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Hydrodynamic Modelling of Porous Media
2-Experimental Model of Filter

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Abstract

A study has been conducted to examine several selected water filter's media, namely, crushed silica, crushed anthracite coal, glass beads, crushed pocilinaite and crushed garnet. Large spread of particle size (0.5mm-2mm), porosity (35%-60%) and physical temperature (20^oC-80^oC) were tested in order to validate the experimental modeling of both Darcy's and Forchheimer's laws parameters.

Typical constants of head loss in porous media as functions of velocity using Forchheimer's model ($\Delta H/\Delta L= aV+bV^2$) has been correlated by using hydraulic conductivity coefficients a,b* which are experimentally evaluated at based conditions ($T_w=25^oC$) ,(n=50%),(d=1mm).*

The basic conclusion of a wide ranges of experiments show those empirical equations for different selected porous media and filter's characteristics in the forms.

$$\frac{a}{a*}=(\frac{n}{50\%})^{1.4}\left(\frac{1}{d}\right)^{2}(1.028)^{(25-T)} \quad \textit{or} \quad \frac{a}{a*}=2.64(\frac{n^{1.4}}{d^{2}})(1.028)^{(25-T)} \quad \textit{and} \quad \frac{b}{b^{*}}=(\frac{n}{50\%})^{1.333}(\frac{1}{d}) \quad \textit{or}$$

$$\frac{b}{b*}=2.514(\frac{n^{1.333}}{d}).$$

الخلاصــــــة

أجريت هذه الدراسة لفحص عدة أنواع أوساط منتقاة من المرشحات المستخدمة في تصفية المياه والمتمثلة بالحبيبات المنكسرة من (السليكا الرمل، الفحم، خبث الزجاج، البورسلنتات والغرانيت). شملت التجارب أحجام جزيئات (0.5mm-2mm) ومساميتها (35%-60%) ودرجة الحرارة (20^oC-80^oC) والتي اختبرت لتطابق النموذج العملي لنموذج Darcy و Forchheimer .

اعتمدت الثوابت المثالية لقيم الخسائر في الأوساط المسامية كدالة للسرعة باستخدام نموذج Forchheimer ($\Delta H/\Delta L= aV+bV^2$) والتي قيمت باستعمال معاملات التوصيل a, b* والتي تم إجرائها تحت ظروف قياسية(Tw=25oC) (n=50%),(d=1mm) .*

تم استنتاج المعادلات التجريبية من التجارب الواسعة التي أجريت على الأوساط المسامية والمرشحات والتي كانت كالتالي:

$$\frac{a}{a*}=(\frac{n}{50\%})^{1.4}\left(\frac{1}{d}\right)^{2}(1.028)^{(25-T)}$$

أو

$$\frac{a}{a*}=2.64(\frac{n^{1.4}}{d^{2}})(1.028)^{(25-T)}$$

و

$$\frac{b}{b^{*}}=(\frac{n}{50\%})^{1.333}(\frac{1}{d})$$

أو

$$\frac{b}{b*}=2.514(\frac{n^{1.333}}{d}).$$

1. Introduction

The simple linear relationship between flow and head loss by Darcy have been demonstrated by Forchheimer[1] for condition under which the relationship between flow and head loss in porous media doesn't follow. Forchheimer [1] proposed the nonlinear behavior of underground hydraulic gradients under certain conditions as follows:

$$\frac{\Delta H}{\Delta L} = av + bv^2 \quad \text{(1)}$$

where:

v: superficial (approach) velocity $= \frac{Q}{A} \quad (LT^{-1})$;

a: hydraulic conductivity coefficient related to linear head loss (TL^{-1});

b: hydraulic conductivity coefficient related to nonlinear head loss ($T^2 L^{-2}$)

Blake [2] developed empirical relationship using a plot of friction factor vs. Reynolds number for flow through tower packing at relatively high Reynolds numbers as the following form.

