

## Comparative study of different types of an active filter

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### Abstract

*This paper proposes different types of an active filters design. A comparative study between the theoretical analysis and practical design of different types of analog filters is presented.*

*In this paper a comparison is made between Mat lab simulation toolbox for the Butterworth, Chebyshev, Elliptical filter and a practical simulation using Electronic workbench (EWB).*

*In general, the results prove that a good agreement between the practical (experimental) and computer simulation (theoretical) by using the available Mat lab toolboxes.*

### الخلاصة

يصف هذا البحث دراسته مختلفه لتصاميم مجموعه أنواع من المرشحات الفعاله. كذلك هناك دراسة مقارنة ما بين النتائج النظرية والعملية. وفي هذا البحث تم التطرق الى أنواعا مختلفة من المرشحات الفعاله منها طريقة Butterworth و Chebyshev وطريقة Elliptical. وذلك لأنواع مختلفة من التمرير المنخفض, العالي, الحزمي ونوع الحزمي ذو التوقف المحدود. وقد تم استخدام حقيبة (Mat lab) لأغراض التصميم. تم دراسة تأثيرات درجة تصميم المرشح سواء للنوع الواحد أو ما بين الأنواع المختلفه وذلك لأغراض التحليل والتصميم. تم استخدام تقنية الاستجابة التردديه لغرض هذه المقارنة. عموما " أثبتت النتائج مدى التقارب ما بين الواقع النظري والعملية.

### 1- Introduction

Filter is a device that passes electric signals at certain frequencies or frequency ranges while preventing the passage of others.

The type of filters: active and passive filter (low - pass, high - pass, band - pass and stop band - pass).

Transfer function can be selected in accordance with certain mathematical rules so that corresponding low – pass filter curves all have( 3dB ) point at 1 rad (W=1) each curve represents a set of element values for an LC or active filter. this filter and its response are said to be "normalized" to (1) rad., the general technique for designing filters is to first convert the filter requirement to normalized low–pass require, and select a satisfactory low– pass filter, then renormalized to required frequency range, if a high-pass, band–pass or band – stop filter is desired.[1,2]

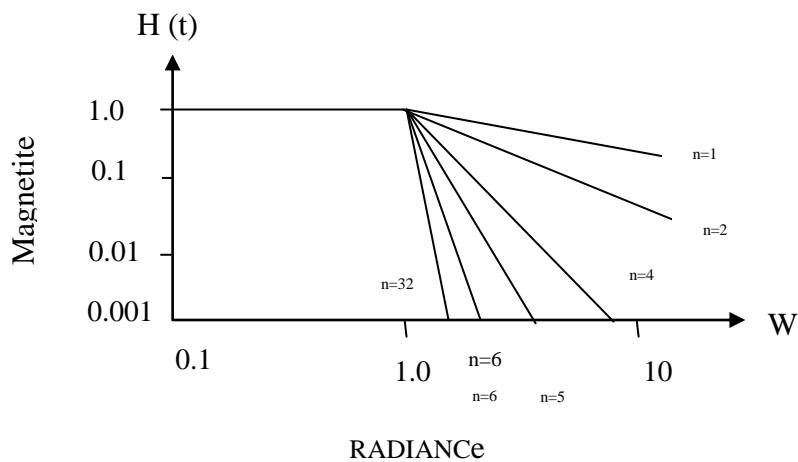
## 2. Filters types and design

### 2.1. Butterworth filter design

If offers the ability to design any filter type must be choosing the suitable order (n) which depends on the required pass band gain and stop- band attenuation. [3,4,5,6]

$$|H(f)| = \frac{1}{\sqrt{1+(\frac{f}{f_c})^{2n}}} \dots (1)$$

Where (n) is the order of the filter (number of poles), H (f) is the magnitude, f is stop-band cutoff frequency, f<sub>c</sub> is the cutoff frequency, as shown in Fig. (1).



**Fig (1) Normalized low pass Butterworth filter response curves**

Any filter is first designed proto type L.P.F then converted to the desired type using analog – to – analog transformation .as shown in Fig(2).

