DOI: http://doi.org/10.32792/utq.jceps.09.01.02

Modulation of A Brusselator Model

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Abstract:

The dynamics of a Brusselator model under AC modulation are studied. Varieties of dynamics are noticed to occurs. These dynamics ranged from simple periodic, sustained and damped ones to types of self-pulsing (or breathing), as a result of variation of frequency of modulation.

Keywords: *Brusselator model, AC modulation, period dynamics, self-pulsing*.

Introduction:

The Brusselator is a simple model of a hypothetical chemical oscillator. It is a reaction-diffusion equation frequently arise in the study of chemical and biological systems and to analyze the behavior of a nonlinear oscillator associated with a chemical system.

The Brusselator system was written and studied in different manners, such as the occurrence of temporal oscillation [1], pattern formation in three dimensional reactions – diffusion systems [2], activation depletion mechanism [3], Turing pattern [4], chemical oscillations with the second law of thermodynamics [5], coherence resonance by noise recycling [6], stochastic resonance [7], etc.

In this article we study the dynamics of a Brusselater system under the effect of modulation.

Mathematical:

To carry out the study of the Brusselator system we follow the work of Ma et al. [6]. The Mathematical system can be written in the absence of noise as follows:

$$\dot{X} = A - (B+1)X + X^2Y \qquad (1)$$
$$\dot{Y} = BX - X^2Y \qquad (2)$$

X and Y are non-dimensional concentrations variables and A and B>0 are reactant concentrations. The over dot represent differentiation with respect to time.

To find the initial conditions i.e $X_\circ$ and $Y_\circ$ or the fixed point $(X_\circ, Y_\circ)$ we can set $\dot{X} = \dot{Y} = 0$ at steady state situation .By setting left hand sides of equations (1) and (2) equal to zero:

$$0 = A - (B+1)X_\circ + X_\circ^2 Y_\circ \qquad (3)$$
$$0 = BX_\circ - {X_\circ}^2 Y_\circ \qquad (4)$$

We get from equation (4)

$$Y_\circ = \frac{B}{X_\circ} \qquad (5)$$

Substitute equation (5) into equation (3) we get:

$$0 = A - (B+1)X_\circ + X_\circ^2 \frac{B}{X_\circ}$$
$$X_\circ = A \qquad (6)$$

Substitute equation (6) into equation (5) we get:

$$Y_\circ = \frac{B}{A} \qquad (7)$$

So the fixed point is $(X_\circ, Y_\circ) = (A, \frac{B}{A})$

Results:

The set of equations (1,2) given above were solved via the fourth order Range-Kutta numerical method and the matlab system. Suitable initial conditions were chosen, based on the steady state condition $(X_\circ, Y_\circ)$ where $X_\circ = A, Y_\circ = \frac{B}{A}$. To study the effect of AC modulation condition of the Brusselator model rewrite A and B as:

$$A = C\sin\omega t \qquad (8)$$
$$B = E\, sin\omega t \qquad (9)$$

where C and E are the amplitudes of A and B respectively,ω is the modulation angular frequency ($\omega = 2\pi f$ and f is the modulation frequency).To study them, we do the following steps:

i. Started by the introducing the definition Eq. (8) to Eq. (1) and varying C from 0.1A to A in a step 0f 0.1A and for each C the value of angular frequency ω was varied from 0.001 to 100 and monitoring the temporal variation of X(t).

For any C value and increasing ω the system i.e., X(t), shows oscillatory behavior that ends-up with DC value at X(t) =1. This DC value breaks to oscillatory one of low frequency then period one of high frequency appears. Then decaying of oscillation occurs. For C≥0.3A self pulsing behavior appears as can be seen in Fig. (1).

ii. By introducing the definition (Eq (8)) to Eq (2) only, the same behavior seen in the previous case (i) happen occurs but for C≥ $0.5A$ and $\omega = 0.1$. The rest X(t) dynamics as the same seen the previous case.

iii. By introducing the definition Eq (9) to Eq (1) only. Once more the same behavior of X(t) against time reoccurs.

iv. By introducing the definition Eq (9) to Eq (2) only. No changes occur except the broken of the regular oscillations seen in same previous cases to non-harmonic one as can be seen in figure (3).