$$\frac{\Delta H}{\Delta L} = C_b \; \mu^{0.2} \; \gamma^{0.8} \left[\frac{(1-n)^{1.2}}{n^3}\right] S_v^{\,1.2}\, V^{1.8} \quad \text{(2)}$$

Burke and Plummer [3] used dimensional analysis to develop streamlined equation that fit quite well a variety of experimental data collected from the literature.

$$\frac{\Delta H}{\Delta L} = K_2 \left(\frac{1}{g}\right)\left[\frac{(1-n)}{n^3}\right] S_v V^2 \quad \text{(3)}$$

where:

K_2: *is constant;*

γ: *specific weight* ($ML^{-2}\,T^{-2}$);

C_b: *Blake constant.*

Also notable were Kozeny [4] and White [5] who applied similar principles to characterize the nonlinear resistance of porous media to the flow of gases at high Reynolds number. Nutting [6] defined the specific resistance and Wyckoff et. al. [7] popularized Nuttings specific resistance parameter, resulting in its wide use whenever the modeling of the flow of fluids underground was undertaken. Fancher and Lemis [8], Fair and Hatch [9], Wallis and White [10] and Wallis [11] developed a powerful model for predicting Darcy resistance from the characteristics of the porous media. Forchheimer's model (Eq.1) will also shown that this is the best model which characterizes the flow through the porous media over the range of conditions of importance in water filtration. Rose and Risk [12] have argued that an equation of the following form often fits the data as well or better:

$$\frac{\Delta H}{\Delta L} = mv^{z} \qquad (4)$$

where: m, v: are coefficients.

To create an equation for losses through porous media over a wide range of flow conditions. Ergun and Orning [13] added to Kozeny [4] term for linear losses to the new term for kinetic losses. They argued that the transition form the dominance of viscous to kinetic effects for most packed systems is smooth indicating that there should be a continuous function relating pressure drop to flow rate. The following is the empirical equation that resulted:

$$\frac{\Delta H}{\Delta L} = C_1\left[\frac{\mu}{\rho_c^a}\right]\left[\frac{(1-n)^2}{n^3}\right] S_v^2\, V + C_2\left[\frac{1}{g}\right]\left[\frac{(1-n)}{n^3}\right] S_v V^2 \qquad (5)$$

Ergun [14] then substituted the diameter of a sphere d_S having the specific surface S_V in Equation (5) producing the following result:

$$\frac{\Delta H}{\Delta L} = 150\left[\frac{\mu}{\rho_g}\right]\left[\frac{(1-n)^2}{n^3}\right]\left[\frac{1}{d_s^2}\right] V + 1.75\left[\frac{1}{g}\right]\left[\frac{(1-n)}{n^3}\right]\left[\frac{1}{d_s}\right] V^2 \qquad (6)$$

The constants C_1=150 and C_2=75 were found from various sized spheres and also from sand and pulverized coke. Many experimenters have attempted to use Reynolds concept to determine the upper limit of the validity of Darcy's law by Franzini [15], Hubbert [16], and Scheidegger [17].

Irmay [18], Ward [19], Sunada [20], Wright [21], Fair et. al. [22], Beavers and Sparrow [23], Ahmed Sunada [24], Bear [25] and MacDonald [26] worked to show that Forchheimers nonlinear equation could be derived from the first principles beginning with the Navier-Stokes equation. Recently, modeling progresses of flows within unconsolidated, granular media rely mostly on experimental works using homogeneous, spherical, artificial media (Comiti and Renaud, [27]).

In civil engineering, these researches are applied to the study of internal flows within earth and rock structures (Martin [28]; Burchart and Christensen [29]; Shih [30]; Hansen [31]; Hamilton [32]; Wahyudi [33]; Bingjun et. al. [34] and to the problems of similarities of flow parameters in centrifuged geotechnical small-scale models, within which very large hydraulic gradients are often found (Babendreier [35]; Burchart [29]; Khalifa et. al. [36]).