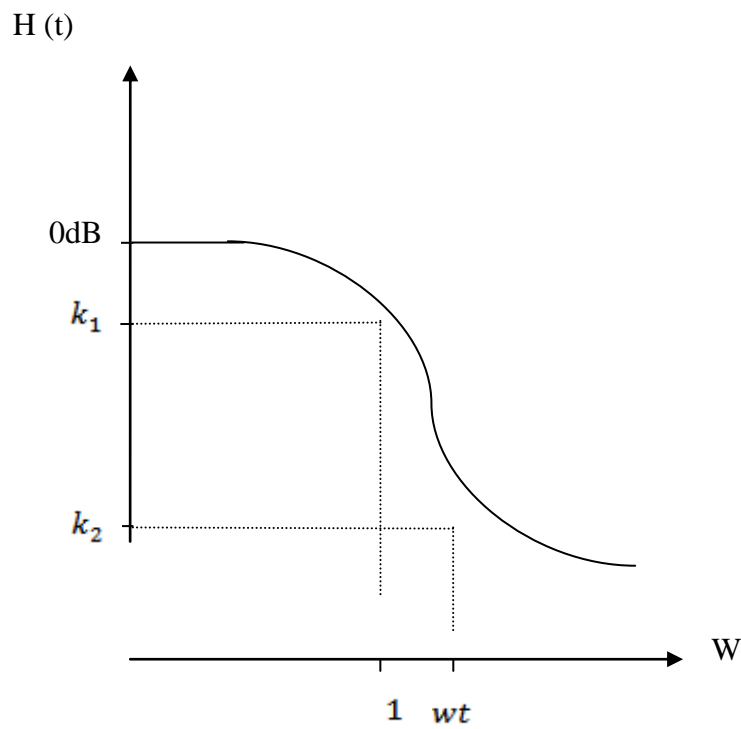
- Where: K<sub>1</sub> = pass band gain.
- K<sub>2</sub> = stop band gain.
- W<sub>c</sub>= cutoff frequency.

$W_r$ = rejected frequency.

$$K_1 = 10 \log \left[ 1 + \left( \frac{1}{wc} \right)^{2n} \right] \dots\dots\dots(2)$$

$$K_2 = 10 \log \left[ 1 + \left( \frac{wr}{wc} \right)^{2n} \right] \dots\dots\dots(3)$$

$$\therefore n = \left[ \frac{\log_{10} \left( \frac{10^{-k_1/10} - 1}{10^{-k_2/10} - 1} \right)}{2 \log_{10} \left( \frac{1}{w_r} \right)} \right] \dots\dots\dots(4)$$



