A Computer Aided Design for Internal Spur and Helical Gears

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Abstract

This research describes an interactive program to design internal gear pairs. From a specification, the program first performs a kinematic analysis to determine tooth numbers and to satisfy centre distance requirements, and then proceeds to calculate the face width. It is possible to call up any design standard in order to stress the gears; that used here is the British standard. Calculated face widths are compared with practical design and, if outside accepted bounds, the program will alter tooth numbers, or module or request a change in gear material.

A complete design is provided, which includes all the necessary information for manufacture. The design specification places tight tolerances on centre distances and/or gear ratios; the program automatically makes the required centre distance arid tooth profile corrections. On every occasion the program will find a satisfactory solution, or guide the user towards one. The program is flexible and allows the experienced designer maximum freedom to direct the design. The program contains many built-in databases, such as standard tooth cutters and material properties, and will draw the gear pair if required.

The research describes the design method and the structure of the program and a sample design is given and compared with a manual design to the same requirement.

الخلاصـــــة

يقدم البحث برنامج فعال لتصميم أزواج المسننات الداخلية، ويعمل هذا البرنامج على تحليل كينماتيكي لتحديد عدد الأسنان وضمان الأبعاد المركزية وبالتالي حساب عرض وجه السن وبشكل عام يستند أي تصميم على أنظمة قياسية (حيث تم هنا استخدام النظام البريطاني). إن حسابات عرض وجه السن هي مقرونة مع التصاميم العملية وفي حالة الخروج عن هذه المعطيات المقبولة فأن البرنامج سوف يعمل على تعديل عدد الأسنان أو زاوية السن أو طلب إجراء تغيير في معدن المسنن. يتضمن التصميم المتكامل جميع المعلومات الضرورية والخاصة بالتصنيع.

ترتكز المواصفات التصميمية على تفاوتات ضيقة للمسافات المركزية أو نسب المسننات، إن البرنامج يعمل بشكل آلي على تصحيح مسافة المركز والمقطع الجانبي للسن. ويضع الحلول المناسبة للمستخدم لمرونته ويعطي للمبرمج أو المصمم الحرية الكبيرة للتغيير بالتصاميم. يحتوي البرنامج أيضاً على كثير من قواعد البيانات مثل عدد قطع الأسنان القياسية ومواصفات المادة المستخدمة، وكذلك رسم زوج المسننات إن تطلب الأمر.

يوضح البحث طريقة التصميم وهيكلية البرنامج وعينة من التصميم وتمت المقارنة مع التصميم اليدوي لنفس المتطلبات.

1. Introduction

The program IGD (Internal Gear Design) is part of a suite of programs as shown in **Fig.(1)**, aimed to providing transmission engineers with a powerful design tool. The programs have been wholly developed for use on microcomputers and the gear software forms a part of a much larger system for designing power transmission drives as shown in **Fig.(2)**, which includes shaft finite element programs, fatigue analysis, bearing ratings and so on [1]. While the programs have been written for use by experienced engineers, the software makes use of an extensive knowledge base allowing non-specialised users to achieve satisfactory designs.

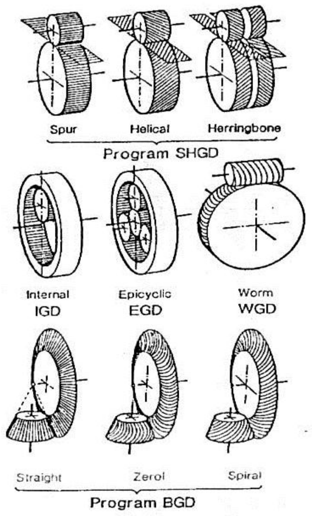


Figure (1) Gear pair design software*

* ***SHGD (Spur, Helical and Gear Design), BGD (Bevel Gear Design), EGD (Epicyclic Gear Design), WGD (Worm Gear Design), IGD (Internal Gear Design).***