Trussell and Chang [37] developed the constants (a and b) for three filter media (glass beads, crushed silica sand and crushed anthracite coal). Summary of design parameters for selected filter media is tabulated in **Table (1)**.

Table (1) Summary of design parameters for selected filter medium

Medium	coefficient		Typical porosity	Reference
	a	a		
Crushed anthracite	3.5-5.3	210-245	Trussell and Chang. (1999)	47-52
Crushed sand	2-2.5	110-115	Chang et. al. (1999)	40-43
Glass beds	1.3-1.8	130-150	Rumpf and Gupte (1971)	38-40

2. Experimental Set-Up and Method

The experimental set-up, the schematic diagram and the photograph of which are presented in **Fig.(1)** and **Plate (1)**, respectively, consists of both a hydraulic device and a measurement device.

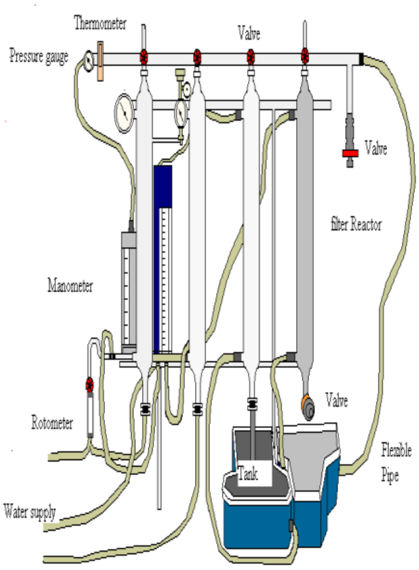

Figure (1) Experimental system

Plate (1) Photograph of system

The hydraulic device consists of a pump inverting the flow of water from a 400-liter tank through the material placed in the testing cell. When flowing out of the cell, water is forced back into the tank. The temperature is monitored at a quasi-constant value: 24-26 ^{0}C. Temperature sensors are placed in the tank, at the entrance and at the exit of the test cell. Then the values of the water density, ρ, and of the water viscosity ,µ,are quasi constant during the tests (ρ =998.2kg /m ^{3} and µ=1.008 *10 $^{-3}$ Pa s).

Some valves are used to select the pressure tappings and the differential pressure gauge suiting the best measurement range. The measurement bench consists of three flow meters and four differential pressure gauges regulated by a valve set. The different measurement ranges of these devices are complementary and can be used to cover all the measurements accurately. The sand was packed as homogeneously as possible using the pluviation method with a constant 1.2 m height of fall. Satisfactory repeatability of the bulk densities was achieved throughout the experiments .The bed porosity is calculated knowing the cell volume and the mass of sand.

3. Empirical Models in Porous Media at Forchheimer’s Regime

There are four regimes of flow in porous media. Darcy regime is the first which is limited to Reynolds number below 1. In this regime, flow is creeping flow, i.e., a laminar flow with no significant inertial contribution. Forchheimer regime is the second regime in which initially the flow is steady laminar but as it progresses the initial effects becomes very vital. Initially the head loss is proportional to v with small v^2 dependence but as it progresses the head loss becomes related to v^2 with small dependence on v. Furchheimer regime corresponds to a Reynolds number of approximately 100.

The third regime represents more or less inertial flow to full turbulence in which Reynolds number is between 600 and 800 depending on the media characteristics and flow conditions. The final regime is fully turbulence in which the velocity is randomly fluctuating. In water filter, Reynolds number in excess of 0.5 and less than 50 has to be kept to put them solidly in the Darcy and Forchheimer flow regimes.

The actual head loss through porous media such as under ground aquifer and water filter was often greater than that derived from Darcy’s law, particularly when high flow velocity exists. Forchheimer’s model for hydraulic gradient proposed nonlinear equation as (Eq.1).

