Ibn Al-Haitham Jour. for Pure & Appl. Sci. 34 (2) 2021

Ibn Al Haitham Journal for Pure and Applied Science

Journal homepage: http://jih.uobaghdad.edu.iq/index.php/j/index

The Effect of Individuals Asymptomatic (*Carrier*) on The Dynamical Behavior Of a COVID-19 Virus

Hassan F. AL-Husseiny
Hassan.fadhil.r@sc.uobaghdad.edu.iq
Department of Mathematics, College of Science, University of Baghdad, Iraq.

Ahmed A. Mohsen
aamuhseen@gmail.comDepartment
Department of Mathematics, College of Education for Pure Science (Ibn Al-Haitham), University of Baghdad, Iraq

Xueyong Zhou
xueyongzhou@xynu.edu.cn
School of Mathematics and Statistics, Xinyang Normal University, Xinyang 464000, Henan, P.R. China

Article history: Received,15,June,2020, Accepted 20,Julay ,2020, Published in April 2021

Doi: 10.30526/34.2.2624

Abstract
In this paper, a novel coronavirus (COVID-19) model is proposed and investigated. In fact, the pandemic spread through a close contact between infected people and other people but sometimes the infected people could show two cases; the first is symptomatic and the other is asymptomatic (carrier) as the source of the risk. The outbreak of Covid-19 virus is described by a mathematical model dividing the population into four classes. The first class represents the susceptible people who are unaware of the disease. The second class refers to the susceptible people who are aware of the epidemic by media coverage. The third class is the carrier individuals (asymptomatic) and the fourth class represents the infected individuals. The existence, uniqueness and bounded-ness of the solutions of the model are discussed. All possible equilibrium points are determined. The locally asymptotically stable of the model is studied. Suitable Lyapunov functions are used to investigate the globally asymptotical stability of the model. Finally, numerical simulation is carried out to confirm the analytical results and to understand the effect of varying the parameters of how the disease spreads.
Keywords: n-Coronavirus, Covid-19, Stability, Awareness, Asymptomatic People.

1. Introduction
In November 2002, the severe acute respiratory syndrome coronavirus emerged in China causing global anxiety as the outbreak rapidly spreads, and by July 2003, had resulted in over 8000 cases in 26 countries. The SARS epidemic of 2003 reported 8098 cases with 774 deaths, and was eventually brought under control by July 2003, in a period of 8 months [1,2]. Ahmed et al [3] gives a review of China response in comparison with other SARS-affected countries. Riley et al [4] studied the transmission dynamics of SARS in Hong Kong.

Since December 2019, a novel corona virus, named (COVID-19), emerged in Wuhan, China and the epidemic has globally spread, updated on January 30, 2020, COVID-19 was declared a public health emergency of international concern. It caused more than 200,000 deaths out of 3,177,500 cases in the world among which 1,434,000 in Europe, 1,291,000 in Americas, 148,000 in Western Pacific and other cases in Asia and Africa [5]. The number of the total cases, total death, new confirmed cases and recovery for some countries in the world are shown in **Figure 1**.

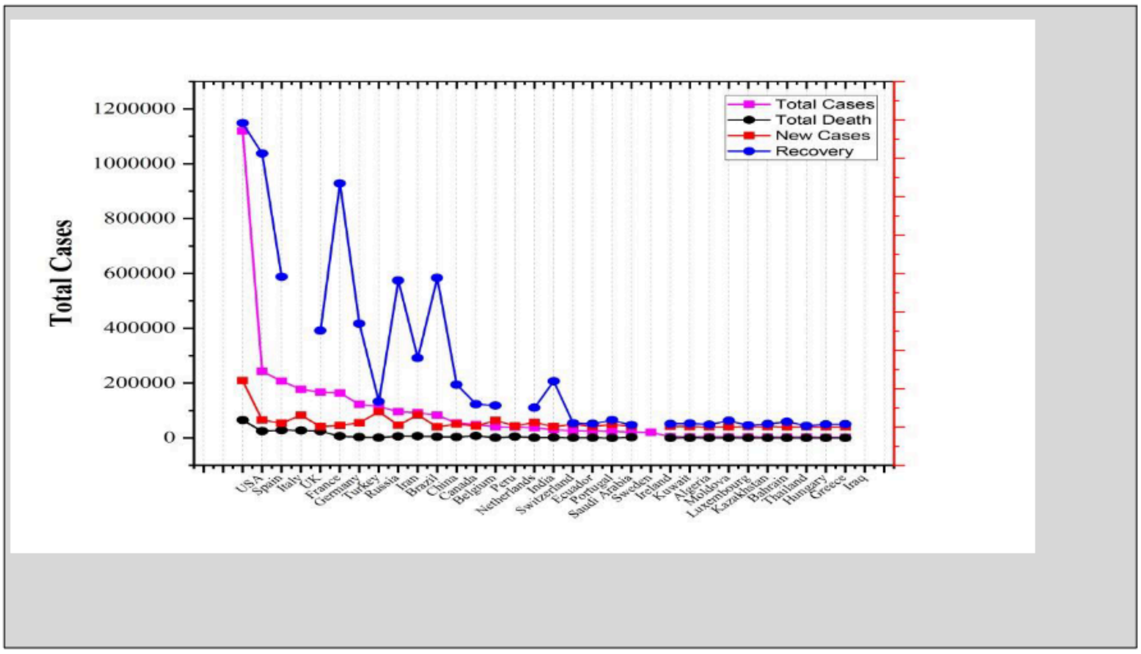

Figure 1. The number of total cases, total death, new confirmed cases and recovery of some countries in world from 20 January to 05 May 2020.

