الهجرة بين الحضر والريف

Internal Immigration between urban & rural Using markov chain

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الخلاصة:
يهدف هذا البحث إلي دراسة الهجرة السكانية بين الريف والمدينة باعتبارها متسلسلة ماركوف ذات الحالتين، على فرض عدم وجود شرط للانتقال من الريف إلى المدينة وبالعكس . ويتم اشتقاق التوزيع الاحتمالي المستقر للسلسلة المحولة إضافة للتوزيعات الاحتمالية لفترة البقاء وزمن الانتظار وزمن العبور من والى كل حالة.

Internal Immigration between urban & rural
Using markov chain

Abstract:
This paper is concerned with Immigration between cities & villages as markov chain , under consider that no constrain to transition .we derive the stationary probability distribution of the series and some related distribution Functions.

1- introduction:

The study of instants at which observations from a stochastic process are greater or less than specified value is of particular interest in hydrology,Battaglia (1981) considered a binary transformation to

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river series,salim(1996) studied a trinary transformation of river flow by mean of well-know methods developed for markov chains,salim & Gannam(1997) studied binary modeling of rainfall time series.

Our aim in this paper is to consider a binary transformation that will make the Immigration series two states; state one (urban) and state two (rural)

2- Formulation of the model

let Xt be a strictly stationary time series observed at regular time intervals . For our study the peoples traveling from the village to the city with out constraints ,we define the transformation of { Xt } of the form

$$jt = \begin{cases} 1 ; \text{ if } Xt \text{ in urban} \\ 0 ; \text{ if } Xt \text{ in rural.} \end{cases}$$

The process { jt } whose dependence upon the past is limited to a finite number of preceding values . Such processes will be formed as a markov chain with two states.

Let

$$a = P(jt = 1 / Jt - 1 = 0$$

$$b = p(jt = 0 / jt-1 = 1)$$

a,b are constant independent of time because the process is stationary this constant are supposed to be strictly positive the transition probability matrix p of (jt) has the following form:

$$P = \begin{vmatrix} 1-a & a \\ b & 1-b \end{vmatrix}$$

3-stationary Distribution

it is well – known that if the chain is ergodic then it has a unique stationary distribution (see Cox and Miller 1965,p108).To check the ergodicity of the chain , we need to show that the transition matrix

P is primitive and irreducible.

Def.1 [3] -: if A is any square matrix then by a suitable permutation applied to both rows and columns we can reduce A to the form

$$A = \begin{vmatrix} A11 & 0 \ldots & & 0 \\ A12 & A22 & 0 \ldots & 0 \\ Am1 & \ldots & & Amn \end{vmatrix}$$

if A are square and irreducible, Where the matrices Aii on the diagonal
itself irreducible then, of course A =A11.

Def .2[3]- :The transition matrix P is Primative,means that it has a simple eigenvalue 1 which exceeds all other eigenvalues in modules

Def:3[3] -:The states of a Markov chain is called positive recurrent if the ultimate return to any state of the chain is a certain event.

Def: 4[3] -:A state of a Markov chain is called periodic if the return to that state possible only at time t, 2t , 3t , … where t >1.

Def :5[3] - A state which is not periodic is call a periodic Essentially it has period 1.

Def:6[3] -An aperiodic state which is positive – recurrent called ergodic

By solving the system $| P- \lambda I | = 0$, we have

$\lambda 1 = 1$

$\lambda 2 = 1 - (a + b)$

if $(a + b) < 1 \longrightarrow$ P is primitive and since P is irreducible

$\rightarrow$ P is ergodic . Also from the digraph of the chian (fig. 1)

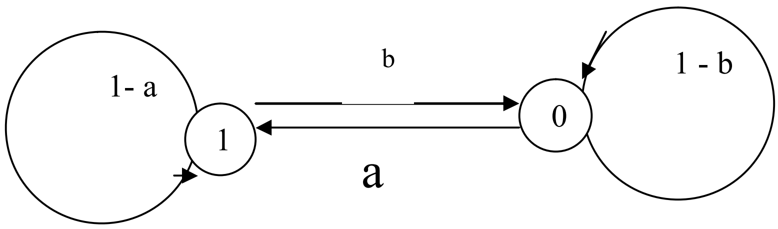

fig (1)

We see that P is aperiodic since the return to each state possible at t=1 . Since P is ergodic, therefore there is a unique stationary distribution which can be obtained from the solution of the system (I – P)T X = 0, where X is a column vector and I is the identity matrix .

