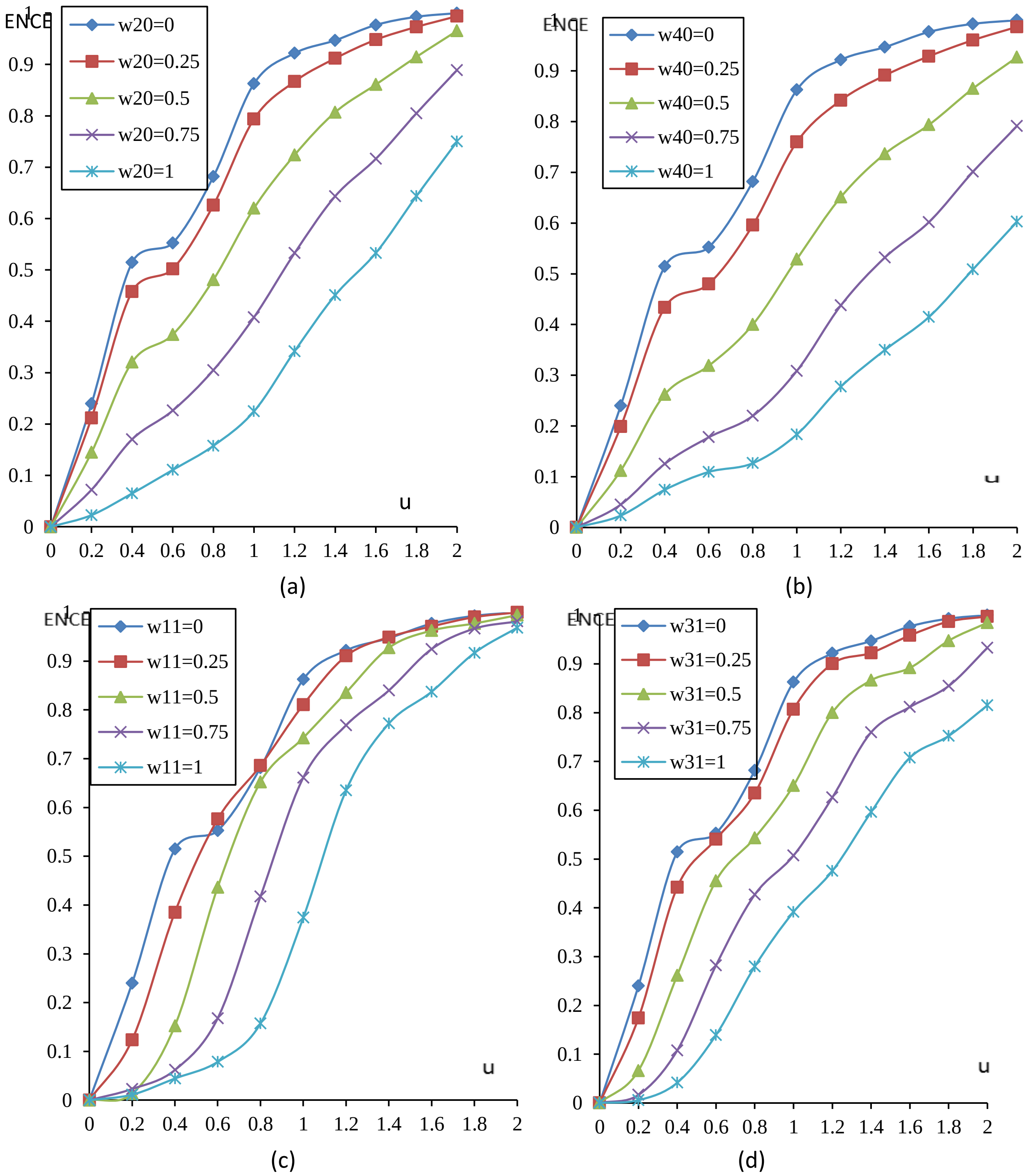


Figure 3:ENCE for annular aperture of obscuration ratio (ε =0.5) with.
a) different values of W_{20} (focus error) b) different values of W_{40} (spherical aberration)
c) different values of W_{11} (tilt error) d) different values of W_{31} (comma aberration).