Hydraulic conductivity depends upon a number of factors, which are summarized as (particle size distribution, particle shape and texture, mineralogical composition, voids ratio, degree of saturation, filter fabric, nature of fluid, type of flow and temperature). The hydraulic conductivity coefficient is influenced by particle characteristics. The smaller the particles, the smaller the voids between them, and therefore the resistance to flow of water increases (i.e. the hydraulic conductivity coefficients decreases) with decreasing particle size. Elongated or irregular particles create flow paths, which are more tortuous than those around nearly spherical particles. Particles with a rough surface texture provide more frictional resistance to flow than do smooth-textured particles. Both effects tend to reduce the rate of flow of water through the filter, i.e. to reduce its hydraulic conductivity coefficients.

Applying Poiseuilles equation in Eq.(5), *a* and *b* can be calculated from the following Equations:

$$a=C_1\{\frac{(1-n)^2}{n^3}\}(\frac{1}{d})^2 \quad \text{.......} \quad (7)$$

$$b=C_2\{\frac{(1-n)}{n^3}\}(\frac{1}{d}) \quad \text{.......} \quad (8)$$

A set of experiments for different water filters materials were carried in the hydraulic laboratory to evaluate the proposed hydraulic conductivity *a** and *b**at based conditions T_w =25°C,n=50%,d=1mm which is tabulated in **Table (2)**.

Table (2) Hydraulic conductivity coefficients at based condition (Tw=25oC),(n=50%),(d=1mm) for different selected porous media

Medium	Sg (specific gravity)	a* (s/m)	b* (s^2/m^2)
Crushed sand	2.6	31.2	1513
Crushed coal	1.4	56.3	1902
Glass beads	2.4	44.8	1082
Crushed garnet	3.8	26.2	1618
Crushed porcilinaite	1.12	36.4	1807

Forchheimer's model constants *a* and *b* (Eq.1)can be corrected and expressed in the forms:

$$\frac{a}{a*}=2.64(\frac{n^{1.4}}{d^2})(1.028)^{(25-T)} \quad \text{.......} \quad (9)$$

$$\frac{b}{b*}=2.514(\frac{n^{1.333}}{d}) \quad \text{.......} \quad (10)$$

From experiments on the selected materials used in water filters **Fig.(2)**, for various grain sizes (**Fig.(3)**), temperatures (**Fig.(4)**) and porosities (**Fig.(5)**). The properties of fluid, which are relevant to hydraulic conductivity coefficient, are density and dynamic viscosity. For water the density ρ_w varies little over the range of temperatures normally experienced (10°C – 80°C), but viscosity μ_w decreases by factor of about 4 over this range. For a laboratory test the standard temperature is 25°C, while the temperature for atypical field hydraulic conductivity test in U.S.A may be 20°C, and in Britain is 10°C.

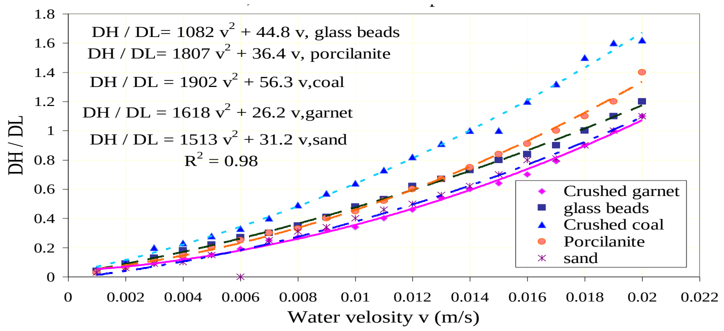

Figure (2) Relationship between pressure loss (head per unit filter depth) with water velocity for different bed materials with d=1 mm, n= 0.5 and water temperature = 25^{0}C