There are some factors that complicate the infection dynamics of novel coronavirus and add challenges to how to control the epidemic. First, the clinical evidence shows that the incubation period of this epidemic ranges from 7 to 14 days. During this period of time, infected individuals may not develop any symptoms and may not be aware of their infection, yet they are capable of transmitting the disease to other people. Secondly, the virus is new and there are no vaccines currently available for treatment. Thirdly, many people do not take into account prevention measures. Fourthly, the origin of the infection is still uncertain although it is widely speculated that wild animals such as bats, civets and minks are responsible for starting up the epidemic [6]. A number of modeling studies have already been performed for the COVID-19 epidemic. Feng et al. [7] studied a COVID-19 model in UK with the effects of media and quarantine.Yang and Wang [8] have proposed a mathematical model for COVID-19 epidemic in Wuhan, China. Wu et al. [9] have introduced epidemic model with (SEIR) type to study the transmission dynamic and global spread of the disease depending on reported data from December 31, 2019 to January 28, 2020. Mohsen et al. [10] have introduced a mathematical model for COVID-19 pandemic involving the quarantine strategy and media coverage effects. Abdulkadhim and Alhusseiny [11] have studied the global stability and bifurcation of a COVID-19 virus modeling with possible loss of the immunity. Kucharski et al. [12] have discussed early dynamics of transmission and control of COVID-19 in a mathematical model.

In this study, the researchers have proposed and studied the mathematical model of the novel coronavirus involving awareness programs. Details about the mathematical modeling of the novel coronavirus are shown in Section 2. Some basic properties
(existence equilibrium points and calculated reproduction number) of the model are discussed in Section 3. The local stability analysis is studied by using Gersgorin theorem in Section 4. The cases of the backward bifurcation occurrence are shown in Section 5. By using Lyapunov functions, the researchers have also studied the global stability of the proposed model at all equilibrium points as shown in Section 6. Finally, the effect of media coverage to raise the people's awareness to the risk of the coronavirus and the effect of the direct contact with carrier individuals on breaking out the COVID-19 among people are shown by numerical simulation of the proposed model.

2. Model Formulation

The researchers have formulated the coronavirus mathematical model involving awareness. It is assumed that the total population denoted by N is subdivided into four compartments: unaware susceptible individuals denoted by (S_u) , aware susceptible individuals denoted by (S_a), asymptomatic individuals (Carrier) denoted by (C), and infected individuals (symptomatic) denoted by (I), Recovery individuals denoted by $R(t)$. Thus, $N = S_u + S_a + C + I + R$. Let (v) be the coronavirus resources in (host). This model describes COVID-19 virus by the following equations:

$$\begin{aligned}
\dot{S_u} &= \psi - \alpha S_u - \beta S_u v - \sigma S_u C - \mu S_u \\
\dot{S_a} &= \alpha S_u - \beta_1(1-\epsilon)S_a v - \sigma_1(1-\epsilon)S_a C - \mu S_a \\
\dot{C} &= \beta S_u v + \beta_1(1-\epsilon)S_a v + \sigma S_u C + \sigma_1(1-\epsilon)S_a C - (\mu+\theta+\gamma+\delta_1)C \\
\dot{I} &= \gamma C - (\mu+\theta+\delta_2)I \\
\dot{R} &= \delta_1 C + \delta_2 I - \mu R \\
\dot{v} &= rv\left(1-\frac{v}{k}\right) - dv
\end{aligned} \tag{1}$$

With initial conditions: $S_u > 0, S_a \geq 0, C \geq 0, I \geq 0, R \geq 0, v \geq 0$. In system (1) the birth rate of individuals is given$\Psi > 0$. The awareness rate is given$\alpha \geq 0$. The parameters $\beta > 0, \beta_1 > 0, \sigma > 0$ and $\sigma_1 > 0$ respectively measure the contact rate between susceptible with carrier, and coronavirus with the prevention of disease that is given a rate of $(0 \leq \epsilon \leq 1)$. The death rate is $\mu > 0$. The death rate due to the disease is $\theta > 0$. After 14^{th} days, symptoms of the disease begin to appear on the carrier that is given a rate of $\gamma > 0$. The recovery rates from carrier and infected individuals are $\delta_i > 0\,, i = 1,2$ respectively. $r > 0, k > 0$ and $d > 0$ respectively, represent the spread rate of the virus, the carrying capacity of coronavirus and the removal rate of virus.

Theorem (1): All solutions of system (1) which initiate in R_+^5 are uniformly bounded.
Proof: Let $w(T) = (N(t),\ v(t))$
where $\mathrm{N(t)} = \mathrm{S_u(t)} + S_a(\mathrm{t}) + \mathrm{C(t)}, +\mathrm{I(t)} + \mathrm{R(t)};$
Then
$$\frac{dW}{dt} = \left(\frac{dN}{dt}, \frac{dv}{dt}\right) = \left(\psi - \mu \mathrm{S_u} - \mu S_a - (\mu+\theta)C - (\mu+\theta)I - \mu R, rv\left(1-\frac{v}{K}\right) - dv\right)$$

We know that

$\frac{dN}{dt} \leq \psi - qN$, where $q = min\{\mu, \mu+\theta, d\}$

Which implies that

$$\lim_{t\to\infty} supN(t) \leq \frac{\psi}{q}$$

While, the last equation of system (1) it follows that

$$\sup\left(rv\left(1-\frac{v}{k}\right)\right) \leq \frac{rk}{4}$$

Hence

$$\frac{dv}{dt} \leq \frac{rk}{4} - dv$$

So that

$$\lim_{t\to\infty} \sup v(t) \leq \frac{rk}{4d}$$

This completes the proof of theorem ■

3. Existence of equilibrium points

It is easy that the infected and recovery population I and R, are related with carrier population only . Hence for fixed value of C, the values of I and R can be determined directly by solving the fourth equation in system (1). Then, we can calculate the values of I and R the following form

$$I = \frac{\gamma C_*}{\mu+\theta} \quad (2a)$$

$$R = \frac{\delta_1 C_* + \delta_2 I_*}{\mu} \quad (2b)$$

Consequently, for simplifying, system (1) can be reduced to the following system, in which we can determine the values of I and R, by solving it,

$$\begin{aligned}
\dot{S_u} &= \psi - \alpha S_u - \beta S_u v - \sigma S_u C - \mu S_u \\
\dot{S_a} &= \alpha S_u - \beta_1(1-\epsilon)S_a v - \sigma_1(1-\epsilon)S_a C - \mu S_a \\
\dot{C} &= \beta S_u v + \beta_1(1-\epsilon)S_a v + \sigma S_u C + \sigma_1(1-\epsilon)S_a C - (\mu+\theta+\gamma+\delta_1)C \\
\dot{v} &= rv\left(1-\frac{v}{k}\right) - dv
\end{aligned} \quad (3)$$