**Fig. (2) Prototype (L.P.F)**

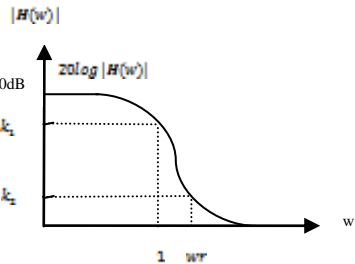
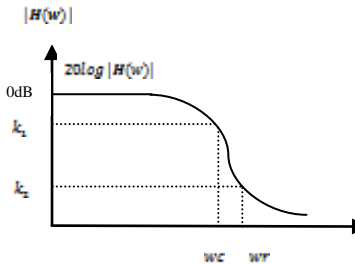
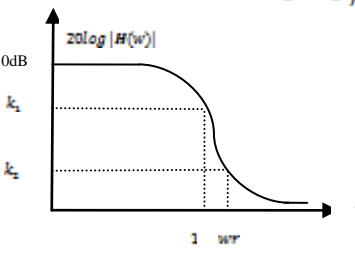
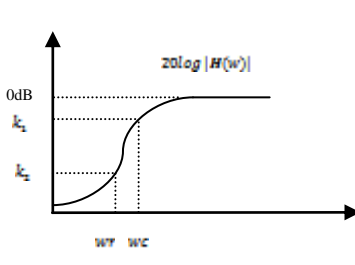
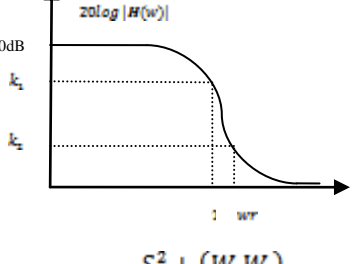
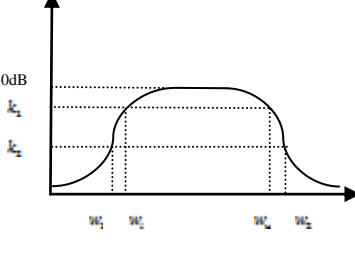
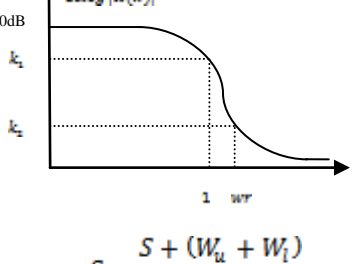
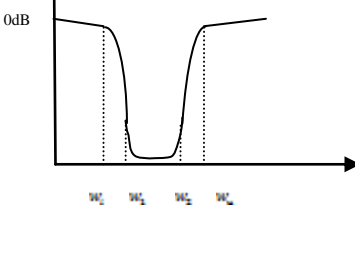
Proto tupe response	Transformed filter response	Design equation
 <p style="text-align: center;"><math>s = s / w_0</math></p>		$W_r = \frac{W_c}{W_c}$
 <p style="text-align: center;"><math>S = \frac{W_c}{S}</math></p>		$W_r = \frac{W_c}{W_r}$
 <p style="text-align: center;"><math>S = \frac{S^2 + (W_u W_l)}{S(W_u - W_l)}</math></p>		$W_r = \min[ A ,  B ]$ $ A  = \frac{-W_1^2 + W_1 W_u}{W_2 (W_u - W_l)}$ $ B  = \frac{-W_2^2 - W_1 W_2}{W_2 (W_u - W_l)}$
 <p style="text-align: center;"><math>S = \frac{S + (W_u + W_l)}{S^2 + (W_l W_u)}</math></p>		$W_r = \min[ A ,  B ]$ $ A  = \frac{W_1 (W_u - W_l)}{-W_1^2 + W_1 W_u}$ $ B  = \frac{W_2 (W_u - W_l)}{-W_2^2 - W_1 W_u}$

Fig.(3) Show analog-to- analog transformation

## 2.2. Chebyshev filter design

There are two types of Chebyshev filters , one containing a ripple in the pass band (type1 ) and the other containing a ripple in the stop – band (type 2 ).

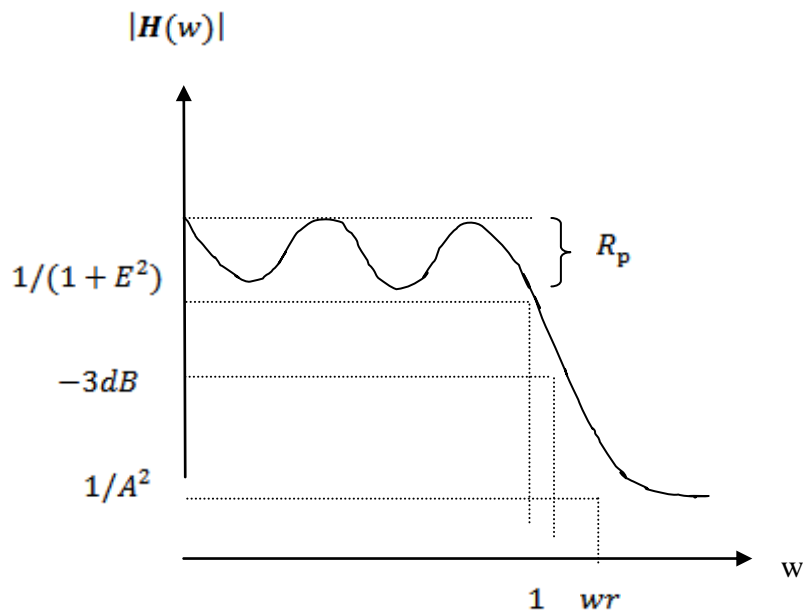
Atype1(unit band width)Chebyshev filter with a ripple in the pass band characterized by the following magnitude squared frequency.

