

FACTORS CONTROLLING MIMO CHANNELS CAPACITY

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ABSTRACT

Multiple-Input, Multiple-Output (MIMO) describes systems that use more than one antenna element at each end of the wireless link. Combining two or more received signals on both the transmit and receive ends, has the most benefit of improving received signal strength, but MIMO also enables better performance in high data rate transmission. Many in the wireless communication world are hoping to utilize MIMO communications to boost capacities, expand bandwidths, increase signal-to-interference ratios, and mitigate fading. This paper investigates many factors that may affect the MIMO channel capacity when perfect channel state information (CSI) is assumed at the receiver, but not always at the transmitter. The results showed that the increase of Rician fading, K factor, antenna correlation, cross-polarization discrimination (XPD) may affect the ergodic and outage capacity of MIMO systems. MIMO was described, and the potential of 2×2 MIMO system in capacity degradation reduction was presented and simulated using Monte Carlo simulation. The simulation results obtained using MATLAB-2009 were presented and discussed.

Keywords:- Ergodic and Outage Capacity, MIMO, LOS, NLOS, Rician fading, K factor, antenna correlation, XPD.

العوامل المسيطرة على سعة قدرة قنوات متعدد المداخل متعدد المخارج

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المخلص

متعدد المداخل، متعدد المخرجات (MIMO) يصف الأنظمة التي تستخدم أكثر من هوائي في نهاية كل وصلة لاسلكية. الجمع بين اثنين أو أكثر من الإشارات في المرسل والمستلمة تعطي أكبر قدرة لتحسين قوة الإشارة المستلمة وكذلك تتيح أفضل أداء في ارتفاع معدل نقل البيانات. معظم الاتصالات اللاسلكية في العالم يأملون في الاستفادة من متعدد المداخل، متعدد المخرجات لتعزيز القدرات وتوسيع نطاق الموجة وزيادة نسبة الإشارة إلى التدخل والتخفيف من الخفوت أو التلاشي. في هذا البحث تمت دراسة العديد من العوامل التي تؤثر على قدرة قناة متعدد المداخل، متعدد المخرجات وعندما يعرف معلومات حالة القناة (CSI) في جهاز الاستقبال، ولكن ليس دائما في الإرسال. لقد أظهرت النتائج أن الزيادة في خفوت ريشين، العامل K ، تشابه الهوائيات والتميز عبر الاستقطاب تقلل من استيعاب قدرة اركوديك واستيعاب قدرة الانقطاع لنظام متعددة المداخل، متعدد المخرجات. لقد تم وصف نظام متعددة المداخل، متعدد المخرجات، وقدم نظام (متعدد المداخل، متعدد المخرجات 5×5) أمكانية في الحد من تدهور القدرات باستخدام محاكاة مونت كارلو. لقد تم عرض ومناقشة نتائج المحاكاة التي تم الحصول عليها باستخدام $MATLAB$ 2009a.

I. Introduction

Multiple Input Multiple Output (MIMO) systems promise a substantial improvement in the wireless system capacity, without requiring more bandwidth. The MIMO channel capacity is defined as the maximum data rate that can be transmitted over the channel with a probability of error almost close to zero. The capacity increases linearly with the number of antennas at both transmitter and receiver^[1]. With the minimum number of transmit and receive antennas for a given fixed signal-to-noise ratio (SNR), the capacity can be improved even if the fades between antenna pairs are correlated^[2]. However, the correlation of a real-world wireless channel may result in a substantial degradation of the MIMO architecture performance and there is a possibility that the line-of-sight (LOS) component may exist within the scattered components. Then, the fading will follow the Rician distribution, degrading the performance of MIMO, compared to Rayleigh fading.

Considering a single user MIMO system with T antennas at the transmitter and R antennas at the receiver, the system can be described as ^[3].

$$\hat{y} = \sqrt{\frac{E_s}{T}} H \hat{s} + \hat{n} \quad \dots(1)$$

where E_s is the total energy available at the transmitter, y is the $R \times 1$ vector of signals received on the R antennas, s is the $T \times 1$ vector of signals transmitted on the T transmit antennas, n is the $R \times 1$ noise vector consisting of independent complex Gaussian distributed elements with zero mean and variance σ^2 , and H is the $R \times T$ channel matrix.

To study the MIMO channel capacity, a channel with a $MR * MT$ matrix is presented as ^[3].

$$H = \begin{bmatrix} h_{1,1}(\tau, t) & h_{1,2}(\tau, t) & \dots & h_{1,M_T}(\tau, t) \\ h_{2,1}(\tau, t) & h_{2,2}(\tau, t) & \dots & h_{2,M_T}(\tau, t) \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ h_{M_R,1}(\tau, t) & h_{M_R,2}(\tau, t) & \dots & h_{M_R,M_T}(\tau, t) \end{bmatrix} \quad \dots(2)$$

The matrix elements are complex numbers that represent the attenuation and the phase shift of the signal arrives at the receiver with a delay of τ sec. Hence, the MIMO system may be described in matrix notation as $y = H \otimes s(t)$, Where $s(t)$ is a vector that represents the signals transmitted from the MT transmit antennas and can be described as $s(t) = [s_1(t) s_2(t) \dots s_{M_T}(t)]^T$ which is a $M_T \times 1$ vector, and $y(t)$ is vector which represents the received signals from the MR receive antennas, and can be described as $y(t) = [y_1(t) y_2(t) \dots y_{M_R}(t)]^T$, which is an $M_R \times 1$ vector.

The MIMO channel capacity is given by Shannon's extended formula as [4].

$$C = \max_{tr(R_{ss}) \leq p} \log_2 \left[\det \left(I + H R_{ss} H^H \right) \right] \quad \dots(3)$$

where, the matrix H^H is the transpose conjugate of the channel matrix H , R_{ss} is the covariance matrix of the transmitted signal vector $s(t)$ and p is the maximum normalized average transmit power.