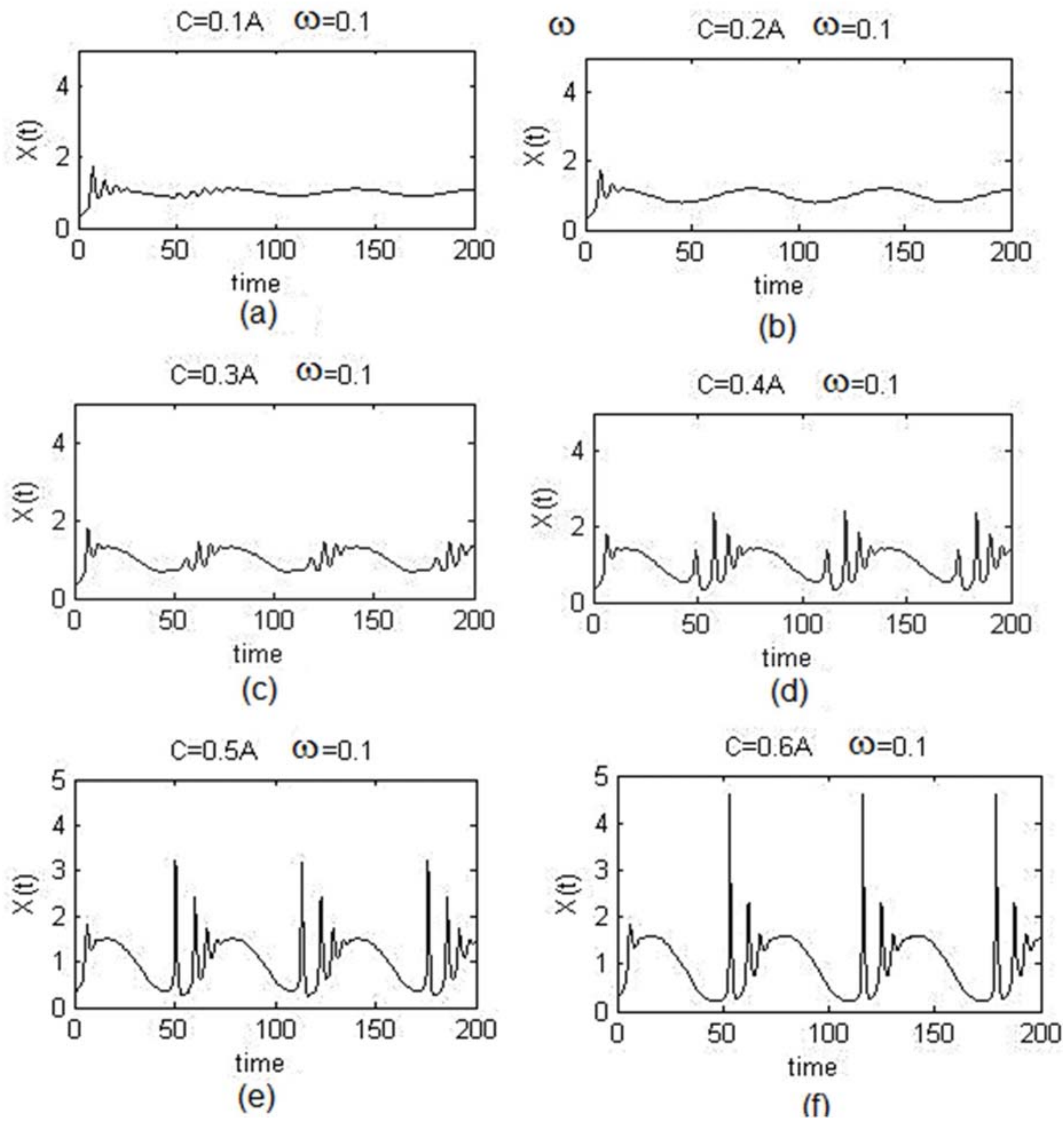