Now, we can compute the reproduction number for the given system (3) that is denoted by $\mathcal{R}_0$, such that

$$\mathcal{R}_0 = \frac{\sigma\psi}{\mu(\mu+\theta+\gamma+\delta_1)} \quad (4)$$

Therefore, system (3) has at most two biologically feasible points, namely, $E_i = (S_{ui}, S_{ai}, C_i, v_i)$, $i = 0, 1$. The existence conditions for each of these equilibrium points are discussed in the following:

- In the absence of COVID-19 virus, that is $S_a = C = v = 0$. Then, system (3) has a unique positive equilibrium point, namely COVID-19 free equilibrium point, which is denoted by $E_0 = (S_{u0}, 0,0,0)$ where

  $$S_{u0} = \frac{\psi}{\mu} \quad (5a)$$

  provided that the following condition holds

  $$\alpha = 0 \quad (5b)$$

- The endemic equilibrium point or COVID-19 equilibrium point, which is denoted by $E_1 = (S_{u1}, S_{a1}, C_1, v_1)$, where

$$S_{u1} = \frac{r\psi}{r\alpha+\beta(r-d)k+r\mu+r\sigma C_1} ;$$
$$S_{a1} = \frac{\alpha r^2\psi}{A_1\, C_1^2+A_2C_1+A_3} ; \qquad (6a)$$
$$v_1 = \frac{(r-d)k}{r}$$

here

$$A_1 = \sigma_1\,(1-\epsilon)r^2\sigma$$
$$A_2 = \beta_1(1-\epsilon)(r-d)kr\sigma + \sigma_1\,(1-\epsilon)r[r\alpha+\beta(r-d)k+r\mu] + r^2\,\mu\sigma$$
$$A_3 = [\beta_1(1-\epsilon)(r-d)k + r\mu][r\alpha+\beta(r-d)k+r\mu]$$

exists under the condition

$$r > d \qquad (6b)$$

Now, substituting the equation (6a) in 3rd equation of system (3), and simplifying that result, we get

$$D_1C^4 + D_2C^3 + D_3C^2 + D_4C + D_5 = 0 \qquad (7a)$$

Where

$$D_1 = -(\gamma+\theta+\mu+\delta_1)r^2\sigma A_1 < 0$$

$$D_2 = \sigma r^2\psi A_1 - (\gamma+\theta+\mu+\delta_1)(r^2\alpha+\beta(r-d)kr+r^2\mu)A_1 \\ -(\gamma+\theta+\mu+\delta_1)r^2\sigma A_2$$

$$D_3 = \beta(r-d)kr\psi A_1 + \sigma r^2\psi A_2 + \sigma_1(1-\epsilon)\alpha r^4\psi\sigma \\ -[\gamma+\theta+\mu-(\gamma+\theta+\mu+\delta_1)r^2\sigma A_2](r^2\,\alpha+\beta(r-d)kr+r^2\mu)A_2 \\ -(\gamma+\theta+\mu+\delta_1)r^2\sigma A_3$$

$$D_4 = \beta(r-d)kr\psi A_2 + \beta_1(1-\epsilon)(r-d)k\alpha r^3\psi\sigma + \sigma r^2\psi A_3 \\ +\sigma_1(1-\epsilon)\alpha r^3\psi(r\alpha+\beta(r-d)kr+\,\mu r) \\ -(\gamma+\theta+\mu+\delta_1)[r^2\alpha+\beta(r-d)kr+r^2\mu]$$

$$D_5 = \beta(r-d)kr\psi A_3 + \beta_1 k\alpha r^2\psi(1-\epsilon)(r-d)[r\alpha+\beta(r-d)k+r\mu] > 0$$

Obviously, the endemic equilibrium point exists unique if and only if the following condition holds

$$D_3 > 0\,, D_4 > 0 \qquad (7b)$$

or

$$D_2 < 0\,, D_3 < 0 \qquad (7c)$$

4. Local stability analysis

In this section, the local stability analysis of the all equilibrium points $E_i,\ i = 0,1$ of system (3) studied as shown in the following theorems.

Theorem (2): The COVID-19 free equilibrium point E_0 of the system (3) is locally asymptotically if the following condition is satisfied

$$\mathcal{R}_0 < 1 \qquad (8a)$$
$$r < d \qquad (8b)$$

Proof: The Jacobian matrix of system (3) at E_0 can be written as

$$J(S_{u0},0,0,0) = \begin{pmatrix} -\mu & 0 & -\sigma S_{u0} & -\beta S_{u0} \\ 0 & -\mu & 0 & 0 \\ 0 & 0 & \sigma S_{u0} - (\gamma+\theta+\mu+\delta_1) & \beta S_{u0} \\ 0 & 0 & 0 & r-d \end{pmatrix}$$

Thus, the eigenvalues of $J(E_0)$ are given by

$$\lambda_1 = \lambda_2 = -\mu < 0$$
$$\lambda_3 = \sigma S_{u0} - (\gamma + \theta + \mu + \delta_1)$$
$$\lambda_4 = r - d$$