$$|H(W)|^2 = \frac{1}{1 + E^2 T_n^2(W)}$$

Where  $T_N (w)$  is the 4th order Chebyshev polynomial, (E) is pass-band ripple (dB) and it generated from the following recursive formula:

$$T_n(w) = 2xT_{n-1}(x) - T_{n-1}(x) \dots n \geq 2$$

with  $\dots T_0(x) = 1, T_1(x) = x$



**Fig (4) Show the low – pass filter in Chebyshev.**

The magnitude of  $|H_n(w)|$  oscillates between 1 and  $1/(1+E)^2$  as shown in fig(4) as (W) goes ( 0 to 1 ) rad , but the periods of oscillations are not equal .

$$K_2 = 20 \log \left( \frac{1}{A} \right) \dots \dots \dots (5)$$

$$K_1 = 20 \log \left( \frac{1}{\sqrt{1 + E^2}} \right) \dots \dots \dots (6)$$

$$g = \sqrt{\frac{A^2 - 1}{E^2}} \dots \dots \dots (7)$$

$$n = \left\lceil \frac{\log_{10}(g + \sqrt{(g^2 - 1)})}{\log_{10}(w_r^2 - 1)} \right\rceil$$

**2.3. Elliptical filter design: -**

Elliptical function filters contain zero as well as poles in the transfer function, these results in infinite rejection at stop – band frequencies near cutoff. The pass – band has ripples similar to Chebyshev filters and the stop – band has return lobes which are all equal in amplitude.

For a given filter order (n).elliptical function filters have the steepest rate of descent to the stop – band theoretically possible Fig (5) compares Butterworth Chebyshev, and elliptic function filters of equivalent complexity.

The steeper rate of roll of is obtained at the expense of return lobes, however these return lobes are acceptable.

We most instances, providing they do not exceed the minimum attenuation required .elliptic function filter section are more complex and critical than the other types however , fewer sections are required for a given attenuation requirement then with other filter families for made rate filter requirements the all pole types are satisfactory.

Where steeper filter requirements are specified. Elliptic function low – pass filters are normalized so that the attenuation of 1 rad is equivalent to the ripple rather than (3dB) Fig (6) illustrates a normalized elliptic function low – pass filter.

A number of terns unique to these filters are used in fig (6) and are defined as follows.

$W_s$  = lowest stop – band frequency of which a dB accurse (in radian).

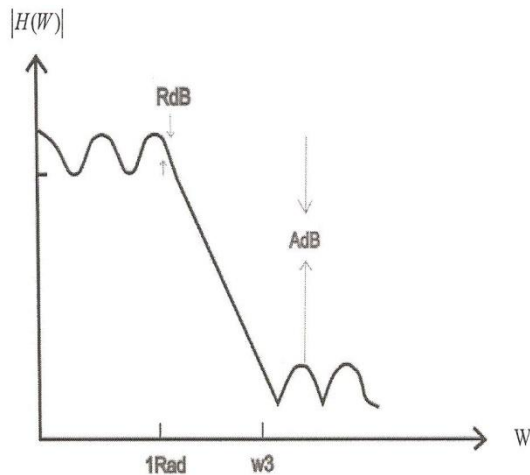
A dB = minimum stop – band attenuation in decibels.

R dB =pass – band ripple in decibels.

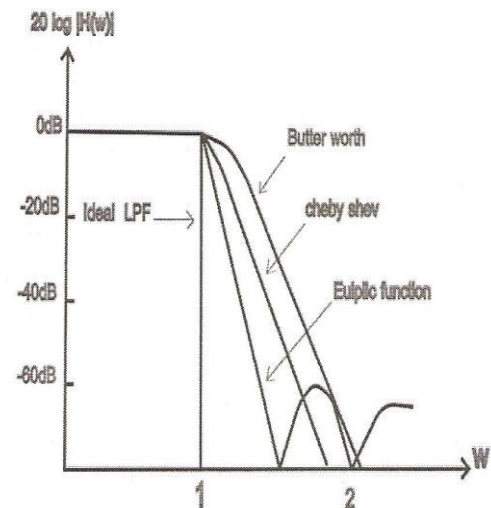
Elliptic function filters are categorized by these parameter in addition to order (n), since ( $W_s$ , A dB, R dB) defined the pass – band and stop – band limits. Normalized frequency response curves are not required. The amplitude response is given by the equation (8):

$$[H(w)] = \frac{1}{1 + E^2 C_n^2 \left(\frac{f}{f_c}\right)} \dots \dots \dots (8)$$

Where: -  $C_n$  = elliptic polynomial of the first kind degree (n)  
 E = constant that sets the pass – band and stop – band ripple.



