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FINITE ELEMENT APPLICATION FOR MODIFIED CAM CLAY MODEL TO ANALYZE SILTY CLAY SOIL UNDER STRIP FOOTING USING MATLAB

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ABSTRACT

In this paper, the nonlinear behavior of soil has been studied. For this purpose, the modified Cam clay model has been employed. The foundation taken for this study is a strip footing of width (B=2.0 m) resting on the surface of silty clay soil. A two-dimensional finite element problem has been taken as a plane strain problem. The displacements and stresses under the strip footing during applied incremental loading sequences has been estimated by a program written in MATLAB. The influences of incremental loading, width of footing, and depth of footing are considered in this paper.

KEYWORDS: ***nonlinear soil, modified Cam clay, MATLAB, finite element method, strip footing***

الملخص

في هذا البحث تم دراسة التصرف اللاخطي للتربة, ولهذا الغرض تم استعمال نموذج طين كام المعدل (modified Cam clay). تم استعمال أساس شريطي بعرض(2م) فوق تربة طينية غرينية. هذا البحث يُقدمُ طريقة العناصر المحددة الثنائية الأبعاد. الإزاحات والإجهادات تحت أساس شريطي أثناء تطبيق تحميل تزايدي متعاقب تم احتسابها باستعمال برنامج تم كتابته بالـ MATLAB. تم في هذا البحث دراسة تأثير التحميل المتزايد، وعرض وعمق الأساس.

1. INTRODUCTION

The finite element method is a powerful tool for analysis of a wide range of problems in engineering. In geotechnical engineering, this method plays a key role in solving nonlinear problems where analytical solutions are not available. The material nonlinearity occurs when the relationship between stresses and strains is not linear. Deformation behavior of soil is influenced by a number of factors, such as physical structure, porosity, density, stress history, loading characteristics, existence and movement of fluid in pores, and time-dependence of soil skeleton and the pore fluid. These factors render the stress deformation behavior of soil highly complex and nonlinear. No available analytical solution can handle them all.

Numerical methods, such as finite element method have been successful in approximating the effects of many of these factors [1].
The application of plasticity theory in soil mechanics has been employed fruitfully over the last 30 years, with a major milestone being the development of critical state models by Roscoe et al. at the University of Cambridge. In recent times, these classical critical state models have been modified in various ways by many researchers to cover different soil types and loading conditions in an attempt to achieve a better predication of experimental data [2].
The modified Cam clay model (Roscoe and Burland [3]) is widely referenced and has been widely used in solving boundary value problems in geotechnical engineering practice (e.g., Gens and Potts [4]; Yu [5]; Potts and Zdravkovic [6]).
In practice, most foundations are flexible. Even very thick ones deflect when loaded by the superstructure loads [7].
The solutions of displacements and stresses for various types of applied loads in homogeneous and nonhomogeneous isotropic/anisotropic full/half-spaces have played an important role in the design of foundations. However, it is well known that a strip load solution is the basis of complex loading problems for all constituted materials. A large body of the literature was devoted to the calculation of displacements and stresses in isotropic media with the Young's or shear modulus being not constant.

2. MODIFIED CAM CLAY MODEL

The modified Cam clay model was proposed by Roscoe and Burland [3] and a description and systematic study of the model can be found in the text by Muir Wood [8]. Formulations of this model suitable for use in finite element analysis can also be found in various texts (e.g., Britto and Gun [9]; Potts and Zdravkovic [6]).
Nonlinear stress-strain behavior may be approximated in finite element analyses by assigning different modulus values to each of the elements into which the soil is subdivided for purposes of analysis.
To describe the stress strain relationships, the Young's modulus E, Poisson's ratio ν, and bulk modulus K are to be used, related to each other by the following relations:

$$E = 3K(1-2\nu) \qquad (1)$$

The elastic behavior of soils is nonlinear and stress dependent. Therefore, the elastic moduli need to be presented in incremental form. For soils modeled using the modified Cam clay model, the bulk modulus K is stress dependent (i.e., K is not a constant). The bulk modulus depends on the mean effective stress p', void ratio e_o, and unloading–reloading line slope κ.
The following equation can be obtained from the equation of the unloading–reloading line used for consolidation analysis, describing the elastic behavior of soil:

$$K = \frac{(1+e_o)\,p'}{\kappa} \qquad (2)$$

Where:

$$p' = \frac{(\sigma_1' + 2\sigma_3')}{3} \qquad (3)$$

σ_1' and σ_3' = the major and minor principal stresses; κ = unloading–reloading line slope, as shown in Figure (1).

