

A Novel Approach to Deducing of the Data- PAPR Relation for OFDM System

Assist. Prof. Dr. Dhafer R. Zaghar
Department of Computer & Software
Engineering / College of Engineering
University of Al-Mustansiriyah / Iraq

Tahreer Sh. Mahmoud
Department of Electrical Engineering
College of Engineering
University of Al-Mustansiriyah / Iraq.

Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is a very attractive technique for high-data-rate transmission in wireless and wired applications. The main drawback of the OFDM technique is its high peak to average power ratio (PAPR). Several approaches exist to reduce PAPR of OFDM symbols. All the previous approaches have a common disadvantages that are not having a scientific or mathematics rules to prove them or to measure the effect of these methods on PAPR.

This paper will propose a useful model for the relation between the input data and output PAPR as main aim. This aim satisfies in two steps:

- 1- Study and analyze the effect of peak value (PV) problem in OFDM system, that intern influence directly in the value of PAPR.*
- 2- Conclude the results in a graphical model to joint the relation between the input data and output PAPR.*

Key Words: OFDM system, PAPR, peak value, graphical model, BFSK.

الخلاصة

ان متعامد التردد بالتقسيم العمودي (OFDM) هي تقنية جذابة للغاية لنقل البيانات العالية في معدل التطبيقات السلكية واللاسلكية. العيب الرئيسي من هذه التقنية هو حصول نسبة عالية بين اعلى قيمة ممكنة ومتوسط القوة (PAPR). وتوجد عدة مناهج للحد من هذه النسبة. لكن جميع المناهج السابقة لها مساوي مشتركة والتي هي لا وجود لقواعد علمية أو رياضية لاثباتها أو لقياس مدى تأثير هذه الأساليب على PAPR. هذا البحث سوف يقترح نموذجا مفيدا للعلاقة بين البيانات المدخلة والإخراج PAPR كهدف رئيسي. يلبي هذا الهدف في خطوتين :

1 - دراسة وتحليل أثر مشكلة قيمة الذروة (PV) في نظام متعامد التردد بالتقسيم العمودي (OFDM) ، التي تؤثر مباشرة في قيمة PAPR.

2 - استخلاص النتائج في نموذج بياني للعلاقة المشتركة بين البيانات الداخلة و الإخراج PAPR .

1- Introduction

OFDM has recently seen rising popularity in wireless applications since it provides an efficient means to mitigate the intersymbol interference (ISI) caused by the channel multipath spread and high data rate transmission. This is a multicarrier technique in which modulating the entire data stream with different subcarriers and each of these subcarriers is orthogonal to each other. An OFDM-based system can provide greater immunity to multipath fading. OFDM has been widely considered for digital communication systems such as wireless local area networks and digital audio/video broadcasting services. It is also being considered for future broadband applications and fourth generation transmission technique ^[1].

An OFDM transmitter can be implemented by using inverse fast Fourier transform (IFFT) and the output of IFFT block is a time domain signal. The output of IFFT (OFDM signals) have an inherent difficulty that it may exhibit a very high peak since it is generated by the addition of several independently modulated signals. The power of these large peaks will be very high compared to the average power of the signal. Hence peak to average power ratio is very high which is considered as the major disadvantage of the OFDM technique. These large peaks cause saturation in power amplifiers which is placed at the front end of the transmitter and leads to nonlinear distortions ^[1].

Several PAPR reduction schemes have been proposed to alleviate this problem. The simplest method of eliminating this high peak is clipping and filtering method ^[2].

The OFDM signal is deliberately clipped at a particular threshold value before amplification in this method ^[3]. The large peaks of OFDM signals occur with a very low probability and hence clipping could be an effective technique for the reduction of the PAPR. However, clipping causes significant in-band distortion and out-of-band noise which will indirectly degrade the bit error rate performance and the spectral efficiency. Filtering is done after clipping in order to eliminate unwanted frequencies caused by the clipping process. Another solution is to use selective mapping method ^[4].

The entire data stream is divided into different blocks of N symbols each. Each block is multiplied with U different phase factors to generate U modified blocks before giving to IFFT block. Each modified block is given to different IFFT block to generate OFDM symbols. PAPR is calculated for each modified block and select the block which is having minimum PAPR ratio. This technique can reduce PAPR considerably. But this technique will increase circuit complexity since it contains several IFFT calculations. Partial transmit sequence technique (PTS) is another method that has been proposed and studied in the literature ^[5].