**Fig ( 5 )Show L.P.F in Elliptical**

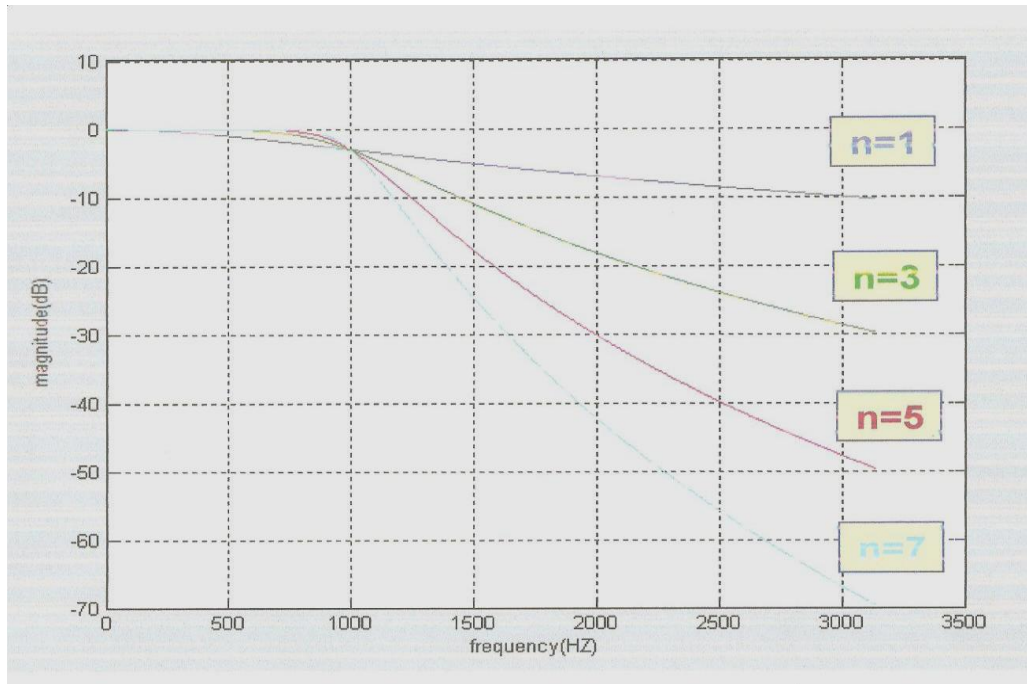


**Fig ( 6 ) Show idea L.P.F , Butter worth L.P.F, Chebyshev L.P.F and Elliptic L.P.F**

### 3.Experimental results

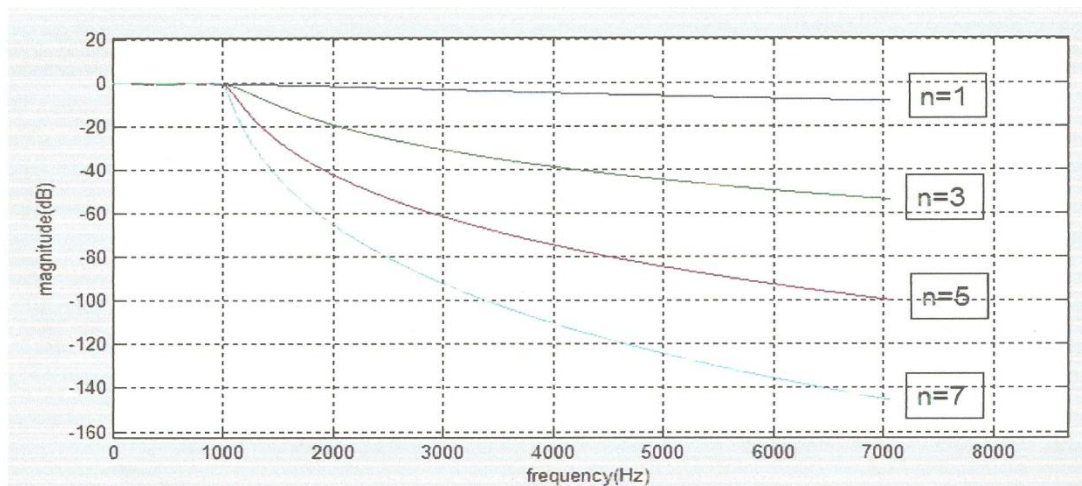
In this experiment , we select the order (  $n = 1,3,5,7$  ) of filter and applying a MATLAB software to analysis and design the low – pass filter using Butterworth type , fig (7) shows the curve of ' LPF ' Butterworth type , and show the effect of multiple (order of filter ) on the frequency response for a low – pass filter using Butterworth type.

