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ON CHARACTERIZATIONS OF PRE-CONTINUOUS AND PRE-IRRESOLUTE MAPPINGS

Lecturer, Ansam Ghazi Nsaif ALBU_Amer
University of wasit
Collage of Basic Education
ansaif@uowasit.edu.iq

Lecture Dr. Sami Abdullah Abed
University of Diyala
College of Administration and Economics
samiaabed@uodiyala.edu.iq

Abstract

In the literature different kinds of mappings between topological spaces have been of this paper if to continue to explore further properties and characterization of PCC and pre-irresolute mappings.

Keywords: PO, PCC, pre-irresolute.

Abbreviations
PO:PO
PC:Pre-Close
PCC: PCC
PL:Pre-Limit
TS:Topological space
OS:Open Set
CS:Closed set
PN:Pre-neighborhood
PD:Pre-derived
IM:Injective Mapping
PI: Pre Irresolute
GM: Graph Mapping

عن الخصائص قبل المستمرة والمتعددة للتطبيقات

م. انسام غازي نصيف
جامعة واسط / كلية التربية الأساسية
ansaif@uowasit.edu.iq

م.د. سامي عبد الله عبد
جامعة ديالى / كلية الإدارة والاقتصاد
samiaabed@uodiyala.edu.iq

الملخص
عن خصائص التطبيقات غير المستمرة والمتقطعة مسبقا
في هذا البحث ، تم استخدام أنواع مختلفة من التطبيقات بين الفضاءات التبولوجية ونستمر في استكشاف المزيد من خصائص وتوصيفات PCC والتطبيقات غير المتقطعة.
الكلمات المفتاحية: PO، PCC، والتطبيقات غير المتقطعة.

Introduction and Preliminaries

Talking about groups is a complex topic that requires special skills and more specific ones. Among these topics is the so-called pre-conquered or PO group, as its idea has a very important role for general topology . The pre-conquered or the PO group has many names, for example the so-called open group and the locally intensive group, as these groups are very useful in covering the characteristics of pre-continuity, continuity and even analyzing them. They are also used in descriptive analyzing context of open-assignment theories and closed-graph theories . This paper is devoted to continue to explore to explore further properties and characterizations of PCC and pre-irresolute mappings. From the beginning, and through the text of the present paper, let us assuming both $X \& Y$ refers to the TSs over it separation occurred, or stated clearly. Assuming that $A \subset \text{space} X$, then; $\text{Cl } A \; \& \; \text{Int } A$ will refer to Closure and Interior of A. Now again, $A \subset \text{space} X$ refer to re-open, under the condition. Complementing of a PO set is called PC . Every open (closed) set is PO (precludes), but the converse is not true, the family of all PO sets of a $\text{space} X$ defined as $PO(X)$.

PCC Mappings

In 1982, A. S. Mashhour et al. presented the symbols of PCC mappings, and giving certain characterizations of PCC mappings. The purpose of this section is to investigate further properties and characterizations of this class of mappings.

Definition 2.1

The mapping $\Im : X, Y$ refer to per-continuous if $\Im^{-1}(\nu)$ is PO in $X \forall \nu \in Y$.

Definition 2.2

Assuming that X is TS, A set $N_x \subseteq X$ is called a PN (Pre-nedb in short) of a point $\chi \in X$ if and only if $\exists$ PO set A such that $\chi \in A \subseteq N_\chi$

Definition 2.3

Assuming that X is TS and $A \subseteq X$, The point $P \in X$ is called a PL of A iff $U \cap A - \{P\} \neq \varphi \, \forall U \in PO(X)$ under the condition $P \in U$. Set of all PL points of A is called the PD set of A defined as A^{pd}, and $(A \cup A^{pd})$ defined as pre-closure of A and is denoted by $Pcl\,A$.

Theorem 2.1

Let $\Im: X, Y$ is mapping, then $X \Leftrightarrow Y$

- $\Im$ is PCC
- $\Im \forall P \in$ and every OS $G \in Y$, $\Im(P) \in G$, $\exists \alpha$ PO set A in X; $P \in A$ and $f(A)$ (A)f(A) $\subseteq G$
- $\forall \chi \in X$, each neighborhood N of $f(\chi)$ $\exists \alpha$ PN V of χ
- Pcl[f^{-1}(B)](cl B) for each subset B of Y.

