A Novel Approach to Solving the OHESW Problem for Multilevel Inverters

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Abstract

A new method is presented to compute the switching angles in a multilevel inverter using the Optimized Harmonic Elimination Stepped-Waveform (OHESW) technique so as to produce the required fundamental voltage while at the same time not generate higher order harmonics. Previous work has shown that the transcendental equations characterizing the harmonic content of the inverter output can be converted to polynomial equations which are then solved using the method of Resultants from Elimination theory. However, when there are several DC sources, the degree of the polynomials are quite large making the computational burden of their resultant polynomials via elimination theory quite high. The proposed method with fast recursive algorithm is derived that provide the exact on-line solution to the OHESW problem.

The proposed algorithm optimization technique is applied to a multilevel inverter to determine optimum switching angles for eliminating low order harmonics while maintaining the required fundamental voltage to drive an induction motor. The proposed method contributes to the existing methods because it not only generates the desired fundamental voltage, but also completely eliminates any number of harmonics. The complete solutions for (5-15) level OHESW switching patterns to eliminate the (3^{rd}-13^{th}) harmonics are given.

Key-Words: Harmonic Elimination, OHESW, Polynomial.

الخلاصة

تمَ أبداء طريقة جديدة لحساب زوايا التشغيل في عاكس متعدّد المستويات باستخدام تقنية الموجة الخطوية المثالية لحذف التوافقيات لتوليد الفولتية الأساسية المطلوبة وفي نفس الوقت عدم توليد التوافقيات ذات المرتبة العالية. لقد وَجدت الدراسات السابقة بأن المعادلات المبهمة الغير الخطية والممَيزة للمحتوى التوافقي لخرج العاكس يمكن تحويلها إلى معادلات جبرية متعددة الحدود وعندئذ يمكن حلها باستخدام طريقة المحصلات من نظرية الحذف. ومع ذلك فعند زيادة عدد مصادر الفولتية المستمرة, فأن درجة هذه المعادلات تكون كبيرة مما يجعل العبء الحسابي لإيجاد المحصلة لها كبيراً نوعا ما.

تمَ اشتقاق الطريقة المقترحة بمساعدة خوارزمية متكررة وسريعة وتجَهز مشكلة الموجة الخطوية المثالية لحذف التوافقيات بالحل المباشر والتام. تمَ تطبيق التقنية المثالية للخوارزمية المقترحة على عاكس متعدّد المستويات لتحديد زوايا التشغيل المثالية وذلك لحذف التوافقيات ذات المرتبة المنخفضة وفي نفس الوقت أبقاء الفولتية الأساسية المطلوبة لسوق محرك حثي. لقد ساهمت الطريقة المقترحة مع الطرق الموجودة لأنها لا تُولدّ الفولتية الأساس فحسب وإنماً تحذف أيضا أي عدد من التوافقيات بشكل تام. تمَ أعطاء الحلول التامة لنماذج تشغيل الموجة الخطوية المثالية بحذف التوافقيات ذات المستويات (5-15) لحذف التوافقيات ذات المراتب (3-13).

1. Introduction:

A multilevel inverter is a power electronic device built to synthesize a desired AC voltage from several levels of DC voltages [(1)]. Multilevel inverters are uniquely suited for utility applications because of the high VA ratings possible with these inverters [(2)]. The devices in a multilevel inverter have a much lower *dv/dt* per switching, and they operate at high efficiencies because they can switch at a much lower frequency than the traditional PWM controlled inverters. Here a fundamental frequency switching scheme (rather than PWM) is considered because; this results in significantly lower switching losses.

The key issue for multilevel inverter modulation is the Harmonics Elimination (HE). The Optimized Harmonic Elimination Stepped Waveform (OHESW) technique [(3)] is very suitable for a multilevel inverter circuit. By employing this technique along with the multilevel topology, the low Total harmonic Distortion (THD) output waveform without any filter circuit is possible.

The Multilevel Fundamental Switching (MFS) method inherently provides the opportunity to eliminate certain higher order harmonics by varying the times at which certain switches are turned “on” and turned “off”, which is also called varying the switching angles.

It was mentioned earlier that an increase in the number of DC voltages in a multilevel inverter results in a better approximation to a sinusoidal waveform and provides the opportunity to eliminate more harmonic contents which will make it easier to filter the remaining harmonic content. As a result, filters will be smaller and less expensive.

A key issue in the MFS scheme is to determine the switching angles (times) so as to produce the fundamental voltage and not generate specific higher order harmonics [(1,4)].

Iterative technique was used to solve for the switching angles for the MFS scheme [(3)], though such an approach did not guarantee finding all the possible solutions. In [(5)], a Genetic algorithm approach was used to solve for the switching angles. In Kato [(6)], a Homotopy technique was used to solve the HE equations for a single DC source inverter. [(7)] had shown that the transcendental equations characterizing the harmonic content of the MFS scheme can be converted into polynomial equations which were then solved using the method of Resultants from Elimination theory. However, if there are several DC sources, the degrees of the polynomials in these equations are large.

As a result, one reaches the limitations of the capability of contemporary computer algebra software tools to solve the system of polynomial equations using elimination theory (by computing the resultant polynomial of the system), even if the idea of Symmetric Polynomials and Power Sums [(1,8)] are used to simplify these equations, they could become too complex and need more time to solve.

To conquer this problem, the switching angles computation of the MFS scheme with general number is solved by using the proposed fast *recursive* on-line algorithm.

The work [9] presents a contribution to the theory of optimal traditional PWM and gives algorithm for efficient on-line calculation of single-phase PWM switching patterns.