In case of unknown channel at the transmitter, the signals to be transmitted are equi-powered at transmit antennas, and the power (P_k) allocated to each of the MT elements is equal to (P/M_T). In that case the R_{ss} matrix of equation (3) equals the identity matrix (I), and the capacity can be expressed as in the following equations [4,5].

$$C = \log_2 \left[\det \left(I + \frac{P}{M_T} H H^H \right) \right] \quad \dots(4)$$

$$C = \sum_{k=1}^n \log_2 \left(1 + \frac{P}{M_T} \varepsilon_k^2 \right) \quad \dots(5)$$

Equation (5) implies that the MIMO channel capacity can be expressed by the sum of the capacities of $n = \text{rank}(H)$ SISO channels, each having power gain ε_k^2 and transmit power of (P/M_T).

When the conditions of the environment permit the use of a Rayleigh model and the antennas of the transmitter and the receiver are sufficiently separated, the elements of the channel

matrix H can be modeled as zero mean circularly symmetric complex Gaussian (ZMCSCG) random variables, with unit variance. The resulting matrix is symbolized H_w and is referred as spatially white matrix. The capacity formula under the assumptions of Rayleigh channel and equal power allocation is.

$$C = \log_2 \left[\det \left(I + \frac{P}{M_T} H_w H_w^H \right) \right] \quad \dots(6)$$

The ergodic capacity, and the outage capacity of a MIMO channel are used to study the stochastic channels which is a random variable. The ergodic capacity of a MIMO channel is the ensemble average of the information rate over the distribution of the elements of the channel matrix H [1]. The outage capacity quantifies the level of capacity performance guaranteed with a certain level of reliability. For example, $q\%$ outage capacity, $C_{out;q}$, indicates that the system can achieve minimum capacity level $C_{out;q}$ with probability $(1 - q)\%$ [2].

II. Channel Capacity At The Transmitter

In case of unknown channel (no CSI) at the transmitter, the ergodic capacity is given by [3].

$$C = E_H \left[\log_2 \left(\det \left(I + \frac{\rho}{M_T} H H^H \right) \right) \right] \quad \dots(7)$$

where $E_H\{\cdot\}$ denote the expectation over H and the operator H^H indicates the Hermitian of the matrix H . Using singular value decomposition (SVD), eq.(7) can be decomposed as.

$$C = E_\lambda \left\{ \sum_{i=1}^k \log_2 \left(1 + \frac{\rho}{T} \lambda_i \right) \right\} \quad \dots(8)$$

where $E_\lambda\{\cdot\}$ denote the expectation over λ , k , ($k \leq m$) is the rank of H , and λ_i ($i = 1, 2, \dots, k$) denotes the positive eigen values of $H H^H$.

When the channel state information CSI are known at the transmitter [4].

$$C = E_H \left\{ \sum_{i=1}^k \log_2 (\mu \lambda_i)^+ \right\} \quad \dots(9)$$

Where μ is chosen to satisfy.

$$\frac{\rho}{N_o} = \sum_{i=1}^k (\mu - \lambda_i^{-1})^+ \quad \dots(10)$$

and “+” denotes taking only the positive terms.

III. Correlated Rician Fading Channel Model

For Rician fading the elements of H are non-zero mean complex Gaussians. Hence we can express H in matrix notation as [17].

$$H = aH^{sp} + bH^{sc} \quad \dots(11)$$

where the specular and scattered components of H are denoted by superscripts sp and sc respectively, $a > 0$, $b > 0$ and $a^2 + b^2 = 1$. H^{sp} is a matrix of unit entries denoted as HLOS. If there is no correlation at the transmitter or at the receiver side then the entries of H^{sc} are independent and identically distributed (i.i.d.) zero mean circularly symmetric complex Gaussian (ZMCSCG) random variables with unit variance, usually denoted by H_w . If there is correlated fading then the H^{sp} matrix can be modeled as [17].

$$H^{sp} = R_r^{1/2} H_w R_t^{1/2} \quad \dots(12)$$

where R_t and R_r are the correlation matrix at the transmitter and at the receiver side, respectively. The correlation matrix R is defined as [17].

$$r_{ij} = \begin{cases} r^{j-i}, & i \leq j \\ r^*_{ji}, & i > j \end{cases}, |r| \leq 1 \quad \dots(13)$$

where “*” denotes the complex conjugate. The Rician factor, K is defined as a^2/b^2 . Thus, the above H matrix can be written as [17].

$$H = \sqrt{\frac{K}{K+1}} H_{LOS} + \sqrt{\frac{1}{K+1}} R_r^{1/2} H_w R_t^{1/2} \quad \dots(14)$$

where K is the Rician K-factor of the channel and is the ratio of the total power in the fixed component of the channel to the power in the fading component^[17] when assumed the channel is assumed to be perfectly known to the receiver. Furthermore, we assume an ergodic block fading channel model where the channel remains constant over a block of consecutive

symbols, and changes in an independent fashion across blocks^[7,8]. Therefore, in the Rician channel case, the channel matrix can be represented as a sum of the line-of-sight (LOS) and non-line-of-sight (NLOS) components^[17].

$$H = H_{LOS} + H_{NLOS} \quad \dots(15)$$

Replacing $K=0$ in equation (15), the non-line-of-sight (NLOS) components will be.

$$H_{NLOS} = R_r^{1/2} H_w R_t^{1/2} \quad \dots(16)$$

where R_r is the $M \times M$ correlation matrix of the receive antennas, R_t is the $N \times N$ correlation matrix of the transmit antennas, and H_w is a complex $N \times M$ matrix whose elements are zero-mean i.i.d. complex Gaussian random variables.

For a particular channel model, we consider the case of $M = N = \nu$ and model H as^[17].

$$H = c_{LOS} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + c_{NLOS} \begin{bmatrix} 1 & \zeta_R \\ \zeta_R & 1 \end{bmatrix}^{1/2} H_w \begin{bmatrix} 1 & \zeta_T \\ \zeta_T & 1 \end{bmatrix}^{1/2} \quad \dots(17)$$

where the constants c_{LOS}^2 and c_{NLOS}^2 characterize the powers of the LOS and NLOS components of H and jointly determine the SNR and the power ratio α of these components. Here, the parameters ζ_R and ζ_T characterize the receive and transmit correlations, respectively^[17].