Cascading filter (increase the order) increase the ultimate slop, but does improve the sharpness of the knee .All have not a very sharp 'knee' at the corner (cut off ) frequency (1000 Hz ) and do not really approach the ideal 'brick wall' filter characteristic the MATLAB program is shown appendix ( A1 ) is amplitude.



**Fig (7) Show the curve of Butterworth L.P.F  $n = (1,3,5,7)$  by MATLAB**

In this experiment, we show the effect of multiple (order of filter) on the frequency response for a low – pass filter using Chebyshev type. We select the order ( $n = 1, 3, 5, 7$ ) of filter and applying a MATLAB software to analysis and design this type of filter, fig (8) shows the curve of 'LPF' Chebyshev type using different order.

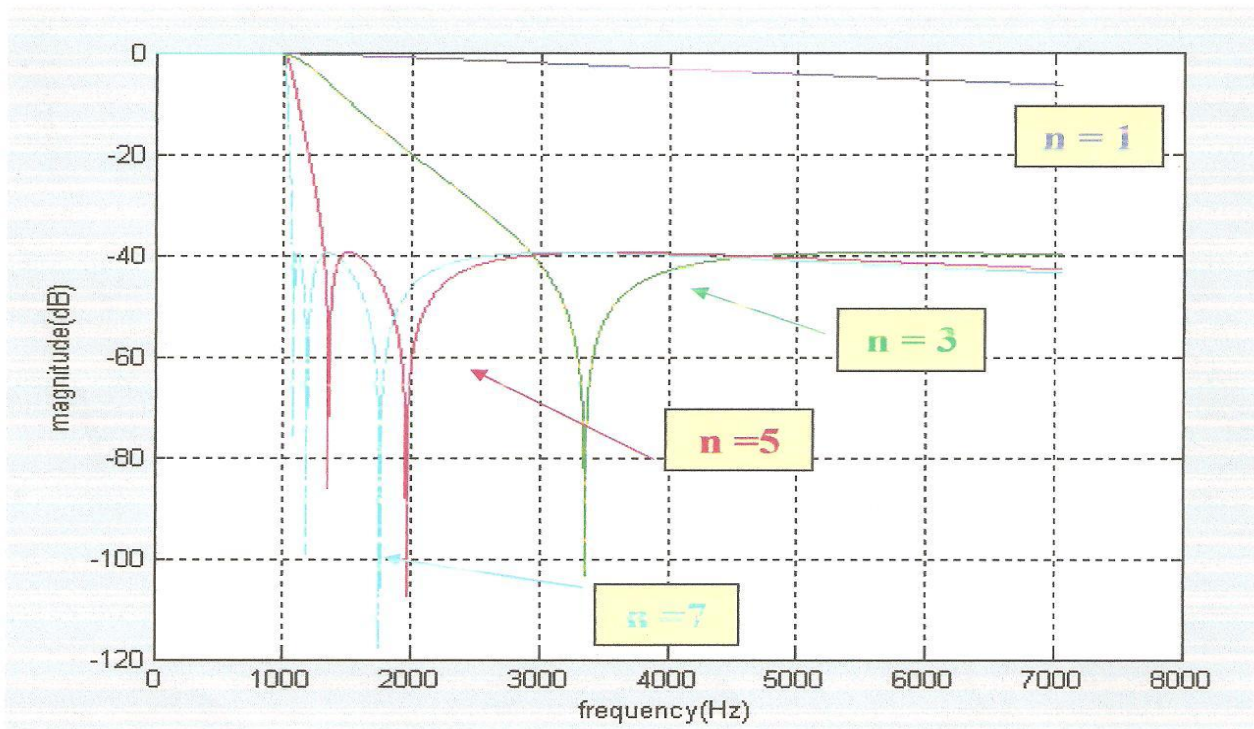


**Fig (8) Show the curve of Chebyshev 1 L.P.F  $n = (1,3,5,7)$**

In this experiment, we select the order ( $n = 1, 3, 5, 7$ ) of filter and applying a MATLAB software to analysis and design of the low –pass filter using Elliptical type.



