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Advance Study in the Generalized Circular Polarization and Scattering from Bi-Isotropic Objects

Al-Majdi Kadhum
Al-Mustansiriya University
College of Engineering
Computer and Software Eng. Dep.

Abstract

This paper discusses Different constitutive relations of generalized circular polarization in chiral media by using Maxwell's equations. In a chiral medium for fields with generalized circular polarizations, the Maxwell equations and constitutive relations have the same form in the achiral medium, but the equivalent material parameters differ for field with different circular polarization. Thus a wave of circular polarization (left or right) propagates with the same wave number regardless of its direction of propagation. In this paper, electromagnetic scattering characteristics by general bi-isotropic objects are investigated based on the surface integral equations. By applying the surface equivalent principle, electromagnetic fields inside a homogeneous bi-isotropic region can be represented in terms of equivalent electric and magnetic currents distributed over its boundary surface.

Keywords: Chiral media; Maxwell equations; circular polarization; left-hand circularly polarized; right-hand circularly polarized.

الخلاصة

في هذا البحث تم مناقشة العلاقات الأساسية المختلفة المولدة للاستقطاب الدائري في وسط chiral باستخدام معادلات ماكسويل. في الوسط chiral للمجالات المتولدة من الاستقطاب الدائري تكون معادلات ماكسويل لها نفس صيغة وسط achiral لكن معامل معدلات المواد يختلف مع اختلاف الاستقطاب الدائري. وبهذه الطريقة فان موجات الاستقطاب الدائري (اليمين واليسار) تنتشر بنفس الطول ألموجي بغض النظر عن اتجاه انتشار الموجة. في هذا البحث تم التوصل ايضا إلى مميزات الاستطارة الكهرومغناطيسية في وسط bi-isotropic بناءا على معادلات تكامل السطوح. بواسطة تطبيق مبدأ تساوي السطوح يكون تمثيل المجال الكهرومغناطيسي داخل منطقة الوسط المتجانس bi-isotropic في حدود مكافئة للتيارات الكهربائية والمغناطيسية الموزعة فوق حدود السطح.

Introduction

Since the beginning of the nineteen century, the geometric concept of Chirality had numerous implications in a variety of fields such as chemistry, optics, particle physics and most recently, electromagnetics. Chirality or handedness was found to be the origin of the physical phenomenon of optical activity [1-6]. Various forms of constitutive relation have been formulated to describe the electromagnetic characteristics of chiral media and bi-isotropic media. Details descriptions of these constitutive relations and discussions on the constitutive parameters are been given in [7-10].

A chiral material is one that has a distinct left or right handedness about its structure, arising from hand molecules or from handed scattering particles such helices. The electromagnetic consequence is that left hand circularly polarized wave's travel with difference velocity and absorption from right hand waves. Thus chirality is a generalization of the more familiar phenomenon of optical activity [4,6,11]. The modern history of chiral media in electromagnetic started in 1979 ref. [1] they presented a solid theoretical explanation. Considerable researchers have been devoted to numerical analysis of three dimensional arbitrary shaped bi-isotropic objects. Most of them focused on chiral scatterer, a special case of bi-isotropy. Dmitrenko, et [12] analyzed chiral bodies by using a T-matrix method, whereas this method suffers the deficiency of stable convergence, especially when the object to be modeled has a surface of complex shape.

This paper concerns with the some constitutive relations of circular polarization in chiral media. Also several widely used constitutive relations are discussed. This paper particularly is dedicated to the analysis of the scattering by general bi-isotropic objects based on the surface integral equation (SIE) method.

This paper is organized as following. In the following section, constitutive relations and analysis of chiral media are discussed. Sections III presents the generalized circular polarization in chiral media. Section IV, the scattering characteristics of general bi-isotropic objects is discussed in details. Numerical results of bi-isotropic sphere are given in section VI and the final section gives the concluding remarks.