The capacity of a channel with correlation can be written as^[17].

$$C = \log_2 \det \left(I_{N_R} + \frac{SNR}{N_T} R_r^{1/2} H^H R_t^{1/2} \right) \quad \dots(18)$$

When $N_T = N_R$, and SNR is high, this expression can be approximated as.

$$C = \log_2 \det \left(I_{N_R} + \frac{SNR}{N_T} H_u H_u^H \right) + \log_2 \det(R_r) + \log_2 \det(R_t) \quad \dots(19)$$

The last two terms are always negative since $\det(R) \leq 0$.

IV. Effect Of Cross Polarization Discrimination (XPD) On MIMO Capacity

The Cross Polarization Discrimination XPD tells us one antenna discriminates its polarization from the other antenna^[4]. The channel models assume that the antennas at the base station and terminal transmit and receive with identical polarizations. The use of antennas with differing polarizations at the transmitter and receivers leads to a gain (or power) and correlation imbalance between the elements of $H^{[1]}$. The polarization matrix H describes the degree of suppression of individual co- and cross-polarized components, cross correlation, and cross coupling of energy from one polarization state to the other state. Two polarization schemes are typically and practically used: horizontal/vertical ($0^\circ/90^\circ$) or slanted ($+\xi^\circ/-\xi^\circ$). The elements of the matrix \tilde{H} which are denoted as $\tilde{h}_{i,j} (i, j = 0,1)$ are zero-mean circularly symmetric complex Gaussian random variables whose variances depend on the propagation conditions and the antenna characteristics.

For 2×2 antenna system and when the transmitter and receiver use antennas with $\pm 45^\circ$ polarization, the diagonal elements of H correspond to transmission and reception on the same polarization, while the off diagonal elements correspond to transmission and reception on orthogonal polarizations. The power in the individual channel elements is assumed to be $[1, \alpha, \alpha, 1]$.

$$\varepsilon \left\{ \left| \tilde{h}_{0,0} \right|^2 \right\} = \varepsilon \left\{ \left| \tilde{h}_{1,1} \right|^2 \right\} = 1 \quad \dots(20)$$

$$\varepsilon \left\{ \left| \tilde{h}_{0,1} \right|^2 \right\} = \varepsilon \left\{ \left| \tilde{h}_{1,0} \right|^2 \right\} = \alpha \quad \dots(21)$$

where $0 < \alpha \leq 1$ is directly related to the XPD (or separation of orthogonal polarizations) for the channel components. Good discrimination of orthogonal polarizations amounts to small values of α and vice versa. The relation between the channel XPD and α is, thus, given by^[1].

$$XPD = \frac{1-\alpha}{\alpha}, \quad 0 < \alpha \leq 1 \quad \dots(22)$$

The capacity in the absence of XPD $C_{\alpha=1}$ and in the presence of perfect XPD $C_{\alpha=0}$ can be given by^[1].

$$C_{\alpha=0} \approx 2 \log_2 \left(1 + \frac{SNR}{2} \right) \quad \dots(23)$$

$$C_{\alpha=1} \approx \log_2(1+2SNR) \quad \dots(24)$$

V. Simulation Results and Discussion

In this section, the effect of many factors on the MIMO channel capacity will be presented and discussed considering correlation at both communication ends in all cases.

Figures 1, 2, and 3 shows the Ergodic and 1% outage capacity of 3*3 MIMO system and 2*2 MIMO system when the Rician fading component are equal to (1, 1.5 and 2) respectively and when Rician K factor vary from 1 to 20 in step of 5 for a given fixed value of SNR from 10 dB to 20 dB when the receiver has perfect CSI but the transmitter does not(equal power allocation). From these figures, it is can be noticed that, as the SNR increased or number of antennas M increased, the Ergodic and outage capacity increase linearly with the number of antennas and logarithmically with the SNR., in addition, as the value of Rician K factor increased, the ergodic and the outage capacity will decrease. However, the loss is more at 2*2 MIMO system due to the increase in Rician K factor that emphasizes the deterministic part of the channel. The deterministic channel is of rank 1 and so the capacity decreases. From eq(14) when K = 1, only the channel matrix can be represented as non-line-of-sight (NLOS) components or pure Rayleigh fading i.i.d channel $H = R_r^{1/2} H_w R_t^{1/2}$, and when K= 20, the channel matrix can be represented as a sum of the (LOS) and NLOS components

$$H = \sqrt{\frac{20}{21}} H_{LOS} + \sqrt{\frac{1}{21}} R_r^{1/2} H_w R_t^{1/2}. \text{ i.e., the ergodic and outage capacity of the system will be}$$

affected by the LOS and it will degrade as the power from the line of sight component increases. Also, from the figures, It can seen that for the 3*3 MIMO and 2*2 MIMO channel, the ergodic capacity is higher than outage capacity for all values of SNR and when the value of Rician fading increases, the ergodic and the outage capacity decrease. However, the loss is more at 2*2 MIMO channel. This is because the increase in correlated fading parameter emphasizes that the fades are less independent and thus reduces the rank of the random channel, so the capacity decreases.

Table(1) shows the values of Ergodic capacity and 1% Outage capacity in b/s/Hz for 2*2 MIMO system when the Rician fading component is equals to 1, 1.5 and 2 when Rician K factor vary from 1 to 20 in step of 5 at SNR = 20 dB.