The present work will be to develop and extend the proposed algorithm in [9] which is used only for 3-level traditional PWM inverter to be compatible with multilevel inverter for solving the OHESW problem in [3] and [7] on-line and in real time.

2. Cascaded H-bridges Multilevel Inverter:

The so-called multilevel; starts from three levels. As the number of levels reach infinity, the output THD approaches zero. There are three main capacitor voltage synthesis-based multilevel inverters, i.e. diode-clamped, capacitor-clamped, and cascaded H-bridges. The cascaded H-bridges multilevel inverter is a relatively new inverter structure. This new inverter does not require any transformers, clamping diodes, or flying capacitors, which are required in today's multilevel inverters. It is proposed to solve all the problems of the multilevel inverters as well as conventional multi pulse or PWM inverters [2]. A cascaded H-bridges multilevel inverter is simply a series connection of multiple H-bridge inverters. Each H-bridge inverter has the same configuration as a typical single-phase full-bridge inverter. By cascading the AC outputs of each H-bridge inverter, an AC voltage waveform is produced. Fig. 1 provides an illustration of a single-phase cascaded H-bridges multilevel inverter using 3-Separate DC Sources (SDCS); (*s*=3, where *s* is the no. of SDCSs).

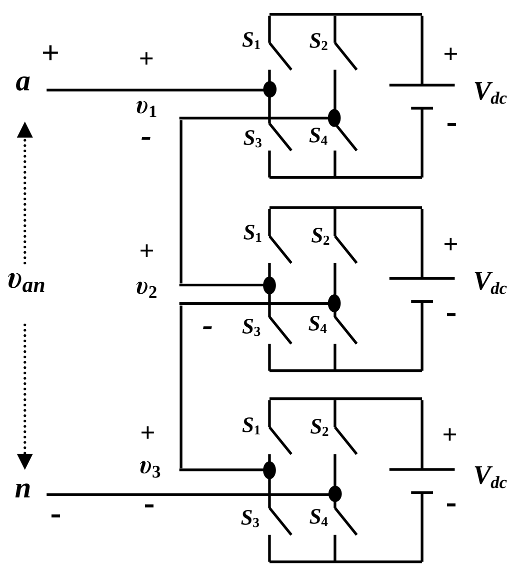


Fig. 1: Cascaded H-bridges Multilevel Inverter using 3-DC Sources

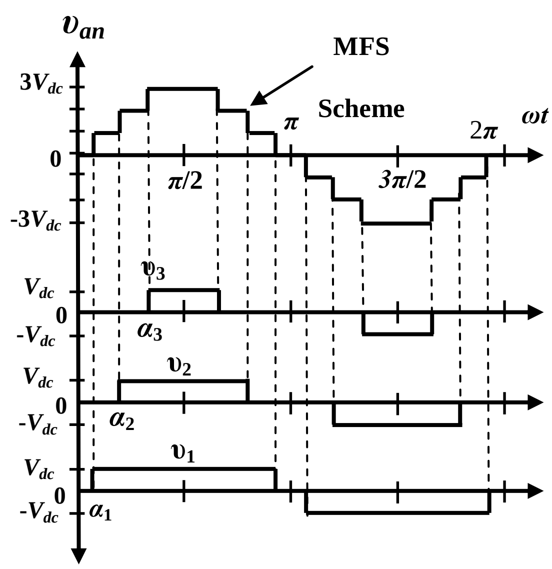


Fig. 2 : Voltage Output of Cascaded H-bridges Multilevel Inverter

By closing the appropriate switches, each H-bridge inverter can produce three different voltages: $+V_{dc}$, 0, and $-V_{dc}$. When switch S_1 and S_4 of one particular H-bridge inverter in Fig. 1 are closed, the output voltage is $+V_{dc}$. When switches S_2 and S_3 are closed, the output voltage is $-V_{dc}$. When either the switches S_1 and S_2 or the switches S_3 and S_4 are closed, the output voltage is 0.

Fig. 2 also illustrates the idea of "levels" in a cascaded H-bridges multilevel inverter. In this figure, one notices that three distinct DC sources can produce a maximum of 7-distinct levels in the output phase voltage of the multilevel inverter. This is used to verify the concept of the OHESW technique. More generally, a cascaded H-bridges multilevel inverter using *s*-SDCS can produce a maximum of $2s+1$ distinct levels in the output phase voltage [(3)].

3. Mathematical Model of the Multilevel Inverter Switching:

Basically, the concept of the OHESW technique is to combine the idea of the Selective Harmonic Elimination PWM (SHEPWM) with the quarter-wave symmetric idea concept [(3)].

The OHESW shown in Fig. 2 is assumed to be the quarter-wave symmetric. The Fourier series of the quarter-wave symmetric *s* H-bridge cell multilevel waveform is written as follows:

$$\upsilon_{out}(\omega t) = \upsilon_{an} = \sum_{n=1}^{\infty} \frac{4V_{dc}}{n\pi} \left[\sum_{k=1}^{s} \cos(n\alpha_k) \right] \sin(n\omega t) \qquad (1)$$

, where α_k is the optimized switching angles, which must satisfy the following condition: $0 \le \alpha_1 \le \alpha_2 \le ... \le \alpha_s \le \pi/2$, k is the k^{th} switching angle, n is the harmonic order, s is the number of SDCS and V_{dc} is the DC voltage source.

The method to solve the optimized harmonic switching angles will be explained in this section. From Eq. (1), the amplitude of all odd harmonic components including fundamental one, are given by:

$$a_n = \frac{4V_{dc}}{n\pi} \sum_{k=1}^{s} \cos(n\alpha_k) \qquad (2)$$

The amplitude of DC component and all even harmonics equal zero. Thus, only the odd harmonics in the quarter-wave symmetric multilevel waveform need to be eliminated. The switching angles of the waveform will be adjusted to get the lowest output voltage THD.