Theory and Constitutive Relations

In this section, a homogeneous isotropic chiral medium is characterized by three (complex) parameters. These are the electric permittivity ε, the magnetic permeability μ and the chirality measure χ .thus the constitutive relations we use in this study are given by

$$D = \varepsilon E - i\chi H$$
$$B = \mu H + i\chi E \tag{1}$$

Where E , H are the electric and magnetic fields, and D, B the electric and magnetic flux densities, respectively. The correspondence between the electric induction $D(r,t)$ and electric field strength $E(r,t)$ given by [13]

$$D(r,t) = \varepsilon E(r,t) \tag{2}$$

Where the electric permittivity ε depends on the properties of the medium requires. In the general case equation (2) that the electric displacement $D(r,t)$ depends only on $E(r,t)$ determined at the same point and moment. One refinement consists in the fact that the electric displacement $D(r,t)$ depends not only on $E(r,t)$ but also on its time derivative. For an arbitrary dependence of $E(r,t)$ on t, Equation (2) is not valid. However, when the field follows the harmonic dependence, i.e. there are time-independent vectors $D(r)$ and $E(r)$ (which are complex amplitudes):

$$E(r,t) = \mathrm{Re}\left[E(r)\exp(i\omega t)\right]$$
$$D(r,t) = \mathrm{Re}\left[D(r)\exp(i\omega t)\right] \tag{3}$$

Then Equation (2) is correct, but the coefficient ε depends on the frequency, $\varepsilon = \varepsilon(\omega)$ this frequency dependence may not be taken into account only when the Fourier spectrum of these quantities is rather narrow, i.e. the process is close to the harmonic one. Further we will consider only the harmonic processes and refer to $E(r)$ and $D(r)$ as the electric field strength and displacement, respectively. The complex amplitudes $H(r)$ and $B(r)$, i.e. the magnetic field strength and induction, are introduced in a similar manner, the other deals with spatial dispersion, i.e. with the fact that the electric displacement $D(r)$ depends not only on $E(r)$ but also on its spatial derivatives. In the media where this effect is substantial, Equation (2) is not correct for arbitrary spatial dependences of the fields. Only when the fields vary in space as in a plane wave, this formula still holds, but ε depends on the direction of the normal N to the wave front:

$$D(r) = \varepsilon(N)E(r) \tag{4}$$

The permittivity $\varepsilon(N)$ is a tensor and not a scalar even in an isotropic medium. For arbitrary dependences of the fields on r, the first spatial derivatives of $E(r)$ enter the electric displacement $D(r,t)$ only through the combination $rotE(r)$ [14]. Since the fields $D(r,t)$ and $E(r)$, as well as $B(r)$ and $H(r)$ for harmonic oscillations in points with no extraneous currents satisfy the homogeneous Maxwell equations

$$\mathrm{rot}\,H = i\kappa D, \quad \mathrm{rot}\,E = -i\kappa B, \quad \kappa = \frac{\omega}{c} \tag{5}$$

The constitutive relations linking these vectors can be written in the symmetric form excluding their explicit derivatives:

$$D = \varepsilon E - i\chi H \ , \ B = \mu H + i\chi E \tag{6}$$

Here ε , μ and χ are the material constants, which do not depend on the field structure [1-3]. There are some other forms of these relations in the literature, which are equivalent in essence.

The cross-terms arising in Equation (6) can be explained without considering the non-local dependence of D on E (and correspondingly B on H). The term proportional to H , which is included in D , means that a current induced by an alternating magnetic field in the elements of a chiral medium causes not only a magnetic dipole moment but also an electric dipole moment. Due to the reciprocity requirement, the alternating electric field induces in such elements the current which in turn gives rise to both the electric and magnetic dipole moments, i.e. the magnetic flux density is also proportional to E . In media with no absorption affects the material constants ε , μ and χ are real. Notice that the coefficients of the cross- terms in Equation (6) are complex conjugated, since the medium properties should be otherwise nonreciprocal. We assume the constants ε , μ , and χ to be scalar, i.e. we consider isotropic chiral media, which are most interesting for radio physics. In the next section we consider some formal properties of solutions to the homogeneous Maxwell equations satisfying constitutive relations in Equation (6).

Generalized Circular Polarization

The electrodynamics behaviors of any homogeneous medium can be naturally characterized by the field structure pertinent to eigenwaves, which can propagate in the medium along an axis z so that all the components of the waves depend on z via the factor $\exp(-ihz)$. In an isotropic unbound medium the axis z can be any straight line.