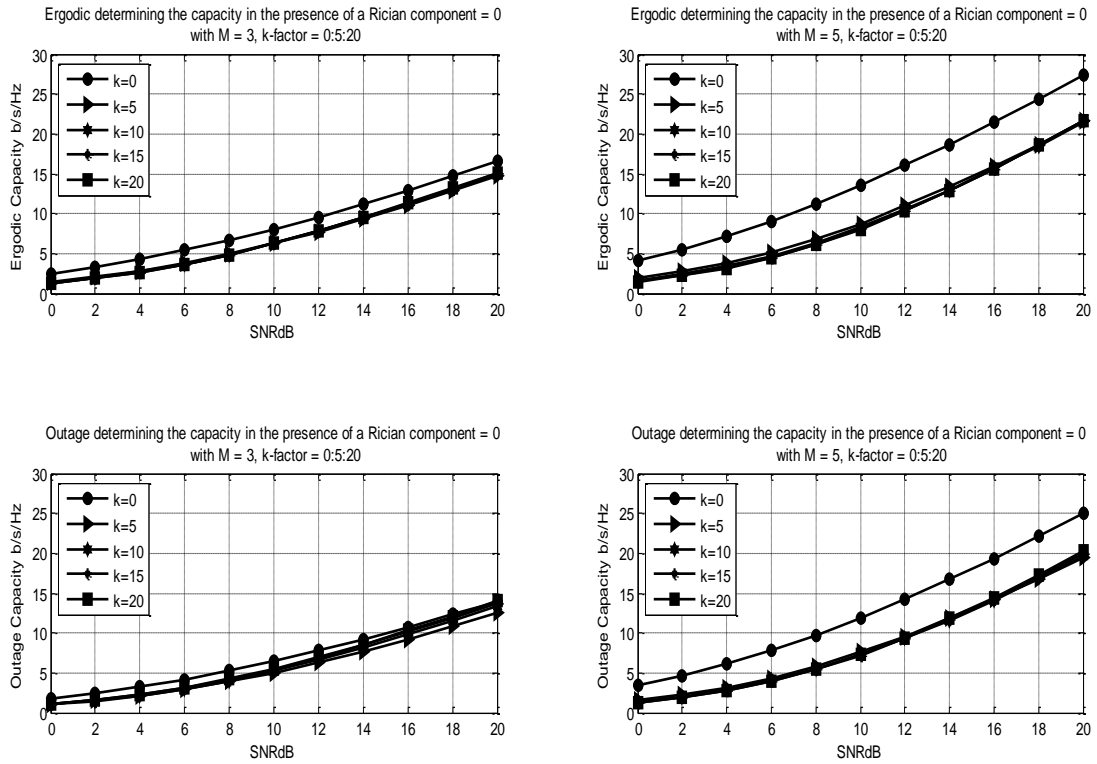


Figure (1). Ergodic and 10% Outage capacity of 3x3 MIMO system and 5x5 MIMO system with zero Rician fading component

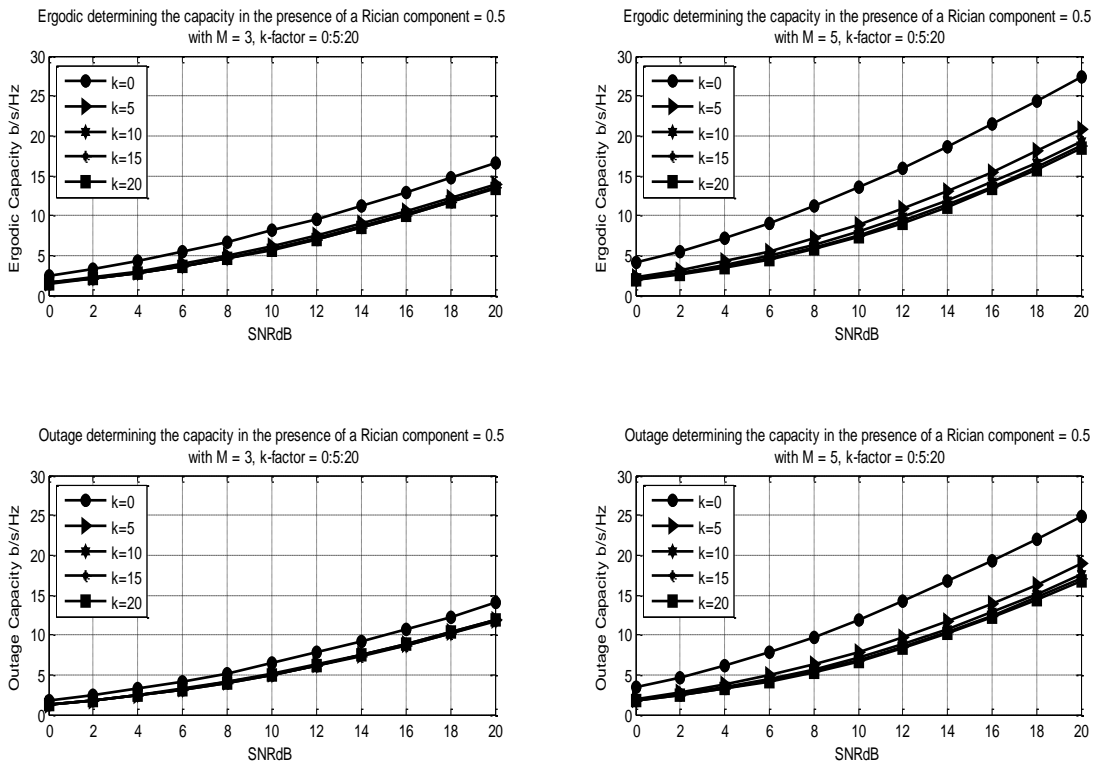


Figure (2). Ergodic and 10% Outage capacity of 3*3 MIMO system and 5*5 MIMO system with 0.5 Rician fading component

