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Some Estimations for the Parameters and Hazard Function of Kummer Beta Generalized Normal Distribution

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Abstract:

Transforming the common normal distribution through the generated Kummer Beta model to the Kummer Beta Generalized Normal Distribution (KBGND) had been achieved. Then, estimating the distribution parameters and hazard function using the MLE method, and improving these estimations by employing the genetic algorithm. Simulation is used by assuming a number of models and different sample sizes. The main finding was that the common maximum likelihood (MLE) method is the best in estimating the parameters of the Kummer Beta Generalized Normal Distribution (KBGND) compared to the common maximum likelihood according to Mean Squares Error (MSE) and Mean squares Error Integral (IMSE) criteria in estimating the hazard function. While the practical side showed that the hazard function is increasing, i.e. the increment in staying the teachers in the service, they will be exposed to a greater failure rate as a result of the staying period which decreases in its turn.

Key words: **Kummer Beta Generalized Normal model, distribution, mean squares, Integrated mean squares error, maximum Likelihood method.**

1, Introduction

Reliability and survival analysis have one essential characteristic, which is a measure of the length of life, whether it is for a machine, system, or organism. The basic idea of reliability is to know the behavior of failure for a certain period in order to reduce production and maintenance costs, especially in the field of survival theory, this topic is especially important in the environment of modern industry and technology because Focuses on the life span of equipment and systems or on the failure of these systems or their durability within a period of time, from here the need to adopt the theory of reliability in studying and analyzing phenomena according to statistical data and thus carrying out the estimation process based on statistical methods, including the method of greatest possibility. There is also a clear need for extended forms of distributions such as exponential generalized (EG), beta generalized (BG), Kumaraswamy Generalized (KWG), a highly versatile set of distributions for analyzing different types of data. These families have been extensively studied in statistics and some authors have developed several proprietary EG, BG and KwG models. However, the beta, kumaraswamy and exponential generators also do not provide maximum elasticity (right and left) of probability density functions (pdfs). For this reason, it is not suitable for analyzing real data with high levels of asymmetry. Therefore, he proposed a Kummer beta generalized, which is one of the distributions that provides more flexibility in extreme cases[2;2012].

The aim of this research is to estimate the hazard function (failure rate) and the parameters of the Kummer Beta Generalized Normal distribution to be compared by different estimation methods, which are both the common and improved maximum likelihood method in order to obtain the best estimation method by relying on the Mean squares Error (MSE) and Mean squares Error Integral (IMSE) criteria.

1. The theoretical side

2.1 The Concept of Kummer Beta

Kotz and Ng introduced in 1995 a distribution that provides greater potential for radicalization, a new class of generative distributions called Kummer Beta Generalized Distrbution (KBGD), which is a combination of beta distribution Added to it is the parameter of the shape (c) and its boundary between (0,1). Its main benefit is to provide more flexibility to the extreme (right and / or left) sides and thus make it suitable for data analysis with a high degree of asymmetry. It can be used in generating new distributions for normal original distributions. The following four distributions (Gumbel, Gamma, Webuill Geometric, and Normal) will be worked on, among many other known distributions. The normal moments for any of the newly generated distributions can be expressed as linear functions since the weighted probability moments of the base distributions are very versatile and can be used to analyze different types of data sets. [11;2012;pp.154] .

1.2. Kummer Beta Generalized Normal Distrbution (KBGND)

The normal distribution is the most common model in real data applications. That's when the number of views is large, it can act as an approximation to other models. So several authors have proposed new generalizations based on normal distribution to model real data sets (Kumaraswamy Normal). The cumulative distribution function (cdf) of the distribution can be determined through the general formula for the Kummer Beta Generalized Distribution (KBGD) as follows: [6;2002;PP.502-504].

$$F_{KBG}(x) = K \int_0^{G(x)} t^{a-1}\,(1-t)^{b-1}\,e^{-ct}dt \qquad \dots(1)$$

Where a> 0 and b> 0 are shape parameters that working skew, thus enhancing the weight variance of the tails, while the parameter -∞ <c < ∞ compresses pdf to the left or right, i.e. it gives the weights of the maximum limits of the PDF functions [12; 2015; pp. 510]

$$K^{-1} = \frac{(\Gamma a)(\Gamma b)}{(\Gamma a + b)} F_1(a; a+b; -c)$$

$$F_1(a; a+b; -c) = \frac{\Gamma(a+b)}{(\Gamma a)\,(\Gamma b)} \int_0^1 t^{a-1}(1-t)^{b-1}\ \exp(-ct)\,dt$$

$$F_1(a; a+b; -c) = \sum_{k=0}^{\infty} \frac{(a)_k(-c)^k}{(a+b)_k k!}$$

The general formula for the pdf function corresponding to formula No. (1) can be expressed as follows:

$$f_{KBG}(x) = K\,g(x)\,G(x)^{a-1}\,[1-G(x)]^{b-1}\,\exp[-c\,G(x)] \qquad \dots(2)$$

$$G(x; \mu, \sigma) = \Phi(\frac{x-\mu}{\sigma})$$

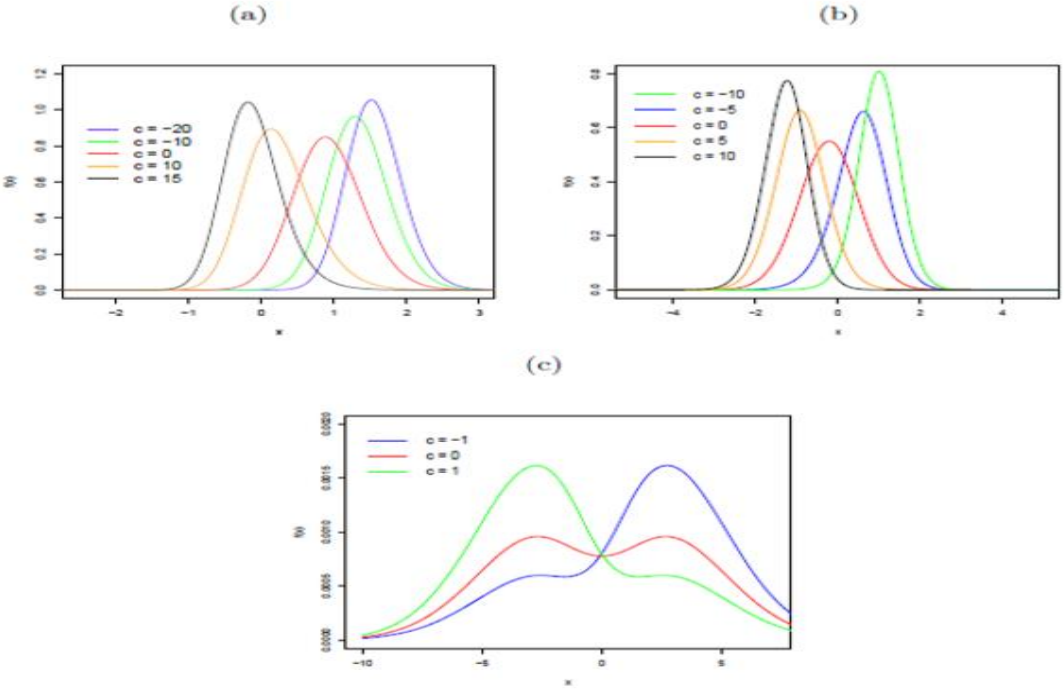
$$g(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma}}\ \exp\left(-\frac{(\chi-\mu)^2}{2\sigma^2}\right) = \frac{1}{\sigma}\,\phi\left(\frac{x-\mu}{\sigma}\right) \qquad \dots(3)$$