In an achiral medium $(\chi = 0)$ the eigenwaves are, for example, two linearly polarized plane waves:

$$E_x = \exp(-ihz) \ , H_y = \frac{1}{\eta}\exp(-ihz) \tag{7a}$$

$$E_y = \exp(-hz) \ , H_x = -\frac{1}{\eta}\exp(-ihz) \tag{7b}$$

Where

$$h = k n, \quad n = \sqrt{\varepsilon \mu} \quad \eta = \sqrt{\frac{\mu}{\varepsilon}} \tag{8}$$

These waves have the same propagation constants h and any linear combination of the waves is the eigenwave too. If the coefficients of this linear combination are complex, then the wave need not be linearly polarized.

In a chiral medium $(\chi \neq 0)$, the waves corresponding to Equation (6) cannot exist independently, only two of their linear combinations are eigenwaves

$$\left.\begin{aligned} &E_x = \exp(-ih_+ z), \; E_y = -i\exp(-ih_+ z) \\ &H_x = \frac{i}{\eta}\exp(-ih_+ z), \; H_y = \frac{1}{\eta}\exp(-ih_+ z) \\ &E_x = \exp(-ih_- z), \; E_y = i\exp(-ih_- z) \\ &H_x = -\frac{1}{\eta}\exp(-ih_- z), \; H_y = \frac{1}{\eta}\exp(-ih_- z) \end{aligned}\right\} \tag{9a}$$

The propagation constants of these waves are different and

$$h_{\pm} = k(n \pm \chi) \tag{9b}$$

The wave corresponding to Equation (9a) is left-hand circularly polarized, while the other wave Equation (9b) has right-hand circular polarization. In these waves the electric and magnetic fields are coupled by the relations

$$H_{\pm} = \pm \frac{i}{\eta} E_{\pm} \tag{10}$$

In a chiral medium any fields E_+, H_+, and E_-, H_- satisfying Equation (10) can exist independently. These fields are naturally referred to as the fields with generalized circular polarization [15]. The upper sign in Equation (10) corresponds to the left-hand circular polarization, the lower sign to the right-hand one.
Any field E, H can be written as the sum of two fields with generalized circular polarization:

$$E = E_+ + E_-, \quad H = H_+ + H_- \tag{11}$$

where

$$E_{\pm} = \frac{1}{2}(E \pm i\eta H), \quad H_{\pm} = \frac{1}{2}\left(H \pm \frac{i}{\eta}E\right) \tag{12}$$

Substituting Equation (10) into Equation (5), we arrive at

$$D_{\pm} = \varepsilon_{\pm} E_{\pm}, \quad B_{\pm} = \mu_{\pm} H_{\pm} \tag{13}$$

where

$$\varepsilon_{\pm} = \varepsilon\left(1 \pm \frac{\chi}{\eta}\right), \quad \mu_{\pm} = \mu\left(1 \pm \frac{\chi}{\eta}\right) \tag{14}$$

Thus, in a chiral medium for fields with generalized circular polarization the Maxwell equations and constitutive relations have the same form as in an achiral medium, but the equivalent material parameters differ for the fields with different circular polarizations [5-16]. Notice that according to Equation (10) the Maxwell equations are reduced to the single first-order equation

$$rot_{\pm}E = \pm k_{\pm} n \tag{15}$$

where

$$n_{\pm} = n \pm \chi \tag{16}$$

The first equation in (13) has the same meaning as Equation (3), but it applies not to locally plane waves but to the fields following Equation (10), and the quantities $\varepsilon_{\pm}$ for these fields depend only on the point considered, in contrast to $\varepsilon(N)$. Thus, the E_{+} circular component propagates forward with wave number k_{+} and backward with k_{-}, and the reverse is true of the E_{-} component. The forward-moving component of E_{+}, that is, $E_{R^{+}}$ and $E_{R^{-}}$, are both right-polarized and both propagate with the same wave number k_{+}. Similarly, the left-polarized wave $E_{L^{+}}$ and $E_{L^{-}}$ both propagate with k_{-}. Thus, a wave of circular polarization (left or right) propagates with the same wave number regardless of its direction of propagation. This is a characteristic of chiral media difference from another media.