Amplitudes of any harmonics can be set by solving a system of nonlinear equations obtained from setting Eq. (2) equal to pre-specified values. In the optimal HEPWM method [(10)], the fundamental component is set to required amplitude and n-1 low-order harmonics are set to zero. This is the most common approach in electric drives since low-order harmonics are the mostdetrimental to motor performance.

This task of designing a waveform, the first n Fourier series coefficients of which match those of a desired waveform has been the subject of many papers [3,6,11]. Often, the Newton iteration method in these papers was used to solve the system of nonlinear Eqs. (2). This method is computationally intensive for on-line calculations and the storage of off-line calculations leads to high memory requirements. Another approach is to simplify the nonlinear HE equations in order to obtain real-time approximate solutions using modern DSPs [10].

The current paper uses the recursive algorithm for solving the OHESW problem without any approximations in the problem statement. The developed algorithm will allow for real-time generation of switching patterns with high order.

The OHESW problem, as it is considered here, is the design of a waveform $\upsilon_{out}(\omega t)$ so that it's first Fourier coefficients h_k are equal to prescribed values in Eqs. (2). Therefore, the OHESW problem gives rise to the following design equations [3]:

$$\begin{aligned}
&\cos\alpha_1 + \cos\alpha_2 + \cos\alpha_3 + \ldots \cos\alpha_n = h_1 \\
&\cos 3\alpha_1 + \cos 3\alpha_2 + \cos 3\alpha_3 + \ldots \cos 3\alpha_n = h_3 \\
&\vdots \\
&\cos(2n-1)\alpha_1 + \cos(2n-1)\alpha_2 \\
&\quad + \cos(2n-1)\alpha_3 + \ldots \cos(2n-1)\alpha_n = h_{2n-1}
\end{aligned} \tag{3}$$

Given the n values, $h_k = k\pi a_k / 4V_{dc}$, we have n equations and n unknowns; we would like to find the n unknowns $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$, with $0 \le \alpha_1 \le \alpha_2 \le \ldots \le \alpha_n \le \pi/2$.

4. Transformation of the OHESW Problem:

Using the trigonometric identities; $\cos nt = T_n(\cos t)$, where T_n is the n^{th} Chebyshev polynomial, changing the variables; $x_i = \cos\alpha_i$, and re-arrangement the necessary equations listed in [9] to be compatible with OHESW problem, we get:

$$\begin{aligned}
&T_1(x_1) + T_1(x_2) + T_1(x_3) + \ldots T_1(x_n) = h_1 \\
&T_3(x_1) + T_3(x_2) + T_3(x_3) + \ldots T_3(x_n) = h_3 \\
&\vdots \\
&T_{2n-1}(x_1) + T_{2n-1}(x_1) + T_{2n-1}(x_3) + \\
&\qquad \ldots T_{2n-1}(x_n) = h_{2n-1}
\end{aligned} \tag{4}$$

As the odd-indexed Chebyshev polynomials are odd polynomials, the Eqs. (4) can be writing:

$$\sum_{i=1}^{n} T_{2k-1}(x) = \sum_{i=1}^{n}\sum_{j=1}^{k} c_{k,j} \cdot x^{2j-1} = h_{2k-1} \quad 1 \le k \le n \qquad \text{or}$$

$$\sum_{j=1}^{k} c_{k,j} \cdot s_{2j-1} = h_{2k-1} \qquad 1 \le k \le n \tag{5}$$

, where $s_j = \sum_{i=1}^{n} x_i^{\,j}$ are the sums of powers [1] of $\{x_i\}$. Eq. (5) forms a set of n linear equations for s_{2j-1}, $1 \le j \le n$. Once the values s_{2j-1} are obtained by solving the linear system of Eq. (5), one has the following problem. Given $\{s_1, s_2, \ldots, s_{2n-1}\}$, find the solution $\{x_1, x_2, \ldots, x_n\}$ to the following system of nonlinear equations:

$$\begin{aligned}
&x_1 + x_2 + \ldots + x_n = s_1 \\
&x_1^{\,3} + x_2^{\,3} + \ldots + x_n^{\,3} = s_3 \\
&\vdots \\
&x_1^{\,2n-1} + x_2^{\,2n-1} + \ldots + x_n^{\,2n-1} = s_{2n-1}
\end{aligned} \tag{6}$$

Once x_i are obtained, the original variables α_i can be found by letting $\alpha_i = \arccos x_i$. Yet it is necessary to order α_i appropriately such that $0 \le \alpha_1 \le \alpha_2 \le \ldots \le \alpha_n \le \pi/2$.

However, the design Eqs. (6) are nonlinear, so obtaining the desired solution $\{x_i\}$ is not so straightforward. In the following sections, this nonlinear system of equations will be closely examined. In Section V, a systematic procedure is given to obtain the solutions.