Theorem 2.2

A mapping $\Im: X, Y$ is PCC iff $\Im$

$(A^{pd}) \subseteq \Im(A) \cup (\Im(A))\, d$, $\forall A \subseteq X$

Proof

Necessity

Assume that $A \subseteq X$, $a \notin APd$

Also, assume that $\Im(a) \notin \Im(A)$

Let N be a neighborhood of $\Im(\alpha)$

Since $\Im$ is PCC, then by Theorem 2.1., $\exists$ pre- nbd U of α such that $\Im(U) \subseteq N$

From $\alpha \in A^{Pd} \Rightarrow U \cap A \neq \varnothing$

Fix $b \in U$ & $b \in U$ & $b \in A$

Therefore

$\Im(b) \in N$ & $\Im(b) \in \Im(A)$

Since $\Im(a) \notin \Im(a) \notin \Im(A)$

Therefore

$\Im(A)$, therefore $(b) \neq \Im(a)$

Thus every neighborhood of $\Im(a)$ contains an element of $\Im(\alpha)$ contains an element of $\Im(A)$ different from $\Im(a)$

Hence it concluded $\Im(b) \in (\Im)\,(A))d$

This proves necessity

Sufficiency

Let $\Im$ is not PCC

Then by Theorem 2.1, $\exists a \in X$ and a neighborhood N of $\Im(a)$ such that every pre-nbd U of α contains at least one element $b \in U$ such that $\Im(b) \notin N$

Put

$A=\{b\in X: \Im(b)\notin N\}$

Then

$\alpha\notin A$ because $\Im(a)\in N$

Therefore $f(\alpha))]=\varnothing$

Thus $\Im(A^{Pd})$ is not contained in $\Im(a)\notin(f((A)^{d}$ because $\Im(A)\cap[N-(f(\alpha))]=\varnothing$

Thus

$\Im(A^{pd})$ is not contained in $\Im(A)(\Im((A))d$

This is a contradiction to our hypothesis, hence $\Im$ is PCC

Theorem 2.3

Let $\Im: X, Y$ be an injective mapping

Then $\Im$ is PCC iff $\Im(A^{pd})\subseteq(\Im((A))d, \forall A\subseteq X$

Proof

Necessity

Assume that $A\subseteq X$, $a\in A^{pd}$ & N be a neighborhood of $\Im(a)$

Since $\Im$ is PCC, then by Theorem 2.1., $\exists$ pre-nbd U of a; $\Im(U)\subseteq N$

But $\alpha\in A^{pd}$

Hence $\exists\in$element $b\in U\cap$; $b\neq a$; then $\Im(b)\in\Im(A)$

Since $\Im$ is an injection, $\Im(b)\neq\Im(a)$

Thus every neighborhood N of $\Im(a)$ contains an clement of $\Im(A)$ different from $\Im(a)$; consequently $\Im(a)\in\Im(a)\in(\Im((A))^{d}$

So $\Im(A^{Pd})\subseteq(\Im((A))^{d}$

Sufficiency follows from theorem 2.2.

Definition 2.4

Assume that X be a TS and $A\subseteq X$, A point $\chi\in X$ is called a pre-interior point of A iff $\exists U\in PO(X)$; $\chi\in U\subseteq A$. The set of all pre-interior points of A is called the pre-interior of A and take the symbol pInt A.

Theorem 2.4

Assume that X be a TS and $A\subseteq X$, then Point A=X=pcl(X–A)

Proof

Obviously,

PInt $A\subseteq A$,

Thus

$X-A\subseteq X-$Pint A,

$\Rightarrow Pcl(X-A)\subseteq Pcl(X-\text{pint } A)$

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i.e., $pcl(X–A) \subseteq X–Pint\ A$
Hence
$Pint\ A \subseteq X–pcl\ (X–A)$
On opposite side, if $\chi \in X–pcl(X–A)$,
Then $\chi \notin pcl\ (X–A)$
Hence $\exists U\chi \in PO\ (X); \chi \in U_\chi$ & $U_\chi \cap (X–A)=\varnothing$
Then
$\chi \in U\chi CPO\ (X)$ & $U\chi \subseteq A$, and so, $\chi \in Pint\ A$
This clarify that $X\text{-}pcl(X–A) \subseteq pint\ A$
Thus, $pint\ A= X–pcl(X–A)$

Theorem 2.5

A mapping $\Im: X, Y$ is PCC iff $\Im^{-1}(Int\ B)=\Im^{-1}(Y–B))\ \ X–\Im^{-1}(cl\ (Y–B))$ Since $\Im$ is PCC we have by Theorem 2.1, $pcl\,\Im^{-1}(Y–B) \subseteq \Im^{-1}\ (cl(Y–B)$
Hence
$\Im^{-1}(int\ B) \subseteq X–pcl\,\Im^{-1}(B)$
Applying theorem 2.4, $\Rightarrow \Im^{-1}(Iny\ B) \subseteq \Im^{-1}(B)$