In PTS technique, an input data block of N symbols is partitioned into disjoint subblocks. The subcarriers in each subblock are multiplied by a phase factor. The phase factors are selected such that the PAPR of the subblocks is minimized. Optimization techniques used to select the phase factors in order to achieve the above objective. Each of the subblocks having the

minimum PAPR and hence the combined signal of the different subblocks is having the minimized PAPR. Tone reservation method of the OFDM signal appears as an attractive solution for reducing the PAPR of OFDM signals. In this method some OFDM subcarriers are reserved. These reserved subcarriers don't carry any information, are only used for reducing PAPR. This method restricts the data vector, and the peak reduction vector to lie in disjoint frequency subspaces. The gradient algorithm is one of the good solution used in this method to reduce PAPR ratio with low complexity. The basic idea of the gradient algorithm has come from clipping. Clipping the peak tone to the target clipping level can be interpreted as subtracting impulse function from the peak tone in the time domain. Impulse function is time shifted to the peak tone location, and scaled so that the power of the peak tone should be reduced to the desired target clipping level, when subtracting this impulse function from the original one [6].

2- OFDM System Block Diagram

Figure (1) shows an OFDM system block diagram. At the transmitter, input binary data are first mapped to some symbols by modulation BPSK, and these symbols are modulated into N orthogonal subcarriers. Subcarriers are sampled with sampling rate N/T_u , where T_u is useful OFDM symbol duration. Finally, samples on each subcarriers are summed together to form an OFDM symbol [7, 8].

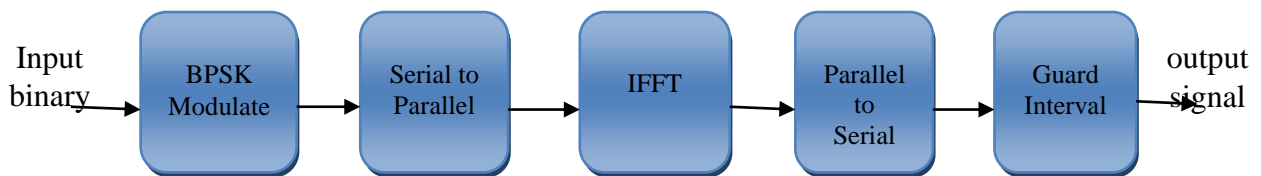


Fig. (1) OFDM Block Diagram

It is well known that IDFT can be implemented efficiently using inverse fast Fourier transform (IFFT); IFFT is used to convert the frequency domain signals to time domain signals. Then, a guard interval is inserted in front of the transmitted symbol to eliminate intersymbol interference (ISI). Then, the transmitted symbols are passed through the channel [7, 8].

3- Peak Average Power Ratio (PAPR)

When the phase of different subcarriers adds up to form large peaks, an important complication comes in OFDM system. This problem is called Peak Average Power Ratio (PAPR) and it is defined for each sampled OFDM signal by the following formula [9]:

$$\mathcal{X}_n = \frac{\max_k |x_n[k]|^2}{E \{ |x_n[k]|^2 \}} \dots\dots\dots (1)$$

In OFDM system, PAPR can have very high values for certain input sets of sample ($x_n[k]$) and overload non-linear characteristics of systems, causing inter-modulations among different carriers and undesired out-of-band radiation.

4- Peak Value (PV) Reduction Technique

The process of reducing the value of PAPR complicated so we will try to focus on reducing the peak value (PV), which has a direct impact on the value of PAPR and because the random data, the mean value do not affect significantly on the PAPR .Through study and observation the output signal of OFDM system we noticed that there is an increase and decrease in peak value (PV), it was noticed that through the change in transmitted data gets changed in peak in number or value, and thus influenced greatly in the value of PAPR.

High peak-to-average ratio means the peak signals power much greater than average signal power.OFDM signal is summation of many sinusoid, in worst may all add.It is known that the transmitted data is completely random, but this is not accurate. The true that the original data is orbituary , but the encryption process with the process of adding bits to the data gives the system the ability to control the format of data and send it, even if partially., it became clear to us that the data is part of the basis and cause of change in output signal of the OFDM system and the possibility to control the signal output to get less peak value (PV).Therefore, we start to study effect of data on OFDM output signal (O_s)

$$O_s = e^{jft} \sum_{i=0}^{N-1} d_i e^{j\Delta f i t} \dots\dots\dots (2)$$

Where O_s the output signal of OFDM transmission

N=number of subcarrier= 2^m

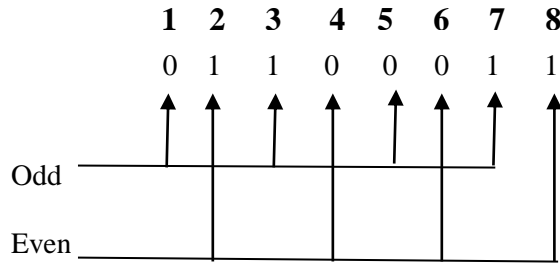
m=(1,2 ,3,.....)