Fig (9) shows the curves of L.P.F Elliptical type, and shows the effect of multiple (order of filter) on the frequency response for a low – pass filter using Elliptical type.



**Fig (9) Show the curve of Elliptic L.P.F  $n= (1,3,5,7)$**

In this experiment, we show the comparative from of different types of L.P.F filter.

Butterworth, Chebyshev, Elliptical with design specification shown be

First:

We need to design Butterworth L.P.F with the design specifications:

Cut off frequency = 1000 Hz.

Filter order = 5 or  $n = 5$ .

Second:

We need to design Chebyshev L.P.F with the design specifications:

Cut off frequency = 1000Hz.

Pass – band ripple = 0.5 dB.

Filter order = 4 or  $n = 4$ .

Third:

We need to design Elliptical L.P.F with the design specifications:

Cut off frequency = 1000 Hz.

Pass – band ripple = 0.28 dB.

Stop – band ripple = 39.48 dB.

Filter order = 3 or  $n = 3$ .

Fig. (10) Show the different types of L.P.F filters. Butterworth response from equation(9):

$$|T_n(jw)| = \frac{1}{\sqrt{1+w^{2n}}}. \quad \dots\dots\dots(9)$$

We see this one property of Butterworth response is for all (n).

$$|T_n(j1)| = \frac{1}{\sqrt{2}} = 0.707\dots or\dots(-3.dB) \quad \text{Where } T_n \text{ is the 4th order Butterworth filter.}$$

In the Butterworth response

$f = 1000$  Hz normalized frequency  $W (1)$  identifies the half – power frequency, while in the Chebyshev response at the same frequency or ( $w=1$ ) identifies the end of the ripple band. These two frequency are different unless  $e=1$  (max = 3dB) as shown in fig (10), the half – power frequency for the Chebyshev response is given by Eq. (10).

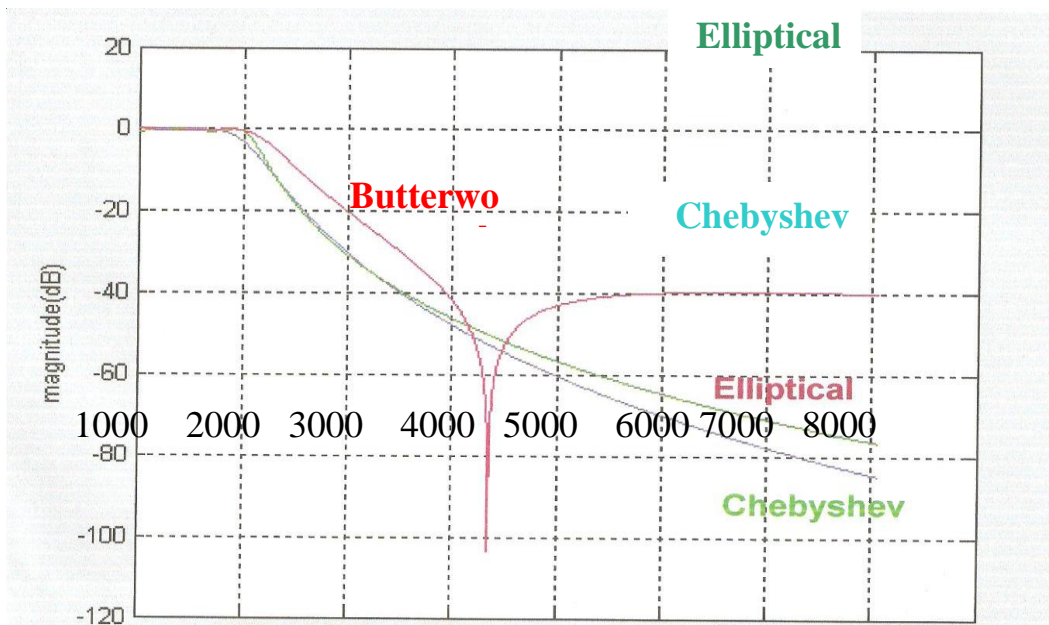
$$W_{n\alpha} = \cosh \frac{1}{n} \cosh^{-1} \left( \frac{1}{e} \right) \quad \dots\dots\dots(10)$$