Sufficiency

Assume that B be an open-set of Y, then B=lnt B
Hence by hypothesis $\Im^{-1}(B) \subseteq pint\,\Im^{-1}(B)$, but $pint\,\Im^{-1}(B)$
Therefore,
$\Im^{-1}(B)=plnt\,\Im^{-1}(B)$ is a PO set and hence $\Im$ is PCC

Theorem 2.6

Assume that $\Im: X, Y$ mapping, $g: X \times Y$ be the GM given by $g(\chi)=(\chi, \Im(\chi))\ \forall\ \chi \in X$, if g is PCC, then $\Im$ is PCC

Proof

Assume that $\chi \in X$ & G is any OS contains (χ), and then $X \times G$ is an OS in $X \times Y$ containing $g\ (\chi)$

Since g is pre continuous, $\exists$ PO set U contains χ such that $g(U) \subseteq G$, by theorem 2.1, it follows that $\Im$ is PCC.

Remark 2.1

The converse of theorem 2.6 is not true as shown by the following example

Example 2.1

Let $X=\{a,b,c\}$ & $\tau=\{\varnothing,\{a\}, \{B\}, \{a,b\},X\}$
Define mapping $\Im:(X,\tau)\ (x,\tau)$ as following:
$F(a)=b,\ \Im(b)=a,\ \Im(c) = a$

Then $\Im$ is PCC, but g:(X ,τ) (XxX, τχτ) is not PCC, due to A={(a, a), (b, a), (c,a)} is open in Xx X, but g^{-1}(A)={b,c} is not PO in X

Pre-irresolute mapping

Herein, it is considered the mappings for which inverse images of PO sets are PO. An investigation of some new properties and characterizations of such mappings are described below.

Definition 3.1

A mapping $\Im: X, Y$ is called PI [10] if $\Im^{-1}$(V) is PO in X for any PO subset V of Y.

Theorem 3.1

A mapping $\Im: X, Y$ is PI, iff $\forall\chi$ in X and each PO set V in Y with $\Im$ (χ)χV, $\exists$ a PO set U in X; $\chi\in U$, $f(U)\subseteq V$

Proof

Necessity

Assume that $U=\Im^{-1}(V)$

Since $\Im$ is PI, U is PO in X

Also

$\chi\in \Im^{-1}(V)=U$ as $\Im(\chi)\in V$

Also we have $\Im(U)=\Im(\Im^{-1}((V))\subseteq V$

Sufficiency

Assume that $V\in PO(Y)$ & $U=\Im^{-1}(V)$

We had shown that U is PO in X

So, $\chi\in V$, then by hypothesis, there U $\chi\in PO(X)$; $\chi\in U\chi$ and $\Im$**(U**χ)$\subseteq \Im^{-1}(\Im(U\chi)\subseteq\Im^{-1}(V)=U$

Thus U = $\cup\chi\in U$ UX, it follows that U is PO in X, hence $\Im$ is PI

Theorem 3.2

A mapping $\Im: X, Y$ is pre-irresolute iff inverse image of every PN of $\Im(\chi)$ is a PN of χ

Proof

Necessity

Assuming that $\chi\in X$ & B be pre-nbd of $\Im(\chi)$, then $\exists U\in f$-1 (U) $\subseteq\Im^{-1}(B)$

Since $\Im$ is PI, then $\Im^{-1}(U)\in PO(X)$

Hence $\Im^{-1}$(B) is a pre-nbd of χ

Sufficiency

Assuming that $B\in PO(Y)$

Putting

$U=\Im^{-1}(B)$ & $\chi\in U$

Then $\Im(\chi)\in$, But B, being PO is pre–nbd of $\Im(\chi)$, therefore, by hypothesis, $U=\Im^{-1}(B)$ is a pre–nbd of χ

Hence by definition, $\exists U\chi\in PO(X)$; $\chi\in U\chi\subseteq U$

Therefore

$U=\cup_{\chi\in} UU_{\chi}U\ \chi$

It following that U is a PO in X, so $\Im$ is PI

Theorem 3.3

Mapping $\Im: X$, is PI iff $\forall\chi$ in X and each PN V of $\Im(\chi)$, $\exists$ PN U of χ; $\Im(U)\subseteq V$

Proof

Necessity

Assuming $\chi\in X$&V be a pre–nbd of $\Im(\chi)$,

Then $\exists B_f(\chi)\in PO(Y)$;

$\Im(\chi)\in B\Im(\chi)\subseteq \Im^{-1}(V)$, by hypothesis $\Im^{-1}(B\Im(\chi))\in PO(X)$.

