Finite Element Calculation of Leakage Reactance in Distribution Transformer Wound Core Type Using Energy Method

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Abstract

This paper presents the accurate energy method for calculation of leakage reactance of wound core transformers. It takes into consideration the curvature of the winding. The energy technique procedures for computing the leakage reactance are based on finite element analysis. This method is very efficient compared with the classical design methodology which based on magnetic circuit theory.

The electromagnetic stored energy is obtained by leakage magnetic flux computing based on finite element method (FEM), and then leakage inductance can be computed.

The Finite Element model of distribution transformer with non-linear magnetic characteristic for iron core is built using software "ANSYS".

Two dimensional (2D) and three dimensional (3D) finite element modeling of distribution transformer have been used to analyze the leakage field,

The obtained results have shown that the 3D model provides higher accuracy in the prediction of the leakage reactance than 2D model, with respect to the test value, due to the better representation of the transformer geometry especially the portions of the coil out of the core window(End and Curvature region),Therefore, the adoption of this energy technique during the design phase is able to enhance transformer manufacturer ability to predict the transformer leakage reactance, thus resulting to better performance and reducing the design time and cost before manufacturing .

Two types of analyses are performed, including static and transient analysis. Finally, the transformer leakage reactance is calculated and compared with the value obtained from the actual test. The results obtained are very similar with the test values.

الخلاصة:

يقدم هذا البحث طريقة الطاقة الدقيقة لحساب مفاعلة التسرب في محولات التوزيع ذات القلب الحديدي الملفوف وتأخذ بنظرألاعتبار المناطق المنحنية في الملف. وتعتمد إجراءات تقنية الطاقة لحساب مفاعلة التسرب على طريقة تحليل العنصرالمحدود (FEA) .هذه الطريقة فعالة جدا بالمقارنة مع منهجية التصميم الكلاسيكي التي تعتمد على نظرية

الدائرة المغناطسي يتم الحصول على الطاقة المغناطيسية المخزونة من خلال حساب الفيض المغناطيسي المتسرب ومن ثم حساب المحاثة. بناء موديل المحول بخصائص لاخطية للقلب الحديدي بطريقة العناصر المحدودة *(FEM)* وبأستخدام برنامج هندسي *(ANSYS)*. في هذا البحث أستخدم موديلات ثنائية الابعاد وثلاثية الابعاد لمحول التوزيع لتحليل الفيض المغناطيسي المتسرب. وأظهرت النتائج التي تم التوصل أليها ,إن موديل الثلاثي الابعاد يوفر أعلى قدرة من الدقة في التنبوء بقيمة المفاعلة التسريبية من موديل ثنائي الابعاد نسبة الى القيمة الفحص, ويرجع ذلك الى التمثيل الجيد للشكل الهندسي للمحول وخاصة لاجزاء الملفات التي تقع خارج نافذة القلب الحديدي (مناطق نهاية الملف والمناطق المنحنية). لذلك فأن أعتماد طريقة الطاقة المخزونة أثناء مرحلة التصميم تساعد الشركة المصنعة للمحولات على التنبوء بقيمة المفاعلة التسريبية وتحسين الأداء وتقليل الوقت والكلفة قبل تصنيع المحول. تم إجراء نوعين من التحليلات، بما في ذلك تحليل (الساكن) و (العابر) واخيرا تم حساب مفاعلة التسرب ومقارنتها مع قيمة التي تم الحصول عليها من الفحص الفعلي للمحول وكانت النتائج متطابقة جدا مع قيمة الفحص.

1-Introduction

In power systems, the transformer is one of the essential elements, and the distribution transformers are the heart of every electrical distribution system, and its failures can cause serious problems in electric utility operation. Furthermore as transformer remain energized for all the 24 hours of the day, whether they are supplying any load or not, constant losses occur in it for the whole day, and copper losses are different during different period of the day. Therefore both efficiency and the voltage drop in a transformer on load are chiefly affected by its leakage reactance, which must be kept as low as design manufacturing techniques would permit [1]. For this purpose it is crucial to be able to calculate leakage reactance.