For the HE problem, the Fourier coefficients of the OHESW waveform $\upsilon_{out}(\omega t)$ should match the Fourier coefficients of a pure sine wave. That is, the values h_{2n-1} appearing in Eq. (3) are given by $h_1 = m$, and $h_{2i-1} = 0$ for $2 \le i \le n$. For this case, the values s_{2i-1} depend on the modulation index m only and are given by:

$$s_{2i-1} = \frac{m}{4^{i-1}} \binom{2i-1}{i-1}, \qquad 1 \le i \le n \tag{7}$$

5. Solving the OHESW Problem:

To solve the OHESW problem, we will first write the polynomial $P(x)$ having roots $\{x_1, x_2 \ldots, x_i \ldots, x_n\}$ as:

$$P(x) = \prod_{i=1}^{n} (x - x_i)$$

, then the logarithmic derivative is given by:

$$\frac{P'(x)}{P(x)} = \sum_{i=1}^{n} \frac{1}{(x - x_i)}$$

Expanding each term in the sum, one gets:

$$\frac{P'(x)}{P(x)} = \sum_{i=1}^{n} \sum_{j=0}^{\infty} \frac{x_i^{\,j}}{x^{j+1}} = \sum_{j=0}^{\infty} \frac{s_j}{x^{j+1}} = \frac{n}{x} + \sum_{j=1}^{\infty} \frac{s_j}{x^{j+1}} \tag{8}$$

, where s_j are the sums of the root powers and $s_0 = n$. Integrating Eq. (8) gives:

$$\ln P(x) = n \ln x - \sum_{j=1}^{\infty} \frac{s_j}{j\, x^j}$$

Raising e to the power of ln $P(x)$ and using the last equation gives:

$$P(x) = x^n \exp\left(-\sum_{j=1}^{\infty} \frac{s_j}{j\,x^j}\right) \tag{9}$$

In order to generalize procedure, we will obtain an expression similar to Eq. (9), but having only odd s_i. To this end, note that:

$$P(-x) = (-1)^n x^n \exp\left(-\sum_{j=1}^{\infty} \frac{s_j (-1)^j}{j\,x^j}\right)$$

, therefore:

$$\frac{P(x)}{P(-x)} = (-1)^n \exp\left(-\sum_{j=1}^{\infty} \frac{s_j}{j\,x^j}\left(1-(-1)^j\right)\right), \quad \text{or} \quad \frac{P(x)}{P(-x)} = (-1)^n \exp\left(-2 \sum_{j=1,odd\ j}^{\infty} \frac{s_j}{j\,x^j}\right)$$

, then:

$$P(x) = (-1)^n P(-x) G(1/x)$$

, where

$$G(x) := e^{V(x)}, \qquad \text{and} \qquad V(x) := -2\left(s_1 x + \frac{s_3}{3} x_3 + \frac{s_5}{5} x_5 + \ldots\right)$$

Let:

$$\tilde{P}(x) = (-1)^n P(-x) \tag{10}$$

, then:

$$P(x) = \tilde{P}(x) G(1/x) \tag{11}$$

, where $\tilde{P}(x)$ is the monic polynomial related to $P(x)$ by negating the roots of $P(x)$.

Eq. (11) is the counter-part to Eq. (9). Likewise, by setting like powers of x equal, we can obtain equations that relate p_k and s_i, where p_k is the polynomial coefficients of $P(x)$. However, in order to do this, we need to expand $G(x) = e^{V(x)}$ into a power series of x. Such a power series for $e^{V(x)}$ can be obtained using the following algorithm. Let:

$$V(x) = \sum_{i=0}^{\infty} v_i x^i\ , \qquad \text{and} \qquad G(x) = e^{V(x)} = \sum_{i=0}^{\infty} g_i x^i$$

If v_i for $0 \le i \le j$ are known, then the values of g_i for $0 \le i \le j$ are given by:

$$g_0 = e^{v_0} \tag{12}$$

$$g_i = \frac{1}{i}\sum_{k=1}^{i} k v_k g_{i-k} \qquad 1 \le i \le j \qquad (13)$$

When the first n odd values of s_i are known, then v are known for $0 \le i \le 2n$. [$v_{2i} = 0$ for $0 \le i \le n$ and $v_{2i-1} = -2s_{2i-1} / (2i-1)$ for $1 \le i \le n$]. Therefore, using the relations in Eqs. (12) and (13), we obtain g_i for $0 \le i \le 2n$, and consequently, we can write out Eq. (11), matching like powers of x, to obtain linear equations from which p_k can be obtained. For example, we write out the expressions for n = 3 (7-level inverter). That is, we are given s_1, s_3, and s_5, and our goal is to find the corresponding monic 3rd degree polynomial $P(x)$; $P(x) = x^3 + p_1x^2 + p_2 x + p_3$.

6. A Recurrence Algorithm for *P*(*x*):

The notation $P_n(x)$ will be used to emphasize the dependence of $P(x)$ on n. Specifically, $P_n(x)$ denotes the monic degree-n polynomial associated with the OHESW problem, with coefficients $p_{n,k}$:

$$P(x) = x^n + p_{n,1}x^{n-1} + \ldots + p_{n,n}$$

With this notation, a recurrence relation:

$$P_{n+1}(x) = x P_n(x) + C_n P_{n-1}(x) \qquad (14)$$

can be used to compute $P_n(x)$. For the OHESW problem, the initial conditions can be taken to be $P_0(x) = 1$ and $P_1(x) = x\text{-}m$. The coefficients C_n in the recursion can be computed using the following formula:

$$C_n = -\frac{\sum_{k=0}^{n}(-1)^k g_{2n+1-k}\, p_{n,k}}{\sum_{k=0}^{n-1}(-1)^k g_{2n-1-k}\, p_{n-1,\,k}} \qquad (15)$$

The coefficients $p_{n+1,k}$ are then determined recursively as [this implements Eq. (14)]:

$$p_{n+1,k} = p_{n,\,k}, \qquad k = 1$$

$$p_{n+1,k} = p_{n,k} + C_n \,.\, p_{n-1,\,k-2}, \qquad k = 2, \ldots, n$$

$$p_{n+1,k} = C_n \,.\, p_{n-1,\,k-2} \qquad k = n + 1 \qquad (16)$$

The Recursive algorithm for computing can be summarized as follows:

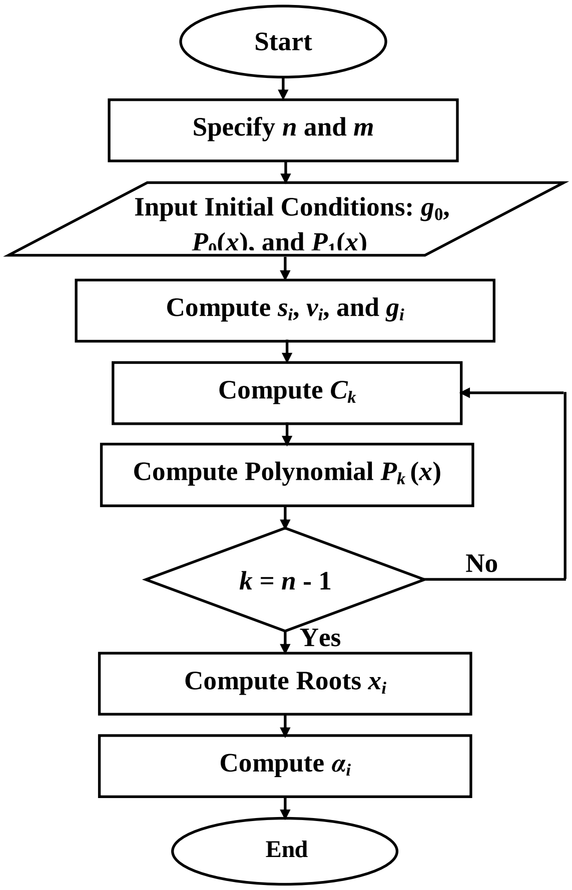


Fig. 3: Flowchart of a Fast Recursive Algorithm for OHESW Technique

Given n and m, the polynomials $P_n(x)$ for are recursively computed as follows:

1) Set $g_0 = 1$ and for $k = 1$ to $2n$, find g_i from Eq. (13).

2) Set $P_0(x) = 1$ and $P_1(x) = x - m$ and for k =1 to n-1 let:

$$C_k = -\frac{\sum_{i=0}^{k} (-1)^i g_{2k+1-i} \, p_{k,i}}{\sum_{i=0}^{k-1} (-1)^i g_{2k-1-i} \, p_{k-1,i}}, \qquad \text{and} \qquad P_{k+1}(x) = x P_k(x) + C_k P_{k-1}(x)$$

Find the roots x_i of $P_n(x)$.

3) Set $\alpha_i = \arccos x_i$, $i = 1, \ldots, n$. Sort the angles α_i.

Fig. 3 illustrates the flowchart of the recursive algorithm for on-line calculation of the optimized switching angles.

Using a computer algebra system, such as Maple or Mathematica, this recursive algorithm allows one to obtain $P_n(x)$ as an explicit function of m.

7. Simulation Results:

To perform the necessary calculations, a computer software package Maple was used. For organizing and analyzing all of the collected optimized switching angles α, the software package MATLAB were utilized. Using MATLAB, the collected switching angles were organized into look-up tables to be used later in simulations as shown in Table 1. Also, MATLAB was used to generate plots of α and *THD* versus *m*. The *THD* mathematically calculated by [11]:

$$THD = \frac{\left(\sum_{k=2}^{\infty} h_k^{\ 2} \right)^{0.5}}{h_1} \tag{17}$$

The inverter is loaded by single-phase capacitor-run induction motor with the following ratings: 175Watt, 220V, 1.22A, and 1275Rpm.

The best compromise between efficiency and quality of the inverter operation is achieved by the optimal switching pattern technique.

The solutions of the optimum switching angles α versus *m* for the MFS scheme are shown as in the Fig. 4. Note that not all the range of *m* has a solution, for example, in the case of 7-level (l = 7 or n = 3) OHESW scheme, there are solutions in the intervals: $m \in$ [1.65-2.07] and [2.41,2.45] only. On the other hand, for $m \in$ [0,1.64], [2.08-2.4], and [2.46-3] there are no solutions that solve the Eqs. (2).

The figure also shows the solution of α for 15-level OHESW. It can also be seen that the range of *m* decreases with increasing inverter voltage levels *l*. All the optimized switching angles (at *THDmin*) for different values of voltage levels *l* are illustrated in the Table 1.

Fig. 5 shows the plot of the voltage *THD* with *m* for two different values of *l*. It can be conclude from the figure that the increasing of *l* causes decreasing *THDmin* [See Fig. 6] until we get the lowest one; *THDmin*=6.4554 at *m*=4.925 for 15-level inverter. Fig.7 shows the instantaneous output voltage OHESW for both 7- and 15-level and with minimized *THD*. If the number of *l* is higher, a near-sinusoidal staircase voltage can be generated with only fundamental frequency switching.

The voltage spectra at *THDmin* corresponding to the instantaneous output voltage waveforms in Fig. 7 are illustrated as in the Fig. 8 which shows two examples, one for 7-stepped waveform and the other for 15-stepped waveform. The first shows that the 3^{rd} and 5^{th} harmonics are eliminated. Therefore the 7^{th} (350Hz) harmonic will appear in the spectra as a first harmonic. The second spectra show that the 3^{rd}, 5^{th}… 13^{th} harmonics are eliminated. Therefore the 15^{th} (750Hz) harmonic will appear in the spectra as a first harmonic. It can be seen that increasing *l*, causes increasing the eliminated harmonics. The normalized values of the first 15^{th} harmonics' are listed in Table 1.