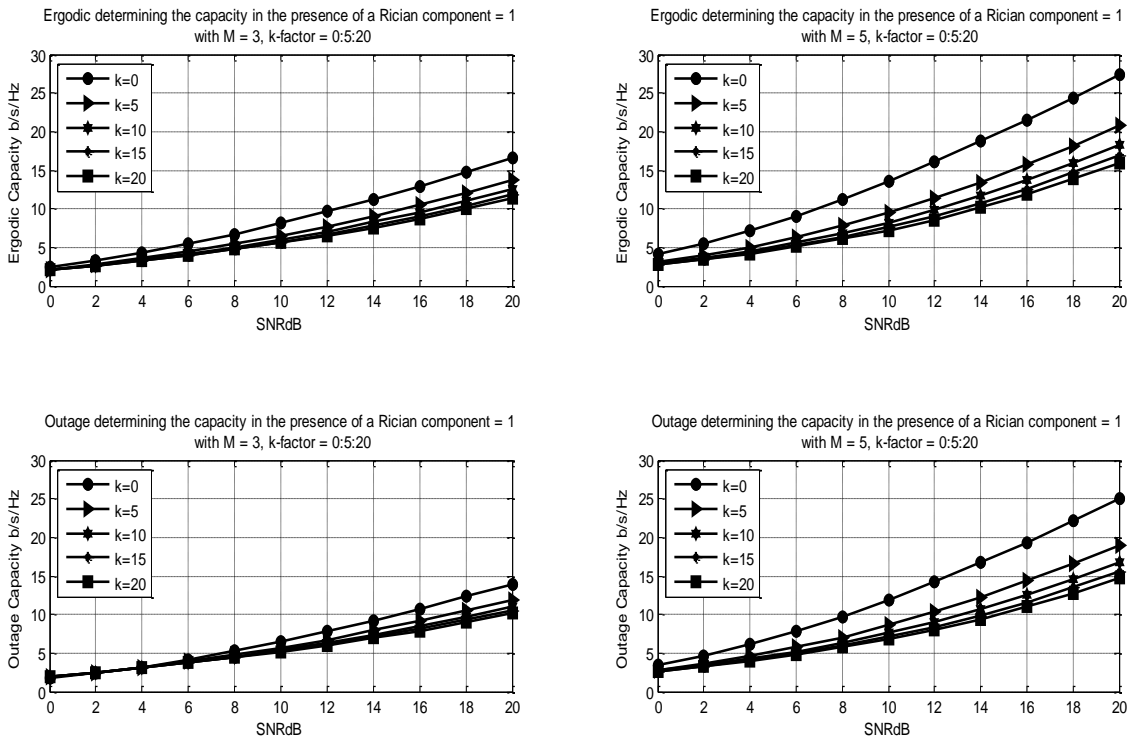


Figure (3). Ergodic and 10% Outage capacity of 3*3 MIMO system and 5*5 MIMO system with 1 Rician fading component

Table (1). Ergodic – 10% outage capacity with different values of Rician Factor

K factor	Rician fading component = 0		Rician fading component = 0,0		Rician fading component = 1	
	Ergodic capacity	10% Outage capacity	Ergodic capacity	10% Outage capacity	Ergodic capacity	10% Outage capacity
0	27,4102	24,9004	27,3943	24,8917	27,1919	24,7483
0	21,7632	20,3170	20,7672	18,8886	20,7008	18,0437
10	21,7328	20,0241	19,2634	17,0748	18,3007	17,8416
10	21,0927	19,7022	18,7207	17,0830	17,9724	10,0986
20	21,0329	19,4327	18,4283	17,8388	17,0034	15,7768

Figures 4 and 5, show the Ergodic and 10% Outage capacity of 3*3 MIMO system and 0*0 MIMO system in the absence of XPC (i.e., the antennas discriminate no interference between each other's polarizations) with and without fading correlation and when there is a correlation between the antenna elements only at the receiver with correlation values are (0,2, 0,4, 0,6, 0,8) at a given fixed value of SNRdB from 0 to 20 dB when the receiver has perfect CSI but the transmitter does not(equal power allocation). From the figures, it is clear that, the capacity of 0*0 MIMO system is higher than the capacity of 3*3 MIMO system for all values of SNR in all figures and the capacity of uncorrelated MIMO system is higher than the capacity of correlated MIMO channel. For the correlated MIMO channel, when the correlation among the antenna elements decreases the capacity increases. This result is expected, since smaller angular spread leads to higher correlation and consequently lower capacity (i.e., we observe a very large ergodic capacity loss for the case of high correlation) and the effect is more significant at higher SNR.

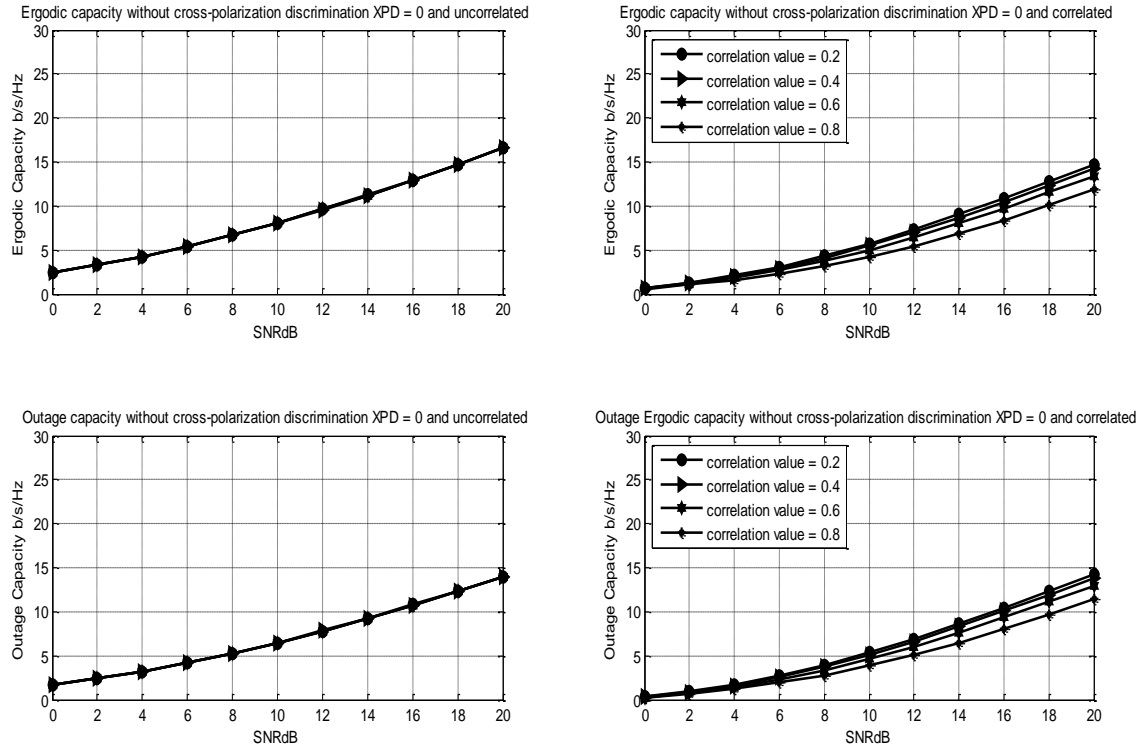


Figure (4). Ergodic and 10% Outage capacity of 3*3 MIMO system in the absence cross polarization discrimination with uncorrelated and correlated MIMO system