The internal involute gear can have either spur or helical teeth (double helical are also used) cut on the inside of the annular ring. Internal gears are most commonly used in planetary or epi-cyclic gear trains but they can also be used as a compact ordinary reduction unit, as shown in **Fig.(1)**. Thus internal gears are used particularly where space is limited and when the rotational direction of the input and output shafts must be maintained. For a given tooth size they are stronger in bending and the contact stresses are lower than the equivalent external gear, giving higher power ratings, less wear and greater pitting resistance [2]. In general, the contact ratio is larger for a given gear ratio than the equivalent external gears and this together with the convex-concave contact, results in smoother and quite running [2]. A further advantage is that the shaft centre distance may be kept small, often allowing only a single pair of internal gears to be used where otherwise two sets of external gears are needed. A disadvantage of internal gears is that the annular gear is more expensive to manufacture than external wheels. Otherwise, internal gears are limited to much the same constraints as external gears in terms of permissible reduction ratios (about 10:1) and pitch line velocities (about 13 m/sec for helical gears) [2].

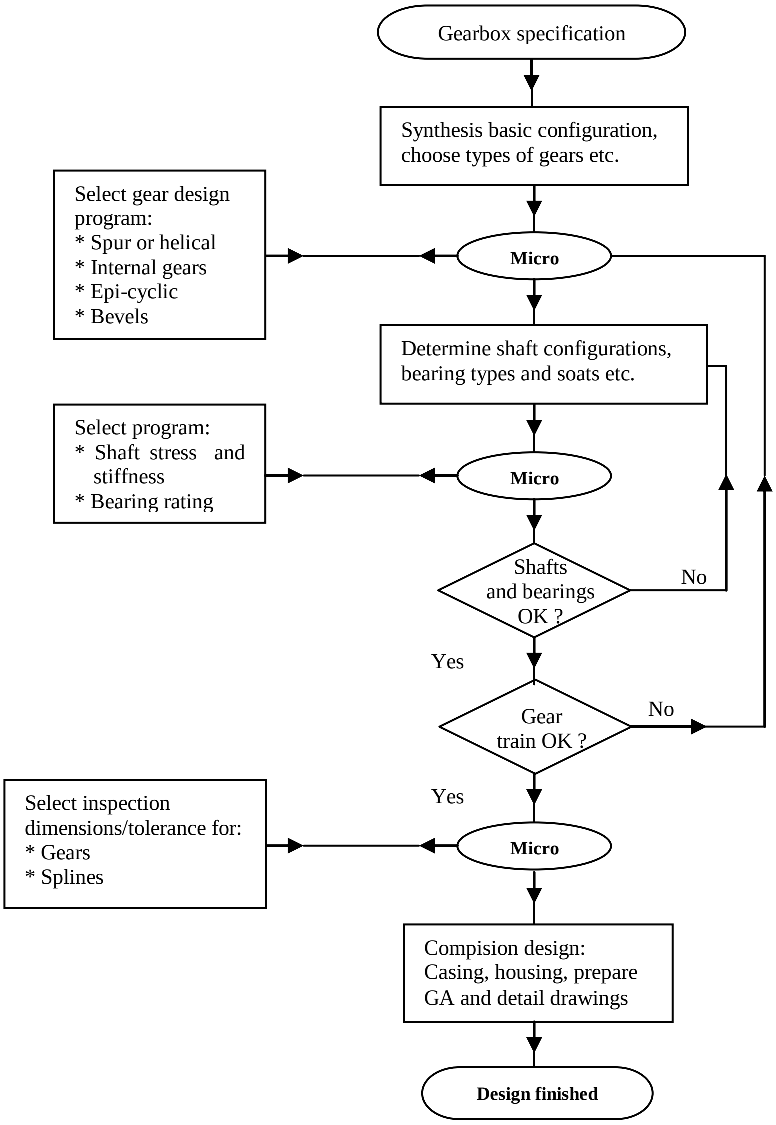


Figure (2) Computer aided design of transmission drives [2]

2. Theory

It would be stated that there is no universal method of designing gears. The choice of tooth profile, the numbers of teeth and whether standard or non-standard cutters are to be used depends entirely on the designer. Design standards for spur and helical gears, such as that used in Britain, BS 436 [2], only suggest which tooth sizes should be used and their proportions, and provide a method of gear stressing.