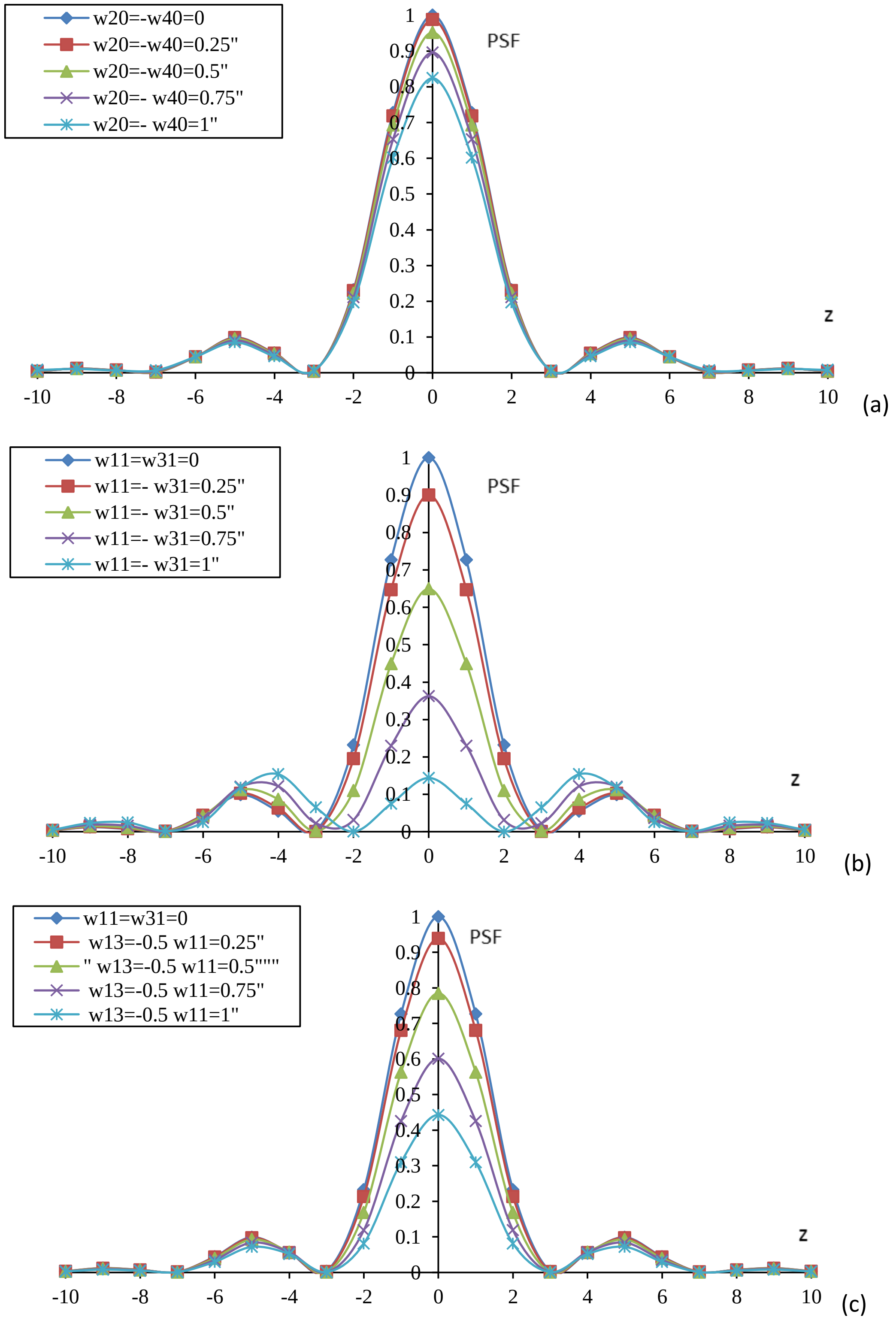