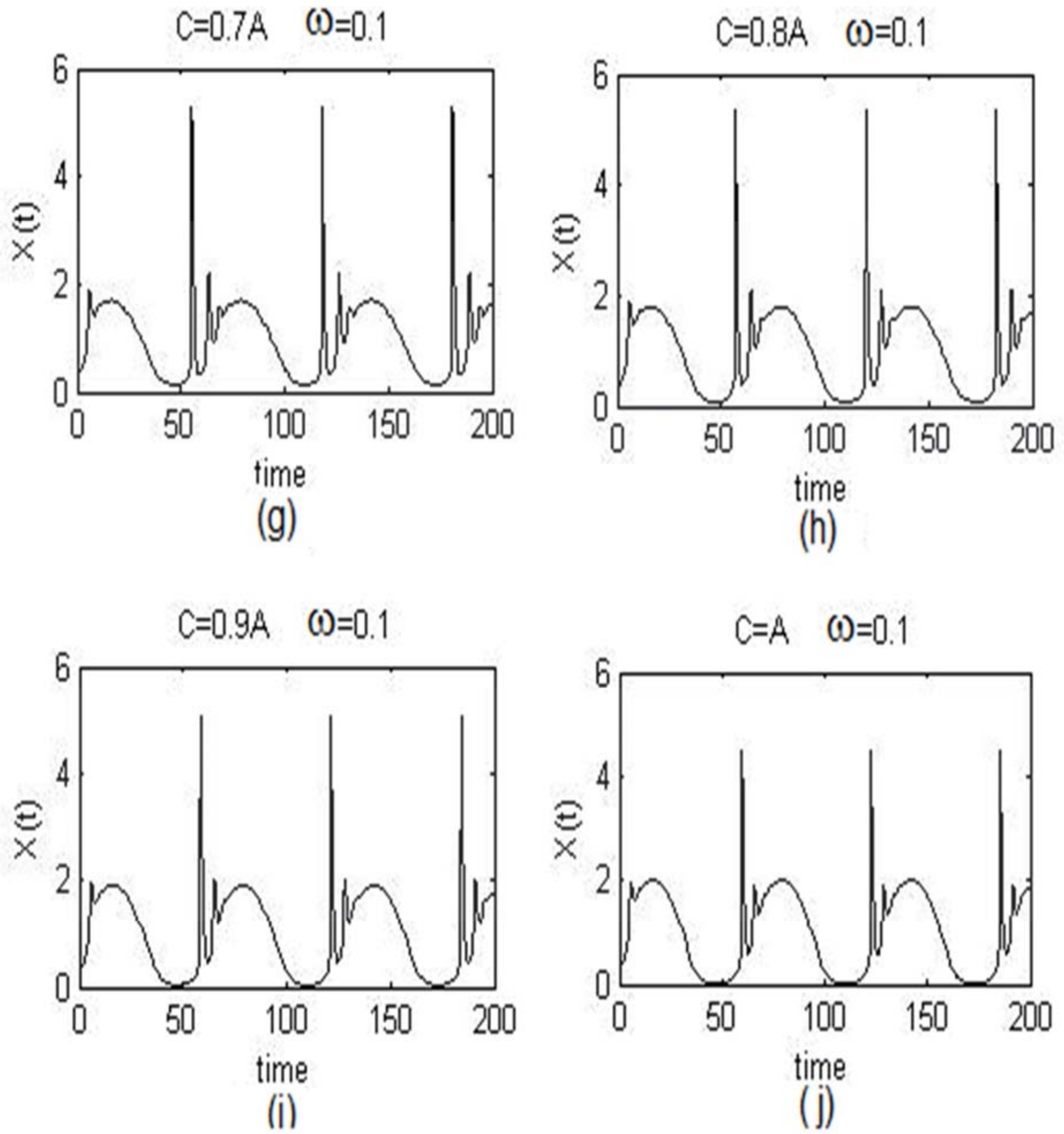