The transformers manufacturing industry improve transformer efficiency and reliability. Transformer efficiency is improved by reducing load and no-load losses and transformer reliability is improved mainly by the accurate evaluation of the short-circuit reactance and the resulting forces on transformer windings under short-circuit, since these enable the avoidance of mechanical damage and failures during short-circuit tests and power system faults[2].

The Leakage reactance is one of the important characteristic parameters of transformers. It is necessary and very helpful to calculate the reactance's accurately in the transformer design before making it, and Leakage reactance calculations play an important role in designing geometry of transformers. The design parameters may be varied as such that the required short circuit Leakage reactance is determined [3].

The calculation of leakage reactance is performed in many papers by using different analytical methods [4],[5],[6] and numerical methods[7],[8],[9], but most of the analytical methods are not accurate, especially when the axial length of HV and LV windings are not equal.

There are different techniques for the leakage-reactance evaluation in transformer; the most common technique is the use of the flux leakage elements and estimation of the flux in different parts of the transformer [2], [10].

The images technique (Rogowski method) [7, 11] which have been established in the first half of the last century .The base of this method is considering the image of every turn of the winding with the effect of iron core taken into account. The Main weaknesses of image

method are Incapability to calculating reactance when the axial lengths of HV and LV windings are not equal and with unbalanced windings, and assume that μ= ∞ in all calculations [12].

In 1928 Roth [11], [13] introduced a considerably more advanced method of calculation. He extended Rogowski's analysis by using a double-Fourier-series solution to calculate the leakage reactance for irregular distribution of windings. The advantage of this method is that it is applicable to uniform as well as non-uniform ampere-turn distributions of windings. The ampere-turn distribution was transformed into a double Fourier series axially and radically, which could be solved analytically and the disadvantage of this method is failed to take into account the field curvature.

In 1956 L. Rabin's [11] presented a solution for axi-symmetric fields, more suitable for numerical calculations. He also used Fourier series representation of the ampere-turn distribution, but only in the axial direction. The field was considered to be unbounded in the radial direction. In this method the effect of winding curvature is taken into account and became more suitable.

During recent decades the development of the philosophy of transformer design has been a logical extension of the use of computers and numerical techniques enabling one to model accurately the geometrical complexities as well as the nonlinear material characteristics for problem analysis. Numerical modeling techniques are now-a-days well established for transformer analysis and enable representation of all important features of these devices [14]. Among the numerical techniques, the most popular method for the solution of electromagnetic field problems is the finite element method (FEM). The main advantage of the FEM is its ability to deal with complex geometries, as well as properties of the materials and it yields stable and accurate solutions [15].

Finite element analysis (FEA) is now very important tool during the transformer design phase, when the manufacturer needs to check the correctness of the transformer leakage reactance or short-circuit impedance. The transformer leakage reactance determination using FEA had already been done in [3], [16], [17], [18].

In the present paper, finite element techniques are used for the magnetic field analysis of three phase, wound core, distribution transformers. The analysis focuses in the leakage field evaluation for calculation of leakage inductance in transformer using the electromagnetic stored energy in the winding and surrounding air volumes. The proper modeling and post-processing operation are of great importance. In this paper two dimensional and three dimensional finite element modeling of the three phase distribution transformer have been used to analyze the leakage field. Just one half of the whole model of the three phase transformer was modeled for 2D modeling and quarter of the whole model for 3D modeling.

By using magneto-static analysis, the magnetic vector potential of the model nodes was calculated, and then the flux distribution over the model was obtained. Then, in the post-processing stage, by using the energy storage method, the leakage reactance of the transformer windings was calculated.

The transformer that was considered in this paper is a 400 kVA, (D/Y) connected, rated voltages (11000/ 416 V), three-phase, wound core, oil-immersed, distribution transformer. The main design parameters of this transformer were taken from the design documents from the manufacturing company (Diyala Company of Electrical Industries)[19].