Where  $W_{n\alpha}$  normalize frequency.

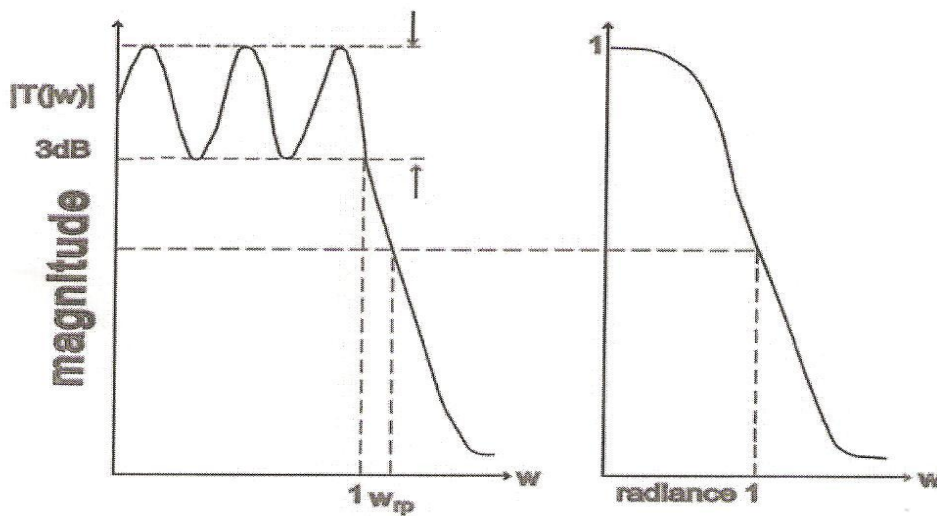
We have observed that the Chebyshev response has a sharper cut off rate than the Butterworth response, and this suggests that a new response with equal ripple in both the pass–band and the stop–band (Elliptical filter). If we compared the Butterworth, Chebyshev, Elliptical under our specification, we show the Butterworth response it is necessary only that ( $n=5$ ), for Chebyshev response that ( $n=4$ ), and for elliptical response it is necessary only that ( $n=3$ ).

This is significant improvement fig (10) shows the comparative form of different types L.P.F filters.

We can comprise between two response Butterworth and Chebyshev filters, as shown in fig (11).



**Fig (10) Shows the curves of Butterworth  $n=5$ , Chebyshev  $n=4$  and Elliptical  $n=3$  L.P.F MATLAB**



**Fig(11) Comparison between two responses Butterworth and Chebyshe**

#### **4. Conclusion**

1. Due to first experiment, A multipole filter can give any desired ultimate slop , but simple cascading still gives a softness, as shown in fig (7).
2. Due to second experiment, we conclude that the Chebyshev response provides a sharp (knee ) but has a ripple in the amplitude compared with Butterworth response which gives very flat amplitude, also there is a shifting in cut off frequency ( 1000 ) Hz in Chebyshev response at (-3 dB) , compared with Butterworth response , as shown in fig (8).
3. Due to third experiment, we conclude that the Elliptical response provides more sharp but has a ripple in the amplitude and in stop – band frequency compared with Chebyshev and Butterworth ,also there is a more shifting in cut off frequency(1000 ) Hz in Elliptic response at( -3 dB)compared with Chebyshev and Butterworth response , as shown in fig (9).
4. Due to fourth experiment, we conclude that if we want to design L.P.F in three methods we need an order design in Butterworth response (n =5) and for Chebyshev response (n= 4) and for Elliptical response (n = 3) we require to realize the specifications, as shown in fig (10, 11).

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