Scattering of General Bi-Isotropic Objects

According to the surface equivalence principle, the scattered electric $\vec{E}_{\circ}$ and $\vec{H}_{\circ}$ in free space can be expressed in terms of equivalent electric currents and magnetic currents placed over the surface of the bi-isotropic object, written by

$$\vec{E}_{\circ}(\vec{J}_d, \vec{M}_d) = -L(\vec{J}_d) - K(\vec{M}_d) \tag{17}$$

$$\vec{H}_{\circ}(\vec{J}_d, \vec{M}_d) = K(\vec{J}_d) - \frac{1}{\eta_{\circ}^2} L(\vec{M}_d) \tag{18}$$

where $\vec{J}_d$ and $\vec{M}_d$ are the equivalent electric and magnetic currents on the exterior surface of the bi-isotropic object under analysis. $\eta_{\circ}$ is the wave impedance of free space. L and K are integro-differential operators, defined as below [17],

$$L(\vec{X}) = j\omega\mu_{\circ}\left\{\int_s \vec{X}(\vec{r}\,')G(\vec{r},\vec{r}\,')d\vec{r}\,' + \frac{1}{k_{\circ}^2}\int_s \left[\nabla\cdot\vec{X}(\vec{r}\,')\right]\nabla G(\vec{r},\vec{r}\,')d\vec{r}\,'\right\} \tag{19}$$

$$K(\vec{X}) = \int_s \vec{X}(\vec{r}\,')\times\nabla G(\vec{r},\vec{r}\,')d\vec{r}\,' + \frac{1}{2}\hat{n}_S \times \vec{X}(\vec{r}\,') \tag{20}$$

and

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$$G(\vec{r}, \vec{r}') = \frac{e^{-jk|\vec{r} - \vec{r}'|}}{4\pi|\vec{r} - \vec{r}'|} \tag{21}$$

where the propagation constant of free space $k_\circ = \omega\sqrt{\mu_\circ \varepsilon_\circ}$ $\varepsilon_\circ$ and $\mu_\circ$ are the permittivity of free space, respectively. The angular frequency is $\omega = 2\pi f$ (f is the operated frequency). The analyzed bi-isotropic object has a boundary surface S and $\hat{n}_S$ is defined as the unit vector outward S. Note that in equation (20), the left second term in the operator K is called as the residue term[17] that guarantees the continuity of the field as the observation point approaches the source point. If the observation point is not on S, the residue term equals to zero.

The expressions of electric and magnetic fields inside the bi-isotropic region are relatively complex due to the introduction of bi-isotropic constitutive relations, namely,

$$\vec{D} = \varepsilon \vec{E} + \xi_r \sqrt{\varepsilon\mu}\vec{H} = \varepsilon \vec{E} + (\chi_r - j\kappa_r)\sqrt{\varepsilon\mu}\vec{H} \tag{22}$$

$$\vec{B} = \zeta_r \sqrt{\varepsilon\mu}\vec{E} + \mu\vec{H} = (\chi_r + j\kappa_r)\sqrt{\varepsilon\mu}\vec{E} + \mu\vec{H} \tag{23}$$

where χ_r, κ_r are Tellegen and Pasteur parameters, respectively. A bi-isotropic medium with $\chi_r = 0$ and $\kappa_r \neq 0$ is named Pasteur medium while the one with $\chi_r \neq 0$ and $\kappa_r = 0$ named Tellegen medium. It should be noted that the limit condition of $\chi_r^2 + \kappa_r^2 < 1$ must be satisfied and otherwise the nature of the medium is radically changed. To represent the fields in the bi-isotropic region, a field splitting scheme is applied as referred to [18]. At first $\vec{E}_d$, $\vec{H}_d$ in the homogeneous bi-isotropic medium are split into two independent and uncoupled wave-fields, namely, (plus) wave-fields $\vec{E}_+$, $\vec{H}_+$ and (minus) wave-fields $\vec{E}_-$, $\vec{H}_-$

$$\vec{E}_d = \vec{E}_+ + \vec{E}_- \tag{24}$$

$$\vec{H}_d = \vec{H}_+ + \vec{H}_- \tag{25}$$

Each pair of wave-fields $\vec{E}_+(\vec{H}_+)$ and $\vec{E}_-(\vec{H}_-)$ is related with respective medium characterized by $\varepsilon_+(\varepsilon_-)$, $\mu_+(\mu_-)$, $\eta_+(\eta_-)$ and $k_+(k_-)$ which are defined by