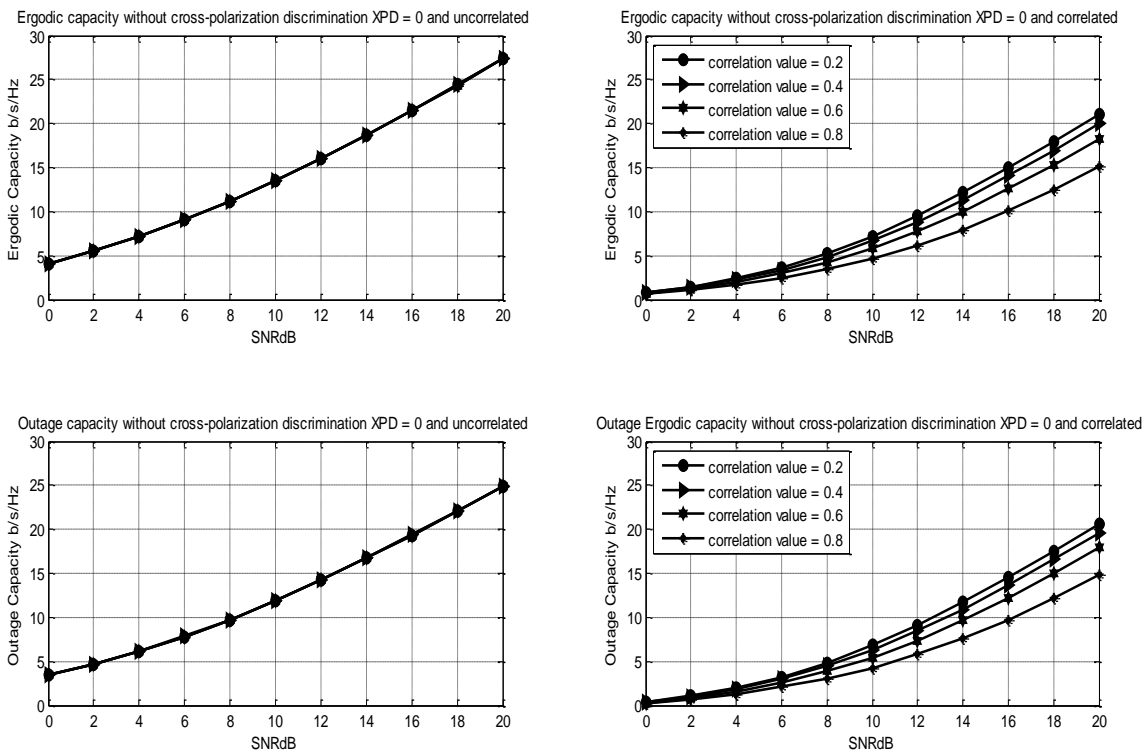


Figure (6). Ergodic and 10% Outage capacity of 2x2 MIMO system in the absence cross polarization discrimination with uncorrelated and correlated MIMO system

Table (7). shows the values of Ergodic capacity and 10% Outage capacity in b/s/Hz for 2x2 MIMO system in the absence of XPD with uncorrelated and correlated MIMO system at SNR = 20 dB.

Table (7). Ergodic – 10% outage capacity for correlated and uncorrelated MIMO

Correlated values	Ergodic capacity		Outage capacity	
	Uncorrelated 2x2 MIMO system	correlated 2x2 MIMO system	Uncorrelated 2x2 MIMO system	correlated 2x2 MIMO system
0.2	27,4239	21,0093	24,9053	20,6093
0.4	27,4219	20,0297	24,9016	19,6297
0.6	27,4128	18,2882	24,8947	17,8882
0.8	27,4070	10,1971	24,8930	14,7971

Figures 6, 7, and 8 show the Ergodic and 10% Outage capacity of 3x3 MIMO and 2x2 MIMO system when cross polarization discrimination is investigated with α values are (0.2, 0.4, 0.6, 0.8) with uncorrelated and correlated antenna elements when the correlated values are

($\alpha, \beta, \gamma, \xi, \eta, \zeta$) at a given fixed value of SNRdB from 0 to 20 dB and when the receiver has perfect CSI but the transmitter does not (equal power allocation). From all figures, it is clear that, when the cross polarization discrimination is investigated, the capacity of 2×2 MIMO system is higher than the capacity of 3×3 MIMO system for all values of SNRdB in all figures. The capacity is decreased as α increased for uncorrelated fading channel but when the channel fading is correlated the α values cannot effect on the capacity (i.e., all curves of the capacity are close to each other for all values of α in the same figure) and when the correlation among the antenna elements increases the capacity are decreases. Figure 4, shows the 10% Outage capacity of 2×2 MIMO system with cross polarization discrimination for uncorrelated and correlated MIMO system.

Table(3) shows the values of Ergodic capacity and 10% Outage capacity in b/s/Hz for 2×2 MIMO system when XPD present with uncorrelated and correlated antenna at SNR = 20 dB.