 The transformer models are analyzed with *ANSYS 11* software electromagnetic packages that solves problems of electromagnetic fields in two and three dimensions based on the FEA. The *"ANSYS"* Package provides an excellent and accurate analysis tool.

The calculation of Leakage reactance in the transformer winding were studied by using two types of analysis (static and transient) for the non-linear transformer models.

For the validation of the model, the obtained results of the solution are compared with the results obtained from the actual routine tests performed to the transformer at the factory.

2- Leakage Reactance

The definitions of leakage inductance is based on an academic consideration of the electromagnetism, that not all the magnetic flux generated by AC current excitation on the primary side follows the magnetic circuit and link with the other windings complete. Some flux leaks from the core and returns to the air, winding layers and insulator layers.

 This flux exists in the spaces between windings and in the spaces occupied by the windings. The magnitude of this leakage flux is the function of the number of turns in the windings, the current in the windings, and the geometry of the core and windings. [20] [21].

3- Leakage reactance calculation

The leakage reactance of a transformer is one of the most important specifications that have significant impact on its overall design and the Leakage reactance calculations play an important role in designing geometry of transformers.

There are different techniques for the leakage-reactance evaluation in transformer using different analytical and numerical methods. But most of the analytical methods are not accurate, especially when the axial length of HV and LV winding are not equal.

3-1 Analytical methods

 Several methods have been applied to determine the leakage field distribution and the leakage reactance in transformer. Most of them are based on magnetic field calculations for simplified configurations. Among analytical methods, the most popular method for the leakage-reactance evaluation in transformer is the classical method.

In the classical method the leakage flux can be calculated by using the concept of equivalent magnetic circuits and this method was based upon simplifying assumptions of the leakage field being unidirectional and without curvature.

This method has certain limitations: the effect of core is not taken into account. It is also not take into account axial gaps in windings and asymmetries in ampere-turn distribution.

The transformer manufacturers are often employed this method in order to simplify the time and complexity of the calculations required in automated design process.

The classical method is first approach for reactance calculation is based on the fundamental definition of inductance in which inductance is defined as the ratio of total leakage flux (λ) to a current (I) and the leakage flux for a two-winding transformer, based on the above assumptions is [11]:

$$L = \frac{\lambda}{I} = \frac{N\emptyset}{I} \qquad \text{------ (1)}$$

$$\lambda_P = \mu_0 N_P^2 I \frac{L_{mt\,(P)}}{L_C}\left(\frac{a}{2} + \frac{d_P}{3}\right) \quad \text{------ (2)}$$

$$\lambda_S = \mu_0 N_S^2 I \frac{L_{mt\,(S)}}{L_C}\left(\frac{a}{2} + \frac{d_S}{3}\right) \quad \text{------(3)}$$

All considered parameters in above equations shown in Fig.1 which shows a part section of a transformer taken axially through the Centre of the wound limb and cutting the primary and secondary windings. The principal dimensions are marked in the figure, as follows:

L_{mt} is Mean length of primary and secondary turns

L_C is axial length of windings (assumed the same for primary and secondary)

a is the radial spacing between windings

ds is the radial depth of the secondary winding next to the core

d_P isthe radial depth of the primary winding (outer winding)

N_P is the number of turn of the primary

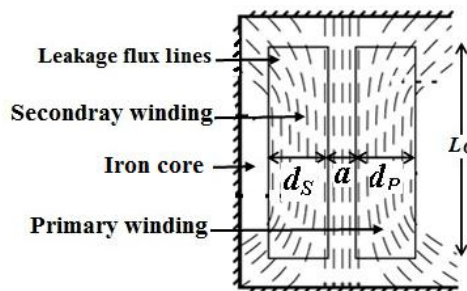N_S is the number of turn of the primary

Fig.1 part section of a transformer

Using the following equation:

$$X = \frac{2\pi f \lambda}{I} \qquad \text{------ (4)}$$

And reflecting leakage reactance between windings to the primary *side yields*

$$X_L = X_p + \left(\frac{N_p}{N_s}\right)^2 X_s \qquad \text{------ (5)}$$

Equation 5 will be as follows

$$X_L = 2\pi f \mu_0 N_P^2 \frac{L_{mt\,(P)}}{L_C}\left(\frac{a}{2}+\frac{d_P}{3}\right) + \left(\frac{N_p}{N_s}\right)^2 2\pi f \mu_0 N_S^2 \frac{L_{mt\,(S)}}{L_C}\left(\frac{a}{2}+\frac{d_S}{3}\right) \qquad \text{------- (6)}$$

If it is assumed that $L_{mtp} = L_{mts}$ (meaning that the length of each turn of primary and secondary windings are equal). Equation 6 can be simplified as follows:

$$X_L = 2\pi f \mu_0 N_P^2 \frac{L_{mt\,(P)}}{L_C}\left(a + \frac{d_P+d_S}{3}\right) \qquad \text{------ (7)}$$

The magnitude of this leakage flux is a function of the geometry and construction of the transformer. This is the conventional equation used in References [1], [8],[11]. Furthermore, in the engineering applications, the value of leakage reactance can show the percentage of leakage reactance voltage and rated voltage, which is written as

$$\%IX = \frac{IX_L}{V_{Rated}} \times 100 \qquad \text{------- (8)}$$

In case of rectangular winding of "wound core" transformer shown in Fig. 2, the calculation of leakage reactance in the axial and radial direction as follows[15][22].

$$\%IX_a = \frac{8 \times f \times I_p \times T_P^2 \times S_a}{L_a \times V_P} \times 100 \qquad \text{------- (9)}$$

$$\%IX_r = \%IX_a \times \frac{S_r}{S_a} \times \frac{L_a}{L_r} \qquad \text{------- (10)}$$

$$S_a = A_g + \frac{A_P + A_S}{3} \qquad \text{-------- (11)}$$

$$L_a = \frac{a+d_s+d_p}{3} + \frac{h_P+h_S}{2} \qquad \text{-------- (12)}$$

$$S_r = \frac{1}{3} \times L_{mt} \times h_S \times \left[\frac{\frac{1}{2}(h_s - h_p)}{H_S}\right]^2 \qquad \text{----- (13)}$$

$$L_r = 1.3 \times (a + d_s + d_p) \qquad \text{-------(14)}$$

And the total per cent leakage reactance

$$\%IX_L = \%IX_a + \%IX_r \qquad \text{-------- (15)}$$
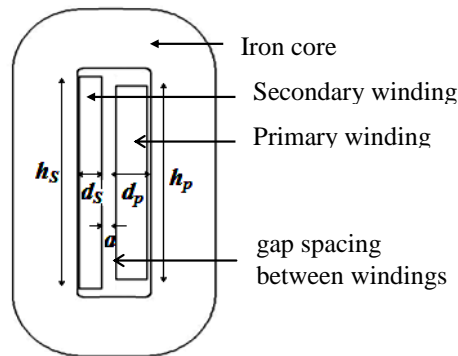

Fig.2 Part section of wound core transformer

Where

$\%IX_a$, $\%IX_r$: Leakage reactance in the axial direction and radial direction respectively.

f: Rated frequency , I_P : Phase current of primary winding , T_P : Turn of primary winding

L_a :Leakage flux length in axial direction. , L_r :Leakage flux length in radial direction

A_P: Cross section area of primary winding. , AS: Cross section area of secondary winding

A_g: Cross section area of gap spacing between windings

S_r : Equivalent leakage area of winding in radial direction

S_a : Equivalent leakage area of winding in axial direction

h_P , h_S : Height of primary and secondary windings

L_{mtp} :Average mean turn of primary &secondary windings

3-2 Numerical method

Transformers involve magnetostatic problems. These problems can be solved by analytical and numerical techniques. The limitation of the analytical techniques as well as the progress of computers has facilitated the development of numerical techniques for the solution of electromagnetic field problems. The most important numerical techniques are the following: (Finite difference method), (Boundary element method), Finite element method

Among the numerical techniques, the most popular method in the solution of magnetostatic problems is the Finite Element Method (FEM). The main advantage of FEM is that any complex geometry can be analyzed since the FEM formulation depends only on the class of problem and is independent of its geometry. Another advantage is that it yields stable and accurate solutions [15] [23].