Hence, a pdf is obtained for the distribution of (KBGND) by substituting G (x) and g (x) and they express ((pdf) and (cdf) for the original distribution in equations (1) and (2) as follow

$$F(x) = K \int_0^{\Phi(\frac{X-\mu}{\sigma})} t^{a-1}\,(1-t)^{b-1}\,\exp(-c\,t)\,dt \qquad \dots(4)$$

$$f(x) = \frac{K}{\sigma}\,\phi\left(\frac{X-\mu}{\sigma}\right)\left[\Phi\ \left(\frac{X-\mu}{\sigma}\right)\right]^{a-1}\left[1-\Phi\ \left(\frac{X-\mu}{\sigma}\right)\right]^{b-1}$$
$$\exp\left[-c\,\Phi\ \left(\frac{X-\mu}{\sigma}\right)\right] \quad -\infty < x < \infty \qquad \dots(5)$$

Where the random variable x∈ R and the μ∈ R is a location parameter, σ> 0 is a scale parameter and the a,b are the positive shape parameters and c ∈R which is a shape parameter with real value, φ (•), Φ (•) are the pdf and the cdf special The standard normal distribution, respectively. It is said that x is a random variable that Kummer Beta Generalized Normal Distributes and denotes it with the symbol X ∼ KBGND (a, b, c, μ = 0, σ = 1), a distribution that recognizes different degrees of flatness and asymmetry. The normal distribution represents only a special case of the KBGND distribution. This (pdf) function contains three form parameters a, b and c, which allows for a high degree of flexibility. Parameter c controls the weights of the tail to the maximum extent of



the distribution. Some distributions extend reality, also the KBGND distribution contains sub-models such as the Beta normal distribution (BN) when (c = 0) and the exponential normal distribution (EN) when (c= 0,b =1). KBN plots (pdf) are shown for the values of the parameters specified in Figure (1) that the new (pdf) function forms are much clearer than its sub-models [12;2015; pp.60].

Figure (1) The probability density function of Kummer Beta Generalized Normal distribution for some specific females [12; 2015; pp. 512]
(a) KBN(8,2,c,0,1), (b) KBN(1.5, 2,c,0,1) and (c) KBN (0.1,0.1,c,0,1)

That the (pdf) function of the normal distribution is a linear combination of the (pdf) functions of the exponential normal distribution (EN) when b = 1 and c = 0, so many characteristics of the KBGND distribution can be obtained by knowing those properties of the exponential normal distribution (EN) where the expansions in equations (4) and (5) can be derived using the concept of exponential distributions X ~ EN (a, μ, σ) with a series of derivations, and we get the cdf distribution (KBGND).

$$F(x) = \sum_{r=0}^{\infty} b_r \left[\Phi\left(\frac{x-\mu}{\sigma}\right)\right]^r \qquad \dots.(6)$$

$$b_r = \sum_{j,k=0}^{\infty} w_{j,k}\, s_r(\mathrm{a}+\mathrm{j}+\mathrm{k})$$

$$s_r(\mathrm{m}) = \sum_{k=r}^{\infty}(-1)^{k+r}\binom{m}{k}\binom{k}{r} \quad ; \quad w_{j,k} = \frac{K\,(-1)^{j+k}\,c^j}{j!(a+j+k)}\binom{b-1}{k}$$

2.3. Classical Method In Estimation

The method are based on the assumption that the parameter to be assessed is fixed, and the method to be addressed in this research is the Maximum Likelihood and it reflects the relationship between the parameters and the data as follows:

2.3.1 Maximum Likelihood Method

The first person to use the greatest weighting method was the researcher (Fisher) in (1922) and it is one of the most important methods that have been widely used in estimating the parameters of probability distributions. The sample size is large and more accurate than other methods of estimation. It is sufficient and has the least possible variance and the most important characteristic of non-variance (Invariant), especially when increasing the sample size [2; 2012; pp.22].
The estimate can be defined in this way as the values of the parameters that make the maximum probability function at its extreme end, since if $x_1, x_2, \dots, x_n$ are observations of a random sample drawn of size (n) from a population with a known probability density function f(x_i, a, b, c, μ, σ), the probability function is denoted by the symbol (L) as the common probability function, i.e. [12; 2015; pp. 510-514]:

The maximum likelihood function of the (KBGND) distribution is as follows:

$$L(a,b,c,\mu,\sigma) = \frac{\Gamma(a+b)^n}{(\Gamma a^n)(\Gamma b^n)}\left[\prod_{i=1}^{n}\Phi\left(\frac{x_i-\mu}{\sigma}\right)\right]^{a-1}\left[\prod_{i=1}^{n}\left\{1-\Phi\left(\frac{x_i-\mu}{\sigma}\right)\right\}\right]^{b-1}$$
$$\frac{1}{\sigma^n\left(\sqrt{2\pi}\right)^n}\exp\left(\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}\right)\exp\left(-c\prod_{i=1}^{n}\Phi\left(\frac{x_i-\mu}{\sigma}\right)\right)$$

For the purpose of estimating the possibility function and converting it to linear form by taking the normal logarithm of both sides of the equation:

$$L(\theta) = n\log\Gamma(a+b) - n\log(\Gamma a) - n\log(\Gamma b) - \frac{n}{2}\log(2\pi) - n\log\sigma + (a-1)\sum_{i=1}^{n}\log\left[\Phi\left(\frac{xi-\mu}{\sigma}\right)\right] + (b-1)\sum_{i=1}^{n}\log\left[1-\Phi\left(\frac{xi-\mu}{\sigma}\right)\right] - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(xi-\mu)^2 - c\sum_{i=1}^{n}\Phi\left(\frac{xi-\mu}{\sigma}\right)$$

And to find the estimated values for all the parameters, which make the function of the Maximum Likelihood as large as possible, through which the maximum limits of the function will be calculated, as follows:

$$U_a(\theta) = 0 = \frac{n}{K}\frac{\partial K}{\partial a} + \sum_{i=1}^{n}\log\left[\Phi\left(\frac{xi-\mu}{\sigma}\right)\right] \quad \dots(7)$$

$$U_b(\theta) = 0 = \frac{n}{K}\frac{\partial K}{\partial b} + \sum_{i=1}^{n}\log\left[1-\Phi\left(\frac{xi-\mu}{\sigma}\right)\right] \quad \dots(8)$$

$$U_c(\theta) = 0 = \frac{n}{K}\frac{\partial K}{\partial c} - \sum_{i=1}^{n}\Phi\left(\frac{xi-\mu}{\sigma}\right) \quad \dots(9)$$

$$U_\mu(\theta) = 0 = \frac{n\mu}{\sigma^2} + \frac{1}{\sigma^2}\sum_{i=1}^{n}x_i - \frac{(1-a)}{\sigma\sqrt{2\pi}}\sum_{i=1}^{n}\left\{\frac{\exp\left[-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right]}{\Phi\left(\frac{x_i-\mu}{\sigma}\right)}\right\} + \frac{(b-1)}{\sigma\sqrt{2\pi}}\sum_{i=1}^{n}\left\{\frac{\exp\left[-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right]}{1-\Phi\left(\frac{x_i-\mu}{\sigma}\right)}\right\} + \frac{c}{\sigma\sqrt{2\pi}}\sum_{i=1}^{n}exp\left[-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right] \quad \dots(10)$$

$$U_\sigma(\theta) = 0 = -\frac{n}{\sigma} + \frac{1}{\sigma^3}\sum_{i=1}^{n}(x_i-\mu)^2 - \frac{(a-1)}{\sigma^2\sqrt{2\pi}} + \frac{(b-1)}{\sigma^2\sqrt{2\pi}}\sum_{i=1}^{n}\left\{\frac{(x_i-\mu)\exp\left[-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right]}{1-\Phi\left(\frac{x_i-\mu}{\sigma}\right)}\right\} + \frac{c}{\sigma^2\sqrt{2\pi}}\sum_{i=1}^{n}(x_i-\mu)exp\left[-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right] \quad \dots(11)$$

