Some Results on fuzzy sub NASNR-BCK algebra

بعض النتائج حول ضبابية شبه الحلقة القريبة غير التجميعية مع جبر BCK

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Abstracts:

In this research , we define a fuzzy NASNR-BCK algebra, where we study and prove some results and a properties, then we give some examples.

الخلاصة:

في هذا البحث عرفنا الضبابية الجزئية في النظام الجبري NASNR-BCK algebra حيث درسنا وبرهنا عدد من النظريات وأعطينا بعض الأمثلة.

1)Introduction:

L.A.Zedeh[1] in 1965 introduced the notion of fuzzy sets at present this concept has been applied to many mathematical branches,such as group,semigroups, function analysis and so on .The notion of BCK algebra was introduced first in 1966 by Y-Iami and K-Iseki [2],in the same year K-Iski[3] introduced the notion of BCK-algebra .In 2015 introduced Sajda and Azal NASNR-BCK algebra and study sub NASNR-BCK algebra and some type of ideals on it[4].In this paper we define a fuzzy sub NASNR-BCK algebra where we prove some results and give some examples.

2)Basic Concepts:

In this section we recall some definitions ,which we needed them in this research

Definition(2.1)[5]

Let X be a non empty set with binary operation , *, and 0 is constant . An algebraic system(X,*,0) is called **a BCK algebra** if it satisfies the following conditions :

1) $((x*y)*(x*z))*(z*y)=0$
2) $(x*(x*y))*y=0$
3) $x*x=0$
4) if $x*y=0$ and $y*x=0$ then $x=y$
5) $0*x=0$.

$\forall x,y,z\in X$

Remarks(2.2)[6]

Let X be a BCK algebra then:

•A partial ordering "$\leq$ " on X can be defined by $x \leq y$ if and only if $x*y=0$.

•ABCK-algebra X has following properties :

1) $x*0=x$
2) if $x*y=0$ and $y*z=0$ imply $x*z=0$
3) if $x*y=0$ implies $(x*z)*(y*z)=0$ and $(z*y)*(z*x)=0$
4) $(x*y)*z=(x*z)*y$
5) $(x*y)*x=0$
6) $x*(x*(x*y))=x*y$
7) if $(x*y)*z=0$ implies $(x*z)*y=0$
8) $[(x*z)*(y*z)]*(x*y)=0$
9) $[((x*z)*z)*(y*z)]*[(x*y)*z]=0$.   for all $x,y,z\in X$

Definition (2.3)[4]

Let (X,**.**,*,0) be a non-empty set with two binary operation "*" and "**.**" and 0 is constant element in X satisfying the following condition:

a) (X,**.**) is a semigroup

b) (X , *, 0) is a BCK algebra

c) (x **.** y) * z=(x * z) **.** (y * z) , for all x ,y,z$\in$ X which is called the distributive law of * over **.**

d) 0**.**x=x**.**0=x , for all x$\in$X

Then (X , **.** , * , 0) is called **a Non Associative Seminear-Ring With BCK Algebra** , we refer to by **NASNR-BCK algebra**.

Example(2.4)[4]

Let X={0,1,2,3} with two binary operations . and * are defined by the following tables:

*	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	1	0	0
3	3	1	0	0

.	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then X isa NASNR- BCK algebra.

Definition (2.5)[4]

Let (X ,. , * , 0) is a NASNR-BCK algebra , a nonempty subset P of X is said to be **a Non Associative sub Seminear-Ring With BCK Algebra** if (P , **.** , * , 0) is itself a NASNR-BCK algebra, we denoted by sub NASNR-BCK algebra. Note that every sub NASNR-BCK algebra is NASNR-BCK algebra.

Definition(2.6)[7]

Let X be a non-empty set **a fuzzy subset** μ on X is a function μ: X→ [0, 1].

Definition *(2.7)* [1]

Let μ and v be a fuzzy sets on X . Define the fuzzy set $\mu \cap v$ as follows:

$(\mu \cap v)(x)=\min\{\mu(x), v(x)\}$ for all x$\in$X .

Definition (2.8) [1]

Let μ and v be a fuzzy sets on X . Define the fuzzy set $\mu \cup v$ as follows:

$(\mu \cup v)(x) = \max\{\mu(x), v(x)\}$ for all x$\in$X.