d_i =sequence data input

For simplicity we divided (O_s) into two parts, *fixed part (fixed function)*, which is include two sets of frequencies ($2n\Delta f, (2n + 1)\Delta f$), group on a semi-cycle ($\Delta f, 3\Delta f, 5\Delta f, \dots$) and group complete cycle ($2\Delta f, 4\Delta f, 6\Delta f, \dots$). For the purpose of study and the absence of a significant impact between two groups will be separated. The second part of the (O_s) is

random data, can be divided into two parts *odd* and *even* according to the above frequencies can be illustrated by the following example:

Let data sequence = [0 1 1 0 0 1 1], (8-bit) divided to *odd* & *even*



Then the equation (3) will write as:

$$O_s = O_{so} + O_{se}$$

$$O_s = e^{jft} \sum_{i=0}^{N/2-1} d_{2i} e^{j\Delta f 2it} + e^{jft} \sum_{i=0}^{N/2-1} d_{2i+1} e^{j\Delta f (2i+1)t} \dots (3)$$

5- Analysis and Minimize of Peak Value (PV)

To understand how the peak value (PV) increases or decreases we take some examples to study and analysis this situation, it is consider the basic element and influential in PAPR, the example one is the summation of many separate sinusoids, which is look like OFDM signal, these are available subcarriers (BPSK modulation) assuming all ones (all positive) shown in figure (2) not how the value of peak is high, the example two is the summation of many separate sinusoids, which is look like OFDM signal, these are available subcarriers (BPSK modulation) assuming zeros and ones (positive with negative) shown in the same figure (2) not how the value of peak is reduce.

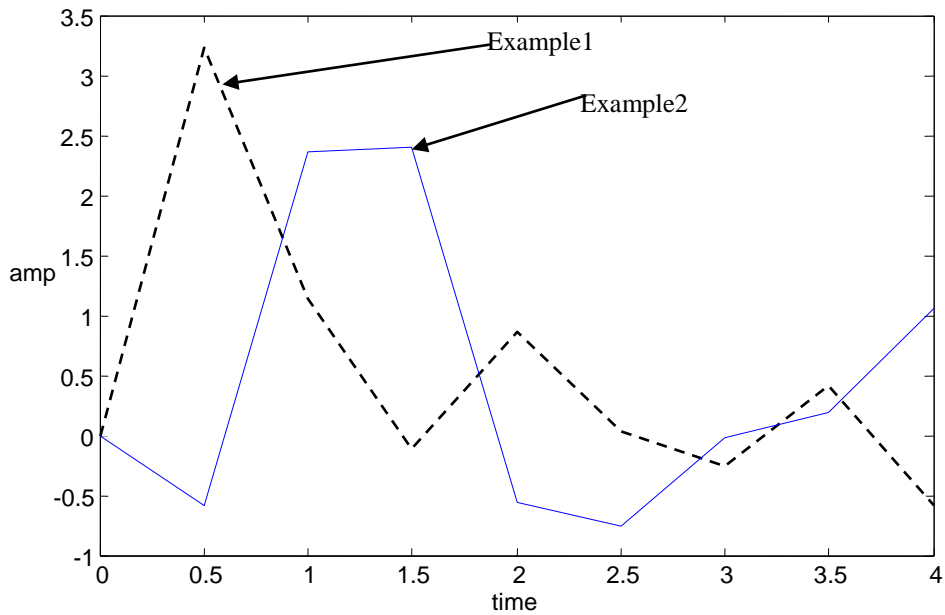


Fig. (2) $ex1=sint+sin2t+sin3t+sin4t$, $ex2=sint+sin2t-sin3t-sin4t$

From previous two examples we note that the value of samples which is controlling the variation of the output signals. As a result of repetition and the expansion of the experiment we guess three proposals which all reduce value of the peak signal power that is intern influence directly in the value of PAPR:-

1-Phase Balance:- it is a first notice to minimize the peak signal power, by making the number of zeros equal the number of ones for the data sequence. This condition applied on the odd (even) part.

$$O_s = \sum_{i=0}^{N-1} d_i = 0 \quad \dots \dots (4)$$

This proposal will redistribute the PV as in figures (3), (4) and (5). Not how the peak redistributes from data all ones to phase balance.