As an example, when the gears are stressed the standards provide no guide as to whether the face width-to-diameter ratios are satisfactory; all these decisions being left to the designer and very often form the basis of manufacturing companies' expertise. **Figure (3)** shows an internal gear pair and the terminology used in this research. The first stage of design starts by calculating the minimum number of teeth required on the pinion to avoid interference. Which is a function of the gear induction ratio and tooth pressure angle only? If less teeth are used, this will require tooth modification to avoid undercutting. Using the computed minimum number of teeth as a starting point, the number of pinion and annulus teeth can be determined to give the specified gear ratio. If the centre distance is specified, the module must be calculated and the nearest tooth size chosen. If a standard tooth cannot give the required centre distance either the tooth numbers must be changed and the whole process repeated, or the tooth profile coefficient must be calculated. This value must be checked to ensure that the tooth modification is within acceptable limits. If a helical gear pair is to be designed, the specified centre distance can usually be obtained by changing the helix angle. Changing the helix angle does not normally create problems in manufacture as this only necessitates altering the angle of the bobbing or cutting tool [2].

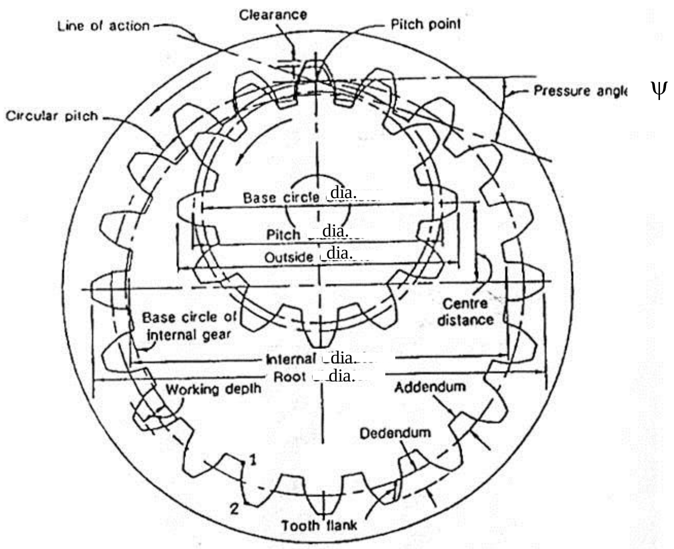


Figure (3) Internal gear pair geometry and nomenclature [2]

The second stage is to compute the contact ratio. Should this be less than 1.4, the designer can either increase the pinion teeth or modify the gear teeth addendum. If the pinion teeth are increased, the module must be recalculated and the centre distance checked again. This seemingly simple procedure can often involve a large number of calculations before a successful result is obtained. At this stage the kinematic analysis is complete, although if the design fails later on in the stress stage, it quite frequently arises that the designer must go back and selects a new tooth size.

The third stage is to determine the gear pair faces width. This involves selecting the gear pair materials and stressing the gears according to national design standards. In nearly all standards, for example ISO, BS and AGMA, this requires both the pinion and annulus to be stressed on the basis of bending strength and wear (ISO also considers scuffing), so that at least four calculations are needed. The maximum computed face width then becomes the face width for the gear pair. Wear calculations require the life of the gear train to be specified. If the gears run at a fixed speed the life can be simply stated, but should the operating power and speed cycle alter throughout the day, this requires an equivalent life to be calculated.