There are partial derivatives of K with respect to a, b, and c

$$\frac{\partial K}{\partial a} = -\frac{\left\{[\psi(a)-\psi(a+b)]\ {}_1F_1(a,a+b,-c) + \frac{\partial\ {}_1F_1(a,a+b,-c)}{\partial a}\right\}}{B(a,b)[.\ {}_1F_1(a,a+b,-c)]^2}$$

$$\frac{\partial K}{\partial b} = -\frac{\left\{[\psi(b)-\psi(a+b)]\ {}_1F_1(a,a+b,-c) + \frac{\partial\ {}_1F_1(a,a+b,-c)}{\partial b}\right\}}{B(a,b)[.\ {}_1F_1(a,a+b,-c)]^2}$$

$$\frac{\partial K}{\partial c} = \frac{a\ {}_1F_1(a,a+b,-c) + \frac{\partial\ {}_1F_1(a,a+b,-c)}{\partial a}}{(a+b)B(a,b)\ {}_1F_1(a,a+b,-c)}$$

$$\frac{\partial\ {}_1F_1(a,a+b,-c)}{\partial a} = -[\psi\,(a)-\psi(a+b)]\ {}_1F_1(a,a+b,-c) - \sum_{k=0}^{\infty}\frac{(a)_k(-c)^k}{k!\,(a+b)_k}$$
$$[\psi(a+b+k)-\psi(a+k)]$$

$$\frac{\partial\ {}_1F_1(a,a+b,-c)}{\partial b}$$
$$= \psi(a+b)\ {}_1F_1(a,a+b,-c) + \sum_{k=0}^{\infty}\frac{(a)_k(-c)^k}{k!\,(a+b)_k\psi(a+b+k)}$$

$$\frac{\partial}{\partial\mu}\ \Phi\{(x-\mu)/\sigma\} = -\phi\{(x-\mu)/\sigma\}$$

$$\frac{\partial}{\partial\sigma}\ \Phi\{(x-\mu)/\sigma\} = -\phi\{(x-\mu)/\sigma^2\}$$

$$*\ \psi(x) = \frac{d\log\Gamma(x)}{dx}$$

System Equation cannot be solved by ordinary methods. For the purpose of solving it, we use one of the numerical methods for solving nonlinear equations, such as the Newton - Raphson method.

2.4. Genetic Algorithm Methods In Estimation

It is one of the methods of artificial intelligence to obtain an optimal solution to the problem under study in an effective and fast manner. His idea came from Professor of Computer Science John Holland of the University of Michigan (Michigan University), who aimed to update the concept of the natural development process and design industrial systems with similar characteristics. Natural systems and his constant ambition to improve the performance of computational systems has made genetic algorithms more effective in solving optimization issues, which have no solution when applying other known traditional methods, due to the fact that genetic algorithms reduce and shorten a lot of effort and time for system and program designers. In resolving these issues, taking into account the specificity of each issue in terms of the size and type of data used, the nature of the objective function and the restrictions imposed [14; 2008; p.120].

First: The Beginning

It is represented by a random population of chromosomes called (study space). In other words, it is a set of solutions to the problem under study [15; 2005; p. 245].

Second: Initialization

It is represented by the generation of random chromosomes according to the size of the community, depending on the nature of the problem to be solved, meaning the process of creating the primary generation [7; 2015; p.775].

Third: Selection

The formation of the new generation involves choosing suitable chromosomes from the old generation that are created according to the values of the evaluation function for the purpose of having chromosomes with the highest value of the evaluation function in the new generations. Roulette wheel selection, match selection, etc. [13; 2012; p. 40].

Fourth: Reproduction

It is the process of selection and the generation of a new generation of individuals (chromosomes) according to the principle of survival of the fittest, and then a crossover and mutation process to produce children (the next generation) [5; 2015; p.168].

Fifth: crossover

Mating takes place between each of the two chromosomes to produce the new generation (the offspring), after selecting the good chromosomes from the first generation depending on the (mother) chromosomes. It takes the good characteristics from them and among its methods is hybridization with one point and two points and others [1; 2016; p.1033]

Sixth: Mutation

The mutation process takes place with random changes in its chromosomal pigment after the creation of the new generation (children) and with the possibility of the mutation, which leads us to preserve the good qualities between genes in one chromosome and reach the solution faster, in which the exchange between the chromosomes occurs and when there is no mutation then the process of reproduction of chromosomes takes place (Parents) directly without crossbreeding taking place [5;2015;p.169].

Seventh: Termination

When one of the factors is present, the genetic algorithm ends [5; 2015; p. 332].
Obtaining the optimal solution, reaching the required number of generations, when a certain value is available.

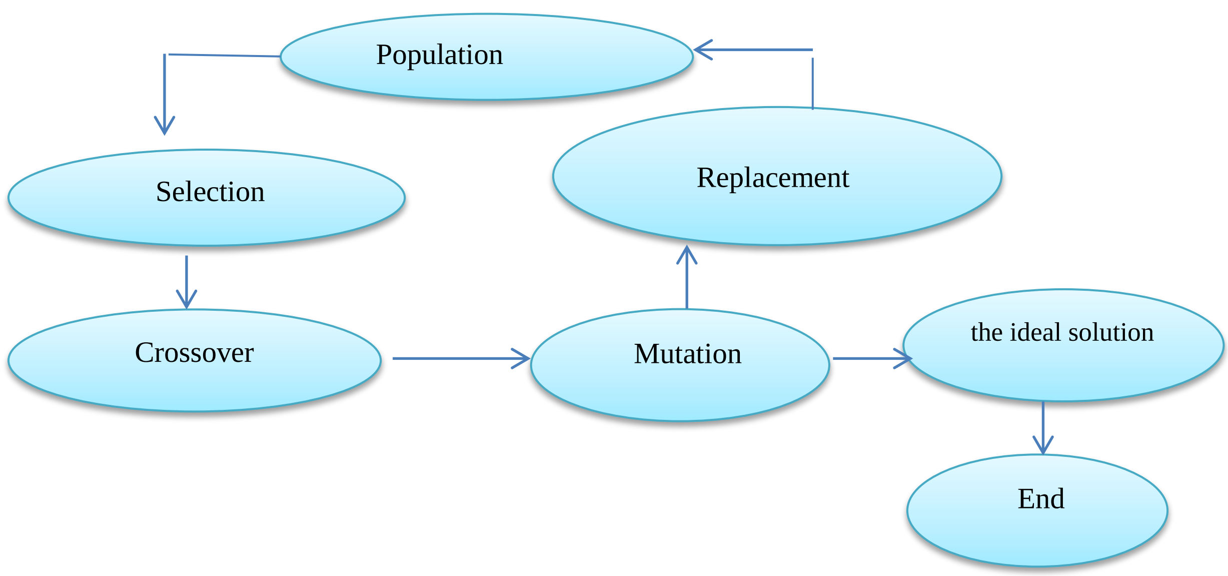


Figure (2) illustrates the general diagram of the algorithm

3. The experimental side

3.1 Describe the stages of the simulation experiment

The simulation experiments included the following stages of applying methods of estimating the survival function and distribution parameters (KBGND) in this research.