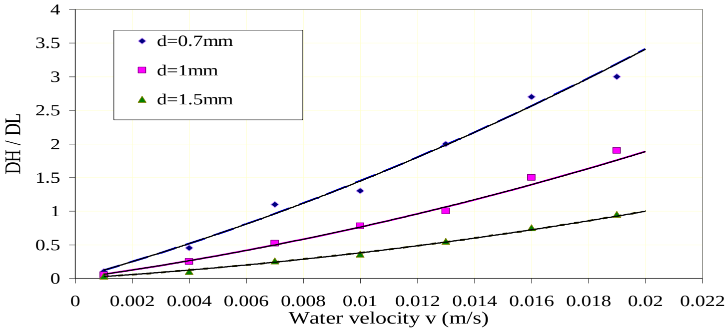

Figure (3) Effect of sand grain size on pressure drop (head per unit depth of sandfilter) for different water velocity at temperature =9^{0}C with porosity = 0.57

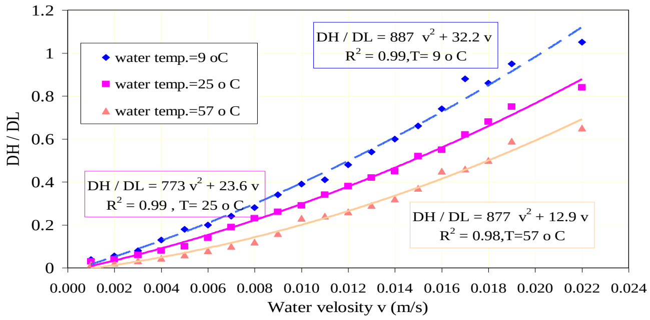

Figure (4) Relationship between pressure loss (head per unit sand filter depth) with water velocity for different water temperature with d=1.5 mm, n= 0.57

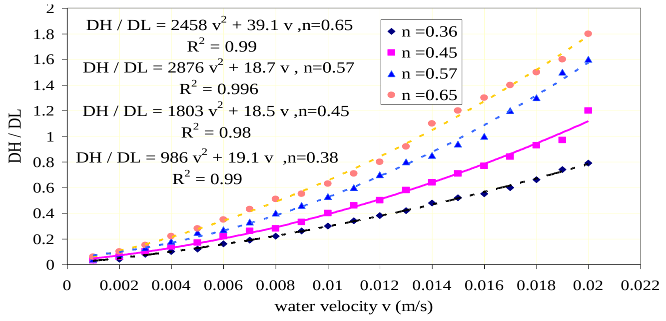

Figure (5) Relation ship between pressure loss (head per unit sand filter depth) with water velocity for different sand filter porosities with d=1 mm, n= 0.57 and water temperature =25^{0}C

Pressure losses (head per unit depth of different material) porosities, grain sizes and water temperature are tabled in **Tables (3)** to **(12)**.

Table (3) Pressure loss (Head per unit depth of filter ΔH/ΔL) for different values of grain sizes temperature and porosity with n=0.46, d=1.5 mm for porcilinaite

Velocity m/s	ΔH/ΔL		
	T = 9 °C	T = 25 °C	T = 57 °C
0.002	0.049105	0.033105	0.016205
0.004	0.106815	0.074815	0.041015
0.006	0.173131	0.125131	0.074431
0.008	0.248052	0.184052	0.116452
0.01	0.331579	0.251579	0.167079
0.012	0.423712	0.327712	0.226312
0.014	0.52445	0.41245	0.29415
0.016	0.633793	0.505793	0.370593
0.018	0.751743	0.607743	0.455643
0.02	0.878297	0.718297	0.549297

Table (4) Pressure loss (Head per unit depth of filter ΔH/ΔL) for different values of grain sizes temperature and porosity with n=0.38, d=0.7mm crushed coal