Assuming $U=\Im^{-1}(V)$, then it follows that U is a pre-nbd of χ & $\Im(U)=\Im(\Im^{-1}(V))\subseteq V$

Sufficiency

Assuming $V\in PO(Y)$

Putting $B=\Im^{-1}(V)$ & $\chi\in B$, then $\Im(\chi)\in V$ is a pre-nbd of $\Im(\chi)\in V$, thus V is pre–nbd of $\Im(\chi)$, therefore by hypothesis $\exists$ pre-nbd $U\chi$ of χ; $\Im(U\chi)\subseteq B$

$B\chi\in PO(X)$; $\chi\in B\chi\subseteq U\chi$, hence $\chi\in B\chi\subseteq B$, $B\chi\in PO(X)$

Thus

$B=\cup\chi\in B\ B\chi$. It follows that B is pre open in X, and so $\Im$ is PI

Theorem 3.4

Mapping $\Im: X, Y$ is PI iff $\forall B\subseteq Y$, pcl $(\Im^{-1}(B))\subseteq\Im^{-1}(pcl(B))$

Proof

The easy proof is omitted

Theorem 3.5

Mapping $\Im: X, Y$ is PI iff, $\forall B\subseteq Y$, pcl $(\Im^{-1}(B))\subseteq\Im^{-1}(pcl\ (B))$

Proof

The easy proof is omitted

Theorem 3.6

Mapping $\Im: X, Y$ is PI iff, $\forall B\subseteq Y$, then by theorem 2.4, we have pint B=Y–pcl(Y–B)

Hence

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$\Im^{-1}$(Plnt B)=$\Im^{-1}$(Y–Pcl(Y–B))
= $\Im^{-1}$(Y) - $\Im^{-1}$(Pcl (Y–B))
= x - $\Im^{-1}$(Pcl(Y–B))
Since $\Im$ is a PI mapping, then by making use of theorem 3.4, pcl $\Im^{1}$(Y–B)$\subseteq$ $\Im^{-1}$(Pcl(Y–B)
Hence $\Im^{-1}$(Pint B)$\subseteq$X–pcl($\Im^{-1}$(Y–B))
Therefore
$\Im^{-1}$(plnt B)$\subseteq$/X–pcl($\Im^{-1}$(B))
$\Im^{-1}$(Plnt B)$\subseteq$X–Pcl(X-$\Im^{-1}$(B)), by theorem 3.4, we get $\Im^{-1}$(plnt B)$\subseteq$plnt$\Im^{-1}$(B)
Hence $\Im^{-1}$(B)$\in$PO(X) & $\Im$ is PI

Theorem 3.7

Mapping $\Im: X, Y$ is PI, iff $\Im$(Apd)$\subseteq$ $\Im$(A)$\cup$($\Im$(A)) pd , for A$\subseteq$ X

Proof

Necessity

Assuming $\Im: X, Y$ be a PI, and assuming A$\subseteq$X & $\alpha\in$Apd, and $\Im(\alpha)\notin\Im$(A) & V be a pre–nbd of $\Im(\alpha)$, since $\Im$ is PI, by making use of theorem 3.3, $\exists$ pre–nbd U of α; $\Im$(U)$\cap$A; $\Im$(b)$\in$ $\Im$(A) & $\Im$(b)$\in$V, since $\Im(\alpha)\notin$ $\Im$(A), we have $\Im$(b)$\neq$ $\Im(\alpha)$. Thus every pre nbd of $\Im(\alpha)$ containing an element of $\Im$(A) different from $\Im(\alpha)$ consequently $\Im(\alpha)$,$\in$($\Im$(A)) pd, this proves the necessity part.

Sufficiency

Assuming $\Im$ is not PI, by Theorem 3.3, $\exists\alpha\in$X & pre-nbd V of $\Im(\alpha)$; $\forall$ pre-nbd U of α contains at least one-element b$\in$U for which $\Im$(b)$\notin$V
Putting A={b$\in$X: $\Im$(b)$\notin$V}, then $\alpha\notin$A, where $\Im(\alpha)\in$V, & $\Im(\alpha)\notin$ $\Im$(A), also $\Im(\alpha)$ $\notin$ ($\Im$(A)) pd since $\Im$(A)$\cap$(V–$\Im(\alpha)$)=$\varnothing$, following $\Im(\alpha)\in$ $\Im$(A pd)-($\Im$(A)$\cup$($\Im$(A))pd) $\neq\varnothing$.
This is a contradiction to condition no. 5, the condition of the theorem is therefore sufficient and the theorem is proved.