The method used in the British Standards Institution formula [2] for determining the strength of gear teeth is essentially the same as the Lewis equation [3], but instead of taking the load to act at the extreme corner of the tooth as in the Lewis method, in the BSI analysis the load is taken to act at the position it occupies at the moment when the whole load comes on to one tooth as shown in **Fig.(4)**. This introduces a strength factor, Y, which is dependent only on the tooth system used, and the number of teeth on the pinion and annulus.

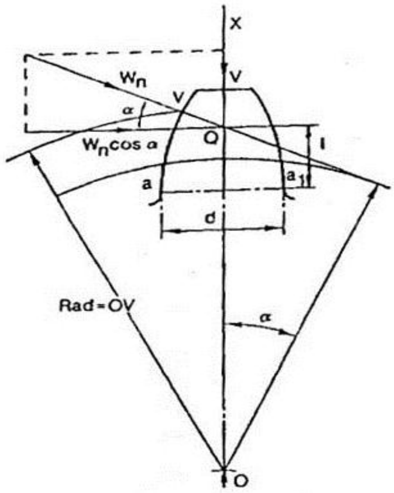


Figure (4) (a) The distance from pitch circle to tooth root, point V where the tooth carries the full load. (b) The dimensions d width of tooth root and l used in determining the strength factor, Y

Figure (5) shows the strength factor given in the standard for external gears [2]. For the internal gear the strength factor should be the same as that of a rack gearing with a pinion having the same number of teeth as the actual pinion, multiplied by $(1+3/T_B)$, where T_B is the number of teeth in the internal gear. In addition a speed factor, X_b, is introduced to account for fatigue loading, which depends on the gear running speed and time.

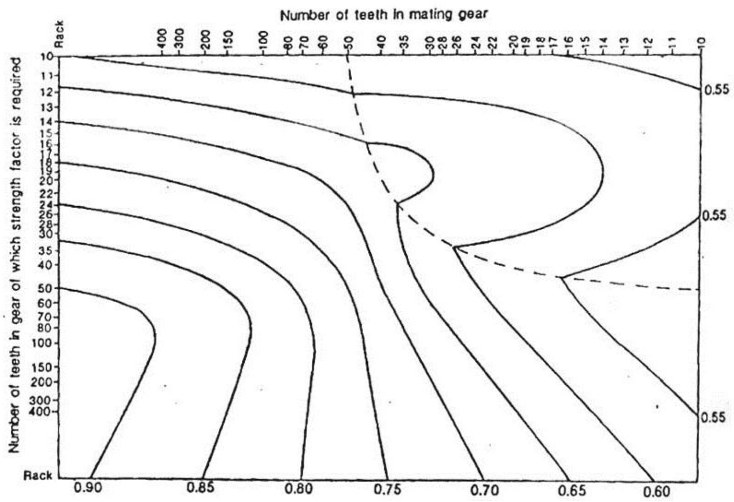


Figure (5) Strength factor (Y) chart for external spur and helical gears, from BS436 [2]

Figure (6) shows the speed factor for bending. Rearranging the BS formula for the bending strength of a tooth gives an expression for the face width b, viz,

$$\mathbf{b = WP / (S_b\, X_b\, Y)} \quad \text{..........} \quad \mathbf{(1)}$$

where:

W = safe load acting tangentially at the pitch circle

S_b = bending stress factor, given in the standard for all common gear materials

P = diametral pitch.