$$\varepsilon_\pm = \varepsilon(\tau \pm \kappa_r)e^{\pm j\vartheta} \tag{26}$$

$$\mu_\pm = \mu(\tau \pm \kappa_r)e^{\pm j\vartheta} \tag{27}$$

$$\eta_\pm = \sqrt{\frac{\mu}{\varepsilon}e^{\pm j\vartheta}} = \eta e^{\pm j\vartheta} \tag{28}$$

$$k_{\pm} = \omega\sqrt{\mu\varepsilon}\left(\tau \pm \kappa_r\right) = k\left(\tau \pm \kappa_r\right) \tag{29}$$

where $\tau = \sqrt{1-\chi_r^2}$ $\left(0 < \tau \le 1\right)$ and $e^{\pm j\vartheta} = \tau \pm j\chi_r$. Since two wave-fields are independently governed by the Maxwell equations $\vec{E}_{+}(\vec{H}_{+})$ and $\vec{E}_{-}(\vec{H}_{-})$ can be obtained using the surface equivalent principle, respectively. The expressed by

$$\vec{E}_{\pm}(\vec{J}_{\pm}, \vec{M}_{\pm}) = -L_{\pm}(\vec{J}_{\pm}) - K_{\pm}(\vec{M}_{\pm}) \tag{30}$$

$$\vec{H}_{\pm}(\vec{J}_{\pm}, \vec{M}_{\pm}) = K_{\pm}(\vec{J}_{\pm}) - \frac{1}{\eta_{\pm}^2}L_{\pm}(\vec{M}_{\pm}) \tag{31}$$

where $\vec{J}_{+}(\vec{M}_{+})$ and $\vec{J}_{-}(\vec{M}_{-})$ are surface equivalent currents placed over the interior surface. The integro-differential operators $L_{\pm}$ and $K_{\pm}$ are defined by

$$L_{\pm}\left(\vec{X}\right) = j\omega\mu_{\pm}\left\{\int_S \vec{X}(\vec{r}\,')G_{\pm}(\vec{r},\vec{r}\,')d\vec{r}\,' + \frac{1}{k_{\pm}^2}\int_S\left[\nabla\cdot\vec{X}(\vec{r}\,')\right]\nabla G_{\pm}(\vec{r},\vec{r}\,')d\vec{r}\,'\right\} \tag{32}$$

$$K_{\pm}\left(\vec{X}\right) = \int_S \vec{X}\left(\vec{r}\,'\right)\times\nabla G_{\pm}\left(\vec{r},\vec{r}\,'\right)d\vec{r}\,' + \frac{1}{2}\hat{n}_S\times\vec{X}\left(\vec{r}\,'\right) \tag{33}$$

$$G_{\pm}\left(\vec{r},\vec{r}\,'\right) = \frac{e^{-jk_{\pm}\left|\vec{r}-\vec{r}\,'\right|}}{4\pi\left|\vec{r}-\vec{r}\,'\right|} \tag{34}$$

as can be seen, the expressions of scattered wave-fields in plus and minus medium induced by $\vec{J}_{+}(\vec{M}_{+})$ and $\vec{J}_{-}(\vec{M}_{-})$ are similar to those of free space except that materials parameters are different. In other word, $\vec{E}_{+}(\vec{H}_{+})$ and $\vec{E}_{-}(\vec{H}_{-})$ can be obtained from the equation (17) and (18) through replacing $(\varepsilon_\circ, \mu_\circ, \eta_\circ, k_\circ, \vec{J}_d, \vec{M}_d)$ by $\left(\varepsilon_{+}(\varepsilon_{-}), \mu_{+}(\mu_{-}), \eta_{+}(\eta_{-}), k_{+}(k_{-}), \vec{J}_{+}(\vec{M}_{+}), \vec{J}_{-}(\vec{M}_{-})\right)$ Here, the relations of between $(\vec{J}_d, \vec{M}_d)$ and $(\vec{J}_{+}(\vec{M}_{+}), \vec{J}_{-}(\vec{M}_{-}))$ can be obtained from the Maxwell equations [18],

$$-\vec{J}_{\pm} = \frac{1}{2\tau}(e^{\pm j\vartheta}\vec{J}_d \pm \frac{j}{\eta}\vec{M}_d) \tag{35}$$

$$-\vec{M}_{\pm} = \frac{1}{2\tau}(e^{\pm j\vartheta}\vec{M}_d \pm j\eta\vec{J}_d) \tag{36}$$