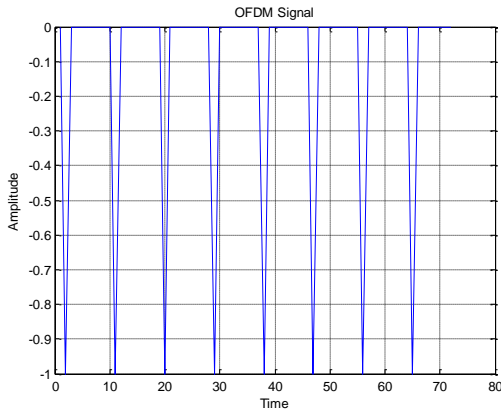


Fig. (3) OFDM Signal

(N=64, BPSK, 8-point IFFT, PAPR=9.5424)

Data sequence (64 bit) = [11111.....]

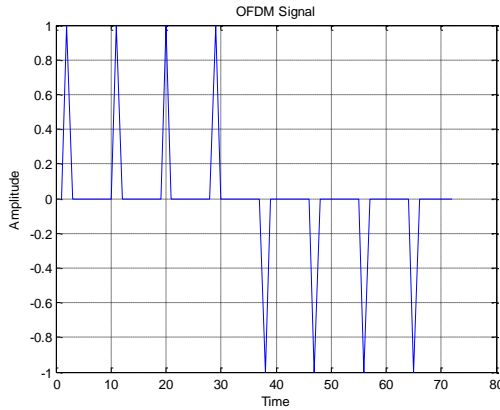


Fig. (4) OFDM Signal for Phase Balance

(N=64, BPSK, 8-point IFFT, PAPR=9.0224)

Data sequence (64 bit) = [0000..., 1111...]

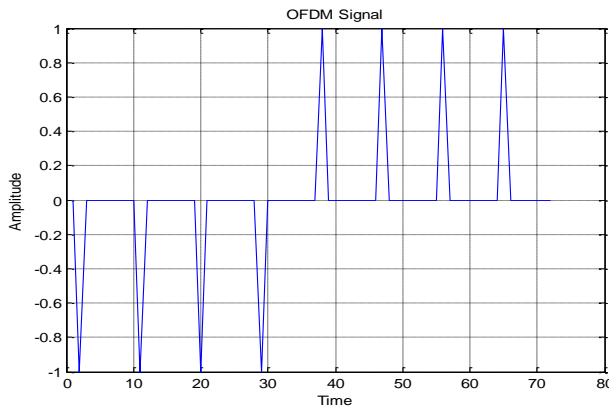


Fig. (5) OFDM Signal for Phase Balance

(N=64, BPSK, 8-point IFFT, PAPR=9.0224)

Data sequence (64 bit) = [1111..., 0000...]

2-Anti-Neighbor:-it is the second notice is proposed to reduce the peak signal power by making each sample of the data is on the contrary of each neighboring sample ($d_i = -d_{i+1}$), so this condition applied on the odd (even) part.

$$O_s = e^{jft} \sum_{i=0}^{N-1} d_i e^{j\Delta f i t} = e^{jft} \sum_{i=0}^{N-1} d_i (e^{j\Delta f i t} - e^{j\Delta f (i+1)t}) \dots \dots \dots (5)$$

This proposal reduces from the peak value (PV) see figures (6) and (7), one of the phase balance and the second after implementation of the proposed.

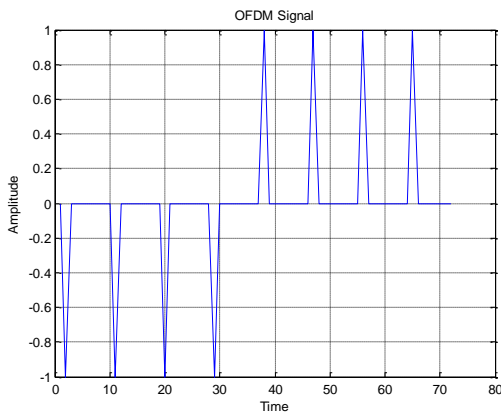


Fig. (6) OFDM Signal for Phase Balance

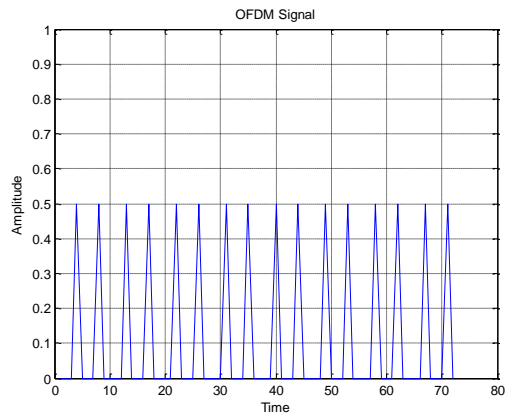


Fig. (7) OFDM Signal for Anti-Neighbor

(N=64, BPSK, 8-point IFFT, PAPR=9.0224) (N=64, BPSK, 8-point IFFT, PAPR=6.5321)

Data sequence (64 bit) = [1111..., 0000...]