Definition (2.9) [7]

Let λ and μ be the fuzzy subsets in a set X **the cartesian product**

$\lambda\times\mu: X\times X \to [0, 1]$ is defined by$(\lambda\times\mu)(x, y) = \min\{\lambda(x), \mu(y)\}$ for all x, y$\in$ X .

Remark(2.10)[8]

Let μ ,v be a fuzzy sets . If $\mu(x) \leq v(x)\ \forall\ x \in X$, then we say that μ is contained in v (or v contains μ) and we write $\mu \subseteq v$ (or $v \supseteq \mu$).If $\mu \subseteq v$ and $\mu \neq v$, then μ is said to be properly contained in v (or v properly contains μ) and we write $\mu \subset v$ (or $v \supset \mu$).

Definition (2.11)[9]

Let v be a fuzzy subset on X , **the strong fuzzy relation** on X that is a fuzzy relation on v is ρ_V given by$\rho_V(x, y) = \min\{ v(x), v(y)\}$.

Remark(2.12)

In this paper we will use $\wedge$ to denote min and $\vee$ to denote max

Definition (2.13) [9]

Let $f: X \longrightarrow Y$ be a mapping of NASNR-BCK algebra and μ be a fuzzy subset of Y . The map μ^f is **the pre-image of μ under f** if $\mu^f = \mu(f(x)), \forall x \in X$.

Definition(2.14)[10]

Let f be a mapping from a set R into a set S and let μ be fuzzy subset on R Then f(μ) ,**the image of μ under f**, is a fuzzy subset of S:

$$f(\mu)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu(x) & \text{If } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases} \quad , \text{for } y \in S.$$

Definition (2.15)[11]

Let λ be a fuzzy subset of a semigroup S. α, β ∈ (0, 1] such that α < β. we define the fuzzy subset λ_α^β of S as follows, $\lambda_\alpha^\beta(x) = (\lambda(x) \wedge \beta) \vee \alpha$, for all x∈ S.

3) Some Results on fuzzy sub NASNR-BCK algebra

In this section we define a fuzzy sub NASNR-BCK algebra ,then we study and prove some of properties.

Definition(3.1)

Let X be a NASNR- BCK algebra, and letμ be a fuzzy subset of X, we say μ is **a fuzzy sub NASNR- BCK algebra** of X if:

$$\left.\begin{array}{l} 1)\ \mu(x*y) \geq \mu(x) \wedge \mu(y) \\ 2)\ \mu(x.y) \geq \mu(x) \wedge \mu(y) \end{array}\right\} \forall x,y \in X.$$

Examples(3.2)

Let X= {0,1,2,3} with two binary operations (*)and (.)be defined as in example [2.4]

$$\text{and } \mu \text{ defined by } \mu(x) = \begin{cases} 0.6 & \text{if } x=0 \\ 0.5 & \text{if } x=1,2,3 \end{cases}$$

Then μ is fuzzy sub NASNR-BCK algebra, since,

$\mu(0.0)= \mu(0) = 0.6 \geq \mu(0) \wedge \mu(0) = 0.6$,$\mu(2.0) = \mu(2) = 0.5 \geq \mu(2) \wedge \mu(0) = 0.5$

$\mu(0.1)=\mu(1) = 0.5 \geq \mu(0) \wedge \mu(1)= 0.5$,$\mu(2.1) = \mu(3) = 0.5 \geq \mu(2) \wedge \mu(1) = 0.5$

$\mu(0.2)= \mu(2) =0.5 \geq \mu(0) \wedge \mu(2)= 0.5$, $\mu(2.2)=\mu(0) = 0.6 \geq \mu(2) \wedge \mu(2)=0.5$

$\mu(0.3)=\mu(3) =0.5 \geq \mu(0) \wedge \mu(3)= 0.5$, $\mu(2.3)= \mu(1)= 0.5 \geq \mu(2) \wedge \mu(3) = 0.5$

$\mu(1.0)= \mu(1) =0.5 \geq \mu(1) \wedge \mu(0) =0.5$, $\mu(3.0)= \mu(3)=0.5 \geq \mu(3) \wedge \mu(0) =0.5$

$\mu(1.1)= \mu(0) = 0.6 \geq \mu(1) \wedge \mu(1) = 0.5$,$\mu(3.1) = \mu(2) = 0.5 \geq \mu(3) \wedge \mu(1) = 0.5$

$\mu(1.2) = \mu(3) = 0.5 \geq \mu(1) \wedge \mu(2)= 0.5$,$\mu(3.2) = \mu(1) = 0.5 \geq \mu(3) \wedge \mu(2) = 0.5$

$\mu(1.3) = \mu(2) = 0.5 \geq \mu(1) \wedge \mu(3) = 0.5$,$\mu(3.3) = \mu(0) = 0.6 \geq \mu(3) \wedge \mu(3) = 0.5$
Then $\mu(x.y) \geq \mu(x) \wedge \mu(y)$
In a Similar way ,we have $\mu(x*y) \geq \mu(x) \wedge \mu(y)$.