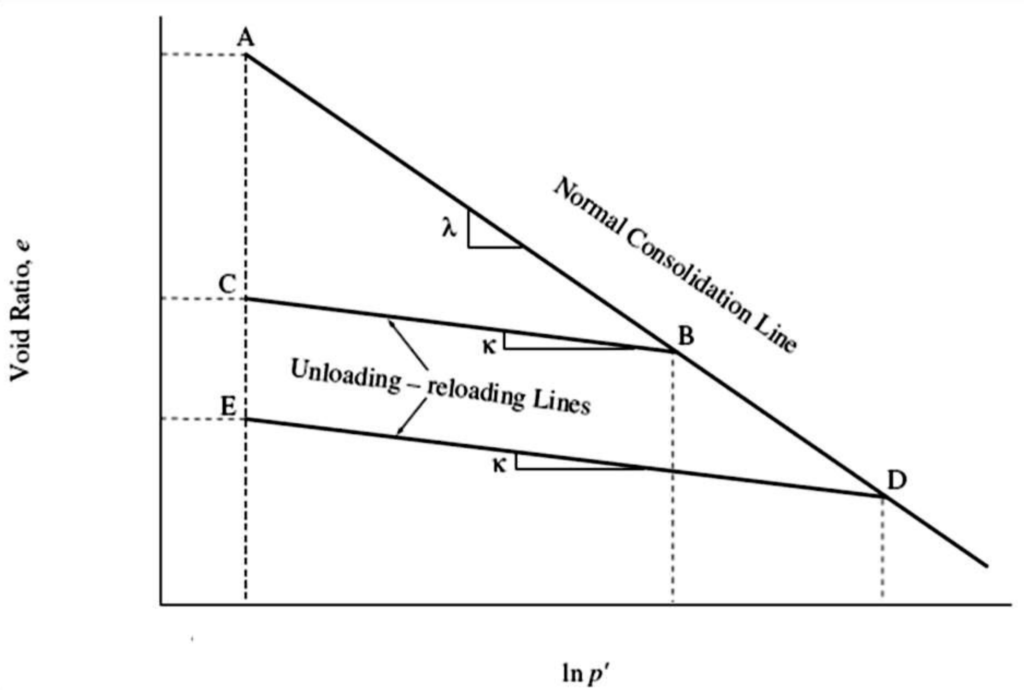

Fig. 1 The relationship between void ratio (e) and ln (p').

Substituting eq. (2) into eq. (1) we can obtain

$$E = \frac{3(1-2\nu)(1+e_o)\,p'}{\kappa} \qquad (4)$$

E is stress dependent, and functions of mean effective stress p', void ratio e_o, unloading–reloading line slope κ, and Poisson's ratio ν.

3. FINITE ELEMENT ANALYSIS

The finite element technique has been a powerful engineering analysis tool, and versatile numerical method of considerable potential for simulating a real problem in the field and the laboratory; because it essentially permits the realistic molding of more aspects of problems than do alternative techniques.

The initial development of the finite element method for aerospace and structural engineering was soon followed by application of the method to problems in soil and rock mechanics **[10]**. Geomechanics is one field in which a significant expansion of the application of finite elements has occurred in the past five decades.

The finite element method is of practical value since it is capable of predicting the deformation, states of stresses and strains, and the localized failure zones around the soil-structure interface throughout the entire range of loading up to ultimate load **[11]**.

Geotechnical engineers have recognized in the finite element method a tool by which many of their complicated analytical problems can be attacked in nearly their full complexity **[12]**.

One of the essential ingredients for a successful finite element analysis of a geotechnical problem is an appropriate soil constitutive model **[13]**.

The advantage of an arbitrary triangular shape is to approximate to any boundary shape **[14]**.So the triangular element shape is considered in this research.