Theorem 3.8

Assuming $\Im: X, Y$ be an IM, then $\Im$ is PI iff $\Im(A^{pd})\subseteq(\Im(A))^{pd}$, $\forall$ A$\subseteq$X

Proof

Necessity

Assuming $\Im$ is PI, A$\subseteq$X, $\alpha\in A^{pd}$ & V be a pre–nbd of $\Im(\alpha)$, since $\Im$ is PI, then by theorem 3.3, $\exists$pre-nbd U of a; $\Im$(U)$\subseteq$V

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But $a \in A^{pd}$, then $\exists$ an element $b \in U \cap A$; $b \neq (a)$; so every pre–nbd V of $\Im(a)$ contains an element of $\Im(A)$ differing from $\Im(a)$; so $\Im(a) \in (\Im(a) \in (\Im(A))^{pd}$

Theorem 3.9

Assuming X & Y is TS. If $A \in PO(X)$ & $B \in PO(Y)$, then $AxB \in PO(XxY)$

Proof

Similar to the method of proving the corresponding results for the other cases of generalized OSs.

Theorem 3.10

Assuming $\Im: X, Y$ be mapping & $g: X \times Y$ to be the GM defined as $g(\chi)=(\chi, \Im(\chi))$, $\forall \chi \in X$.

If g is PI, then f is pre-irresolute

Proof

Assuming $\chi \in X$ & $V \in PO(Y)$; $\Im(\chi) \in V$, then by theorem 3.9, XxV is a PO sub-set of XxY containing $g(\chi)$ and hence by theorem 3.1, $\exists U \in PO(X)$; $\chi \in U$ & $g(U) \subseteq XxV$, by the definition of g we have $\Im(U) \in V$, therefore by theorem 3.1, $\Im$ is PI.

Remark 3.1

Referring to example (2.1), the mapping $\Im:(X,\tau)$ (X,τ) is PI, but $g:(X,\tau)$ $(XxX, \tau x \tau)$ is not, due to A={a,a),(b,a)} is open in XxX & so a is PO, but $g^{-1}(A)=\{b.c\}$ is not PO in X, so g is not PI.

Definition 3.2

A TS X is said to be pre–T_2 if for each two distinct points $\chi, y \in X$, $\exists$ $A, B \in PO(X)$; $\chi \in A$, $y \in B$ & $A \cap B = \varnothing$

Remark 3.2

Every T2-space is a pre-T_2-space but the opposite does not need to be true as illustrated by the next example

Example 3.2

Assuming $X=\{a,b,c\}$ & $\tau=\{\varnothing,\{a\},\{b,c\},X\}$ be a topology on X, then (X,τ) is a pre – T_2- space but it is not a T_2–space .

Theorem 3.11

If $\Im: X, Y$ is a pre-irresolute-injection and Y is pre-T_2, then X is pre-T_2

Proof

Assuming χ & y be two-distinct points of X, since $\Im$ is injective & Y is pre-T_2, $\exists U, V \in PO(Y)$; $\Im(\chi) \in U$, $\Im(y) \in V$ & $U_1 \cap U_2 = \varnothing$, then $\chi_1 \in \Im^{-1}(U_1)$, $\chi_2 \in \Im^{-1}(U_2)$ & $\Im^{-1}(U) \cap \Im^{-1}(U_2) = \varnothing$.

Since $\Im$ is PI, $\Im^{-1}(U_1)$ & $\Im^{-1}(U_2)$ are PO-sets in X.

Putting $V = \Im^{-1}(U_1) x \Im^{-1}(U_2)$, then by theorem 3.10, $(\chi_1, \chi_2) \in V \in PO(XxX)$
It is clearly $V \cap A = \emptyset$, therefore $(\chi_1, \chi_2) \notin$ pcl and hence A is PC in XxX

Theorem 3.12

If $\Im: X, Y$ is PI and Y is pre $-T_2$, then the graph $G(f)$ is PC in XxY

Proof

Assuming $(\chi, y) \notin G(\Im)$, then $y \neq \Im(\chi)$
Since Y is prc-T_2, $\exists$ U, V$\in$ & $U \cap V = \emptyset$
Since $\Im$ is pore-irresolute, then by making use of theorem 3.1, $\exists$ $W \in PO(X); \chi \notin W$ & $\Im(W) \subseteq V$, then $\Im(W) \cap U = \emptyset$, therefore $(x,y) \notin pcl\ (G(\Im))$ & thus $G(\Im)$ is pre-closed in XxY.

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