Data sequence (64 bit) = [00110011....]

3-Odd and Even combination :-

the combines of the two parts together (*odd, even*) to get the ideal situation, transmitted data is characterized as the first sample is the opposite of the third sample and the second reverse the fourth and so on ($d_i = -d_{i+2}$), this proposal gives the best arrangement of data to reduce the peak signal power. There are four forms of representation to this proposal reduce all of peak value (PV), forms are:-

- 1- [0 0 1 1 0 0 1 1] as shown in figure (8).
- 2- [0 1 1 0 0 1 1 0] as shown in figure (9).
- 3- [1 1 0 0 1 1 0 0] as shown in figure (10).
- 4- [1 0 0 1 1 0 0 1] as shown in figure (11).

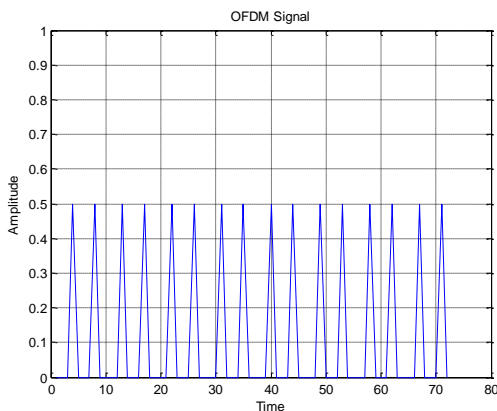


Fig. (8) OFDM Signal for Third Method

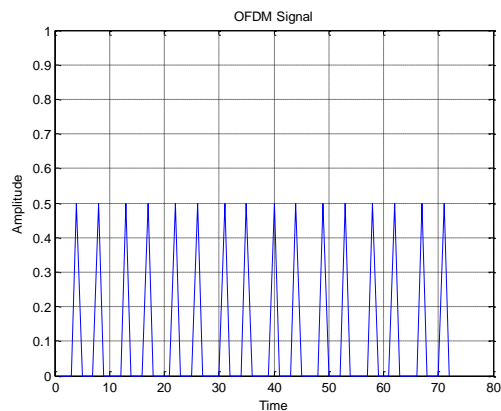


Fig. (9) OFDM Signal for Third Method

(N=64, BPSK, 8-point IFFT, PAPR=6.5321) (N=64, BPSK, 8-point IFFT, PAPR=6.5321)

Data sequence (64 bit) = [00110011....]

Data sequence (64 bit) = [0110110....]

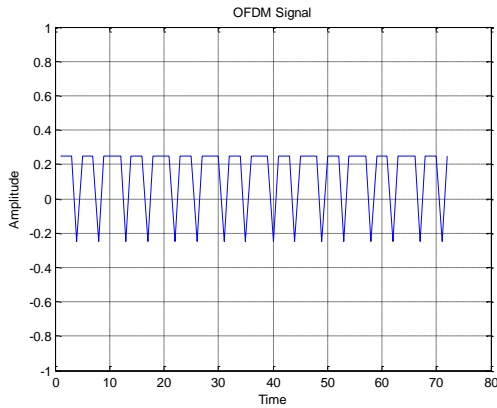


Fig. (10) OFDM Signal for Third Method (N=64, BPSK, 8-point IFFT, PAPR=4.2276) Data sequence (64 bit) = [11001100....]

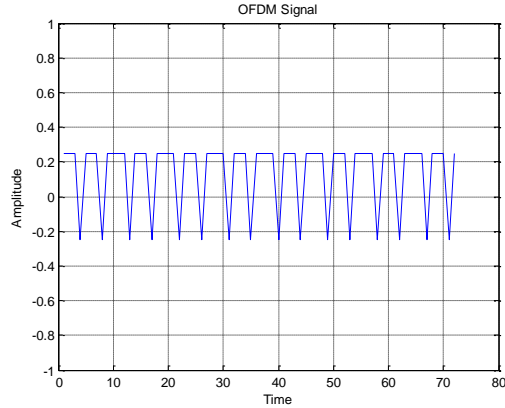


Fig. (11) OFDM Signal for Third Method (N=64, BPSK, 8-point IFFT, PAPR=4.2276) Data sequence (64 bit) = [10011001....]

The figures (10) and (11) as a result of the effect the *odd* part on *even* part the peak value (PV) much less.

6- Model for PV with the Data Distribution

The second step to draw the relation between the data and the PV is appropriate model to show how to get the ideal form of data sequence. The appropriate model to represent the proposed methods with data sequence can be represented as a rectangle shown in figure (12).