In the present study, 3-noded triangles elements with two degree of freedom at each node have been used to model soil. In each increment of the analyses, the stress-strain behavior of the soil is treated as being linear, and the relationship between stress and strain is assumed to be governed by the generalized Hooke's law of elastic deformations, which may be expressed as follows for conditions of plane strain case**[15]**:

$$\begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\tau_{xy} \end{Bmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\gamma_{xy} \end{Bmatrix} \qquad (5)$$

Where:

$\Delta\sigma_x$, $\Delta\sigma y$ and $\Delta\tau_{xy}$ = the increments of stress during a step of analysis.

$\Delta\varepsilon_x$, $\Delta\varepsilon_y$ and $\Delta\gamma_{xy}$ = the corresponding increments of strain.

E = the value of Young's modulus.

ν = the value of Poisson's ratio.

3.1 About MATLAB

MATLAB is an interactive software which has been used recently in various areas of engineering and scientific applications. It is not a computer language in the normal sense but it does most of the work of a computer language. Writing a computer code is not a straightforward job; typically boring and time consuming for beginners. One attractive aspect of MATLAB is that it is relatively easy to learn. It does not require in-depth knowledge on operational principle of computer programming like compiling and linking in most of other programming languages.

The power of MATLAB is represented by the length and simplicity of the code. For example, one page of MATLAB code may be equivalent to many pages of other computer language source codes. In general, MATLAB is a useful tool for vector and matrix manipulations. Since the majority of the engineering systems are represented by matrix and vector equations, we can relieve our workload to a significant extent by using MATLAB.

The finite element method is a well defined candidate for which MATLAB can be very useful as a solution tool. Matrix and vector manipulations are essential parts in the method **[16]**.

The MATLAB programming language is useful in illustrating how to program the finite element method due to the fact it allows one to very quickly code numerical methods and has a vast predefined mathematical library. A simple two dimensional finite element program in MATLAB need only be a few hundred lines of code whereas in Fortran or C++ one might need a few thousand **[17]**.

3.2 The Finite Element Computer Program by MATLAB

A computer program designed by the author was used in the finite element analysis carried out during this research. The program allows for triangle type of elements to be used in the finite element mesh in solving soil problems under plane conditions (strain or stress). The behavior of the soil can be approximated by the modified cam clay model (Roscoe and Burland **[3]**). The model that is considered in this work is nonlinear, isotropic on primary loading with different moduli.

The sign convention for the stresses and the convention for numbering the nodes of elements are shown in Figure (2). The program presents the results of analysis as the displacements of the nodal points, and the value of stresses developed at the centre of each element at the end of each solution increment.

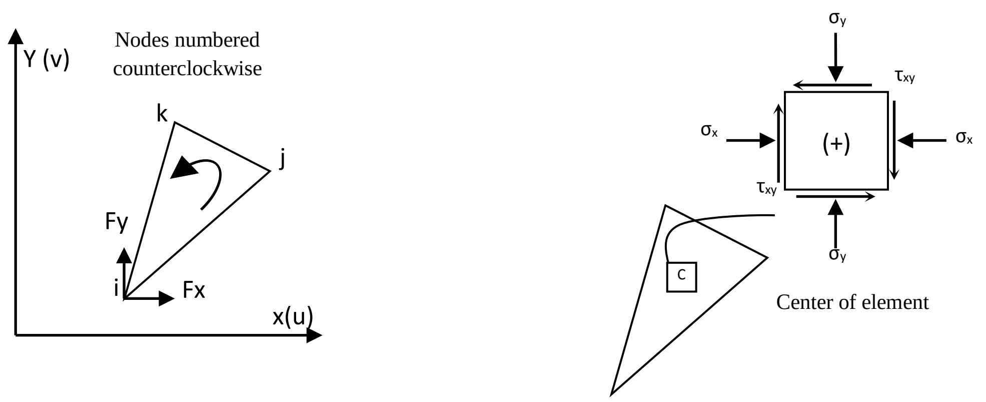

Fig. 2 Sign convention and element numbering.

Figure (3) is a flowchart that illustrates the main features of the solution procedure adopted in the finite element computer program.