The corresponding instantaneous line currents of motor fed by OHESW cascade multilevel inverter at *THDmin* are shown in Fig. 9. The ripple of current decreases with increasing l since increasing l (or n) causes eliminating more harmonics with lower orders, leaving harmonics with higher orders which cause increase of the motor impedance with frequency ($X=2\pi fL$); therefore the harmonic currents decreases. So, the induction motor can be represented as a good low pass filter. As a result, the current waveforms become more sinusoidal with increasing l. By using the voltage spectra and the equivalent circuit of the motor, the current spectra can be calculated as in Fig. 10.

8. Conclusions:

In this paper, a new technique is proposed and applied to a multilevel inverter to determine the optimum switching angles for eliminating the low order harmonics. It can be concluded that:

1. The current trend of modulation control for multilevel inverters is to output high quality power with high efficiency. For this reason, popular traditional PWM methods are not the best methods for multilevel inverter control due to their high switching frequency.

2. The Selective Harmonic Elimination (SHE) method has emerged as a promising modulation control method for multilevel inverters. But the major difficulty for the SHE method is to solve transcendental equations characterizing harmonics, the solutions are not available for the whole modulation index range, and it does not eliminate any number of specified harmonics to satisfy the application requirements.

3. To conquer the problem for the SHE method, the resultant method was used to find all the solutions to the harmonic equations if they exist by converting them into polynomial equations using trigonometric identities. However, increasing the number of switching n (or the order of harmonics to be eliminated) will lead to polynomial equations of higher degree which require several hours to solve. Therefore Resultant theory which proposed in previous work will not be effective of solving these polynomials.

4. The current paper presents a contribution to the multilevel inverters and gives fast algorithm for efficient on-line calculation of OHESW switching patterns and in real time for general number of the output voltage levels (or the number of the switching angles).

5. The proposed algorithm extends the 3-level unipolar optimal PWM switching scheme proposed in the previous work to be also used here for multilevel inverters.

6. By a transformation of variables, the solution to the OHESW problem is given by the roots of n-degree monic polynomial $P_n(x)$.

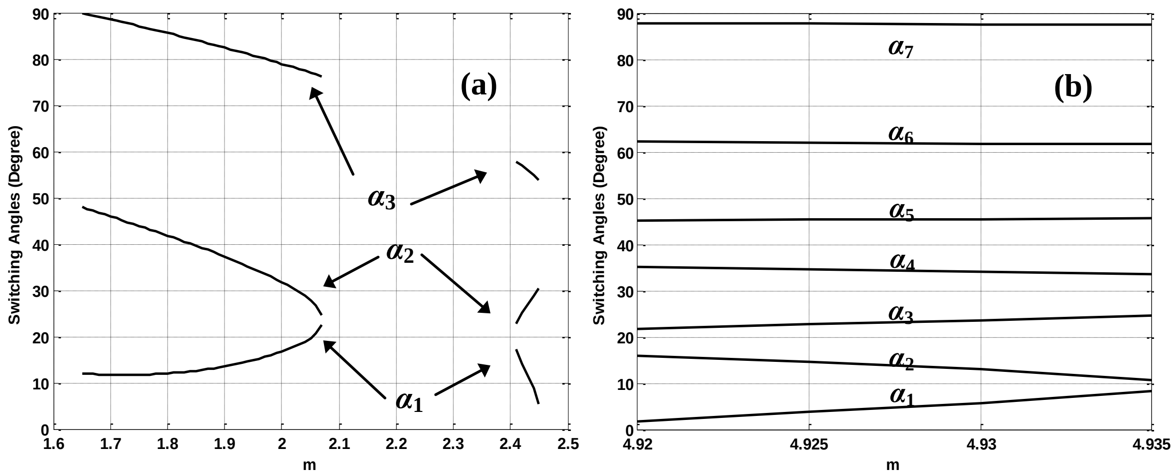


Fig. 4: the Solution of Switching Angles vs. m for Cascade Multilevel Inverter using OHESW Technique with (a) l=7 and (b) l=15

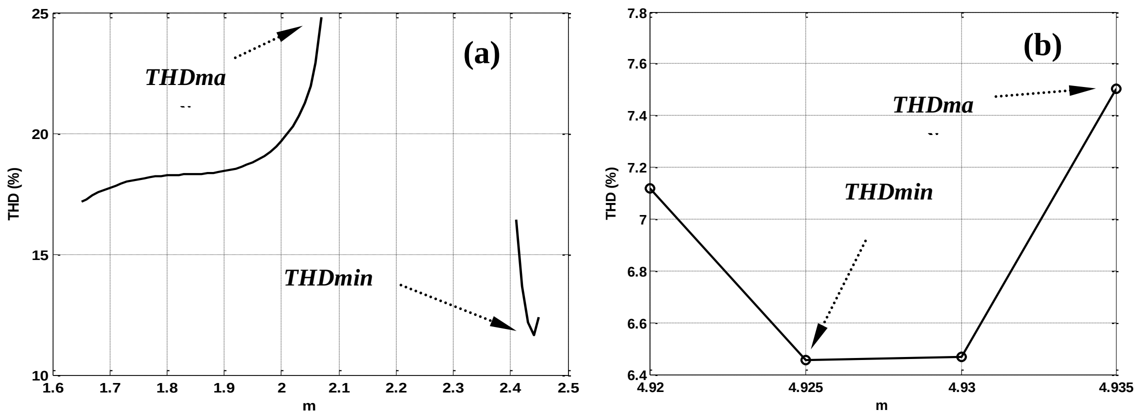


Fig. 5: the Voltage THD vs. m for Cascade Multilevel Inverter using OHESW Technique with (a) l=7 and (b) l=15