The first stage:

It is the stage of choosing the default values, as it is one of the important stages on which the other stages depend, and the default values were chosen as follows:

1- For the different parameters and experiments, they were as shown in the following table:

Table No. (1) for the hypothetical parameters and different experiments

Experiment	A	B	C	μ	σ
I.	2	1	-1	0	1
II.	2	1	1	0	1
III.	2	1	-0.5	0	1
IV.	2	1	0.5	0	1
V.	2	1	-2	0	1
VI.	2	1	2	0	1

2. As for the assumed samples, they were as follows: (n = 20,50,90,250)

The second stage - data generation:

In this stage, the values of the explanatory variable (random) X_i are generated, and this variable follows the uniform distribution, according to the following formula:

X = Rand

$X_i \sim U(0,1), i = 1,2, \ldots \ldots, n$

The explanatory variable that follows the regular distribution is transformed into an explanatory variable (random) that describes The Kummer Beta Generalized Normal model with five parameters (a, b, c, μ, σ) using a statistical mathematical method, and this method is used to generate various explanatory variables that follow the different probability distributions The results were obtained by relying on a program written in (MATLAB).

The third stage:

It is at this stage that the hazard function and parameters of the generalized five-parameter Kummer Beta Normal model are estimated (a, b, c, μ, σ) and shown in the theoretical side according to the following formulas of estimation methods:

1.The Maximum Likelihood Method (ML)

2. The Genetic Algorithm Method (GA)

The fourth stage:

The comparison stage between the estimation methods, where the following scale is used [3;2007; p.156]

1.A measure of the mean square error (MSE)

It calculates all (t_i) of time and the formula for this scale is as follows:

$$MSE = \frac{1}{r}\sum_{i=1}^{r} MSE_i = \frac{1}{r}\sum_{i=1}^{r} \left[\frac{1}{n-P}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2\right]$$

R: represents the number of times the experiment was repeated, and the simulation experiment was repeated the number r = 1000 to obtain the experimental side results.

2. A measure of the mean square error integral (IMSE)

The integral of the total area of (t_i) is reduced by a single value expressing the total time and the formula this scale is as follows:

$$\text{IMESE}(\hat{H}(t)) = \frac{1}{L}\sum_{i=1}^{r}\left\{\frac{1}{n_t}\sum_{J=1}^{n_t}(\hat{H}_i\left(t_J\right) -)H(t_J)^2\right\}$$

$$\text{IMESE}(\hat{H}(t)) = \frac{1}{n_t}\sum_{J=1}^{n_t}(\text{MSE}(\widehat{H(t_i)}))$$

i=1,2…,r 1,2,…..,n = J

r: number of times to repeat the experiment

n_t: the limits of the variable t_i from minimum to upper bound (sample size per experiment (t_i)).

3.2 Discuss simulation experiments

In this Section, the results of simulation experiments will be presented and analyzed to estimate the hazard function of the five-parameter distribution (KBGND) according to the method shown in the theoretical side of the research. Tables to be analyzed in sequence and as follows: Through Tables No. (2), (3), (4), (5), (6) and (7) which are the Kummer Beta generalized normal distribution models (KBGND) and upon Sample sizes (n = 20,50,90,250) and when the default feature values are (c = -1,1, -0.5,0.5, -2,2) respectively, it becomes clear to us.

1.The conventional maximum likelihood method (MLE) is superior to the genetic algorithm method(MLE.GA) and to the level of pdf capacity models by approaching the estimated values to the default values

2.The method of possibility had the maximum likelihood in estimating the hazard function through the statistical scale (IMSE. GA)

3.Results of KBGND analysis show no warping or cluttering data.

Table No. (2) The first model illustrates the mean squares of error (MSE) for the capabilities of the probability density function (PDF) and average integral error squares (IMSE)) for the distribution hazard function (KBGND) using standard and genetic methods and all sample sizes.

Kummer Beta Generalized Normal Distrbution	Sample Sizes	Initial Values	Capabilities (MLE)	Capabilities (GA)	Classic Mse (MLE) pdf Genetic Mse (GA) pdf	IMSE (MLE) h(x) IMSE (GA) h(x)
	N=20	a=2 b=1 c=-1 μ=0 σ=1	$\hat{a}$=0.616976079711081 $\hat{b}$=0.823126541072562 $\hat{c}$= 0.809725959267035 $\hat{\mu}$=0.911368153200935 $\hat{\sigma}$= 1.0465518164324	$\hat{a}$= 0.539294306834710 $\hat{b}$= 0.553941746420953 $\hat{c}$= -0.192960881509742 $\hat{\mu}$= 0.481786150769519 $\hat{\sigma}$= 1.061596563237131	0.024566428950587 1.019647322426541	0.007701784314516 2.801754382730018
	N=50		$\hat{a}$= 2.837537291501524 $\hat{b}$= 0.953524010578584 $\hat{c}$= 0.906842983995543 $\hat{\mu}$= -22.8861111760610 $\hat{\sigma}$=1.018175734991232	$\hat{a}$= 0.572443637606182 $\hat{b}$= 0.542345562324702 $\hat{c}$= -0.0926660[1]39967330 $\hat{\mu}$= 0.527888502887159 $\hat{\sigma}$= 1.023218275621747	2.147240930496e-04 2.288864984713681	0.005588335165101 1.424739725315399
	N=90		$\hat{a}$= 0.817031155574772 $\hat{b}$= 0.972472083361219 $\hat{c}$= 0.95141065239902 $\hat{\mu}$= 4.446902949921229 $\hat{\sigma}$= 1.01031204769842	$\hat{a}$= 0.582161723960323 $\hat{b}$= 0.536711350570246 $\hat{c}$= -0.067199301212972 $\hat{\mu}$= 0.554191180067211 $\hat{\sigma}$= 1.012768392817351	0.003728134977834 4.007086582416011	0.002119759830081 1.060132300785053
	N=250		$\hat{a}$=1.59384906454913 $\hat{b}$= 0.990950159187330 $\hat{c}$= 0.980851288192280 $\hat{\mu}$=55.54940778767874 $\hat{\sigma}$= 1.00365368330170	$\hat{a}$= 0.596588181595424 $\hat{b}$= 0.590610058801330 $\hat{c}$= -0.040139824142943 $\hat{\mu}$=0.516615959540467 $\hat{\sigma}$= 1.004212061241898	0.001485839426273 11.372147744893883	0.001010741298176 0.984645410787141

Table No. (3) The second model illustrates the mean squares of error (MSE) for the capabilities of the probability density function (PDF) and average integral error squares (IMSE)) for the distribution hazard function (KBGND) using standard and genetic methods and all sample sizes.