Remark (3.3)
Let μ be fuzzy sub NASNR-BCK algebra of X .Then $\mu(0)\geq \mu(x)$, $\forall x\in X$.
proof:-
Let μ be fuzzy sub NASNR-BCK algebra ,then
$\mu(x*x) \geq \mu(x) \wedge \mu(x)$ $\forall x\in X$,then
$\mu(0) \geq \mu(x)$ [by 3 of definition 2.1].

Proposition(3.4)

Let I be anon- empty set of NASNR-BCK algebra , if I is a sub NASNR-BCK algebra .Then C_I is a fuzzy sub NASNR-BCK algebra where C_I: $X\rightarrow[0,1]$ define as follows:

$$C_I = \begin{cases} 1 & \text{if } x\in I \\ 0 & \text{if } x\notin I \end{cases}$$

proof:-
Let I be sub NASNR-BCK algebra
1) To proof $C_I(x*y) \geq C_I(x) \wedge C_I(y)$
There are four cases:
a) if $x, y \in I$ so $x*y \in I$, then
$C_I(x) =1$, $C_I(y) =1$ and $C_I(x*y) =1$, then
$C_I(x*y) \geq C_I(x) \wedge C_I(y)$
b) if $x\in I$, $y \notin I$ so $x*y \notin I$, then
$C_I(x)=1$, $C_I(y)=0$ and $C_I(x*y)=0$,then
$C_I(x*y) \geq C_I(x) \wedge C_I(y)$
c) if $x \notin I$, $y\in I$ so $x*y \notin I$, then
$C_I(x)=0$, $C_I(y)=1$ and $C_I(x*y)=0$, then
$C_I(x*y) \geq C_I(x) \wedge C_I(y)$
d) if $x ,y \notin I$, so $x*y \notin I$, then
$C_I(x)=0$, $C_I(y)=0$ and $C_I(x*y)=0$, then
$C_I(x*y) \geq C_I(x) \wedge C_I(y)$
So $C_I (x*y) \geq C_I (x) \wedge C_I (y)$,$\forall x , y \in X$
2)In similar way we have $C_I(x .y) \geq C_I(x) \wedge C_I(y)$,$\forall x , y\in X$
so C_I is a fuzzy sub NASNR-BCK algebra.

Proposition(3.5)
Let A,B be a fuzzy sub NASNR-BCK algebra of NASNR-BCK algebra of X.Then $A\cap B$ is a fuzzy sub NASNR-BCK algebra.
proof:-
Let $x ,y\in X$ and let A and B be a fuzzy sub ANSNR-BCK algebra
1)$(A\cap B) (x*y) = A(x*y) \wedge B(x*y)$
$\geq \{A(x) \wedge A(y)\} \wedge \{B(x) \wedge B(y)\}$[since A,B be a fuzzy sub NASNR BCK algebra]
$= \{A(x) \wedge B(x)\} \wedge \{A(y) \wedge B(y)\}$
$=(A\cap B) (x) \wedge (A\cap B) (y)$

2) $(A\cap B)(x.y) = A(x.y) \wedge B(x.y)$

$\geq \{A(x) \wedge A(y)\} \wedge \{B(x) \wedge B(y)\}$[since A,B be a fuzzy sub NASNR-BCK algebra]

$= \{A(x) \wedge B(x)\} \wedge \{A(y) \wedge B(y)\}$

$=(A\cap B)(x) \wedge (A\cap B)(y)$

<u>Proposition</u> (3.6)

Let μ_1, μ_2 be fuzzy sub NASNR-BCK algebra of NASNR-BCK algebra of X such that $\mu_1 \subseteq \mu_2$ or $\mu_2 \subseteq \mu_1$ then $\mu_1 \cup \mu_2$ is a fuzzy sub NASNR-BCK algebra.