The solution of this system gives the stationary distribution of states which are ;

$\Pi 1 = b / (a + b)$, $\Pi 2 = a / (a+b)$

Where $\Pi i = p$ (state i) ; I = 1,2

4- Distribution of state 1 (urban) and 0(rural):.

The transition to the city occurs when the chain enter state 1,Let Xi denote the number of the transition to the city at the ith step, i,e

$Xi = \{ 1$ if $Jt = 1$ and $Jt\text{-}1= 0$ or $Jt\text{-}1 = 1$
$\{ 0$; otherwise

then the no. of people in the city in the time interval (0,t) , Nc(0,t) may be expressed in the following way:

$$Nc(0,t) = X1 + X2 + \ldots + Xt \qquad \ldots(1)$$

The distribution of Xi depends on the previously occupied state or on the state next will be occupied or on both. now ,let $Mi,j(\theta)$, (i,j =0,1) be the moment generating function (m.g.f) of the variable "Transition from state i to state j in one step "and let $P(\theta)$ denote the matrix with typical elements Pi,j , $Mi,j(\theta)$, with pi,j are the transition probabilities from state i to state j . The matrix $P(\theta)$ plys as similar role to that of the m.g.f in the summation of the independent random variable , xi ,

m.g.f of Nc(0,t) is given by (cox & miller – 1965).

$M_{Nc}(\theta) = (\Pi_0 , \Pi_1) P(\theta)^t (1,1)^t$, where t denote the transpose , Π_0 , Π_1 are defined above.

In our case the m.g.f of the variable take the form :

$M_{0,0}(\theta) = M_{1,0}(\theta) = 1$ and $M_{0,1}(\theta) = M_{1,1}(\theta) = e^{-\theta}$

And so we have

$$P(\theta) = \begin{vmatrix} 1-a & ae^{-\theta} \\ b & (1-b)e^{-\theta} \end{vmatrix}$$

Thus the m.g.f of $X_i = Nc(0,1)$

$$= b / (a+b) + (ae^{-\theta}) / (a+b)$$

and

$P(X_1=1) = a / (a+b)$, $P(X_1 =0)= b/(a+b)$

To evaluate $P(\theta)^t$ the method of canonical decomposition may be used . The two eigenvalues $\lambda_1(\theta)$ and $\lambda_2(\theta)$ are the solution of the determinant equation $| P(\theta) - \lambda I | = 0$, i.e

$$\lambda_1(\theta) = 0.5 [\{(1-a) + (1-b) e^{-\theta} \} +\{(1-a) + (1-b)e^{-\theta} \}^2 -4(1-a-b) e^{-\theta} \}^{1/2}]$$

$$\lambda_2(\theta) = 0.5 [\{(1-a) + (1-b) e^{-\theta} \} -\{\{(1-a) + (1-b)e^{-\theta} \}^2 -4(1-a-b) e^{-\theta} \}^{1/2}]$$

where $\lambda_1(0) = 1$ and $\lambda_2(0) = 1-a-b$ hence we may write $P(\theta)$ in the form $Q(\theta) \Lambda (\theta)$, where $\Lambda (\theta) = diag \{ \lambda_i(\theta)\}$,$Q(\theta)$ is 2x2 matrix where the column $q_i(\theta)$ of $Q(\theta)$ are solution of the equations $p(\theta) q_i(\theta) = \lambda_i(\theta) q_i(\theta)$ (i= 1,2).