	Sample Sizes	Initial Values	Capabilities (MLE)	Capabilities (GA)	Classic Mse (MLE)pdf Genetic Mse (GA) pdf	IMSE (MLE) h(x) IMSE (GA) h(x)
Kummer Beta Generalized Normal Distrbution	N=20	a=2 b=1 c=1 μ=0 σ=1	$\hat{a}$=0.910446840526311 $\hat{b}$=0.958645984663193 $\hat{c}$= 0.955512852824938 $\hat{\mu}$=1.110872368868365 $\hat{\sigma}$=1.010913804991974	$\hat{a}$= 0.774717460446152 $\hat{b}$= 0.781879977508355 $\hat{c}$= 0.416648717474194 $\hat{\mu}$= -0.931224470641678 $\hat{\sigma}$= 1.030120379234974	0.028946558054 0.336551119661	0.006957737341088 2.532425079658575
	N=50		$\hat{a}$= 1.429626614391497 $\hat{b}$= 0.989133650741149 $\hat{c}$=0.978219362633939 $\hat{\mu}$= 1.115409487161177 $\hat{\sigma}$= 1.00458613940166	$\hat{a}$= 0.789109401849544 $\hat{b}$= 0.774263637273031 $\hat{c}$= 0.461046507512499 $\hat{\mu}$= -0.714099430803326 $\hat{\sigma}$=1.011452328779480	3.2801157179 e-06 0.807247192237	0.003939534903545 1.140597857719868
	N=90		$\hat{a}$= 0.957220849049925 $\hat{b}$= 0.993563817354092 $\hat{c}$= 0.988639535642707 $\hat{\mu}$= 1.114903028808857 $\hat{\sigma}$= 1.002543529955820	$\hat{a}$= 0.792996051880074 $\hat{b}$= 0.770479190034758 $\hat{c}$= 0.471291929318287 $\hat{\mu}$=-0.342410814835666 $\hat{\sigma}$= 1.006325671619627	0.003902654859 1.44443075617	0.001517317175149 0.745581569993587
	N=250		$\hat{a}$= 1.138845270918732 $\hat{b}$= 0.997884096019650 $\hat{c}$= 0.995522922847902 $\hat{\mu}$=1.116359702819235 $\hat{\sigma}$= 1.00092960650717	$\hat{a}$= 0.794312356258423 $\hat{b}$= 0.791264289901870 $\hat{c}$= 0.469663753442200 $\hat{\mu}$=-2.493934672930438 $\hat{\sigma}$=1.002147604290741	0.00142219651 4.37633073606	6.69321683582 e-04 0.497411712373352

Table No.(4) The third model illustrates the mean squares of error (MSE) for the capabilities of the probability density function (PDF) and average integral error squares (IMSE)) for the distribution hazard function (KBGND) using standard and genetic methods and all sample sizes.

Kummer Beta Generalized Normal Distrbution	Sample sizes	Initial Values	Capabilities (MLE)	Capabilities (GA)	Classic Mse (MLE) pdf Genetic Mse (GA) pdf	IMSE (MLE) h(x) IMSE (GA) h(x)
	N=20	a=2 b=1 c= -0.5 μ=0 σ=1	$\hat{a}$=0.734391279375519 $\hat{b}$=0.877346738285414 $\hat{c}$= 0.868054077434611 $\hat{\mu}$=6.091591158153983 $\hat{\sigma}$=1.03229348480240	$\hat{a}$=0.61477901819523 $\hat{b}$=0.627026535736365 $\hat{c}$= 0.002500796392417 $\hat{\mu}$= 0.446468156943065 $\hat{\sigma}$= 1.051504222582927	0.02647394732789 0.716035956470048	0.005962034611645 2.714111799891262
	N=50		$\hat{a}$= 2.274243991673785 $\hat{b}$= 0.967771130114621 $\hat{c}$= 0.935400076790311 $\hat{\mu}$= 2.37073936172428 $\hat{\sigma}$= 1.0128353210618	$\hat{a}$= 0.641761944687238 $\hat{b}$=0.616543571378217 $\hat{c}$= 0.084484312027039 $\hat{\mu}$= 0.495423430685971 $\hat{\sigma}$= 1.019453972944949	1.1214894710209e-04 1.641513227657458	0.004451427220762 1.330679474323271
	N=90		$\hat{a}$= 0.873119880749833 $\hat{b}$= 0.980910709925457 $\hat{c}$=0.966305617564201 $\hat{\mu}$= 2.760930632585692 $\hat{\sigma}$= 1.0072447916215	$\hat{a}$= 0.649452711431950 $\hat{b}$=0.611321917605910 $\hat{c}$= 0.104668378043948 $\hat{\mu}$= 0.527242509730920 $\hat{\sigma}$= 1.010712100202781	0.003799431473938 2.894994020219595	0.00158452859655 0.953968545814659
	N=250		$\hat{a}$= 1.411805847947988 $\hat{b}$= 0.99372436938587 $\hat{c}$= 0.986721286647009 $\hat{\mu}$=2.263074559658298 $\hat{\sigma}$= 1.00258614958207	$\hat{a}$= 0.659135575200163 $\hat{b}$= 0.654084336504057 $\hat{c}$= 0.121129707428853 $\hat{\mu}$=0.467927808203867 $\hat{\sigma}$= 1.003558997944382	0.001460995778341 8.408279121991876	7.78309975382e-04 0.811025897123209

Table No. (5) The fourth model illustrates the mean squares of error (MSE) for the capabilities of the probability density function (PDF) and average integral error squares (IMSE)) for the hazard distribution function (KBGND) using standard and genetic methods and all sample sizes

Distribution	Sample Sizes	Initial Values	Capabilities (MLE)	Capabilities (GA)	Classic Mse (MLE) pdf	Genetic Mse (GA) pdf	IMSE (MLE) h(x)	IMSE (GA) h(x)
Kummer Beta Generalized Normal Distrbution	N=20	a=2 b=1 c=0.5 μ=0 σ=1	$\hat{a}$=0.871572023180007 $\hat{b}$=0.940694303201559 $\hat{c}$= 0.936201086150789 $\hat{\mu}$=1.230820102971887 $\hat{\sigma}$=1.015635293449763	$\hat{a}$= 0.73062212121891 $\hat{b}$=0.73918658279134 $\hat{c}$= 0.302467331102381 $\hat{\mu}$= 0.130326735441906 $\hat{\sigma}$= 1.03601594638655	0.028438571174395	0.414826564112862	0.006397797234679	2.581986498129057
	N=50		$\hat{a}$= 1.616126524162826 $\hat{b}$= 0.984416593909859 $\hat{c}$= 0.968764438829263 $\hat{\mu}$=1.21771314670140 $\hat{\sigma}$= 1.00644719351709	$\hat{a}$= 0.748404179365331 $\hat{b}$=0.730692947312624 $\hat{c}$= 0.357019955296566 $\hat{\mu}$= 0.210509171887550 $\hat{\sigma}$= 1.013662809450593	1.672263425284070e-05	0.982582878972082	0.003939702876430	1.191780804553992
	N=90		$\hat{a}$=0.938650519547450 $\hat{b}$=0.990769885501351 $\hat{c}$= 0.983707984601354 $\hat{\mu}$=1.22307511345198 $\hat{\sigma}$= 1.003597965040016	$\hat{a}$= 0.753264399339045 $\hat{b}$=0.726425725570439 $\hat{c}$=0.369813452454675 $\hat{\mu}$= 0.294974802576047 $\hat{\sigma}$= 1.007539800089940	0.003880249070113	1.751449214169342	0.001460421328849	0.800808860692446
	N=250		$\hat{a}$= 1.199117678705190 $\hat{b}$= 0.996965587044178 $\hat{c}$= 0.993579434113881 $\hat{\mu}$=1.217099054669524 $\hat{\sigma}$= 1.001304450920504	$\hat{a}$= 0.756605084919094 $\hat{b}$= 0.752998237961645 $\hat{c}$= 0.372440933508685 $\hat{\mu}$=0.037172558778826 $\hat{\sigma}$=1.002541309504372	0.001430927990958	5.251076585832587	6.7019098389531e-04	0.576503615670536

Table No. (6) The fifth model illustrates the mean squares of error (MSE) for the capabilities of the probability density function (PDF and average integral error squares (IMSE)) for the hazard distribution function (KBGND) using standard and genetic methods and all sample sizes.