proof:-

Let μ_1, μ_2 are a fuzzy sub NASNR-BCK algebra and let $\mu_1 \subseteq \mu_2$

1) $(\mu_1 \cup \mu_2)(x*y) = \mu_1(x*y) \vee \mu_2(x*y)$

$\geq \{\mu_1(x) \wedge \mu_1(y)\} \vee \{\mu_2(x) \wedge \mu_2(y)\}$ [since μ_1, μ_2 are a fuzzy sub NASNR-BCK algebra]

since $\mu_1 \subseteq \mu_2$

$= \{\mu_1(x) \vee \mu_2(x)\} \wedge \{\mu_1(y) \vee \mu_2(y)\}$

$= (\mu_1 \cup \mu_2)(x) \wedge (\mu_1 \cup \mu_2)(y)$

2) $(\mu_1 \cup \mu_2)(x.y) = \mu_1(x.y) \vee \mu_2(x.y)$

$\geq \{\mu_1(x) \wedge \mu_1(y)\} \vee \{\mu_2(x) \wedge \mu_2(y)\}$ [since μ_1, μ_2 are a fuzzy sub NASNR-BCK algebra]

since $\mu_1 \subseteq \mu_2$

$= \{\mu_1(x) \vee \mu_2(x)\} \wedge \{\mu_1(y) \vee \mu_2(y)\}$

$= (\mu_1 \cup \mu_2)(x) \wedge (\mu_1 \cup \mu_2)(y)$

<u>Proposition</u>(3.7)

Let μ_1 and μ_2 are a fuzzy sub NASNR-BCK algebra of NASNR-BCK algebra of X .Then $\mu_1 \times \mu_2$ is a fuzzy sub NASNR-BCK algebra of XxX.

Proof:-

Let $(x_1,x_2), (y_1,y_2) \in$ XxX such that * and .defined by $((x_1,x_2)*(y_1,y_2)) = (x_1*y_1, x_2*y_2)$ and $((x_1,x_2).(y_1,y_2)) = (x_1.y_1, x_2.y_2)$,then

1) $(\mu_1 \times \mu_2)((x_1,x_2)*(y_1,y_2)) = (\mu_1 \times \mu_2)(x_1*y_1, x_2*y_2)$

$= \mu_1(x_1*y_1) \wedge \mu_2(x_2*y_2)$

$\geq \{\mu_1(x_1) \wedge \mu_1(y_1)\} \wedge \{\mu_2(x_2) \wedge \mu_2(y_2)$ [since μ_1 and μ_2 are fuzzy sub NASNR-BCK algebra]

$= \{\mu_1(x_1) \wedge \mu_2(x_2)\} \wedge \{\mu_1(y_1) \wedge \mu_2(y_2)\}$

$= (\mu_1 \times \mu_2)(x_1,x_2) \wedge (\mu_1 \times \mu_2)(y_1,y_2)$

2) $(\mu_1 \times \mu_2)((x_1,x_2).(y_1,y_2)) = (\mu_1 \times \mu_2)(x_1.y_1, x_2.y_2)$

$= \mu_1(x_1.y_1) \wedge \mu_2(x_2.y_2)$

$\geq \{\mu_1(x_1) \wedge \mu_1(y_1)\} \wedge \{\mu_2(x_2) \wedge \mu_2(y_2)\}$[since μ_1 and μ_2 are fuzzy sub NASNR-BCK algebra]

$= \{\mu_1(x_1) \wedge \mu_2(x_2)\} \wedge \{\mu_1(y_1) \wedge \mu_2(y_2)\}$

$= (\mu_1 \times \mu_2)(x_1,x_2) \wedge (\mu_1 \times \mu_2)(y_1,y_2)$

$= (\mu_1 \times \mu_2)(x_1,x_2) \wedge (\mu_1 \times \mu_2)(y_1,y_2)$

<u>Proposition</u>(3.8)

Let v be fuzzy sub NASNR-BCK algebra of NASNR- BCK algebra of X then ρ_V is fuzzy sub NASNR-BCK algebra of X×X,where ρ_V is defined in definition(2.11)

proof:-

Let $x=(x_1,x_2)$ and $y=(y_1,y_2) \in X \times X$ and let v be a fuzzy sub NASNR-BCK algebra of X ,then

1) $\rho_V((x_1,x_2)*(y_1,y_2)) = \rho_V((x_1*y_1),(x_2*y_2))$

$= v(x_1*y_1) \wedge v(x_2*y_2)$

$\geq (v(x_1) \wedge v(y_1)) \wedge (v(x_2) \wedge v(y_2))$ [since v is a fuzzy sub NASNR-BCK algebra]