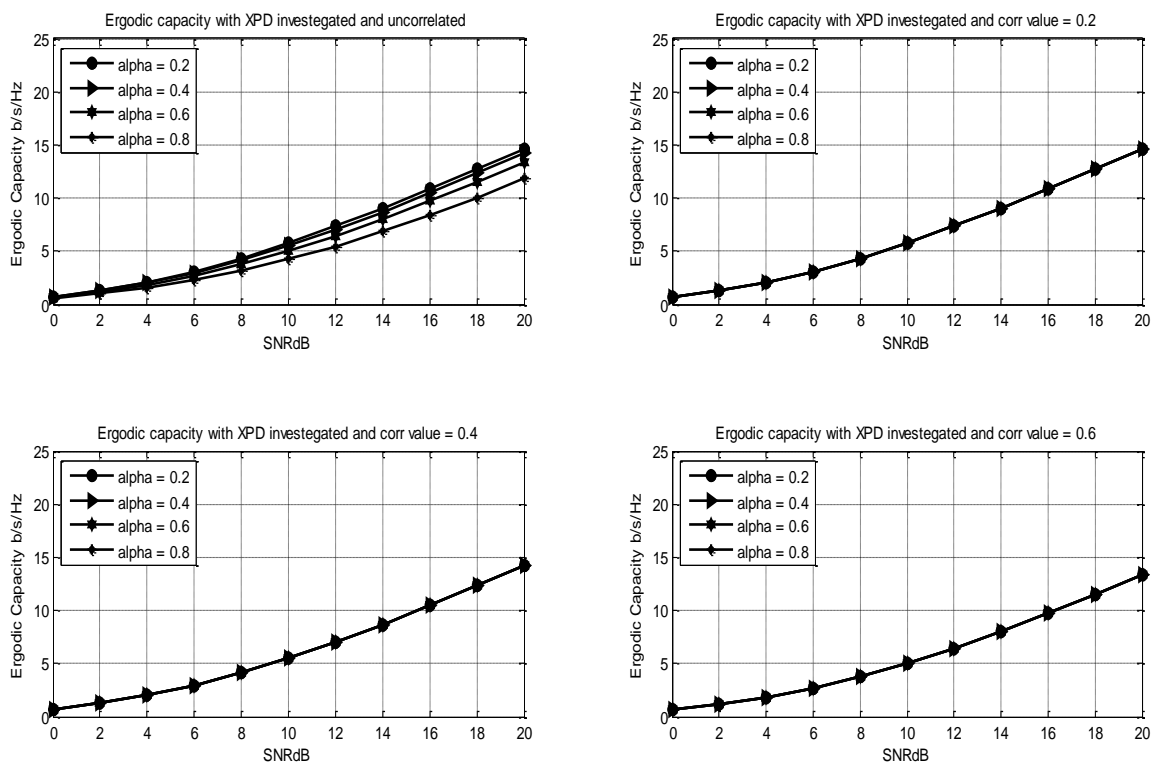


Figure (4). Ergodic capacity of 2×2 MIMO system with cross polarization For correlated and uncorrelated MIMO system

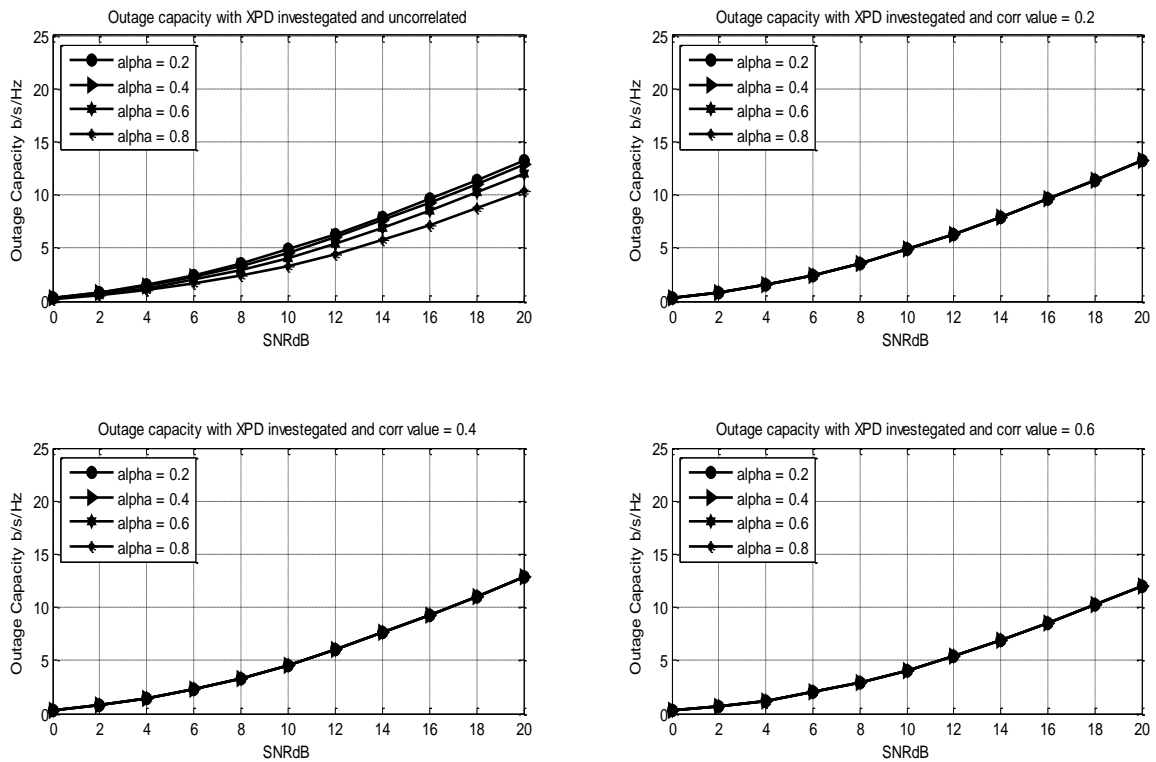


Figure (7). 10% outage capacity of 2*2 MIMO system with cross polarization For correlated and uncorrelated MIMO system