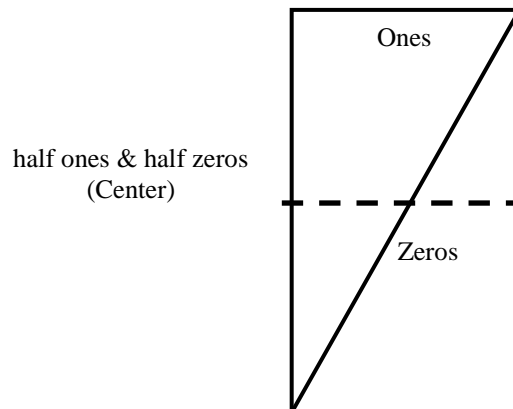


Fig. (12) Rectangle Model for Data Distribution

The rectangle represents the number and position of bits in the data sequence, the rectangle is divided into two halves, half contains the ones and the other half contains the zeros, the edges of the rectangle represents the highest number of zeros or ones and the center of the edges represents the data sequence that contain half ones and half zeros. The number of possibilities for data sequence representation equal $(n!)$ where n represent the number of bits. The main

aim is to get the appropriate model of the data sequence the ones are half of it and the other half of the zeroes and the second aim is the distribution of zeros and ones to get the ideal form of the data, which gives the lowest PV. The first notice (Phase Balance) can be represented by a straight line, is part of the rectangular shape called phase line, the end points of the straight line represent the form of the data sequence (all ones & zeros) gives maximum PV, the center of the line represent phase balance because it represents the data half zeros and other half ones gives a minimum PV and any point on the line has complement data sequence on the other side, shown in figure (13).

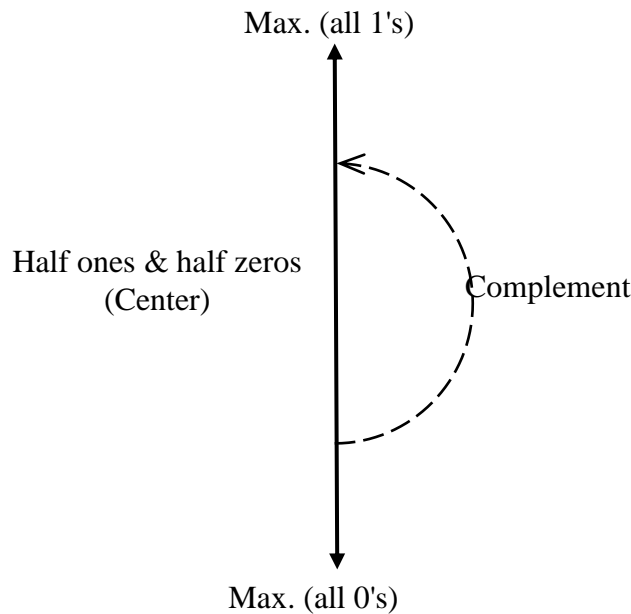


Fig. (13) Phase line

The number of possibilities for data sequence in each point on the phase line is (C_n^k) where n represent the number of bits and k the number ones or zeros. This situation applies to odd part and even part of the OFDM signal, can be illustrated by the following example:

Let data sequence = [0 1 1 0 0 0 0 1], (8-bit), the number of possibilities for data sequence representation on the straight line for (k=3, number of one's)

$$C_n^k = \frac{n!}{k!(n-k)!} = \frac{8!}{3!(8-3)!} = 56$$

While for (k=5, number of zeros)

$$C_n^k = \frac{n!}{k!(n-k)!} = \frac{8!}{5!(8-5)!} = 56$$

The second notice (Anti-neighbor) can be represented as circular called interpolation circular, this proposal represents the only model change positions bits to get the ideal

situation, represent phase balance on the two side for example (1 1 1 1 0 0 0 0) gives maximum peak and anti-neighbor on the other side for example (1 0 1 0 1 0 1 0) gives minimum peak ,any point on a circular have reverses data on other side as shown in figure (14). This situation applies to odd part and even part of the OFDM signal.