It is easy from above to verify that conditions (8a) and (8b) guarantee that the eigenvalue λ_3 and λ_4 are negative respectively. Then, E_0 is locally asymptotically stable. However, it is a saddle point otherwise. ■

Theorem (3): The endemic equilibrium point E_1 of the system (3) is locally asymptotically if the following conditions are hold

$$2\sigma S_{u1} + 2\sigma_1(1-\epsilon)S_{a1} < \gamma + \theta + \mu + \delta_1 \quad (9a)$$
$$k\,[2\beta S_{u1} + 2\beta_1(1-\epsilon)S_{a1} + r] < 2rv_1 + kd \quad (9b)$$

Proof: The Jacobian matrix of system (3) at E_1 can be written as

$$J(S_{u1}, S_{a1}, C_1, v_1) = \begin{pmatrix} b_{11} & 0 & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ 0 & 0 & 0 & b_{44} \end{pmatrix}$$

Here

$b_{11} = -(\alpha + \beta v_1 + \sigma C_1 + \mu)$; $b_{13} = -\sigma S_{u1}$; $b_{14} = -\beta S_{u1}$;
$b_{21} = \alpha$; $b_{22} = -[\beta_1(1-\epsilon)v_1 + \sigma_1(1-\epsilon)C_1 + \mu]$;
$b_{23} = -\sigma_1(1-\epsilon)S_{a1}$; $b_{24} = -\beta_1(1-\epsilon)S_{a1}$
$b_{31} = \beta v_1 + \sigma C_1$; $b_{32} = \beta_1(1-\epsilon)v_1 + \sigma_1(1-\epsilon)C_1$;
$b_{33} = \sigma S_{u1} + \sigma_1(1-\epsilon)S_{a1} - (\gamma+\theta+\mu+\delta_1)$; $b_{34} = \beta S_{u1} + \beta_1(1-\epsilon)S_{a1}$
$b_{44} = \frac{-2rv_1}{k} + (r-d)$; $b_{12} = b_{41} = b_{42} = b_{43} = 0$

Now, according to Gersgorin theorem [13], if the following condition holds:

$$|b_{ii}| > \sum_{\substack{i=1 \\ i\neq j}}^{4} |b_{ij}| = P_i$$

Then all eigenvalues of $J(E_1)$ exist in the region:

$$\Omega = \cup \left\{ U^* \epsilon\, C : |U^* - b_{ii}| < \sum_{\substack{i=1 \\ i\neq j}}^{4} |b_{ij}| \right\}$$

Then, all the eigenvalues of $J(E_1)$ exist in the disc centered at b_{ii} with radius P_i. Thus if the diagonal elements are negative and the condition (9a) holds, all the eigenvalues will be exist in the left half plane and the E_1 of system (3) is locally asymptotically stabile. Clearly conditions (9a)-(9b) guarantee the existence of all eigenvalues in the left half plane and the proof follows. ■

5. Global stability analysis

The propose of this section is to investigate the global stability of COVID-19 virus mathematical model given by equations in system (3) near COVID-19 free and COVID-19 points respectively given by $E_i,\ i = 0,1$. We obtain the result in the following theorems

Theorem (4): E_0 is globally asymptotically stable provided that the following conditions hold:

$$\mathcal{R}_0 < 1 \tag{10a}$$

$$\beta S_{u0} < d - r \tag{10b}$$

Proof: Consider the following function

$$V_0(S_u, S_a, C, v) = \left(S_u - S_{u0} - S_{u0}\ln\frac{S_u}{S_{u0}}\right) + S_a + C + v$$

Clearly, $V_0: R_+^4 \to R$ is a continuously differentiable function such that $V_0(S_{u0}, 0, 0,0) = 0$ and $V_0(S_u, S_a, C, v) > 0,\ \forall\ (S_u, S_a, C, v) \neq (S_{u0}, 0, 0,0)$. Further, we have

$$\frac{dV_0}{dt} = \left(\frac{S_u - S_{u0}}{S_u}\right)[\psi - \beta S_u v - \sigma S_u C - \mu S_u] + [-\beta_1(1-\epsilon)S_a v - \sigma_1(1-\epsilon)S_a C - \mu S_a]$$
$$+[\beta S_u v + \beta_1(1-\epsilon)S_a v + \sigma S_u C + \sigma_1(1-\epsilon)S_a C - (\gamma+\theta+\mu+\delta_1)C]$$
$$+\left[(r-d)v - \frac{r}{k}v^2\right]$$

Now, by doing some algebraic manipulation and using the conditions (8), (10a) and (10b), we get

$$\frac{dV_0}{dt} \leq \frac{-\mu}{S_u}(S_u - S_{u0})^2 - \mu S_a - [(\gamma+\theta+\mu+\delta_1) - \sigma S_{u0}]C - \frac{r}{k}v^2 - [(d-r) - \beta S_{u0}]v$$

Obviously, $\dot{V}_0 = 0$ at $E_0 = (S_{u0}, 0,0,0)$, moreover $\dot{V}_0 < 0$ otherwise. Hence, $\dot{V}_0$ is negative definite and then the solution starting from any initial point that satisfies the conditions (8b) with (10a) and (10b) will approach asymptotically to COVID-19 free equilibrium point. Hence, the proof is complete. ■

Theorem (5): E_1 is global asymptotically stable if $\mathcal{R}_0 > 1$.