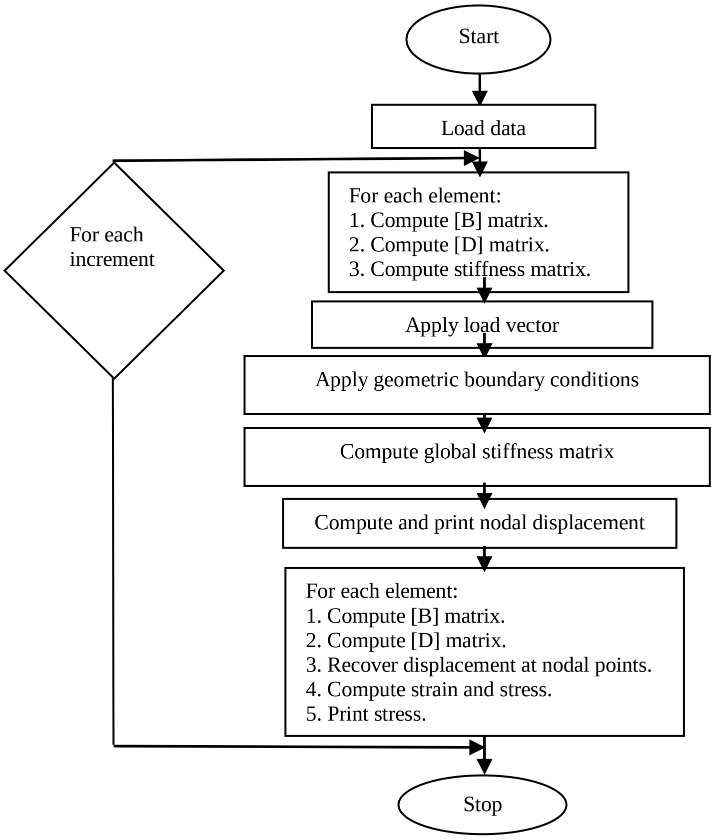

Fig. 3 Simplified flow chart of the finite element program.

3.3 Verification of the Computer Program

The author has used this program in a different problem (Figure 4) presented by another researcher (e.g. Kattan **[18]**).

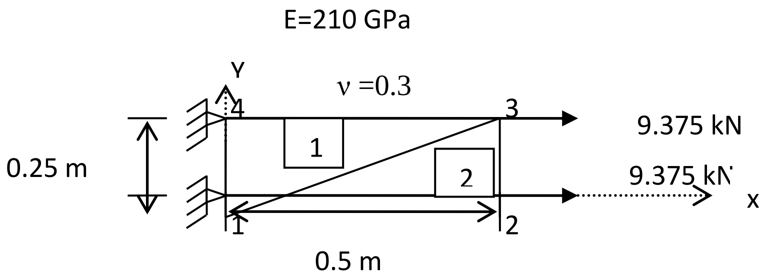

Fig.4. Mesh and data for different problems (after Kattan [18]).

The results obtained by the program in this research were compared with results presented by Kattan **[18]**. In all comparisons, excellent agreement was found between the present program results and those published, as shown in Table 1.

Table 1. Comparison with the theoretical results.

Item considered	Kattan results	Author results
Hor. Disp. of node 2	0.7111E-005	7.1111175E-006
Ver. Disp. of node 2	0.1115E-005	1.1151779E-006
Hor. Stress at elem. 2	2.9856E+003	2.9855885E+003
Ver. Stress at elem. 2	- 0.0036E+003	- 3.6028823E+000

4. PROBLEM GEOMETRY

The case study is treated as plane strain two-dimensional problem for simplicity when analyzed by the finite element method. The shape of elements used is the triangular element because of its suitability to simulate the very important behavior of soils under strip footing.
The basic problem chosen for the parametric study shown in Figure (5.a), involves a soil stratum, 21.0 m thick and 28.0 m width, of a silty clay soil under laid by bedrock and loaded by strip sequence loadings (5, 10, 15, 20, 25, 30 kPa) with base width equal to 2.0 m.
The finite element mesh (Figure 5.b) used consists of 989 nodal points and 1848 triangular two-dimensional elements. The nodal points along the bottom boundary of the mesh are assumed to be fixed both horizontally and vertically. The nodes on the right and left ends of the mesh are fixed in the horizontal direction while they are free to move in the vertical direction. All interior nodes are free to move horizontally and vertically.