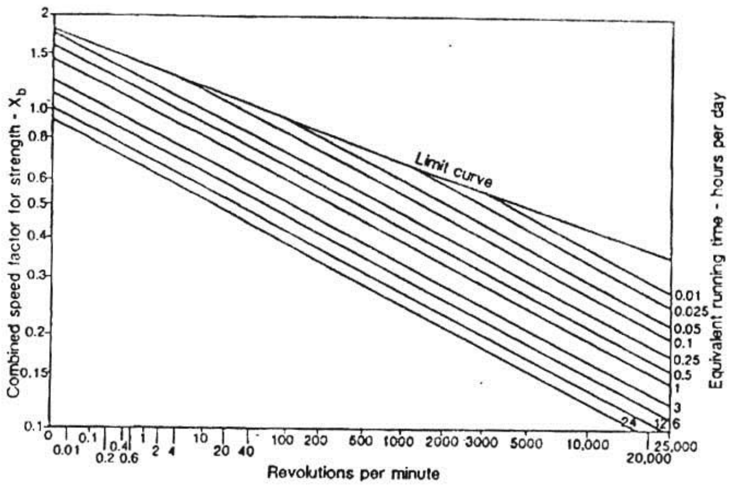


Figure (6) Speed factor chart (in bending), from BS436 [2]

The safe load on a tooth based on surface stresses employs Hertzian contact stress theory, which has been modified in the British Standards to include a speed factor, X_b and surface stress, X_c and a zone factor Z. The zone factor, like the strength factor Y, depends on the numbers of teeth in both mating gears and is found from a chart similar in form to that shown in **Fig.(5)**. For internal gears the zone factor should be equal to that or the same combination of external teeth multiplied by $[(R+1)/(R-1)]^{0.8}$ [3], where R is the gear ratio. The speed factor, for surface stresses, depends on the speed of the gears and the equivalent running time, and is similar to the chart in **Fig.(6)**. The face width based on surface stresses or wear is given by

$$\mathbf{b = W\, K\, (S_c\, X_c\, Z)} \quad \text{..........} \quad \mathbf{(2)}$$

where:

S_c = surface stress factor, as given in the code for gear material, and $K = P^{0.8}$

Apart from strength and wear requirements, the face width must also satisfy rigidity and manufacturing considerations. For spur gears, it is accepted that there is a practical limit on the face width in order to ensure proper tooth contact. This limitation can be taken to be a function of the module or circular pitch. For example, Johnson [4] suggests the use of a face width factor such that

$$\mathbf{b = C_r\, P} \qquad \text{...} \quad \mathbf{(3)}$$

where:

p = circular pitch.

C_r should not exceed 5 or 6, except for profile ground and shaved gears where C_r may be as much as 8. For helical gears equation (3) may be written:

$$\mathbf{b_n = C_{rn}\, P_n} \qquad \text{...} \quad \mathbf{(4)}$$

where:

b_n = face width in the normal plane
C_{rn} = face width factor in the normal plane
P_n = normal circular pitch.

Johnson [4] suggests that the same values for C_{rn} should be used as for spur gears. Dean [5] recommends that the face width should be at least twice the axial pitch so as to obtain reasonable benefit from the helical action. Based on rigidity requirements, the face width should be not less than about one-tenth of the pitch diameter of the annulus, but should not exceed about twice the pitch diameter of the pinion. The limitations on face width may be expressed by the following limit equations:

For spur gears

$$\mathbf{d_B / 10 < b < C_r\, p} \qquad \text{...} \quad \mathbf{(5)}$$

$$\mathbf{b < 2d_A;}$$

$$\mathbf{b < 2P_x;}$$

For helical gears

$$\mathbf{b_n < C_{rn}\, P_n}$$

$$\mathbf{2p_x < b_n < 2d_A} \qquad \text{...} \quad \mathbf{(6)}$$

$$\mathbf{2p_x < b_n < b}$$

$$\mathbf{d_B / 10 < b_n}$$

where:

P_x: axial pitch
d_A = pitch diameter of the pinion
d_B = pitch diameter of the annulus.

When internal gears are designed to a fixed or closely specified centre distance, the exact centre distance may not be possible to achieve without modifying the teeth profiles. If small numbers of teeth are used on the pinion, it is also necessary to give profile-shift modifications to avoid undercutting. This introduces the need for profile shift coefficients, designated k_A and k_B, for the pinion and annulus respectively. Following [6] the profile shift coefficients must satisfy the following conditions:

$$k_A > 0$$

$$-1 < k_B < 0$$

$$k_A + k_B < 0$$

If undercutting is to be avoided, the pinion profile shift coefficient must satisfy the following limit [7].

$$k_A > (17 - T_A)/17$$

where, T_A is the number of pinion teeth. It is necessary to check that the pinion crest width, w. does not become too small and hence too weak, as a result of tooth modifications. The crest width must therefore be calculated and limits set to ensure adequate strength. Finally, checks must be made to ensure that tip interference does not occur. Normally the tip clearance is made to be 0.2 mm or greater.