Proof: For the COVID-19 equilibrium $E_1 = (S_{u1}, S_{a1}, C_1, v_1)$; S_{u1}, S_{a1}, C_1 and v_1 satisfies equations

$$\begin{aligned}
&\psi - \alpha S_u - \beta S_u v - \sigma S_u C - \mu S_u = 0\\
&\alpha S_u - \beta_1(1-\epsilon)S_a v - \sigma_1(1-\epsilon)S_a C - \mu S_a = 0\\
&\beta S_u v + \beta_1(1-\epsilon)S_a v + \sigma S_u C + \sigma_1(1-\epsilon)S_a C - (\mu+\theta+\gamma+\delta_1)C = 0\\
&rv\left(1 - \tfrac{v}{k}\right) - dv = 0
\end{aligned} \tag{11}$$

By applying (11) and putting

$$\frac{S_u}{S_{u1}} = x\ ,\quad \frac{S_a}{S_{a1}} = y\ ,\quad \frac{C}{C_1} = z\ ,\quad \frac{v}{v_1} = u$$

We have

$$
\begin{aligned}
x' &= x\left[\frac{\psi}{S_{u1}}\left(\frac{1}{x}-1\right)-\beta v_1(u-1)-\sigma C_1(z-1)\right]\\
y' &= y\left[\alpha\frac{S_{u1}}{S_{a1}}(x-1)-\beta_1(1-\epsilon)v_1(u-1)-\sigma_1(1-\epsilon)C_1(z-1)\right]\\
z' &= z\left[\frac{\beta S_{u1}v_1}{C_1}\left(\frac{xu}{z}-1\right)+\frac{\beta_1(1-\epsilon)S_{a1}v_1}{C_1}\left(\frac{yu}{z}-1\right)+\sigma S_{u1}(x-1)+\sigma_1(1-\epsilon)(y-1)\right]\\
u' &= u\left[\frac{-rv_1}{k}(z-1)\right]
\end{aligned} \tag{12}
$$

Define the Lyapunov function

$$
\begin{aligned}
V_1 = {} & S_{u1}(x-1-\ln x)+S_{a1}(y-1-\ln y)\\
& +C_1(z-1-\ln z)+v_1(u-1-\ln u)
\end{aligned} \tag{13}
$$

The derivative of V_1 is given by

$$\frac{dV_1}{dt}=S_{u1}\frac{x-1}{x}x'+S_{a1}\frac{y-1}{y}y'+C_1\frac{z-1}{z}z'+v_1\frac{u-1}{u}v'$$

$$
\begin{aligned}
\frac{dV_1}{dt} = {} & (x-1)\left[\psi\left(\frac{1}{x}-1\right)-\beta S_{u1}v_1(u-1)-\sigma S_{u1}C_1(z-1)\right]\\
& +(y-1)[\alpha S_{u1}(x-1)-\beta_1(1-\epsilon)S_{a1}v_1(u-1)-\sigma_1(1-\epsilon)S_{a1}C_1(z-1)]\\
& +(z-1)\left[\beta S_{u1}v_1\left(\frac{xu}{z}-1\right)+\beta_1(1-\epsilon)S_{a1}v_1\left(\frac{yu}{z}-1\right)\right.\\
& \left.+\sigma S_{u1}C_1(x-1)+\sigma_1(1-\epsilon)C_1(y-1)\right]+(u-1)\left[\frac{-rv_1^2}{k}(z-1)\right]
\end{aligned}
$$

Furthermore, by simplifying the resulting terms, we get that

$$
\begin{aligned}
\frac{dV_1}{dt} = {} & \psi\left(2-x-\frac{1}{x}\right)+\ \beta S_{u1}v_1\left(x+u-z-\frac{xu}{z}\right)+\alpha S_{u1}(xy-x-y+1)\\
& +\beta_1(1-\epsilon)S_{a1}v_1\left(y+u-z-\frac{yu}{z}\right)-\frac{rv_1^2}{k}(zu-z-u+1)
\end{aligned}
$$

Since the arithmetical mean is greater than, or equal to the geometrical mean, then, $2-x-\frac{1}{x}\leq 0$ for $x>0$ and $2-x-\frac{1}{x}=0$ if and only if $x=1$; $x+u-z-\frac{xu}{z}\leq 0$ for $x,z,u>0$ and $x+u-z-\frac{xu}{z}=0$ if and only if $x=1, z=u$; $xy-x-y+1\leq 0$ for $x,y>0$ and $xy-x-y+1=0$ if and only if $x=y=1$; $y+u-z-\frac{yu}{z}\leq 0$ for $y,z,u>0$ and $y+u-z-\frac{yu}{z}=0$ if and only if $y=1, z=u$; $zu-z-u+1\leq 0$ for $z,u>0$ and $zu-z-u+1=0$ if and only if $z=u=1$.
Therefore, $V_1'\leq 0$ for $x,y,z,u>0$ and $dV_1/dt=0$ if and only if $x=y=1, z=u=1$, the maximum invariant set of system (3) on the set $\{(x,y,z,u): dV_1/dt=0\}$ is the singleton (1,1,1,1). Thus, for system (3), the COVID-19 equilibrium point E_1 is globally asymptotically stable if $\mathcal{R}_0>1$ by *LaSalle Principle* [14].

6. Numerical Simulation

In this section, we present the numerical simulation results and effect of the parameters of the analytic results of system (3). Using MATLAB version 8 and C++ software, we begin the simulation with a set outbreak data published daily by *WHO* and other sources [8, 15-20], as shown in Table 1 which are described in Section 2.