With the aid of equation (24) and equation (25), one can derive the scattered fields in the bi-isotropic region, given by,

$$\vec{E}_d(\vec{J}_d,\vec{M}_d) = \vec{E}_d(-\vec{J}_+,-\vec{M}_+) + \vec{E}_d(-\vec{J}_-,-\vec{M}_-) \tag{37}$$
$$\vec{H}_d(\vec{J}_d,\vec{M}_d) = \vec{H}_d(-\vec{J}_+,-\vec{M}_+) + \vec{H}_d(-\vec{J}_-,-\vec{M}_-) \tag{38}$$

and

$$\vec{E}_d(-\vec{J}_\pm,-\vec{M}_\pm) = \frac{1}{2\tau}L_\pm\left(e^{\pm j\vartheta}\vec{J}_d \pm \frac{j}{\eta}\vec{M}_d\right) + \frac{1}{2\tau}K_\pm\left(e^{\pm j\vartheta}\vec{M}_d \pm j\eta\vec{M}_d\right) \tag{39}$$

$$\vec{H}_d(-\vec{J}_\pm,-\vec{M}_\pm) = -\frac{1}{2\tau}K_\pm\left(e^{\pm j\vartheta}\vec{J}_d \pm \frac{j}{\eta}\vec{M}_d\right) + \frac{1}{2\tau}\frac{1}{\eta_\pm^2}L_\pm\left(e^{\pm j\vartheta}\vec{M}_d \pm j\eta\vec{J}_d\right) \tag{40}$$

to determine the unknown currents $\vec{J}_d$ and $\vec{M}_d$, the boundary condition is needed to enforce on the surface of the bi-isotropic scatterer, that is, the total tangential fields should be continuous across the surface of the bi-isotropic scatterer. Therefore, a set of combined fields integral equations can be obtained as below,

$$\begin{aligned}\vec{E}^{inc}\Big|_{tan} = \Big[& L(\vec{J}_d) + K(\vec{M}_d) \\ & + \frac{1}{2\tau}L_+\left(e^{+j\vartheta}\vec{J}_d - \frac{j}{\eta}\vec{M}_d\right) + \frac{1}{2\tau}K_+\left(e^{-j\vartheta}\vec{M}_d + j\eta\vec{J}_d\right) \\ & + \frac{1}{2\tau}L_+\left(e^{-j\vartheta}\vec{J}_d + \frac{j}{\eta}\vec{M}_d\right) + \frac{1}{2\tau}K_-\left(e^{+j\vartheta}\vec{M}_d - j\eta\vec{J}_d\right)\Big]\Big|_{tan}\end{aligned} \tag{41}$$

$$\begin{aligned}\vec{H}^{inc}\Big|_{tan} = \Big[& -K(\vec{J}_d) + \frac{1}{\eta_\circ^d}L(\vec{M}_d) \\ & - \frac{1}{2\tau}K_+ e^{+j\vartheta}\left(\vec{J}_d - \frac{j}{\eta}\vec{M}_d\right) + \frac{1}{2\tau}\frac{1}{\eta_+^2}L_+ e^{-j\vartheta}\left(\vec{M}_d + j\eta\vec{J}_d\right) \\ & - \frac{1}{2\tau}K_-\left(e^{-j\vartheta}\vec{J}_d + \frac{j}{\eta}\vec{M}_d\right) + \frac{1}{2\tau}\frac{1}{\eta_-^2}L_-\left(e^{+j\vartheta}\vec{M}_d - j\eta\vec{J}_d\right)\Big]\Big|_{tan}\end{aligned} \tag{42}$$

where $\vec{E}^{inc}$ and $\vec{H}^{inc}$ are the incident electric and magnetic fields in free space and subscript (tan) defines tangential components. In general, the solution of the equation sets (41) and (42) is based on the method of moments (MoM) [2].