3. Computer Program

The design logic for the CAD of internal gear pairs is shown in **Fig.(7)**. The program stalls by asking the user for the complete specification, that is:

(1) Design to metric or imperial units.

(2) Design standard, for example, BSI, AGMA, ISO, etc.

(3) Gear ratio and allowable tolerances on the ratio.

(4) Centre distance (if this is known) with tolerances.

(5) Whether spur or helical gears, and if helical, the helix angle and allowable tolerance on the helix angle.

(6) Input speed.

(7) Power

(8) Pinion and gear material.

(9) Whether cut or ground teeth.

(10) Module definition, i.e. whether first or second preferred modules are permissible, or whether non-standard cutters can be used.

(11) Gear pair life.

At each stage the input data is checked against standard practice so that, for example. if the gear ratio exceeds about 20:1, a warning is given that this is excessive. Similar warnings are given for maximum operating speeds and power. At each stage options exist to help the user or provide additional information, such as a table of numbered gear materials which provides all the necessary data on bending and surface strength factor. The user has only to type in the corresponding number for the material properties to be inserted into the program automatically. The user can display the specification and make any changes before the design begins.

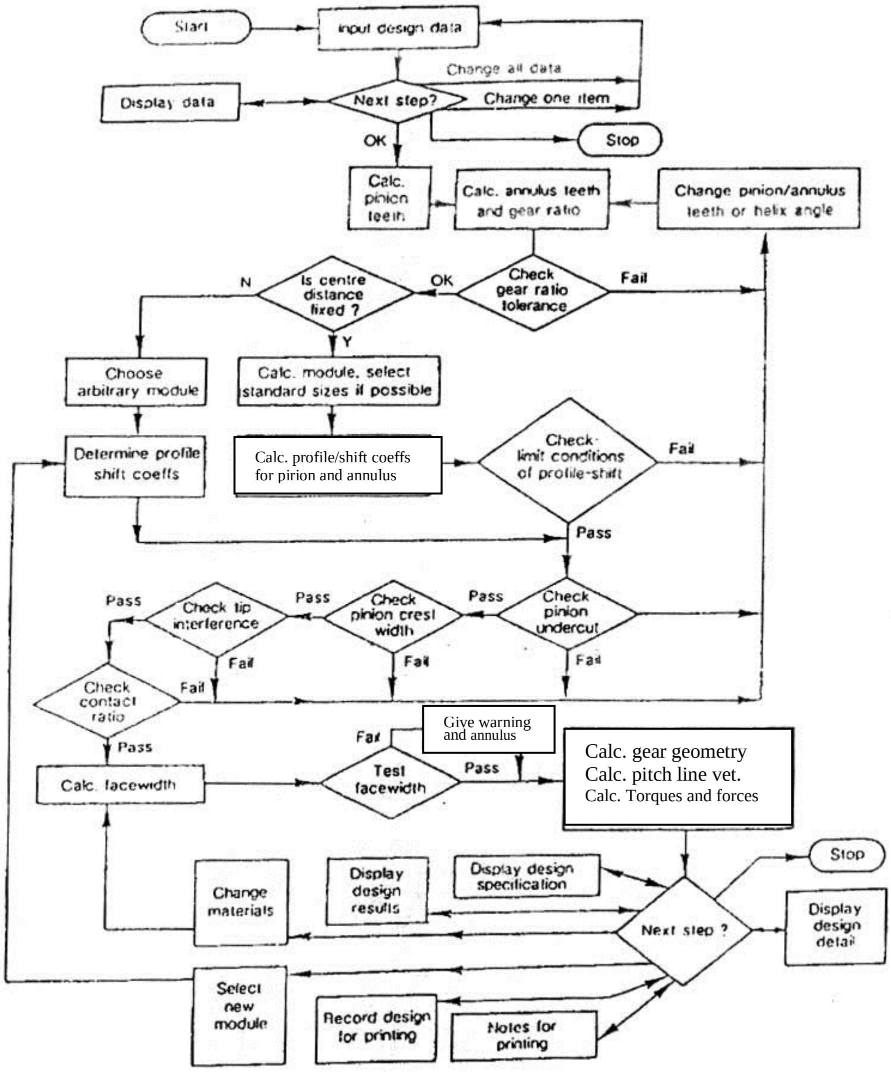


Figure (7) Flow chart block for internal gear pair design

The minimum number of pinion teeth to avoid interference is calculated and displayed on the screen. The user is then asked if this figure is acceptable, may then decide that the computed value is satisfactory, or may change to any desired number. This decision process is typical of the program as a whole. The object is to provide the maximum flexibility to the user, so that the program enables the designer to direct the design rather than leaving the decisions entirely to logic that could be built into the software. The nature of engineering is such that the unusual or non-standard design is occasionally required, and this approach enables any design to be attempted.