Table 1: Definitions and values of parameters in system (3)

Parameter	Definition	Estimated mean values	Reference
ψ	Birth rate	$271.23\ per\ day$	[8]
α	Awareness rate	$\alpha \geq 0\ per\ day$	-
β	Contact rate between S_u and v	$1.555 \times 10^{-8}\ /person/day$	[8,19]
β_1	Contact rate between S_a and v	$3.1 \times 10^{-7}/person/day$	[8,19]
σ	Contact rate between S_u and C	$1.555 \times 10^{-8}/person/day$	[8,19]
σ_1	Contact rate between S_a and C	$3.1 \times 10^{-7}/person/day$	[8,19]
μ	Natural death rate	$3.01 \times 10^{-4}/person/day$	[18]
γ	Transmission asymptomatic to symptomatic	$7\ days$	-
ϵ	Prevention rate	$0.03/person$	-
θ	Death rate due to the disease	$0.01\ per\ day$	[8]
r	Spread rate of the virus	2	-
k	carrying capacity in environment	0.5	-
d	Removal rate of virus	$3\ per\ day$	[20]
δ_1	Recovery rate from carrier	$0.03\ perday$	-

Clearly, the initial point for $I(0) = 25$, we assume that the $C(0) = 10$, $S_a = 10$, $S_u = 15$ and $v = 20$. Using this initial point and simulated the system (1), we get the values of parameters shown in Table 1, whereas the basic reproduction number is estimated $\mathcal{R}_0 = 0.914 < 1$ with condition (8b), we obtain the dynamical behavior of system (3) converge to COVID-19 free equilibrium point $E_0 = (901096,0,0,0,0)$, illustrating its asymptotical stability that is stated in Theorem (2). This is shown in **Figure 2**.

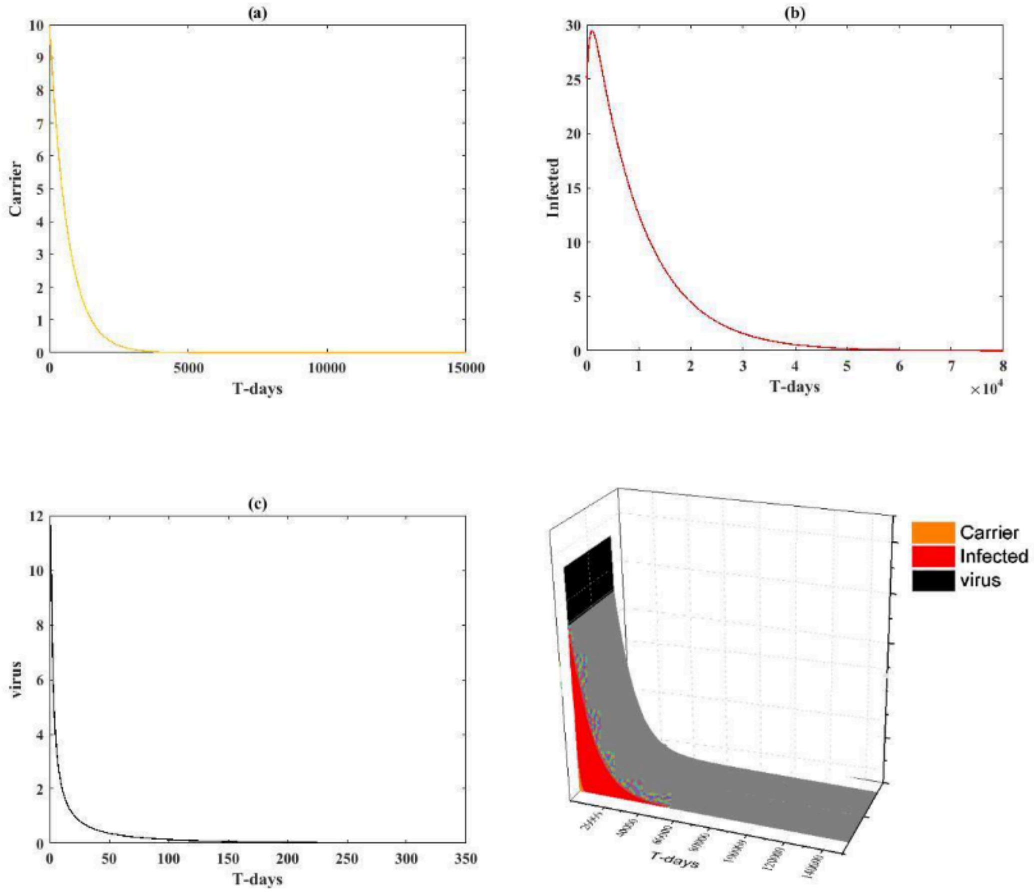

Figure 2: A simulation result for the dynamics of coronavirus model using the parameters from Table 1. The reproduction number $\mathcal{R}_0 = 0.07$ and the behavior coronavirus converge to COVID-19 free equilibrium point.

However, for the same data given by Table 1. with $d = 1$, *(i.e. the condition 6b is holds),* and $\sigma = 1.555 \times 10^{-5}$ *(i.e. the contact rate between S_u and C will increase).*
it is observed that, although system (3) is solved with the same initial point, the dynamical behavior of system (3) converge to COVID-19 equilibrium point $E_1 = (979,2734,1752,24299,0,25)$, illustrating its asymptotical stability that is stated in Theorem (3), and reproduction number is estimated $\mathcal{R}_0 = 7.77 > 1$. This is shown in **Figure 3.**