Kummer Beta Generalized Normal Distrbution	Sample Sizes	Initial Values	Capabilities (MLE)	Capabilities (GA)	Classic Mse (MLE) pdf / Genetic Mse (GA) pdf	IMSE (MLE) h(x) / IMSE (GA) h(x)
	N=20	a=2 b=1 c=-2 μ=0 σ=1	$\hat{a}$=0.199218585482236 $\hat{b}$=0.630213751340321 $\hat{c}$= 0.602197389214694 $\hat{\mu}$= -0.13456302725478 $\hat{\sigma}$=1.097289602386959	$\hat{a}$= 0.340931995660664 $\hat{b}$= 0.361886064429514 $\hat{c}$= -0.706604366075061 $\hat{\mu}$= 0.483170244528603 $\hat{\sigma}$=1.088117695546455	0.015991550260190 / 2.566187638034791	0.031008197061150 / 3.038064243228076
	N=50		$\hat{a}$= 4.841707093405002 $\hat{b}$= 0.902833461362098 $\hat{c}$= 0.805238255114634 $\hat{\mu}$= 0.746926889410653 $\hat{\sigma}$= 1.036703611180480	$\hat{a}$= 0.390728924548924 $\hat{b}$= 0.34783893785083 $\hat{c}$= -0.557057435778646 $\hat{\mu}$= 0.530217746819256 $\hat{\sigma}$= 1.033086219741835	5.0415379660245e-04 / 5.454018914017153	0.015174573504157 / 1.685577962758232
	N=90		$\hat{a}$= 0.617470235433352 $\hat{b}$= 0.942447865898031 $\hat{c}$= 0.898415100359643 $\hat{\mu}$= 0.511130910632314 $\hat{\sigma}$= 1.021016734503459	$\hat{a}$= 0.406337161412902 $\hat{b}$= 0.341761464956040 $\hat{c}$= - 0.516272210629406 $\hat{\mu}$= 0.553229413993043 $\hat{\sigma}$=1.018141277998726	0.003458636356653 / 9.352144620741608	0.07362481883007 / 1.363206779773522
	N=250		$\hat{a}$= 2.241549858194238 $\hat{b}$= 0.981079656012961 $\hat{c}$= 0.959966122961687 $\hat{\mu}$=-0.885139812832689 $\hat{\sigma}$= 1.007340864472400	$\hat{a}$= 0.434872632453765 $\hat{b}$= 0.426498061746205 $\hat{c}$= 0.457100297712145 $\hat{\mu}$=-0.537690845790155 $\hat{\sigma}$= 1.005900548702294	0.001567888019604 / 24.814191759702190	0.002923689082787 / 1.518855589596089

Table No. (7) The sixth model illustrates the mean squares of error (MSE) for the capabilities of the probability density function (PDF and average integral error squares (IMSE)) for the hazard distribution function (KBGND) using standard and genetic methods and all sample sizes

	Sample Sizes	Initial Values	Capabilities (MLE)	Capabilities (GA)	Classic Mse (MLE) pdf Genetic Mse (GA) pdf	IMSE (MLE) h(x) IMSE (GA) h(x)
Kummer Beta Generalized Normal Distrbution	N=20	a=2 b=1 c=2 μ=0 σ=1	$\hat{a}$=0.956208894808371 $\hat{b}$=0.979778066498676 $\hat{c}$= 0.97824597866710 $\hat{\mu}$=1.033491878378178 $\hat{\sigma}$=1.0053530485931	$\hat{a}$= 0.842406663469549 $\hat{b}$=0.847417104864715 $\hat{c}$= 0.591924544331650 $\hat{\mu}$= 1.584688020211009 $\hat{\sigma}$=1.021070301633685	0.029518347169231 0.243719538978404	0.007808473288721 2.457086575243153
	N=50		$\hat{a}$= 1.210085544435416 $\hat{b}$= 0.994686402509527 $\hat{c}$= 0.989349363130866 $\hat{\mu}$= 1.038974067692433 $\hat{\sigma}$= 1.002337954166029	$\hat{a}$= 0.851768075171545 $\hat{b}$=0.841333203830545 $\hat{c}$=0.621177452740476 $\hat{\mu}$= 1.552560742850055 $\hat{\sigma}$= 1.008049674824951	1.361297046404101e-06 0.595978617609271	0.003999501344175 1.063751308264688
	N=90		$\hat{a}$= 0.996852734700418 $\hat{b}$= 0.994444782377085 $\hat{c}$= 1.037461222185263 $\hat{\mu}$= 0.003928731335130 $\hat{\sigma}$=1.001279340969545	$\hat{a}$= 0.854292915984636 $\hat{b}$= 0.838443622720156 $\hat{c}$= 0.627850038735535 $\hat{\mu}$= 1.613956553952055 $\hat{\sigma}$= 1.004452548729170	0.003928731335130 1.072374821864617	0.001627657443024 0.664021460035309
	N=250		$\hat{a}$= 1.067894733138353 $\hat{b}$= 0.998965332163338 $\hat{c}$= 0.997810728759646 $\hat{\mu}$=1.039780502949779 $\hat{\sigma}$= 1.00047538195515	$\hat{a}$= 0.853085377935950 $\hat{b}$= 0.850908263605438 $\hat{c}$= 0.621201605441130 $\hat{\mu}$=1.446584406653078 $\hat{\sigma}$= 1.001533949570222	0.001411793152617 3.293874160798007	6.816365687154284e-04 0.386932401980086

Table No. (8) Proficiency Ratio for estimating KBG parameters using the MSE

Distrbution	Sample Sizes	The number of times the preference MLE)(	The number of times the preference (MLE.GA)	The ratio	Best
KBGND	20	6	-	0.25	MLE
	50	6	-	0.25	MLE
	90	6	-	0.25	MLE
	250	6	-	0.25	MLE
	Total	18	-		MLE

Through Table No. (8) we note

At sample sizes n = (20,50,90,250) the maximum likelihood method (MLE)

More efficient for estimating distribution parameters (KBGND) and for all distribution models

Table (9) the efficiency ratio for estimating the hazard function using the statistical scale (IMSE).

Distrbution	Sample Sizes	The number of times the preference MLE) (	The number of times the preference (MLE.GA)	The ratio	Best
KBGND	20	6	-	0.25	MLE
	50	6	-	0.25	MLE
	90	6	-	0.25	MLE
	250	6	-	0.25	MLE
	Total	24	-		MLE

Through Table No. (9) we note

At sample sizes n = (20,50,90,250) the maximum likelihood method(MLE) was more efficient for estimating the hazard function h (t) and for all KBGND distribution models.

4. Practical Aspect

In this section, the results of the application will be presented and then analyzed to reach the suitability of the real data with the generalized Kummer Beta distribution (KBGND) by conducting tests on the distribution estimations. As the common maximum likelihood method (MLE) will be used for estimation, because the simulation results showed their preference.

The parameters of the probability density function (PDF) and the hazard function of the (KBGND) distribution are to be estimated using the maximum likelihood method (MLE).

4.1 Description of the data

Real data was collected on the size of (197) male and female teachers of the Baghdad Education Directorate Rusafa / 3, and these data represent the teachers' service calculated per day.

4-2 The Match test

The Kolmogorov Smirnov test was used to test the matching data through a program written in Matlab, and the results were as shown in the following:

Ho: data are distributed in a Kummer Beta Generalized Normal Distribution.

H1: The data are not distributed in a Kummer Beta Generalized Normal Distribution.