$=(v(x_1) \wedge v(x_2)) \wedge (v(y_1) \wedge v(y_2))$

$= \rho_V(x_1,x_2) \wedge \rho_V(y_1,y_2)$

2) $\rho_V((x_1,x_2) \cdot (y_1,y_2)) = \rho_V((x_1 . y_1), (x_2 . y_2))$

$= v(x_1 . y_1) \wedge v(x_2 . y_2)$

$\geq (v(x_1) \wedge v(y_1)) \wedge (v(x_2) \wedge v(y_2))$ [since v is a fuzzy sub NASNR-BCK algebra]

$=(v(x_1) \wedge v(x_2)) \wedge (v(y_1) \wedge v(y_2))$

$= \rho_V(x_1,x_2) \wedge \rho_V(y_1,y_2)$

Proposition (3.9)

Let f: X $\longrightarrow$ Y be a homomorphism if μ is a fuzzy sub NASNR-BCK algebra of NASNR-BCK algebra of X ,then μ^f is a fuzzy sub NASNR-BCK algebra, where μ^f is defined in definition(2.13).

proof:-

Let x ,y∈X, then

1) $\mu^f(x*y) = \mu(f(x*y))$

$= \mu(f(x) * f(y))$ [since f is a homomorphism]

$\geq \mu(f(x) \wedge \mu(f(y))$

$= \mu^f(x) \wedge \mu^f(y)$ [since μ be a fuzzy sub NASNR-BCK algebra]

2) $\mu^f(x.y) = \mu(f(x.y))$

$= \mu(f(x) . f(y))$ [since f is a homomorphism]

$\geq \mu(f(x) \wedge f(y))$ [since μ be a fuzzy sub NASNR-BCK algebra]

$= \mu^f(x) \wedge \mu^f(y)$

The converse of (3.9) is true when f is an epimorphism as the following proposition shows.

Proposition(3.10)

Let f: X$\longrightarrow$Y be an epimorphism of NASNR –BCK algebra if μ^f is fuzzy sub NASNR – BCK algebra of X .Then μ is a fuzzy sub NASNR- BCK algebra of Y.

proof:

Let μ^f is a fuzzy sub NASNR-BCK algebra of X and let x ,y∈Y , then ∃ a ,b ∈ X such that f(a)= x, f(b)=y [since f is an epimorphism]

1) $\mu(x*y) = \mu(f(a)* f(b))$

$= \mu(f(a * b))$ [since f is a homomorphism]

$=\mu^f(a*b)$

$\geq \mu^f(a) \wedge \mu^f(b)$ [since μ^f is a fuzzy sub NASNR- BCK algebra]

$= \mu(f(a)) \wedge \mu(f(b))$

$= \mu(x) \wedge \mu(y)$

2) $\mu(x. y) = \mu(f(a) . f(b))$

$= \mu(f(a .b))$ [since f is a homomorphism]

$= \mu^f(a.b)$

$\geq \mu^f(a) \wedge \mu^f(b)$ [since μ^f is a fuzzy sub NASNR- BCK algebra]

$= \mu(f(a)) \wedge \mu(f(b))$

$=\mu(x) \wedge \mu(y)$

Then μ is fuzzy sub NASNR-BCK algebra

Proposition(3.11)

Let μ be a fuzzy sub NASNR-BCK algebra of NASNR-BCK algebra of X.Then the fuzzy set μ^+ defined by $\mu^+(x) = \mu(x)+1-\mu(0)$ is a fuzzy sub NASN-BCK algebra.

Proof:-

Let μ be a fuzzy sub NASNR-BCK algebra and μ^+ is a fuzzy set and x , y∈ X, then

1) $\mu^+(x*y) = \mu(x*y)+1-\mu(0)$

$\geq \{ \mu(x) \wedge \mu(y)\} + 1-\mu(0)$ [since μ is a fuzzy sub NASNR-BCK algebra]

$= \{\mu(x)+ 1-\mu(0)\} \wedge \{\mu(y)+ 1-\mu(0)\}$

$= \mu^+(x) \wedge \mu^+(y)$

2) $\mu^+(x.y) = \mu(x.y)+1-\mu(0)$

$\geq \{ \mu(x) \wedge \mu(y)\} + 1-\mu(0)$ [since μ is a fuzzy sub NASNR-BCK algebra]