Velocity m/s	ΔH/ΔL		
	T = 9 °C	T = 25 °C	T = 57 °C
0.002	0.251043	0.164069	0.072217
0.004	0.517132	0.343184	0.15948
0.006	0.798267	0.537345	0.261789
0.008	1.094447	0.746551	0.379143
0.01	1.405674	0.970804	0.511544
0.012	1.731947	1.210103	0.658991
0.014	2.073265	1.464447	0.821483
0.016	2.42963	1.733838	0.999022
0.018	2.80104	2.018274	1.191606
0.02	3.187497	2.317757	1.399237

Table (5) Pressure loss (Head per unit depth of filter ΔH/ΔL) for different values of grain sizes temperature and porosity with n=0.38, d=1.5 mm for glass beads

Velocity m/s	ΔH/ΔL		
	T = 9 °C	T = 25 °C	T = 57 °C
0.002	0.044197	0.029125	0.013209
0.004	0.092389	0.062245	0.030413
0.006	0.144574	0.099358	0.05161
0.008	0.200755	0.140467	0.076803
0.01	0.260929	0.185569	0.105989
0.012	0.325098	0.234666	0.13917
0.014	0.393261	0.287757	0.176345
0.016	0.465418	0.344842	0.217514
0.018	0.54157	0.405922	0.262678
0.02	0.621716	0.470996	0.311836

Table (6) Pressure loss (Head per unit depth of filter ΔH/ΔL) for different values of grain sizes temperature and porosity with n=0.6, d=1.4 mm crushed garnet

Velocity m/s	ΔH/ΔL		
	T = 9 °C	T = 25 °C	T = 57 °C
0.002	0.065189	0.044197	0.022027
0.004	0.143199	0.101215	0.056875
0.006	0.234031	0.171055	0.104545
0.008	0.337685	0.253717	0.165037
0.01	0.454161	0.349201	0.238351
0.012	0.583458	0.457506	0.324486
0.014	0.725576	0.578632	0.423442
0.016	0.880516	0.71258	0.53522
0.018	1.048278	0.85935	0.65982
0.02	1.228862	1.018942	0.797242

Table (7) Pressure loss (Head per unit depth of filter ΔH/ΔL) for different values of grain sizes and porosity with n=0.46, T=9°C for porcilinaite

Velocity m/s	ΔH/ΔL		
	d=1mm	d=1.5mm	d=2mm
0.002	0.10726	0.049105	0.028429
0.004	0.227429	0.106815	0.063312
0.006	0.360505	0.173131	0.10465
0.008	0.50649	0.248052	0.152441
0.01	0.665384	0.331579	0.206687
0.012	0.837186	0.423712	0.267387
0.014	1.021896	0.52445	0.334541
0.016	1.219514	0.633793	0.408149
0.018	1.430041	0.751743	0.488211
0.02	1.653476	0.878297	0.574728

Table (8) Pressure loss (Head per unit depth of filter ΔH/ΔL) for different values of grain sizes and porosity with n=0.38, T=57°C crushed coal

Velocity m/s	**ΔH/ΔL**		
	d=0.7mm	**d=1.0mm**	**d=1.3mm**
0.002	0.072217	0.036966	0.022809
0.004	0.15948	0.084464	0.053719
0.006	0.261789	0.142495	0.092731
0.008	0.379143	0.211057	0.139845
0.01	0.511544	0.290152	0.195061
0.012	0.658991	0.379779	0.258378
0.014	0.821483	0.479938	0.329797
0.016	0.999022	0.590629	0.409317
0.018	1.191606	0.711852	0.496939
0.02	1.399237	0.843608	0.592663

Table (9) Pressure loss (Head per unit depth of filter ΔH/ΔL) for different values of grain sizes and porosity with n=0.38, T=9°C glass beads

Velocity m/s	**ΔH/ΔL**		
	d=1.0mm	**d=1.5mm**	**d=2.0mm**
0.002	0.097952	0.044197	0.025236
0.004	0.201895	0.092389	0.053467
0.006	0.31183	0.144574	0.084695
0.008	0.427756	0.200755	0.118918
0.01	0.549674	0.260929	0.156137
0.012	0.677583	0.325098	0.196351
0.014	0.811483	0.393261	0.239562
0.016	0.951375	0.465418	0.285768
0.018	1.097259	0.54157	0.334969
0.02	1.249134	0.621716	0.387167