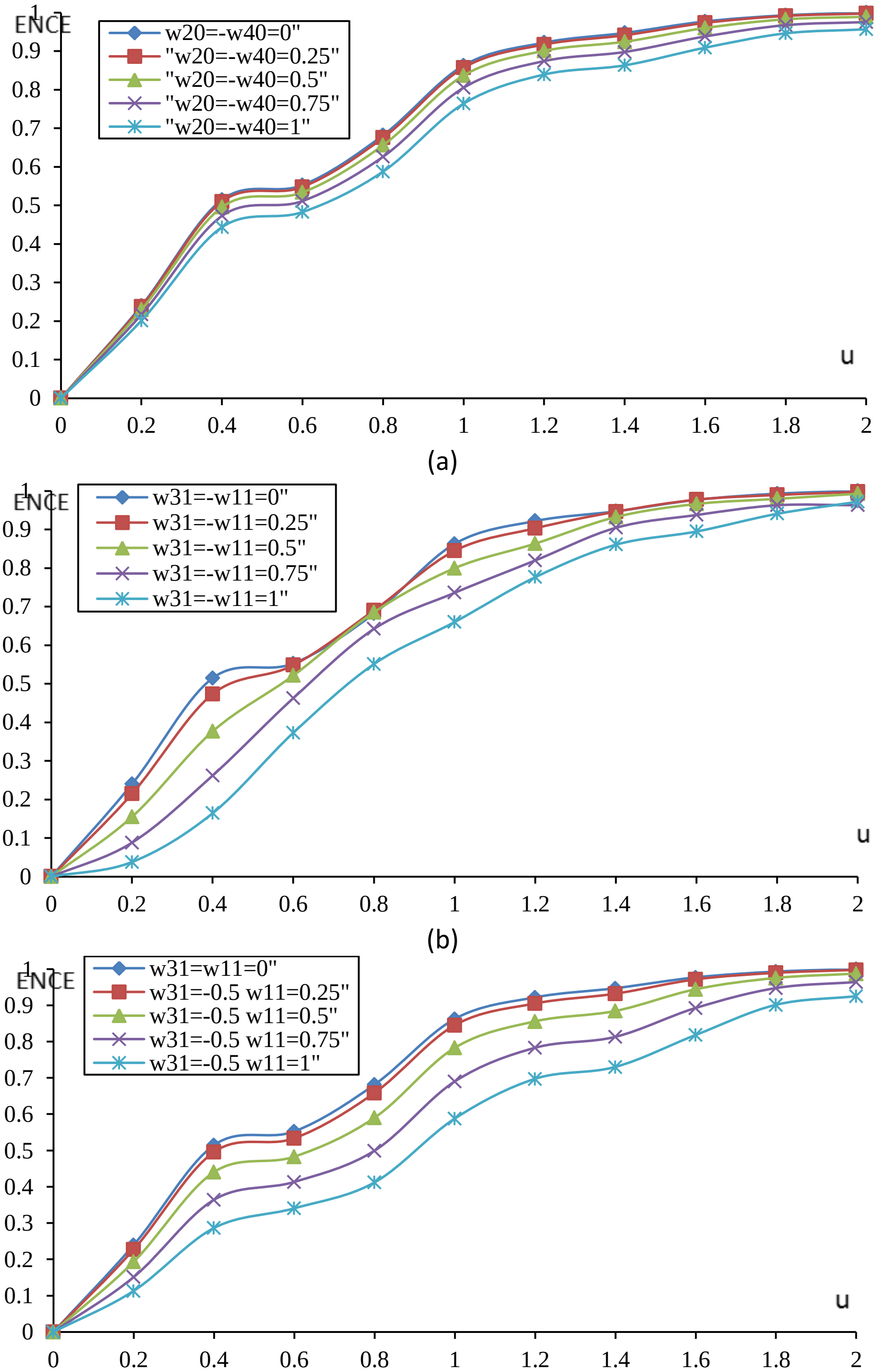