$= \{\mu(x)+ 1-\mu(0)\} \wedge \{\mu(y)+ 1-\mu(0)\}$

$= \mu^+(x) \wedge \mu^+(y)$

Proposition(3.12)

Let f: X ⟶ Y be a homomorphism and let μ be a fuzzy sub NASNR- BCK algebra of X ,then f(μ) is a fuzzy sub NASNR- BCK algebra of Y.

proof:

Let μ be a fuzzy sub NASNR- BCK algebra of X and let $y_1, y_2 \in Y$ and let $x, x_1, x_2 \in X$

1) $f(\mu)(y_1*y_2) = \sup\{ \mu(x): x\in f^{-1}(y_1*y_2) \}$

$\geq \sup\{\mu(x_1*x_2): x_1\in f^{-1}(y_1)\}, x_2\in f^{-1}(y_2)\}$[since$\{ x: x\in f^{-1}(y_1*y_2) \} \supseteq \{ x_1* x_2: x_1\in f^{-1}(y_1), x_2\in f^{-1}(y_2)\}$]

$\geq \sup \{\mu(x_1) \wedge \mu(x_2): x_1\in f^{-1} (y_1)\}, x_2\in f^{-1} (y_2)\}$ [since μ is a fuzzy sub NASNR- BCK algebra]

$\geq \sup \{\mu (x_1): x_1\in f^{-1}(y_1)\} \wedge \sup\{\mu(x_2): x_2\in f^{-1}(y_2)\}$

$= f (\mu)(y_1) \wedge f(\mu)(y_2)$

2) $f (\mu)(y_1.y_2) = \sup \{\mu(x): x \in f^{-1}(y_1.y_2)\}$

$\geq \sup \{\mu(x_1.x_2): x_1\in f^{-1} (y_1)\}, x_2\in f^{-1}(y_2)\}$[$\{ x: x\in f^{-1}(y_1.y_2) \} \supseteq \{ x_1. x_2: x_1\in f^{-1}(y_1), x_2\in f^{-1}(y_2)\}$]

$\geq \sup \{\mu(x_1) \wedge \mu(x_2): x_1\in f^{-1}(y_1)\}, x_2\in f^{-1}(y_2)\}$[since μ is a fuzzy sub NASNR- BCK algebra]

$= \sup \{\mu(x_1): x_1\in f^{-1}(y_1)\} \wedge \sup\{\mu(x_2): x_2\in f^{-1}(y_2)\}$

$= f (\mu)(y_1) \wedge f(\mu)(y_2)$

then f(μ) is a fuzzy sub NASNR- BCK algebra

Proposition(3.13)

Let λ be a fuzzy sub NASNR-BCK algebra of NASNR-BCK algebra of X and $\alpha, \beta \in (0,1] \ni \alpha < \beta$, then λ_α^β is a fuzzy sub NASNR-BCK algebra ,where λ_α^β is defined in definition(2.15).

proof:-

Let λ be fuzzy sub NASNR-BCK algebra of X

1) $\lambda_\alpha^\beta (x*y) = (\lambda(x*y)\wedge\beta)\vee\alpha$ (by definition(2.15))

$\geq\{(\lambda(x) \wedge\lambda(y)\} \wedge\beta)\vee\alpha$ [since λ is fuzzy sub NASNR-BCK algebra]

$=\{(\lambda(x) \wedge\beta) \wedge (\lambda(y) \wedge\beta)\}\vee\alpha$

$=\{(\lambda(x) \wedge\beta)\vee\alpha\}\wedge\{(\lambda(y) \wedge\beta)\vee\alpha\}$

$= \lambda_\alpha^\beta (x) \wedge\lambda_\alpha^\beta (y)$

2) $\lambda_\alpha^\beta (x.y) = (\lambda(x.y) \wedge\beta)\vee\alpha$ (by definition(2.15))

$\geq\{(\lambda(x) \wedge\lambda(y)\} \wedge\beta)\vee\alpha$ [since λ is fuzzy sub NASNR-BCK algebra]

$=\{(\lambda(x) \wedge\beta\wedge\lambda(y) \wedge\beta\}\vee\alpha$

$=\{(\lambda(x) \wedge\beta)\vee\alpha\}\wedge\{(\lambda(y) \wedge\beta)\vee\alpha\}$

$= \lambda_\alpha^\beta (x) \wedge\lambda_\alpha^\beta (y)$

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