D = 0.999999999999128

Tab value = 3.339218917800483e + 02

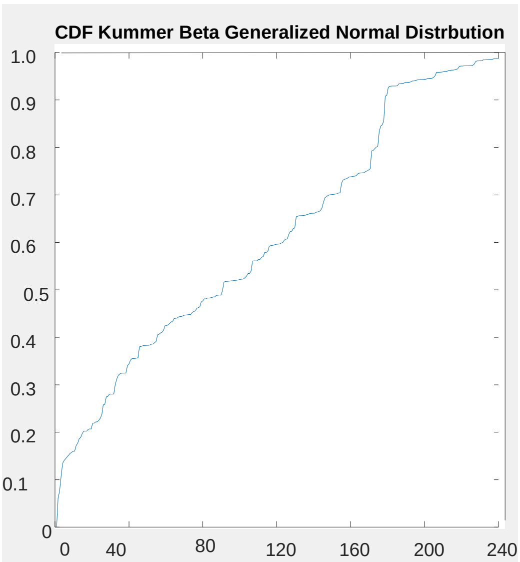
The tabular value is greater than the computed value. This means rejection of the alternative hypothesis and acceptance of the null hypothesis and that the data is distributed in the Kummer Beta Generalized Normal Distribution

Table No. (10) estimating the parameters of the Kummer Generalized Normal Distribution in the case of calculating the service by day

Estimated Parameters	MLE	Var (KN)	MLE(GA)	Var (KN) GA
$\hat{a}$	1.0000002509	1.442182322e+02	1.0005875931975	0.592828554440443
$\hat{b}$	1.0000000488		1.0000284452556	
$\hat{c}$	0.9999997755	Var (H)	0.9995848069162	Var (H) GA
$\hat{\mu}$	0.9999993751	1.083072857e+03	0.9999976600292	0.893476464810911
$\hat{\sigma}$	0.9999950513		0.9999396716341	

Var (KN): is the variance of a function (pdf) of a Kummer beta generalized normal distribution MLE.
Var (H): is the variance of the hazard rate function of the Kummer beta generalized normal distribution by the maximum likelihood method MLE.
Var (KN) GA: is a function variance (pdf) of a Kummer beta generalized normal distribution by the genetic algorithm method(MLE)
Var (H) GA: is the variance of the hazard rate function of the generalized normal beta Kummer distribution by the genetic algorithm method (MLE .GA).

Through the above hypothesis, it becomes clear to us that the data follow the Kummer Beta generalized normal distribution (KBGND), and through the experimental side, the simulation results showed that the normal maximum likelihood method is the best in the case of calculating the service per day for teachers, so this method was used to estimate the parameters (a, b ,c, μ, σ) and as shown in Table (10), and the hazard function was estimated and as shown in Appendix No. (1) KBGND distribution and Figure (4) the behavior of the hazard function as increasing as the value increases (X) *.



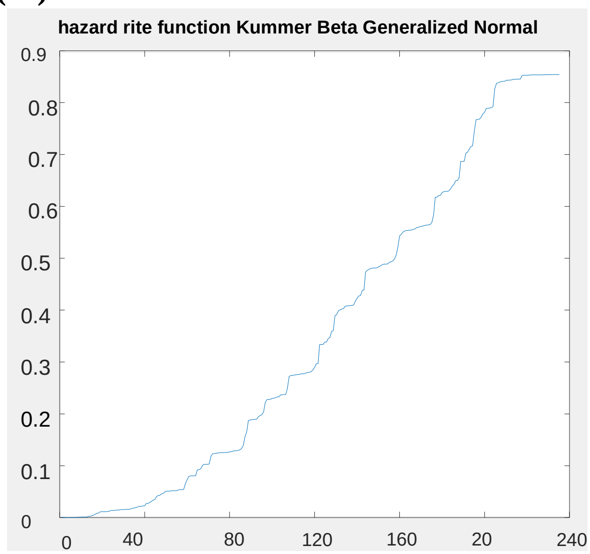


Figure (3): CDF(KBGND)

Figure (4): Hazard Function(KBGND)

5. Conclusions and recommendations

5-1 Conclusion

On the basis of what was extracted from simulation experiments and the results analyzed in the experimental side as well as the practical side of the KBGD distribution, the following conclusions were reached:

The normal maximum likelihood method (MLE) ranked first for the six hypothetical models and default values in estimating the parameters of the KBGND distribution using the MSE comparison standard. This is due to the fact that the data are free from severe skew, since the distribution is normal The effect of increasing the sample size is decreased (MSE).

The simulation results showed that, if the sample size increased, the mean square error integral (IMSE) decreased, and this corresponds to the statistical theory. The application side showed that the data collected from the Baghdad Education Directorate Al-Rusafa / 3 regarding the Teachers Service Account per day follow the KBGND distribution. The application side showed that the hazard function is increasing, ie, increasing the stay of teachers in service is exposed to a greater failure rate as a result of the retention period, which decreases

*** Drawing from the work of the researcher**

2-5: Recommendations and Studies Future

We recommend the standardized method(MEL) to estimate the parameters of the Kummer Beta Generalized Normal Distribution (KBGND).

1.We recommend the use of other estimation methods such as robust estimation methods or moments instead of the maximum likelihood method and other algorithms such as ants, birds, and other algorithms.

2.We recommend extending the study to other distributions of the generalized kummer beta distribution model.

3.Conducting the current study in fields other than educational, for example on social, health, weather or environmental data ... etc.

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Appendix No. (1) Results of the severity ratio of Kummer Beta Generalized Normal Distribution (KBGND)

HX	HX(GA)	HX	HX(GA)	HX	HX(GA)
0.000000066710998	0.000000000000000	0.022896321454564	0.000001631454647	0.058747995817345	0.000001717035483
0.000039568474481	0.000000013570393	0.023182008222047	0.000001632319043	0.062185753463728	0.000001717650092
0.000075546057837	0.000001323220981	0.027267366180448	0.000001642006156	0.023761841041990	0.000001632516408
0.000461785903938	0.000001361235786	0.027705124541771	0.000001644116763	0.024056024383141	0.000001635430874
0.000584728012286	0.000001375480217	0.027980556708832	0.000001650427357	0.067862199541423	0.000001718203463
0.000639445936914	0.000001423009380	0.028484444997534	0.000001651052628	0.069340116313601	0.000001718301738
0.000693453753919	0.000001446541232	0.029280827618691	0.000001653479661	0.081080511801127	0.000001722720337
0.000751372468954	0.000001467459931	0.029627417517312	0.000001661152231	0.085642072156395	0.000001729914830
0.000796083729517	0.000001469551909	0.031166499763722	0.000001668812228	0.086321289644768	0.000001731529968
0.001170561316783	0.000001473767081	0.031287422130340	0.000001669062289	0.092003905263821	0.000001734256652
0.001698541914151	0.000001502746481	0.031408709670629	0.000001670089703	0.092822463032113	0.000001735525480
0.001846727832540	0.000001503894355	0.032144143262397	0.000001671271931	0.100424640804797	0.000001736869241
0.002005808914709	0.000001505119191	0.032268006872165	0.000001671814670	0.101879386384030	0.000001737574204
0.002018515275277	0.000001507700836	0.032392241329385	0.000001675429060	0.102172112293307	0.000001740185605
0.002554914336316	0.000001519823933	0.034777430213896	0.000001676573480	0.102465431624758	0.000001744869535
0.002570521047560	0.000001521645765	0.036305056900316	0.000001678080563	0.103348955080744	0.000001748392886
0.005449419920231	0.000001523297617	0.037954370193290	0.000001697549980	0.103644653022850	0.000001748451495
0.005508555219808	0.000001525304398	0.038378135803703	0.000001699970530	0.103940946841467	0.000001748520682
0.007593100541744	0.000001536020658	0.041900609875819	0.000001701026870	0.104237837023463	0.000001755498579
0.009502736202771	0.000001543818426	0.043860225545363	0.000001705548452	0.107752923402806	0.000001756191930
0.014201516760605	0.000001554727643	0.043914071860574	0.000001707049097	0.107883178853316	0.000001757779903
0.015974581765444	0.000001560801835	0.044700884524768	0.000001710622566	0.108144147998971	0.000001757947986
0.018936081838363	0.000001562525435	0.046094321042988	0.000001711440955	0.108274862027139	0.000001758324922
0.022734489558997	0.000001575279643	0.054688530448263	0.000001713708492	0.161427762570090	0.000001773508851
0.022832438354775	0.000001614023082	0.054722023580986	0.000001715882318	0.183479588766971	0.000001774543664