Table (10) Pressure loss (Head per unit depth of filter ΔH/ΔL) for different values of grain sizes and porosity with n=0.64, T=9°C crushed garnet

Velocity m/s	**ΔH/ΔL**		
	d=1.0mm	**d=1.4mm**	**d=2.0mm**
0.002	0.124177	0.065189	0.03329
0.004	0.266297	0.143199	0.075554
0.006	0.426358	0.234031	0.126794
0.008	0.604362	0.337685	0.187009
0.01	0.800309	0.454161	0.256199
0.012	1.014197	0.583458	0.334365
0.014	1.246028	0.725576	0.421505
0.016	1.495801	0.880516	0.517621
0.018	1.763517	1.048278	0.622712
0.02	2.049175	1.228862	0.736778

Table (11) Pressure loss (Head per unit depth of filter ΔH/ΔL) for different values of porosities with d=1 mm, T=25°C for Crushed Garnet

Velocity m/s	ΔH/ΔL			
	n =0.36	n =0.45	n =0.57	n =0.65
0.002	0.037284	0.050842	0.070673	0.08483
0.004	0.082904	0.112909	0.156723	0.187989
0.006	0.13686	0.1862	0.258151	0.309477
0.008	0.199151	0.270716	0.374956	0.449294
0.01	0.269779	0.366456	0.507138	0.607439
0.012	0.348743	0.47342	0.654698	0.783913
0.014	0.436042	0.59161	0.817636	0.978715
0.016	0.531678	0.721023	0.995951	1.191847
0.018	0.635649	0.861661	1.189644	1.423307
0.02	0.747957	1.013524	1.398714	1.673095

Table (12) Pressure loss (Head per unit depth of filter ΔH/ΔL) for different values of porosities with d=1 mm, T=25°C for Crushed Coal

Velocity m/s	ΔH/ΔL			
	n =0.36	n =0.45	n =0.57	n =0.65
0.002	0.076064	0.103791	0.144384	0.173367
0.004	0.161926	0.220777	0.306845	0.36828
0.006	0.257588	0.350958	0.487383	0.584738
0.008	0.363049	0.494333	0.685997	0.822743
0.01	0.478309	0.650903	0.902688	1.082293
0.012	0.603368	0.820668	1.137455	1.363389
0.014	0.738226	1.003627	1.390299	1.666031
0.016	0.882883	1.199781	1.661219	1.990219
0.018	1.03734	1.40913	1.950217	2.335953
0.02	1.201595	1.631673	2.25729	2.703233

4. Conclusions

Hydrodynamic modeling of viscous flow in porous media was investigated for five selected water filters medias crushed silica, crushed coal, glass beads, crushed porcilinaite and crushed garnet. Typical Forchheimer's model constants *a* and *b* that can be used to estimate head loss for some of the most common design of water filters were correlated.

Hydraulic conductivity coefficients at based condition (T_w=25^{o}C),(n=50%),(d=1mm) *a** and *b** for different selected porous media were evaluated and *a* and *b* correlations for various grain sizes (0.5mm-2mm), porosity (35%-60%) and temperature(20^{o}C-80^{o}C) could be found in the empirical forms:

$\frac{a}{a*} = (\frac{n}{50\%})^{1.4}(\frac{1}{d})^{2}(1.028)^{(25-T)}$ or $\frac{a}{a*} = 2.64(\frac{n^{1.4}}{d^{2}})(1.028)^{(25-T)}$ and $\frac{b}{b*} = (\frac{n}{50\%})^{1.333}(\frac{1}{d})$

or $\frac{b}{b*} = 2.514(\frac{n^{1.333}}{d})$

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