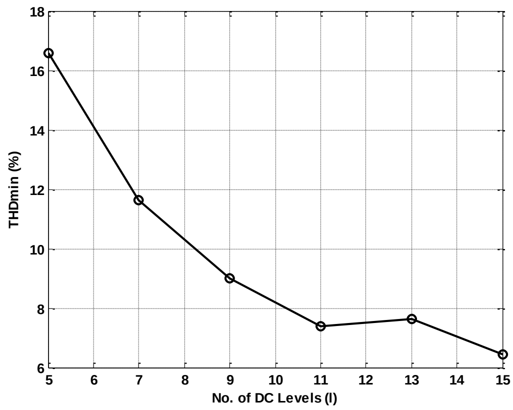


Fig. 6: the Lowest Voltage THD vs. Number of Voltage Levels (l) for Cascade Multilevel Inverter using OHESW Technique

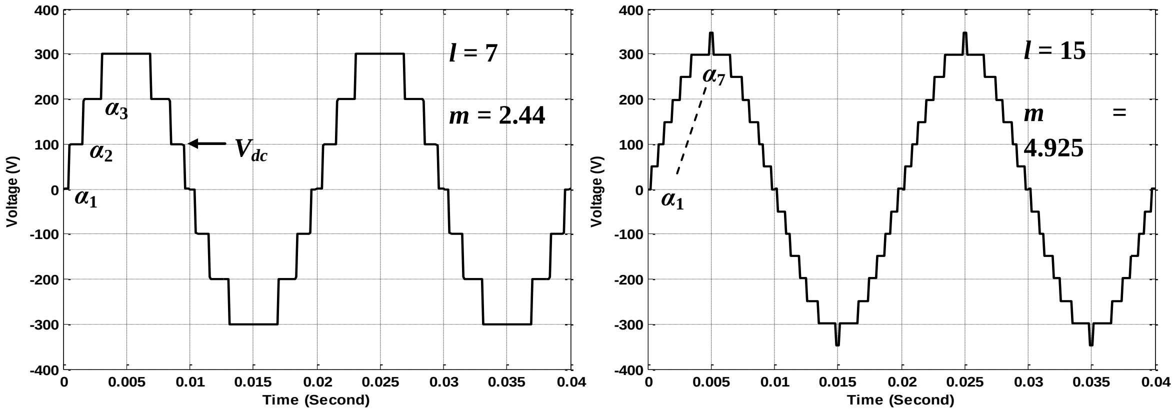


Fig. 7: the Instantaneous Voltage of OHESW Cascade Multilevel Inverter at THDmin with Two Values of Voltage Levels (l)

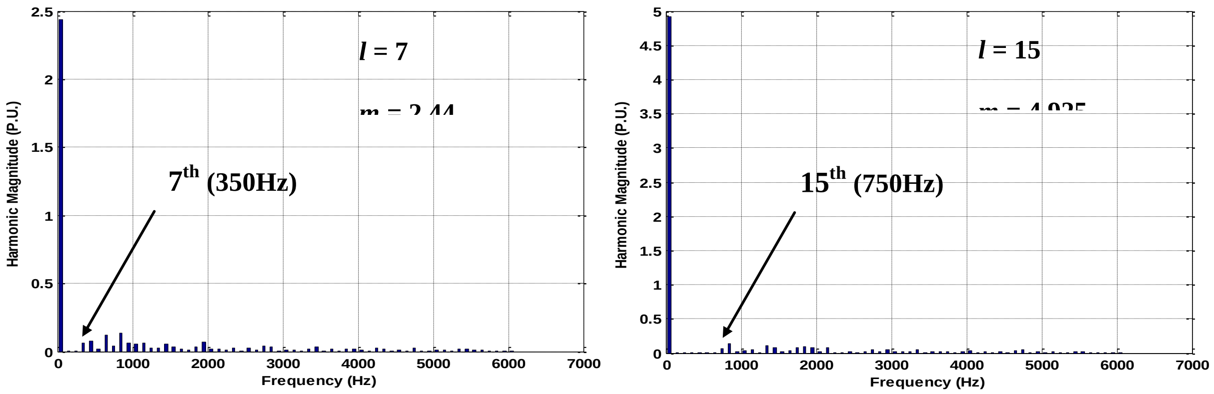


Fig. 8: the Harmonic Content of OHESW for Cascade Multilevel Inverter at THDmin with Two Values of l

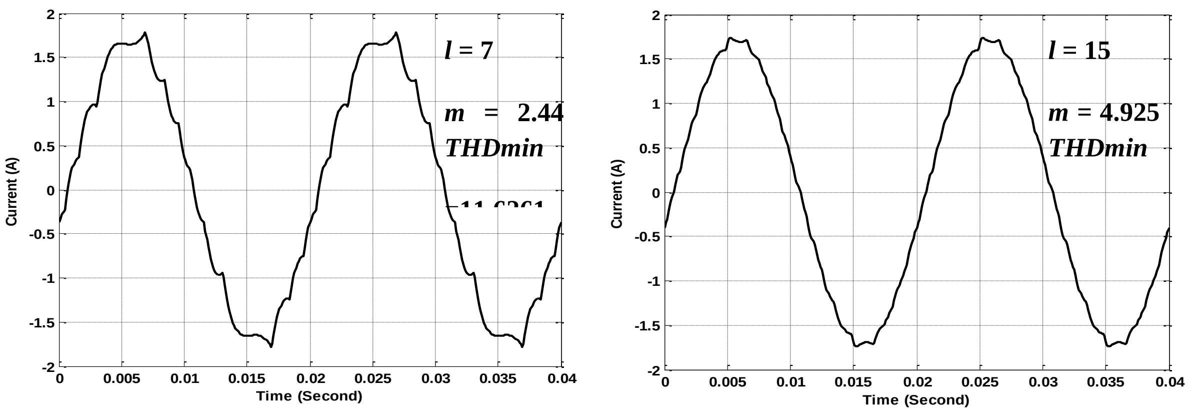


Fig. 9: the Instantaneous Current of Motor Fed by OHESW Cascade Multilevel Inverter at THDmin with Two Values of l