The annulus teeth are computed to give the required gear ratio and, if the centre distance is specified, the module is calculated and then rounded off to the nearest preferred size. The profile shift for pinion and annulus is then calculated. If the tolerances on the centre distance are large, no profile corrections are usually needed. If the tolerances are held to tight limits, the profile modifications may be needed and the profile shift corrections are checked against undercutting, pinion crest width, tip interference and involute interference, as described in the previous section, and finally a check is made on the contact ratio. Should any of these criteria fail, the program will select new tooth numbers, and in the case of helical gears a new helix angle and proceed again to search for a new solution, if no solution is possible the user is directed toward the necessary changes needed in the specification to make a viable design.

4. Sample Design

A number of products have been built using this program [8]. As an example the program results are compared with a design produced manually. The design specifications are as follows:

(1) Power 5.74 kW
(2) Pinion speed 3000 revs/min
(3) Gear ratio 4:1, zero tolerance
(4) Centre distance 60 + 0.05 mm
(5) Materials: pinion, En8, hardened and tempered; annulus, En8, hardened and tempered.
Ordinary cut finish on both gears
(6) Life 10,000 hrs

Table (1) shows the leading gear dimensions for the manual and computer designs. Computer solution 1 is identical to that obtained manually. Computer designs 2 and 3 show the effect of changing material treatment and going to a stronger material, and the effect of design life on the face width. Solutions 4 and 5 show the effect of changes in tooth numbers and life. All the computer results shown are for designs that met the criteria built into the program. Note that the last solution would result in a smaller gear pair.

Table (1) Leading gear dimensions

Design alternative no.	Materials * (pinion annuals)	Working life (hrs)	T_A	T_B	m_n	d_A mm	d_B mm	Centre distance mm	b mm
Manual	En8 H&T	10,000	20	80	2	40	160	60	24.0
1	En8 H&T	10,000	20	80	2	40	160	60	23.6
2	En8 SH	25,000	20	80	2	40	160	60	25.8
3	En24 SH	25,200	20	80	2	40	160	60	16.5
4	En8 H&T	3,600	17	68	2	34	136	51	23.3
5	En8 H&T	10,000	17	68	2	34	136	51	32.7

** Materials: En8 – carbon steels BS 970 En8 H&T (hardened and tempered) ; En8, carbon steels BS 970 En8 SH(surface hardened); En24 SH, alloy steels BS 970 En24 SH (surface hardened).*

Tables (2) and **(3)** show the design results and design details for solution 1. **Figures (8)** and **(9)** are the computer graphics output, which shows the gear pair and pinion tooth detail.