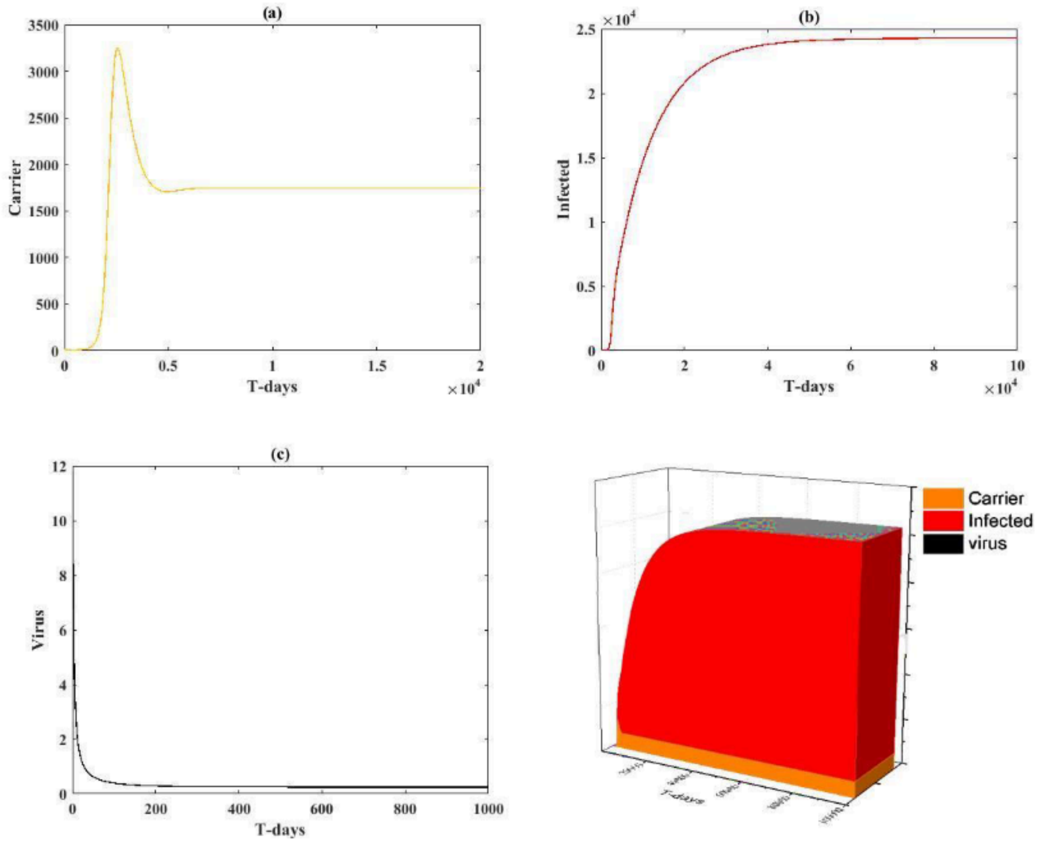

Figure 3: A simulation result for the dynamics of coronavirus model using the parameters from Table 1 with $d = 1$ and $\sigma = 1.555 \times 10^{-5}$. The reproduction number $\mathcal{R}_0 = 7.77$ and the behavior of coronavirus converge to COVID-19 equilibrium point.

Similarly, if the increase effect of the contact rate between S_u *and* v (β), as well as used the values of parameters shown in **Table 1** with $d = 1$, *(i.e. the condition 6b is holds),* and $\beta = 1.555 \times 10^{-4}$ *(i.e. the contact rate between S_u and v will increase).*, we obtain the dynamical behavior of system (3) still converge to COVID-19 equilibrium point $E_1 = ((61636,170389,25,334,0.25))$, illustrating its asymptotical stability that is stated in Theorem (3). This is shown in **Figure 4**.

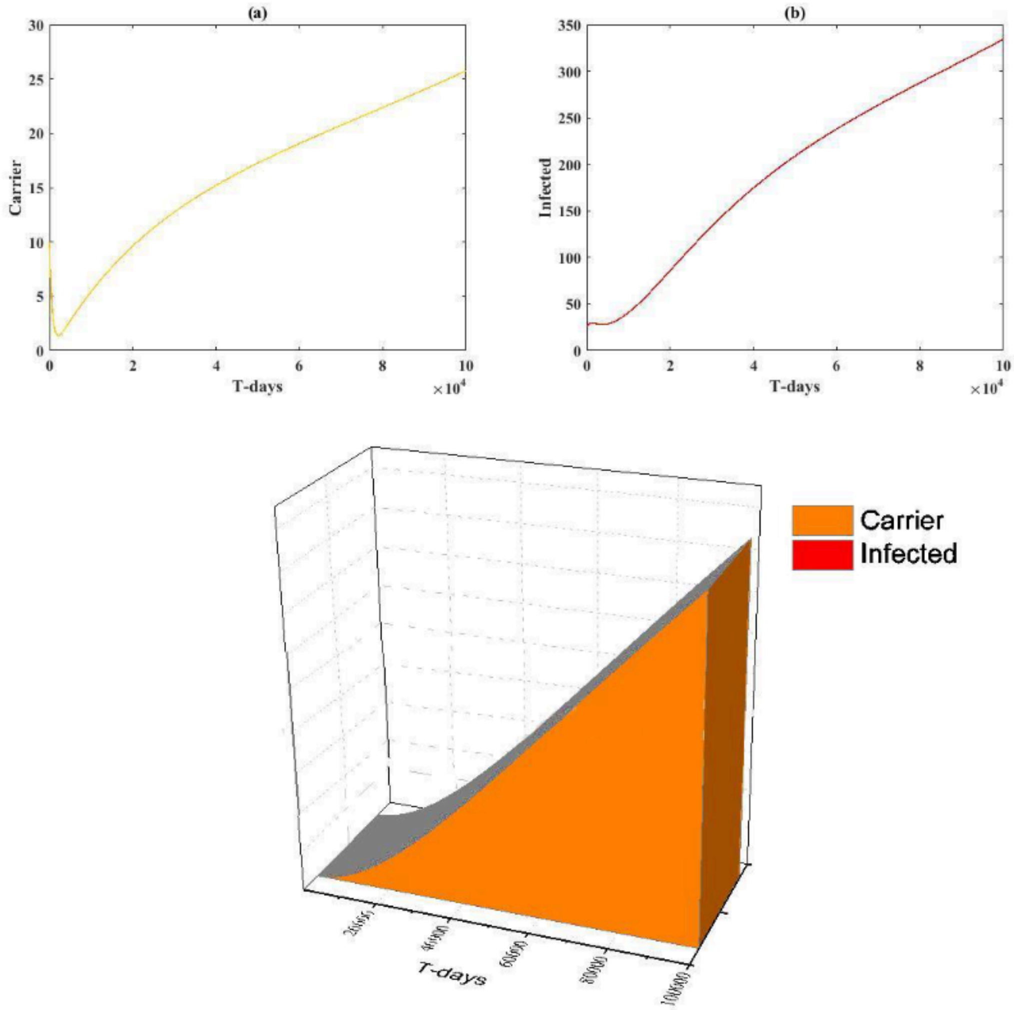

Figure 4: A simulation result for the dynamics of coronavirus model using the parameters from Table 1 with $d = 1$ and $\beta = 1.555 \times 10^{-4}$. The dynamical behavior of coronavirus model converge to COVID-19 equilibrium point.