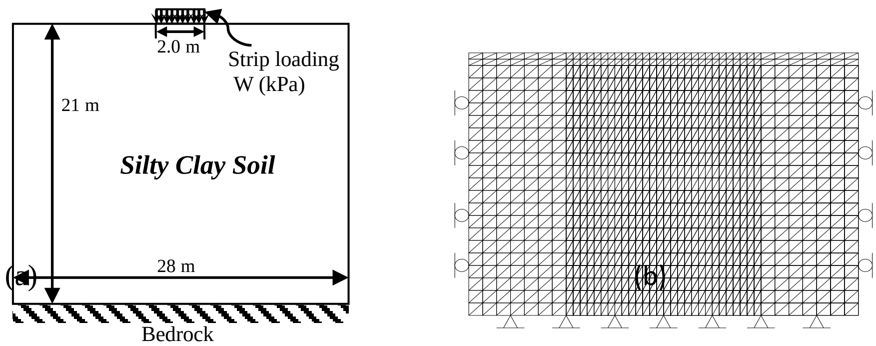

Fig. 5 The basic problem for the parametric study.

5. MATERIAL CHARACTERIZATION

The stratum is silty clay soil considered to have modified Cam clay parameters as reported in Table 2, **[19]**. The elastic behavior of soils is nonlinear and E is not constant but a function of mean effective stress p', void ratio e_o, unloading–reloading line slope κ, and Poisson's ratio ν.

Table 2. Material characteristics.

Unloading–reloading line slope(κ)	**0.032**
Void ratio (e_o)	**1.78**
Poisson's ratio (ν)	**0.4**
The coefficient of lateral earth pressure at rest (K_o)	**0.64**
Effective unit weight (γ')	**8 kN/m^3**

6. RESULTS AND DISCUSSIONS

In this study, a model of silty clay soil was analyzed under uniformly flexible strip loading with variable soil modulus. In order to develop more knowledge about the behavior of soils under strip loading problems, a parametric study is performed by varying the basic problem parameters and comparing these results with the original basic problem results. The results of increasing the load, and changing the depth (D_f) and width (B) of footing are presented as follow:

For uniformly flexible strip loaded area the contact settlement under the strip footing is shown in Figure (6) and the load-settlement curve is plotted for the node lying directly below the footing at its center line as shown in Figure (7). From Figure (6) it can be noticed that the settlement at the center is much larger than the settlement at the edge of the loaded area and the settlement profile is symmetric and parabolic in shape for the load increments. These results agree with the results found by Wu **[20]** and Das **[21]**. Also the vertical displacement increases in direct proportion to the pressure of the loaded area, as shown in Figures (6) and (7), which agrees with that reported by Craig **[22]**.

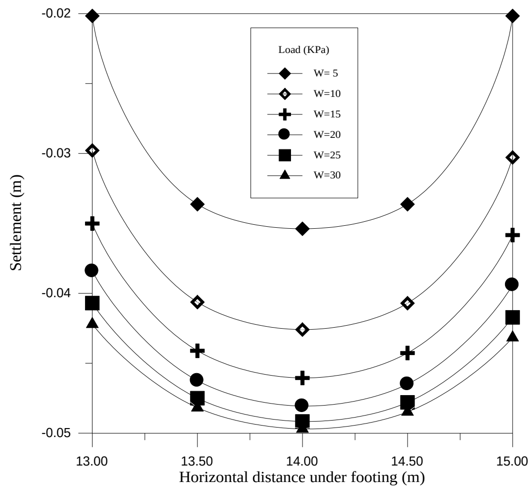

Fig. 6 Contact settlements under the strip Loadings with base width equal to (2.0 m).