Appendix No. (1) Results of the severity ratio of Kummer Beta Generalized Normal Distribution (KBGND)

HX	HX(GA)	HX	HX(GA)	HX	HX(GA)
0.185216099600485	0.000001776209134	0.278259664094357	0.000001853836618	0.498105561764102	0.000001899453701
0.188086119923017	0.000001785515474	0.311110759785270	0.000001856383183	0.544380682995054	0.000001900606028
0.200790705647483	0.000001789507609	0.330349868917371	0.000001856472012	0.548101244029905	0.000001904962960
0.205442788718802	0.000001794509902	0.374320167996337	0.000001857180764	0.548388279811277	0.000001905247013
0.205684060575215	0.000001798622968	0.376826073683265	0.000001858266900	0.549825257544918	0.000001907197278
0.205925597970745	0.000001802616697	0.377035560629610	0.000001858279786	0.550688883784760	0.000001914084200
0.206651805889061	0.000001803026682	0.378715128008541	0.000001858319243	0.551553590213218	0.000001914688131
0.237970075562797	0.000001803411463	0.379346650849262	0.000001861078544	0.552130661465862	0.000001915625550
0.246592366587611	0.000001806502100	0.380401234919285	0.000001864559050	0.554154194568466	0.000001918965498
0.247300047414255	0.000001806921727	0.389792462352220	0.000001866995790	0.554443751389412	0.000001921497480
0.247737982583601	0.000001814955768	0.393909768096066	0.000001867994038	0.555893338903011	0.000001922949519
0.249321225587876	0.000001821456196	0.396529590104833	0.000001868283337	0.557636813260082	0.000001924927331
0.249929925124285	0.000001824910714	0.408506429623523	0.000001872427212	0.559968186450632	0.000001925020670
0.250986520908567	0.000001829796885	0.445351774583372	0.000001872517274	0.560552234165360	0.000001927509817
0.251059039895177	0.000001830052744	0.455362028189106	0.000001873913953	0.562893245855186	0.000001929446876
0.251204360663937	0.000001830881628	0.455855148857458	0.000001875190657	0.569370844534710	0.000001929654446
0.251349758960171	0.000001833414697	0.456105408214626	0.000001878528294	0.579196964562831	0.000001930619788
0.251495234817490	0.000001834928239	0.459567899634560	0.000001883345869	0.593114308029870	0.000001932538889
0.253540059565473	0.000001836032798	0.460562249228385	0.000001885072776	0.593725242187042	0.000001936267805
0.253639611419502	0.000001840829578	0.462556369651538	0.000001887945105	0.667235834068728	0.000001936333668
0.256487803102977	0.000001841508593	0.465561128656430	0.000001887959003	0.667569530281695	0.000001940541029
0.257381748807708	0.000001842998933	0.466063508653509	0.000001891869449	0.667903348148679	0.000001940629147
0.257918985550520	0.000001847477810	0.473144585412964	0.000001892184967	0.676625285425541	0.000001941954156
0.258995441524819	0.000001847987373	0.474418468766549	0.000001892906388	0.676962384669574	0.000001947644879
0.261722305038144	0.000001850097960	0.474673588309561	0.000001897997142	0.690545785992730	0.000001950461993

Appendix No. (1) Results of the severity ratio of Kummer Beta Generalized Normal Distribution (KBGND)

HX	HX(GA)	HX	HX(GA)
0.130886998462242	0.000001762784638	0.856331575158541	0.000001978534883
0.145957442312183	0.000001766106603	0.875743984853729	0.000001979778159
0.158760839025285	0.000001768221468	0.475184170468013	0.000001898732411
0.160852712182567	0.000001771157571	0.693971900051663	0.000001951917566
0.160948418285487	0.000001771728314	0.877738703079929	0.000001986019876
0.161044178603251	0.000001772167408	0.946872144282580	0.000001986658998
0.266286894064590	0.000001851602438	0.954077858734449	0.000001988342670
0.718779544720391	0.000001951988003	0.956997424545983	0.000001993344946
0.720756379039874	0.000001953548862	0.961175461762907	0.000001999925856
0.778663085218998	0.000001956423978	0.961593729962347	0.000002001476389
0.782381252531940	0.000001958366525	0.962430518411591	0.000002003550701
0.797744968928879	0.000001958516511	0.962849038460069	0.000002006146665
0.801457545141513	0.000001958612017	0.964105100689398	0.000002006792584
0.803792154263573	0.000001958925044	0.968717066465118	0.000002008238769
0.805687792808354	0.000001964383263	0.971236891083213	0.000002009292427
0.815201326612294	0.000001967604286	0.975864191613943	0.000002013906007
0.816060952896466	0.000001968400194	0.977127879852764	0.000002014135500
0.816920647791113	0.000001973843193	0.977549269841842	0.000002014198474
0.817780410183260	0.000001973882276	0.978392290043552	0.000002017319828
0.818640238959897	0.000001974848778	0.984302331489700	0.000002020049485
0.820360091214509	0.000001975138329	0.986416799043120	0.000002022704495
0.835230172311825	0.000001975772133	0.989803988363578	0.000002023924646
0.845321785148334	0.000001975915252	0.996168194593426	0.000002027510175
0.854757904762457	0.000001977545967		

بعض مقدرات المعلمات ودالة المخاطرة لتوزيع كومر بيتا الطبيعي المعمم

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مستخلص البحث:

تم تحويل التوزيع الطبيعي الاعتيادي من خلال النموذج المولد (kummer Beta) الى توزيع كومر بيتا الطبيعي المعمم (Kummer Beta Generalized Normal Distrbution) ومن ثم تقدير معلمات التوزيع ودالة المخاطرة باستعمال طريقة الامكان الاعظم (MLE), وتحسين هذه المقدرات من خلال توظيف الخوارزمية الجينية ومن ثم استعمال المحاكاة بافتراض عدد من النماذج وحجوم عينات مختلفة. ومن أهم ما تم التوصل اليه هو ان طريقة الامكان الأعظم الاعتيادية (MLE) هي الأفضل في تقدير معلمات توزيع كومر بيتا الطبيعي المعمم (KBGND) مقارنة بطريقة الخوارزمية الجينية وذلك بالاعتماد على المقياس الإحصائي متوسط مربعات الخطأ (MSE) Mean Squared Error والخطأ التكاملي (IMSE) في تقدير دالة المخاطرة. اما الجانب التطبيقي اظهر بان دالة المخاطرة هي متزايدة، بمعنى أن ازدياد فترة البقاء للمعلمين في الخدمة سيعرضهم الى معدل فشل أكبر نتيجة فترة البقاء التي تتناقص بدورها.

المصطلحات الرئيسة للبحث/ إنموذج كومر بيتا المعمم ، التوزيع الطبيعي ، معيار متوسط مربعات الخطأ التكاملي(IMSE)،طريقة الامكان الأعظم .