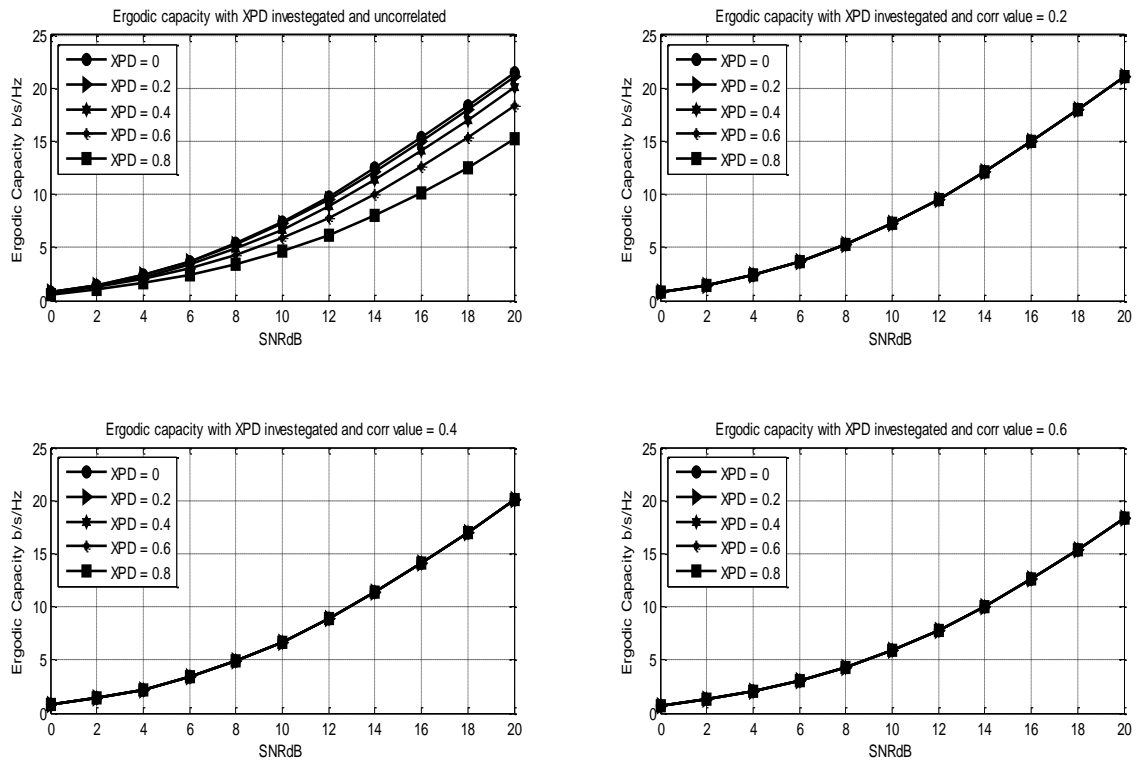


Figure (8). Ergodic Capacity of 2×2 MIMO system with cross polarization For correlated and uncorrelated MIMO system

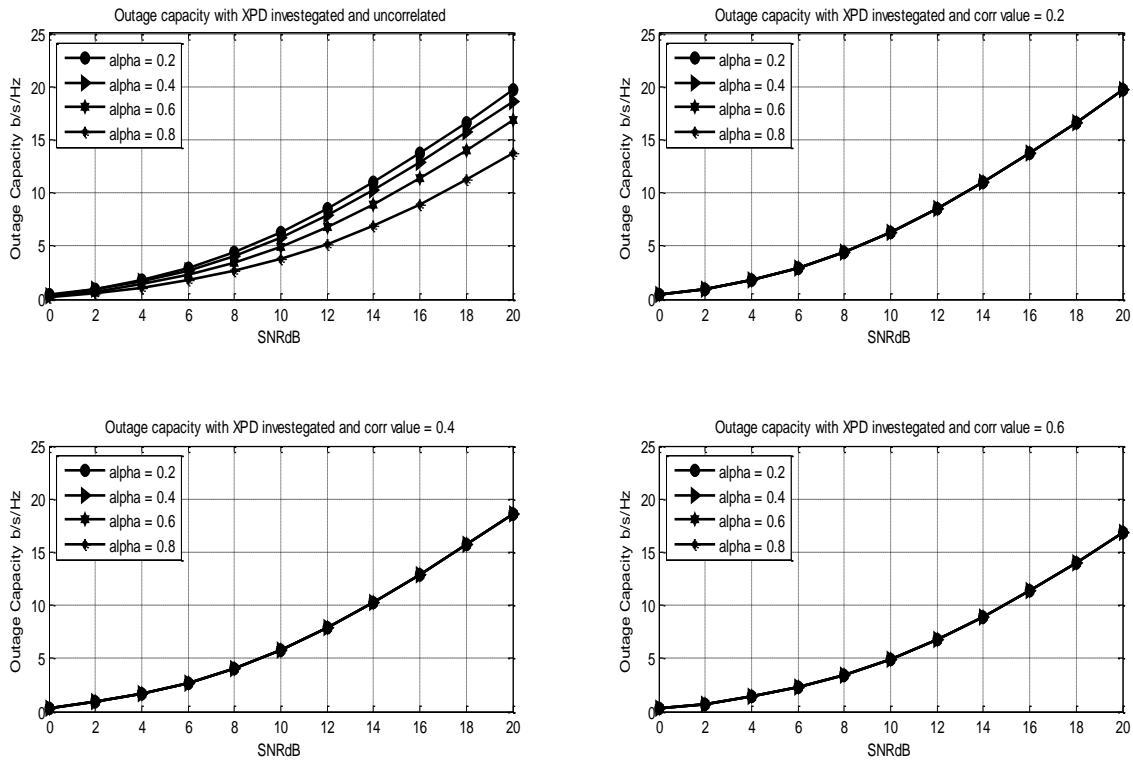


Figure (9). 10% Outage Capacity of 2×2 MIMO system with cross polarization For correlated and uncorrelated MIMO system

Table (3). Ergodic – 10% outage capacity in b/s/Hz for 2*2 MIMO system

α values	Ergodic capacity		Outage capacity	
	Uncorrelated antenna	correlated Value = 0.6	Uncorrelated antenna	correlated Value = 0.6
0.2	21,0093	18,2882	19,6093	16,8882
0.4	20,0297	18,2882	18,6297	16,8882
0.6	18,2882	18,2882	16,8882	16,8882
0.8	10,1971	18,2882	13,7971	16,8882

VI. CONCLUSION

In this paper, we investigated a real life cases which are the Rician fading , Rician K factor, antenna correlation, Line of sight problems and cross-polarization discrimination on the capacity of MIMO system. The simulation results show that the ergodic capacity and outage capacity can be increased as the number of transmitter and receiver increased or SNR increased. The uncorrelated MIMO system performs better than the correlated MIMO system in LOS scenario (i.e., the capacity of uncorrelated MIMO system is higher than the capacity of correlated MIMO system). While the capacity will be degraded as the Rician fading, K factor, antenna correlation , Line of sight problems and cross-polarization discrimination increases.

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