Fig (1):Self- pulsing appear in dynamics of X(t)for C and ω values:

(a) (0.1, 0.1) (b) (0.2 , 0.1) (c) (0.3 , 0.1) (d) (0.4 , 0.1)
(e) (0.5 , 0.1) (f) (0.6 , 0.1) (g) (0.7 , 0.1) (h) (0.8 , 0.1)
(i) (0.9 , 0.1) (j) (A , 0.1)

continue


C=0.7A ω=0.1
X(t)
time
(g)
C=0.8A ω=0.1
X(t)
time
(h)
C=0.9A ω=0.1
X(t)
time
(i)
C=A ω=0.1
X(t)
time
(j)

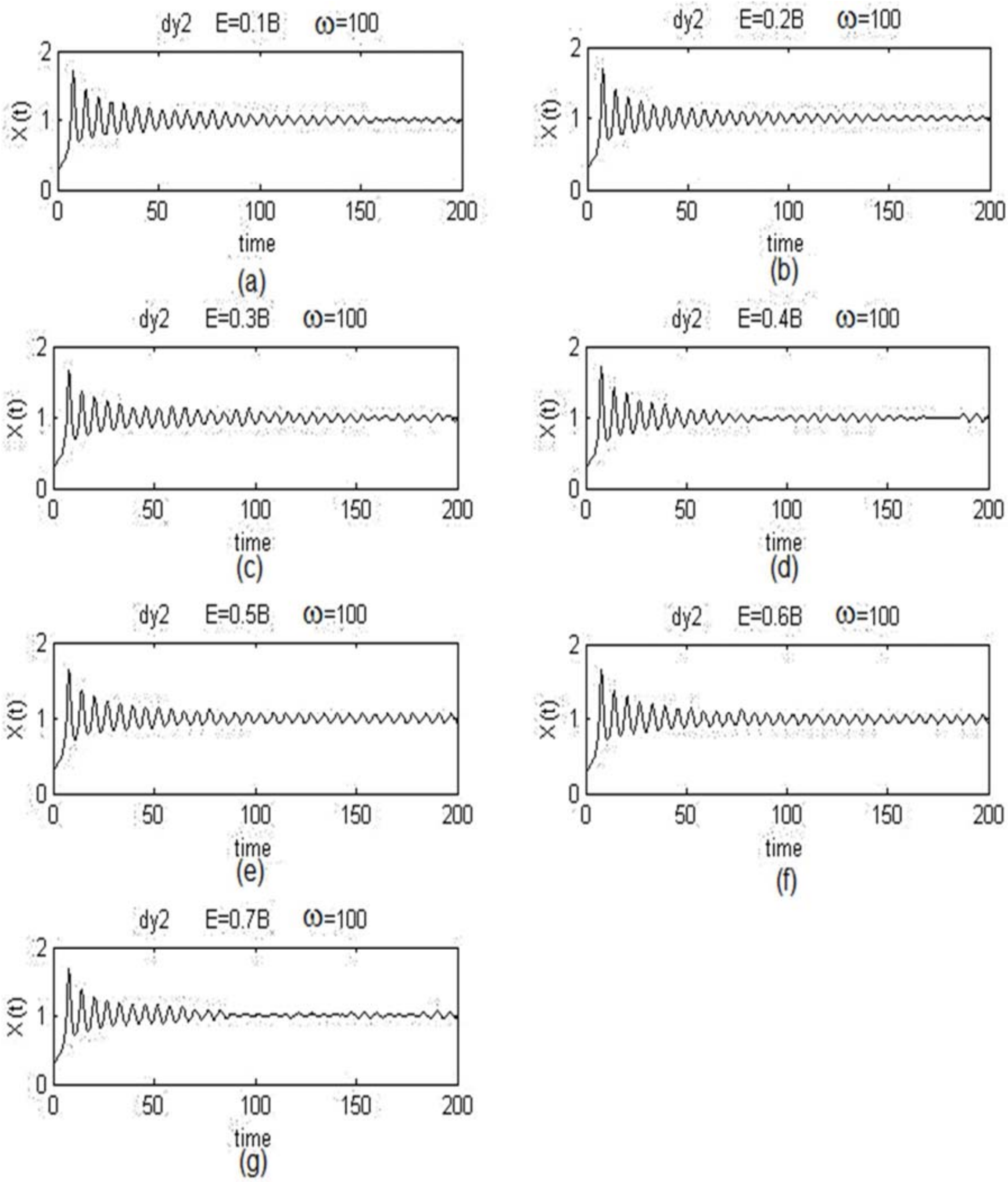


Fig (2):Decaying oscillation in the temporal dynamics of $X(t)$ for (E ,ω) as:

(a) (0.1B , 100) (b) (0.2B , 100) (c) (0.3B , 100)
(d) (0.4B , 100) (e) (0.5B , 100) (f) (0.6B , 100)
(g) (0.7B , 100)