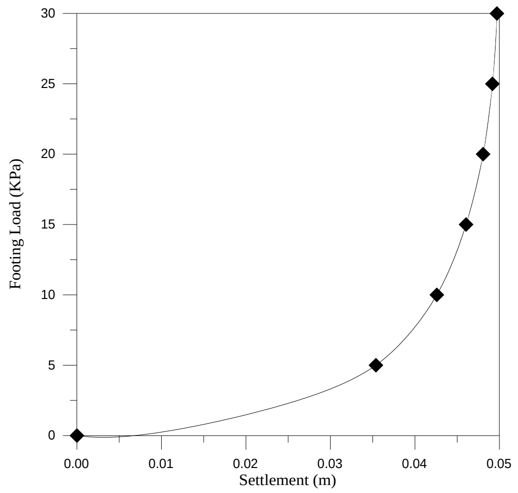

***Fig. 7 Load-settlement curve*.**

The immediate settlement at the center of the loaded area is reduced when the strip footing is placed at some depth ($D_f \leq B$) in the ground, as shown in Figure (8). These results agree with that mentioned by Fox in Bowels' book [7].

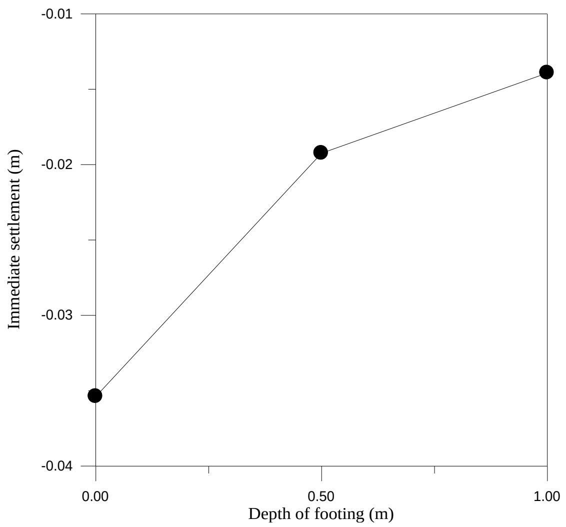

Fig. 8 Immediate settlements at the center of the strip loading (5 kPa) with base width (2m) according to depth of footing.

The vertical displacement (immediate settlement) increases in direct proportion to the width of the loaded area (size of the footing), as shown in Figure (9), which agrees with that reported by Wu **[20]** and Craig **[22]**.

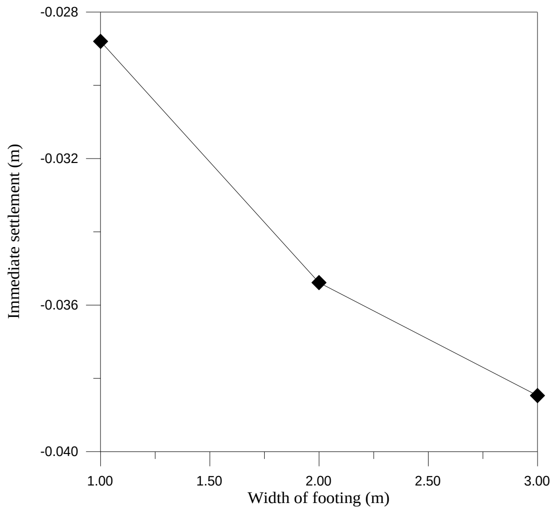

Fig. 9 Immediate settlements at the center of the strip loading (5 kPa) with (D_f =0) according to width of footing.

7. CONCLUSIONS

The results obtained from this study indicate that the computer program can simulate the analysis of the nonlinear behavior of silty clay soil, which had a variable soil modulus and loaded with incremental strip loading.

This paper shows how the computer solutions may be used to improve the prediction of settlements and stresses beneath a strip footing resting on silty clay soil.

Displacements and stresses can be calculated with knowledge of soil stiffness beneath the footing where soil stiffness is stress dependent, soil Poisson's ratio, depth to an incompressible layer, and footing width.

The immediate settlement at the center is much larger than the settlement at the edge of the strip flexible loaded area and the settlement profile is symmetric and parabolic in shape for the load increments. The immediate settlement increases in direct proportion to the pressure and the width of the strip loaded area. The immediate settlement of the strip loaded area decreases when the depth of strip footing increases. The results compare favorably with available published analytical and numerical solutions.

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