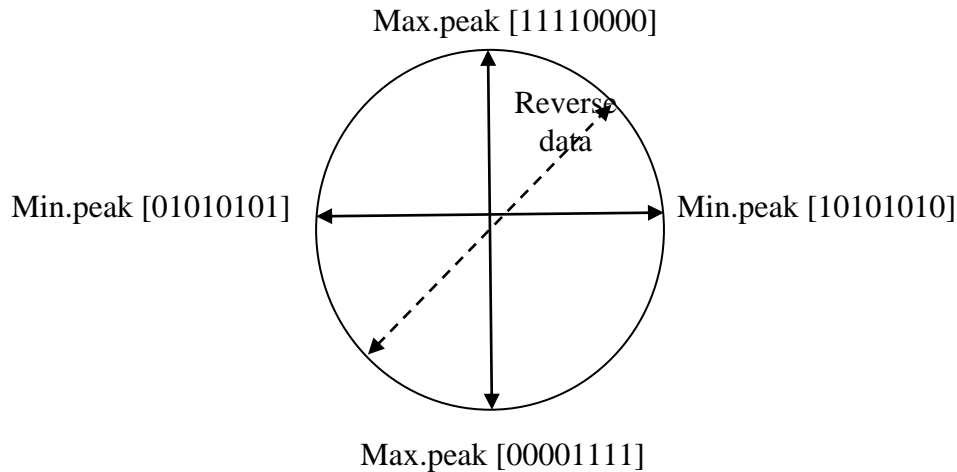


Fig. (14) Interpolation circular

The next step in the model implementation is the combination between straight line model and circular model together to get the largest area of control in the data sequence, this model can be represented in the form of three-dimensional as a *cone* shape, which is as shown in figure (15). Any point on the cone has reverses data sequence horizontally and complement vertically.

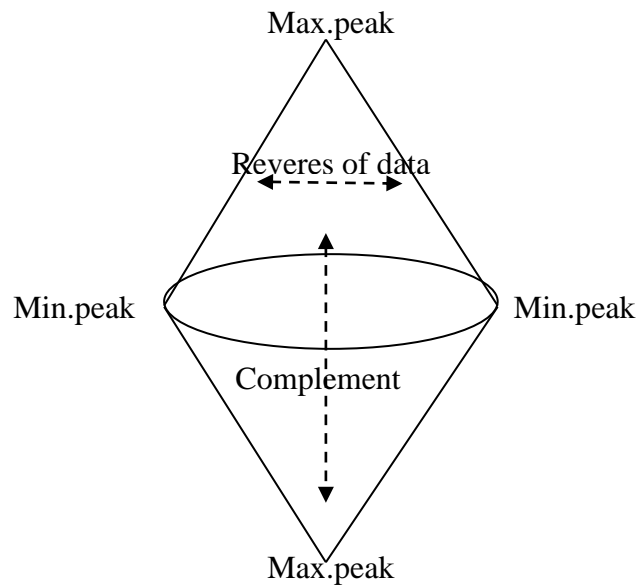


Fig. (15) Cone of data

This model can be represented the ideal situations for the data sequence that all gives less PV which has been explained previously for example (1 1 0 0 1 1 0 0), (0 0 1 1 0 0 1 1), (0 1 1 0 0 1 1 0) and (1 0 0 1 1 0 0 1). This situation applies to odd part also and even part of the OFDM signal, cannot be combined between the odd model and even model in one model is difficult to obtain or study the form of five-dimensional. There are several ways to change the format of the data, the proposed method is to add two bit complement for each two bit in the data sequence, This proposal reduces from the peak value (PV) shown figures (16) and (17), one of the number of zeros equal the number of ones for the data sequence and the other figure after implementation of the proposed, the second experiment shown in figures (18) and (19) one for the random data sequence and the other figure after implementation of the proposed.

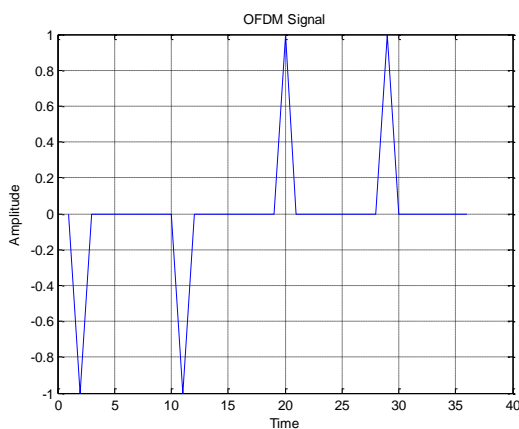


Fig. (16) OFDM Signal

(N=32, BPSK, 8-point IFFT)

Data sequence (32 bit) = [1111..., 0000...]

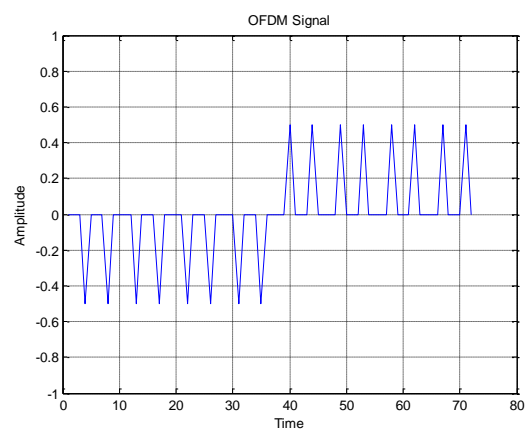


Fig. (17) OFDM Signal

(N=64, BPSK, 8-point IFFT)

Data sequence (64 bit) = [1100..., 0011...]