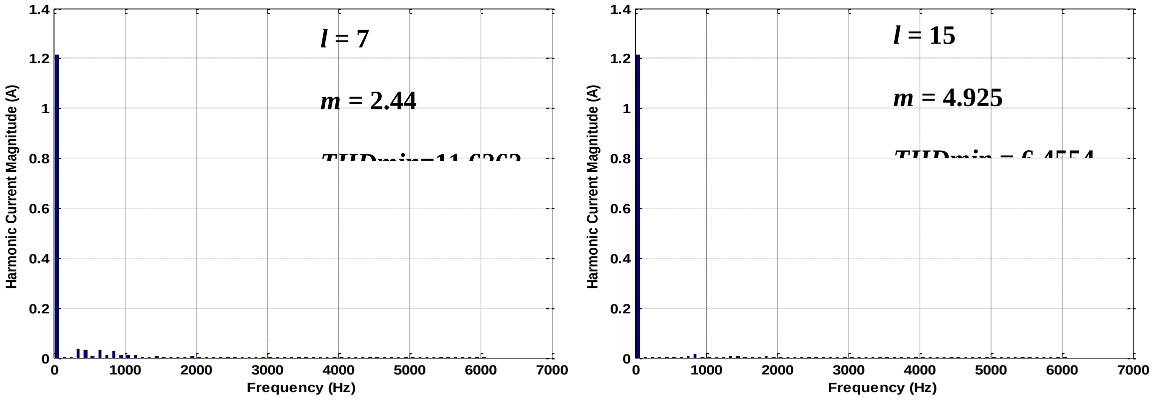
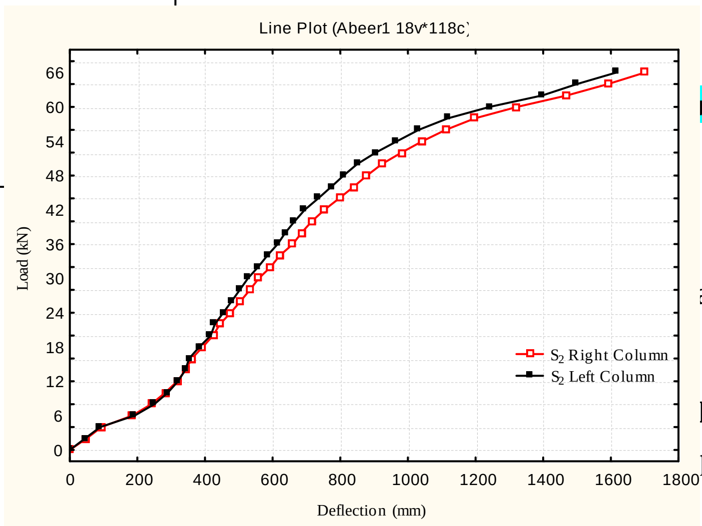


Fig. 10: the Line Current Spectra of Motor Fed by OHESW Cascade Multilevel Inverter at THDmin with Two Values of l

Table 1: the Switching Solution and Output Voltage Harmonics (in p.u.) of a Cascade Multilevel Inverter using the Proposed Algorithm

No. of levels (l)		5	7	9	11	13	15
Switching Angles (Deg.)	α_1	14.6172	8.7666	8.19508	5.67731	5.1996	3.9126
	α_2	45.3828	28.6886	21.0746	16.4853	16.5375	14.5571
	α_3	----	54.9395	37.0305	30.6968	28.4198	22.7573
	α_4	----	----	60.0804	42.0136	41.1376	34.5905
	α_5	----	----	----	63.6953	59.0302	45.2749
	α_6	----	----	----	----	87.2327	62.012
	α_7	----	---	----	----	----	87.647
m		1.67	2.44	3.22	4	4.15	4.925
Harmonic Magnitude (P. U.)	h_1	1.67	2.44	3.22	4	4.15	4.925
	h_3	0	0	0	0	0	0
	h_5	- 0.0784	0	0	0	0	0
	h_7	0.0751	0.0648	0	0	0	0
	h_9	0	- 0.792	- 0.0903	0	0	0
	h_{11}	- 0.1547	0.0154	0.0518	0.1019	0	0
	h_{13}	- 0.1252	0.1203	- 0.0194	- 0.0827	0.0628	0
	h_{15}	0	- 0.0380	- 0.1191	- 0.0698	- 0.1079	- 0.0624
	h_{17}	0.015	- 0.1352	0.0443	0.0652	0.0927	0.1429
	h_{19}	- 0.0346	- 0.0611	0.0677	- 0.0417	- 0.0014	- 0.0248
	h_{21}	0	- 0.0562	- 0.0631	- 0.0206	- 0.0379	0.0301
	h_{23}	0.0749	- 0.0622	- 0.0737	0.0769	- 0.0265	0.0495
	h_{25}	0.063	0.0249	- 0.0925	0.0194	0.1323	0.0118
	h_{27}	0	0.0280	- 0.0909	- 0.0140	- 0.0383	- 0.1037
	h_{29}	- 0.0014	- 0.0531	0.0231	- 0.0858	- 0.0339	0.0732
THD_{min} (%)		16.5924	11.6262	8.9907	7.3873	7.6396	6.4554
THD_{max} (%)		32.9620	24.8098	14.6081	12.1832	9.1754	7.5029



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