Figure 4: PSF for obscured circular aperture(ε =0.5) with different values of balanced aberrations

a) $W_{20}= -W_{40}$

b) $W_{31}=- W11$

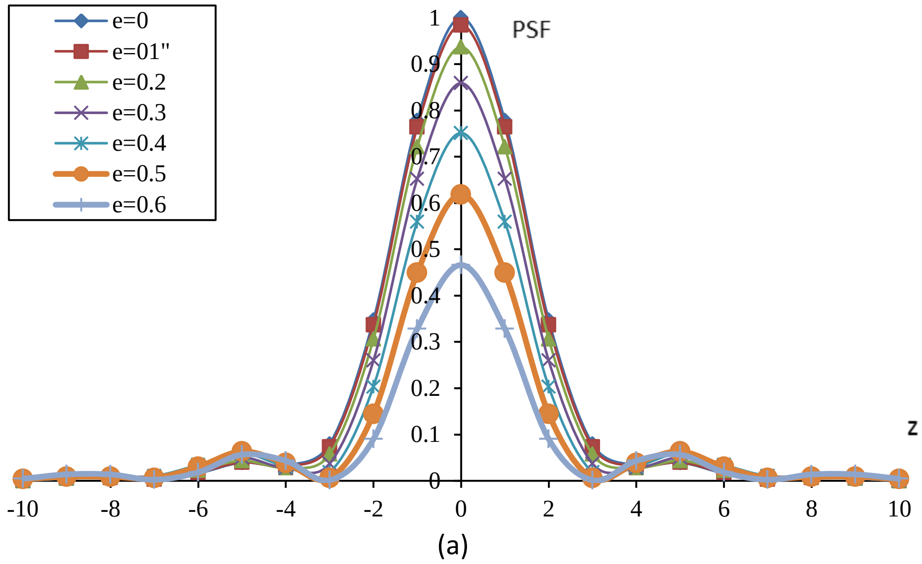
c) $W_{31}=- 0.5W_{11}$



(c)

Figure 5: ENCE for obscured circular aperture(ε =0.5) with different values of balanced aberrations

a) W20= -W40
b) W31=- W11
c) W31=- 0.5W11



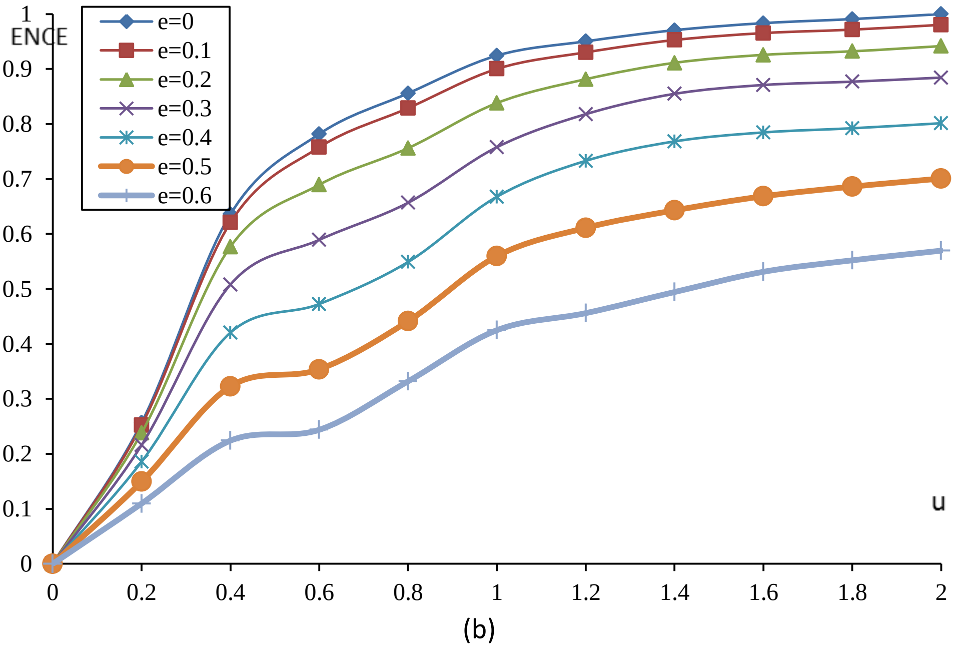


Figure 6: a) PSF for annular aperture with different obscuration ratio
b) ENCE for annular aperture with different obscuration ratio.