Thus:

$$\{P(\theta)\}^n = 1/\{ae^{-\theta}\{\lambda_2(\theta)-\lambda_1(\theta)\}\} \begin{vmatrix} ae^{-\theta} & ae^{-\theta} \\ \lambda_1(\theta)+a-1 & \lambda_2(\theta)+a-1 \end{vmatrix}$$

$$* \begin{vmatrix} \lambda_1^n(\theta) & 0 \\ 0 & \lambda_2^n(\theta) \end{vmatrix} * \begin{vmatrix} \lambda_{21}(\theta)+a-1 & -ae-\theta \\ -\{\lambda 1(\theta)+a-1\} & -ae-\theta \end{vmatrix} \qquad ....(2)$$

there fore : (cox & miller 1965-p136):

$\varphi 1(n)\ (\theta) = E(e\text{-}\theta X\ i / X0 = 1)=$

$$\frac{\lambda_1^{n+1}(\theta)-\lambda_2^{n}(\theta)-(1-a-b)(\lambda_1^{n}(\theta)-\lambda_2^{n}(\theta)}{\lambda 1(\theta)-\lambda 2(\theta)} \qquad ......(3)$$

Where $\varphi^1(n)$ = m.g.f of the number of traveling to orban in n trials .

In obtaining the simplification from (2) and (3) we use the fact that:

$$\left.\begin{aligned} \lambda 1(\theta)+\lambda 2(\theta) &= 1-a+(1-b)e\text{-}\theta \\ \lambda 1(\theta)\,\lambda 2(\theta) &= (1-a-b)e\text{-}\theta \end{aligned}\right\} \qquad ...(4)$$

by differentiating (3) with respect to θ . The derivatives of $\lambda 1(\theta)$, $\lambda 2(\theta)$ at $\theta = 0$ may be found by differentiating the relation (4) and using the fact that

$\lambda 1(0) = 1$, $\lambda 2(0) = 1\text{-}a\text{-}b$.

Asymptotic result may be obtained by noting that in a neighborhood of
θ = 0 . we have :

| λ1(θ) | > |λ2(θ) | since |λ1(0) |> |λ2(0) | hence writing (3) in the form:

$$\phi_1^{(n)}(\theta) = \frac{\lambda_1^{\,n}(\theta)(\lambda_1(\theta)-1+a+b)-\lambda_2^{\,n}(\theta)(\lambda_2(\theta)-1+a+b)}{\lambda 1(\theta)-\lambda 2(\theta)}$$

We see that as n ⟶ ∞

$$\log\phi_1^{(n)}(\theta) \sim\sim n \log \lambda 1(\Theta)$$

Thus asymptotically Xn behaves like a sum of independent random variable . a familiar central limit theorem shown that Xn is asymptotically normal distributed with mean $-n\lambda_1^{/}(0)$ and variance:
n[λ1//(0) – λ1(0)2] and is independent of initial condition [see 3] .we find that :

E(Xn) =~ n(a/(a+b))

$$\text{var} \cong n\,\frac{ab\,(2-a-b)}{(a+b)^2}$$

The exact mean value of Nc(0,t) may be obtaind also from
Nc(0,t) = X1+X2+…+Xt
E Nc(0,t) = ta / (a+b)
Similarly, the no. of people in the villager in the interval (0,t) Nv(0,t) may be expressed in the same way for the city,but
M0,0 (θ) = M1,0(θ) = e-θ and M1,0(θ) = M1,1(θ) = 1 And so:

$$P(\theta) = \begin{bmatrix} (1-a)e^{-\theta} & a \\ be^{-\theta} & (1-b) \end{bmatrix}$$

Thus m.g.f of Xi = Nv(0,1)

$$= \frac{be^{-\theta}}{a+b} + \frac{a}{a+b}$$

λ1(θ) = 0.5 [{(1-a)e-θ + (1 –b) } +{(1-a) e-θ + (1-b) }2

-4 e –θ {1-(a+b)+2ab}1/2]

λ2(θ) = 0.5 [{(1-a) e –θ + (1 –b) } –{{(1-a) e –θ + (1-b) }2

-4 e –θ{1-(a+b)+2ab }1/2]

and as n $\longrightarrow \infty$
log φ1n(θ) ≈ nlog λ1(θ)
and the exact mean value of Nv(0,t) is E[Nv(0,t)] = t.b / (a+b)

5-Some interesting related variables

We define now some interesting variables related to the immigration between the city and village. let Di be the no. of time the process value (people) remains in state i.