Numerical Results and Discussions

First example considers a bi-isotropic sphere with the radius against propagation constant of $k_\circ a = 1.5$ as shown in Figure 1. For comparison with the exact solution, we chose

$\varepsilon_{DBF} = 4\varepsilon_\circ$ and $\mu_{DBF} = \mu_\circ$ then, $\varepsilon = \varepsilon_{DBF}/(1-\xi_r^2)$ and $\mu = \mu_{DBF}/(1-\xi_r^2)$ [19]. Here, $\xi_r = \chi_r - j\kappa_r$ is denoted. Material lossy is not considered in this example. The spherical surface is modeled using 424 triangular patches and totally the 1272 unknown are needed to solve. The normalized bistatic cross-sections (BCS) for the co-polarized component $\sigma_{\theta\theta}$ and the cross polarized components $\sigma_{\theta\theta}$ corresponding to different values of κ_r are given in Figure 2 and Figure 3 when $\chi_r = 0$ .in the same figures we also present the exact solutions based on the modal expansion theory [19]. Good agreements are found between them. The calculated results are also well compared with those from [12] [20].

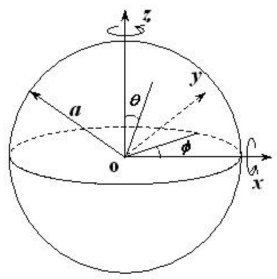

Fig. 1 The geometry of a chiral sphere with the radius of a.

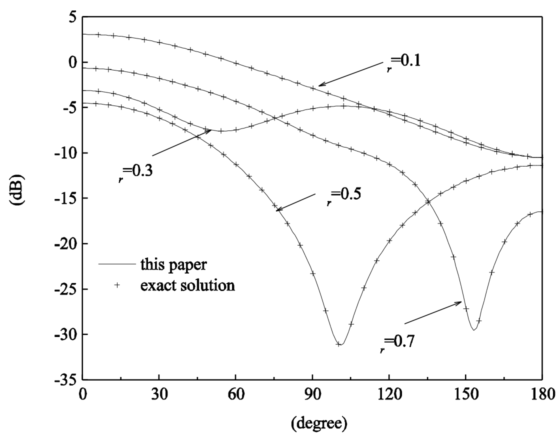

Fig. 2 Bistatic cross sections for co-polarized scattered field component $\sigma_{\theta\theta}$ normalized by $\lambda_\circ^2$ versus the evaluation angle θ for different κ_r when $\chi_r = 0$.

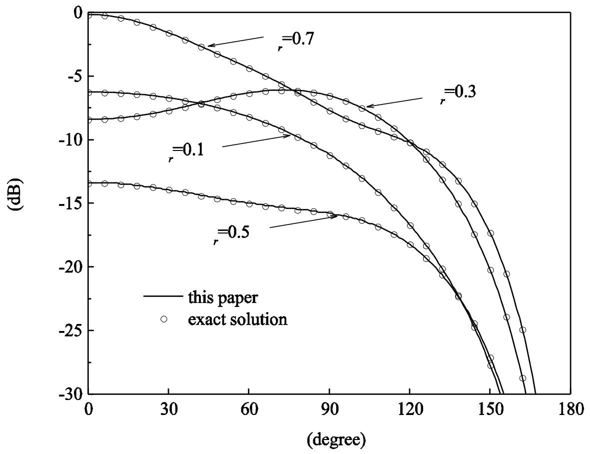

Fig. 3 Bistatic cross sections for cross-polarized scattered field component $\sigma_{\theta\phi}$ normalized by λ_o^2 versus the evaluation angle θ for different κ_r when $\chi_r = 0$.

Conclusion

In this paper, different constitutive relations for chiral media are discussed, a circularly polarized wave scattered by the interface between two chiral media generally produces waves with both circular polarizations. It is known that a wave with left-hand circular polarization normally incident on a plane metallic mirror, reflects from the mirror as a right-hand circularly polarized wave. For an ideally reflective plane interface we find the condition at which such transformation is not the case. According to Equation (10) the component H should also be zero in the total field direction. Thus in a chiral medium for the field with generalized circular polarization the Maxwell equations and constitutive relations have the same form as in an Achiral medium, but the equivalent material parameters differ for the fields with different circular polarization. In this paper, the coupled field integral equations based on the equivalent principle have been formulated for the scattering by general bi-isotropic objects including Pasteur and Tellegen media. The formulations are validated with two analysis examples.

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