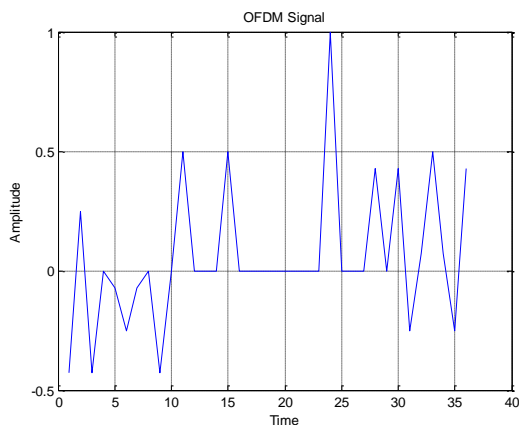


Fig. (18) OFDM Signal

(N=32, BPSK, 8-point IFFT)

Random data

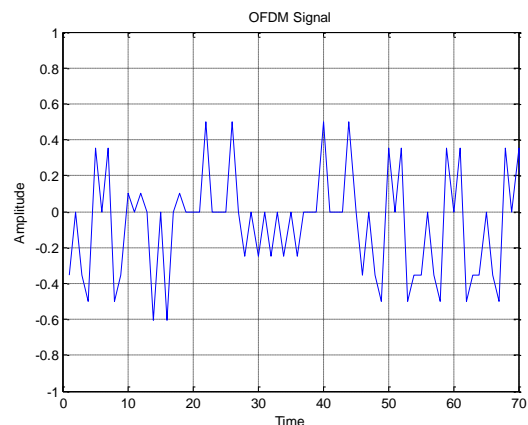


Fig. (19) OFDM Signal

(N=64, BPSK, 8-point IFFT)

after complement process

7- Conclusion

we conclude to be a frame or data sequence can give less or higher value for peak. Therefore, it will be the key to search is to find the perfect formula for data sequence to get the lowest value of the peak that could be occur.

- 1- There is an imbedded relation between the data sequence and the value of PAPR.
- 2- This relation is not clearly to obtain as mathematics equation.
- 3- The graphic model is a good solution to bringing the relation between the data sequence and the value of PAPR.
- 4- The use of the graphic model will help to enhancement the method of PAPR reduction and decrease them costs.
- 5- Using the advantages of the graphic model we can propose a novel methods to reduce PAPR with low cost and low complexity.

References

- [1]- C. Rapp, “**Effects of HPA-nonlinearity on a 4PSK/OFDM signal for a digital sound broadcasting system,**” in Proc. 2nd Eur. Conf. Satell. Commun.Liege, Belgium, Oct. 1991, pp. 179–184.
- [2]- G. Ren, H. Zhang, and Y. Chang, “**A complementary clipping transform technique for the reduction of peak-toaverage power ratio of OFDM system,**” *IEEE Trans Consum. Electron.*, vol. 49, no. 4, pp. 922–926, Nov. 2003.
- [3]- X. Li and L. J. Cimini, Jr., “**Effects of clipping and filtering on the performance of OFDM,**” in *Proc. Globecom, 1997*, pp. 1634–1638. P. Fan and X.-G. Xia, (2002). The IEEE website. [Online]. Available: <http://www.ieee.org/>.
- [4]- Houshou Chen and Hsinying Liang “**A Modified Selective Mapping with PAPR Reduction and Error Correction in OFDM Systems**”. Dept. of Electrical Engineering and Graduate Institute of Communication Engineering National Chung Hsing University, 250, Kuo Kuang Rd., Taichung 402, Taiwan, 11-15 March 2007.
- [5]- L. J. Cimini and N. R. Sollenberger, “**Peak-to-average power reduction of an OFDM signal using partial transmit sequences,**” in *Proc. IEEE ICC*, Vancouver, BC, Canada, 1999, pp. 511–515.
- [6]- ARMSTRONG, J.: “**Peak-to-average power reduction for OFDM by repeated clipping and frequency domain filtering**” *Electronics Letters*, Vol. 38, No. 5, pp. 246-247, 28th February 2002.
- [7]- S. Hara, R. Prasad, " **Multicarrier Techniques for 4G Mobile Communications**", the Artech House Universal Personal Communications Series, Boston · London, 2003.
- [8]- B. P. Cheung, " **Simulation of Adaptive Array Algorithms for OFDM and Adaptive Vector OFDM Systems**", M.Sc thesis, Dept. of Electrical Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, September 3rd, 2002.
- [9]- E. Mabilia and M. Coinchon, “**Study of OFDM modulation**”, Eurecom institute, December1999.