To find the prop. Dist.of Di we observe that
p(Di=k) = P(Jt=I,…J=I,Jk+1 ±I /J0 = I,J1 =I)
I = c , v ; (c = city ,v = village)
by using the markovian property we get;
P(Dc = k) = b (1- b) k-1
P(Dv =k) = a (1- a)k-1 k=1,2,…

Consider now the variable Ti to be the time between two consecutive traveling. Tc and Tv now are equal to the first return time to state 1 or to state 0 respectivly.

Now :
P(TC=k)=P[Nc(1,t) ≠ 1,Nc(t,t+1)=1 / Nc(0,1)=1] and also:
P(Tv=k)= P[Nv(1,t)≠0 , Nv(t,t+1)=0 / Nv(0,1)=0]

Given afirst crossing to urban at step 1,a second one at time (t+1) may occur only if the process assumes once it (2,t) in rural ,let j denoted the step at which this happens ; then if a≠ b)

$$P(Tc=k)=\sum_{j=2}^{k} P(X_j > u.for.i < j; Xm \le u.for.j \le m \le t,.X_{t+1} > u / X_0 \le u, X_1 \ge u$$

$$= ab.\left[\frac{(1-a)^{k-1} - ..(1-b)^{t-1}}{b-a}\right],.k = 2,3,...$$

Where as for a=b
P(Tc=k) = a^2(k-1)(1-a)k-2 , k=2,3

Similarly for village ;a≠ b

$$P(T_v = t) = ab\left[\frac{(1-b)^{k-1} - (1-a)^{k-1}}{b-a}\right] ...k = 2,3,...$$

while for a=b

$$P(Tv = k) = b^2(k-1)(1-b)^{k-2},..kt = 2,3,...$$

Table (1)

The probability mas function the mean and variance of each previous variables

variables	p.m.f	mean	variance
Dc	a(1-a)k-1 , k=1,2,….	(1-a)/a	(1-a)/a2
Dv	b(1-b)k-1 ,k=1,2,…	(1-b)/b	(1-b)/b2
Tc a≠ b a=b	$ab\left[\frac{(1-a)^{k-1} - (1-b)^{k-1}}{b-a}\right],.k = 2,3,...$ a2(k-1)(k-a)k-2 k=2,3,…	(a+b)/ab	$\frac{1-b}{b^2} + \frac{1-a}{a^2}$
Tv a≠ b a=b	$ab\left[\frac{(1-b)^{k-1} - (1-a)^{k-1}}{a-b}\right],.k = 2,3,...$ $b^2(k-1)(k-b)^{k-2}$,k=2,3,…	= =	= =

6-Application:

we applied the present method to the internal immigration in Iraq ,between Urban and Rural . we depend on the data published in the statistical group book for year1998/1999, table no.(15/2).

Table (2)

IRAQI Population by former place of Residence in 1987

To from	Urban	Rural
Urban	10158775	78644
Rural	126258	4561526

Then the Estimated transition matrix is:

$$\hat{P} = \begin{array}{c} \\ 0 \\ 1 \end{array}\begin{array}{cc} 0 & 1 \\ \hline \end{array}$$

State ↓ →	0	1
0	o.986	0.014
1	0.012	0.988

so that the estimated stationary probabilities are $\Pi0= 0.4615$ $\Pi1 = 0.5384$:

the mean no. of immigration is

E(Nv(0,t)) =193 person per day to urban

E(Nc(0,t))= 166 = = = = rural

Also p.m.f of the duration of an excursion in the urban and the rural are

$P(Dc=k) = 0.014(0.986)^{k-1}$, k=1,2,…

$P(Dv=k) = 0.012(0.988)^{k-1}$, k=1,2,…

And the p.m.f of the time between two consecutive traveling are :

$P(Tc=k) = 0.000168[\{(0.986)^{k-1} -(0.988)^{k-1}\}/(-0.002)]$,k= 2,3,…

$P(Tv=k)= 0.000168[\{(0.988)^{k-1} - (0.986)^{k-1} \}/(0.002)]$, k=2,3,..

Then we conclude that the mean of net immigration is 9855 pearsons every year from rural to urban.

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