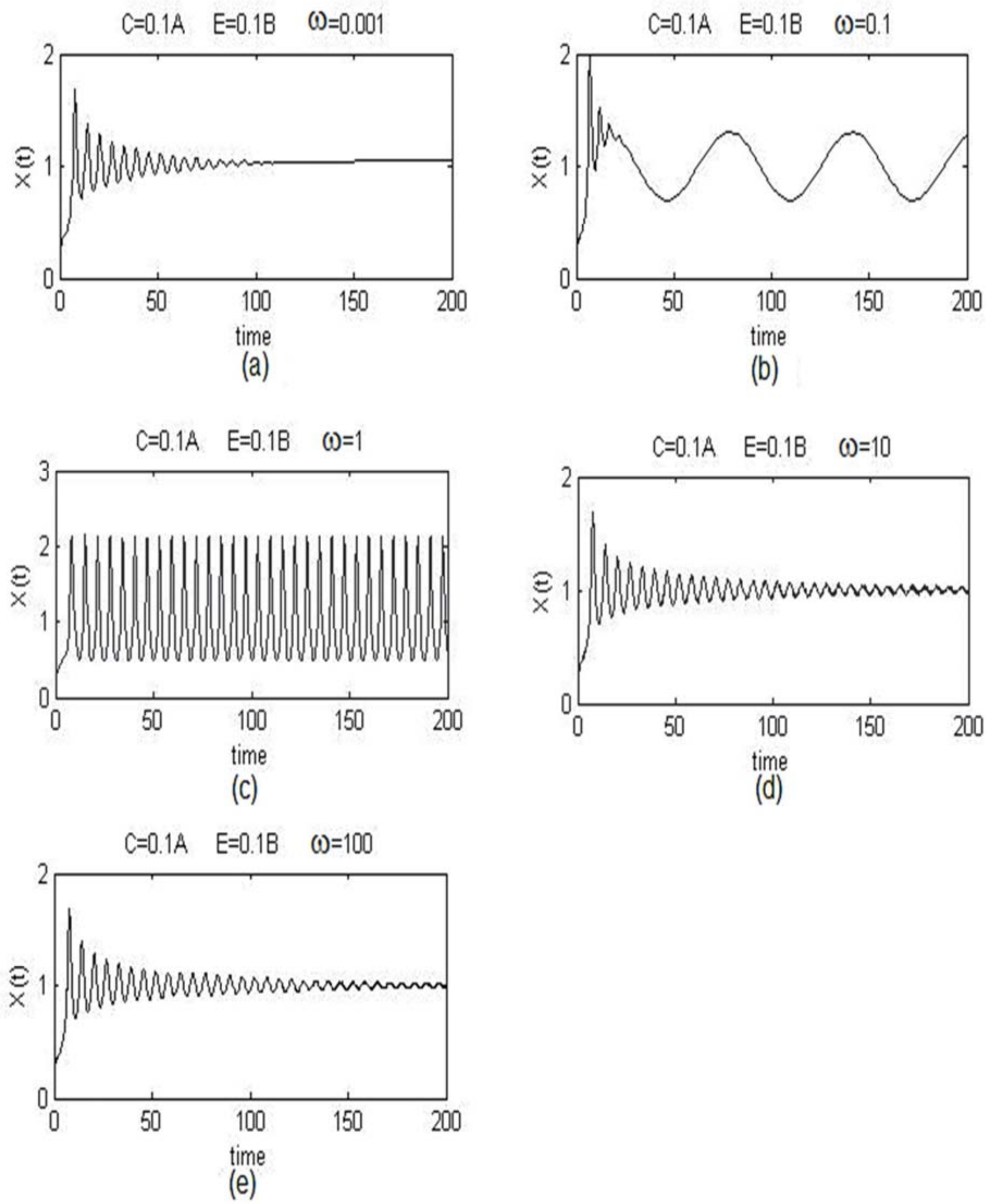


Fig (3):Various dynam ics of $X(t)$ temporal variation for (C,E,ω):

(a) (0.1A, 0.1B, 0.001) (b) (0.1A, 0.1B, 0.1) (c) (0.1A, 0.1B, 1)
(d) (0.1A, 0.1B, 10) (e) (0.1A, 0.1B ,100)

Conclusion:

The AC modulation of a Brusselator model have led to period one damped and sustained oscillations and varieties of dynamics together with self-pulsing effects that usually seen to occur in systems, especially lasers, under the effect of feedback. This procedure dose not lead to any new results in comparison to the previous four cases. The obtained results can be compared well with those obtained by Ma et al. [6], in the presence of noise and even richer than their finding. The varieties of dynamics of X(t) shown in the Fig (3) under AC modulation of low modulation frequency prove that Brusselater model is able to show various signal shapes ranged from damped oscillation to self – breathing.

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