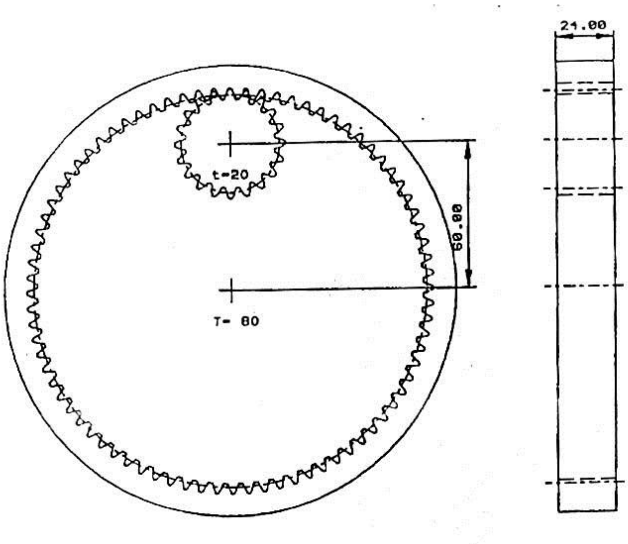
Table (2) Gear design results

Pinion teeth = 20 Module = 2.0 mm Gear ratio = 4.0	Annulus teeth = 80 Error = 0.0
Spur gear Centre distance = 60.0 mm Centre distance extension = 0.0	
Face width = 23.6178 mm Reasonable FW = 6.0-36.0 mm Pressure angle = 20^o	Width/module = 11.81

Pinion wear governs face width; Face width reasonable.

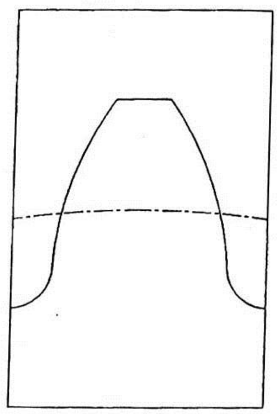
Table (3) Design Details

	Pinion	**Annulus**
Material	En8S:H&T	En8S:H&T
Number of teeth	20	80
P.C.D.	40.0 mm	160.0 mm
Outside diameter	45.0 mm	157.0 mm
Root diameter	36.0 mm	166.0 mm
Base diameter	37.58 mm	150.35 mm
Addendum	2.50 mm	1.50 mm
Dedendum	2.0 mm	3.0 mm
Profile sheet coefficient	0.25 mm	-0.25
Pitch line velocity	6.28 m/sec	6.28 m/sec
C.R. (in transverse)	1.68	1.68
Total contact ratio	1.68	1.68
Speed	3000.0 revs/min	750.0 revs/min
Torque	18.29 Nm	73.18 Nm
Safety factor	24.26	22.56
Tang force	914.72 N	-914.72 N
Radial force	333.0 N	-333.0 N
Axial force	0.0	0.0



Module = 2.000
K1 = .250
K2 = - .250
Ratio = 4.00
Angle = .00
Scale = .7

Figure (8) Sample gear pair graphics



Pinion teeth profile
Crest width calculated = 1.1901
Minimum crest width permitted = 1.0000
Topping is not necessary

Figure (9) Pinion tooth detail

5. Conclusions

The work described is part of a larger program which will also compute the gear inspection dimensions, all manufacturing information such as span gauging dimensions, tooth tolerances on profile, alignment, run-out and tooth composite error and backlash. The example inhand partly indicates the power of using computers for gear design. The average manual time required to produce a satisfactory design is about I day. This can be reduced to around 30 min using CAD, but this will include looking at a large number of solutions. This introduces the prospects of optimizing a design by a manual iterative process using the computer as a quick calculator. Beyond this level, the program can form the basis of a more sophisticated approach to optimization, enabling the user to immediately produce designs having a minimum centre distance or lowest weight, without searching for the best solution.

Finally the internal gear program described here formed the basis of a program to design planetary gear systems.

6. References

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