7. Discussions and conclusions

In this manuscript, a coronavirus model is proposed and discussed. This model has two feasible equilibrium points which are COVID-19 free equilibrium point and epidemic equilibrium point. The results of the proposed work may be used to show and understand the effect of awareness rate, prevention rate of disease and contact rate between asymptomatic people and other. The stability (local and global) of the model has been analyzed at all equilibrium points. It has been obtained that COVID-19 free is locally asymptotically stable if the conditions (8a) and (8b) are hold and it is global stable under the conditions (10a) and (10b) are hold. While, the epidemic point it has been obtained under certain conditions for locally as well as globally asymptotically stable under the conditions (9a-9b) and (11a-11g) are hold respectively. To validate the analytical results, we have executed the numerical simulations for investigating the dynamics of a coronavirus mathematical model. We conclude if the removal rate of virus is increased through used the sterilization materials could us to decrease virus rate and hence, we get the basic reproduction number less than one (i.e. satisfied the condition 10a and 10b). Also caution should be exercised against asymptomatic people who are the source of the risk, so direct contact should be reduced with them. Finally, a rapid health examination to detect the infected is one of the important factors in the control to spread of the epidemic.

Acknowledgements

The authors are thankful to acknowledge the reviewers for their valuable suggestions and comments have contributed to the improvement of the authors work.

References

1. Tan, C. C. SARS in Singapore key lessons from an epidemic. *Annals Academy Medicine Singapore*, **2006**, *35*, 345-349.
2. Tsang, T.; Lam, T., SARS public health measures in Hong Kong. *Respirology Journal*, **2003**, *8*, 46-48.
3. Ahmed, A.; Krumkamp, R.; Reintjes, R. Controlling SARS a review on China response compared with other SARS-affected countries. *Tropical Medicine International Health*, **2009**, *14*, 36-45.
4. Riley, S.; Fraser, C.; Donnelly, C. Transmission dynamics of the etiological agent of SARS in Hong Kong impact of public health interventions. *Science mag Journal*, **2003**, *300*, 1961-1966.
5. WHO novel coronavirus (2019-nCOV) situation reports. Available from https://WWW.Who.covid19.int/detail/01-05-2020.
6. Zhou, P.; Yang, X. L.; Wang, X. G.; Hu, B.; Zhang, L.; Zhang, W. A pneumonia outbreak associated with a new coronavirus of probable bat origin**.** *Nature Journal*, **2020,** *579*, 270–273.
7. Feng, L; Jing, S; Hu, S. Modeling the effects of media coverage and quarantine on the COVID-19 infections in the UK. *Math. Biosci. Eng,* **2020**, *17*, 3618-3636.
8. Yang, C.; Wang, J., A mathematical model for the novel coronavirus epidemic in Wuhan, China, *Math. Bio. And Eng. J.*, **2020**, *17, 3*, 2708-2724. https://DOI:10.3934/mbe.2020148.
9. Wu, J. T.; Leung, K.; Leung, G. M., Now casting and forecasting the potential domestic and international spread of the 2019-nCoV outbreak originating in Wuhan, China: a modeling study, *Lancet*, **2020**, *395*, 689-697.
10. Mohsen, A. A.; Al-Husseiny, H. F.; Zhou, X.; Hattaf, K. Global stability of COVID-19 model involving the quarantine strategy and media coverage effects. *AIMS public Health*, **2020**, *7*, *3*, 587-605. DOI:10.3934/publichealth.2020047.
11. Abdulkadhim, M. M.; Al-Husseiny, H. F., Global stability and bifurcation of a COVID-19 virus modeling with possible loss of the immunity. *AIP conference Proceedings* 2292, 030003, **2020**. DOI.org/10.1063/5.0030669.
12. Kucharski, A. J.; Russel, T. W.; Diamond, C., Early dynamics of transmission and control of COVID-19 mathematical model study. *Lancent infected disease*, **2020**, *20*, 1-7. doi.org10.1016/S1473-3099(20)30144-4.
13. Horn, R. A.; Johnson, C. R., Matrix Analysis. Cambridge University press, **1985**.
14. LaSalle, J. P. The stability of dynamical systems, Society for Industrial and Applied Mathematics, Philadelphia, Pa., **1976**.
15. Wikipedia on the timeline of the 2019-20 Wuhan coronavirus outbreak. Available from https://en.wikipedia.org/wiki/timeline_the _2019\0t1\textendash20_wuhan_coronavirus_outbreak.
16. National Health Commission of china delay reports on novel coronavirus (in China). Available from: http://www.nhc.gov.cn/yjb/pzhgli/new_list.shtml.
17. Sina news real-time reports on novel coronavirus (in Chinese). Available from: https://news.sina.cn/zt_d/yiqing0121.
18. The government of Wuhan homepage. Available from: http://english.wh.gov.cn/.

19. Tang, B.; Wang, X.; Li, Q.; Bragazzi, N. L.; Tang, S.; Xiao, Y., Estimation of the transmission risk of 2019-nCoV and its implication for public health interventions, *Journal of Clinical Medicine*, **2020**, *9*, 462-475.
20. Geller, C.; Varbanov, M.; Duval, R. E., Human coronavirus. Insights into environmental resistance and its influence on the development of new antiseptic strategies, *Viruses Journal*, **2012**, *4*, 3044-3068.