3-2-1 Finite Element Method:

The finite element method (FEM) is a numerical technique for obtaining approximation solutions to boundary value problems of mathematical physics, which can be described by partial equations .The basic step involved in finding the solution usually begins with the sub division of the problem domain into well defined simple sub domains called element.

A variety of element shapes may be used, and different element shapes may be employed in the same solution region. The corners of the finite element are called grid points or nodes. These nodes are assigned to each element and then the interpolation function chosen to represent the variation of the field variable over the element [24].

The finite element model contains information about the device to be analyzed such as geometry (sub divided into finite elements), material, excitations, and constraints. The material properties, excitations and constraints can often be expressed quickly and easily but geometry is usually difficult to be described.

There are generally two types of modeling that are used in analysis: 2D and 3D modelling. While 2D modeling conserves simplicity and allows the analysis to be run on relatively normal computer, the 3D modeling, however, produces more accurate results, and run on the fastest computers. (FEM) is the most commonly used numerical method for reactance calculation of non-standard winding configurations and asymmetrical/ non-uniform ampere-turn distributions, which cannot be easily and accurately handled by the classical method.

Early work on FEA of transformers was presented over four decades ago by P.Silvester and Andersen [13], [25], focused on 2D modeling, due to the restricted performance abilities provided by the early development of personal computers.

The 3D solution becomes necessary, due to nature of the transformer structure (asymmetrical), and 3D analysis is essential for more accurate calculations even though it may be computationally very time consuming.

Many commercial 2-D and 3-D FEM software packages are now available [26] and many manufacturers develop their own customized FEM programs for optimization and reliability enhancement of transformers.

4- Electromagnetic filed in transformers

Transformer is one of the electromagnetic devices whose behavior can be described by field equations. The electromagnetic fields inside the transformer at low frequencies, with displacement current ignored, are described by a subset of Maxwell's equations. A general formulation of electromagnetic field problems in electrical machine has already been presented by many authors [11], [24].

In this section the partial differential equations of the vector and scalar potentials are derived from Maxwell's equations that is required for leakage reactance calculation

$$\nabla \times H = J \qquad \text{(Derived from Ampere law)} \qquad \text{------ (16)}$$

$$\nabla \cdot B = 0 \qquad \text{(Derived from Gauss law)} \qquad \text{------ (17)}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \text{(Derived from Faraday's law)} \text{ ------ (18)}$$

Where H: the magnetic field strength. , J: the current density. , B: the magnetic flux density
   E: the electric field strength.
The equations that describe the material properties are:

$$B = \mu H \quad \Rightarrow \quad H = \upsilon B \qquad \text{------- (19)}$$

$$J = \sigma E \qquad \text{------- (20)}$$

Where υ: the magnetic reflectivity (reciprocal of magnetic permeability μ),
 σ: the electrical conductivity
And the relation between magnetic flux density (B) and magnetic vector potential (A) is:

$$B = \nabla \times A \qquad \text{------ (21)}$$

Substitution of (21) into (16) using relation (19) gives the field equation describing the vector potential [38], [39].

$$\nabla \times (\upsilon \nabla \times A = J \qquad \text{------- (22)}$$

Solving equation (22), magnetic vector potential (A) can be calculated and solving equation (21), magnetic flux density (B) can be calculated.

5-The transformer configuration

The transformer under consideration is a 400 KVA,(delta / star) connected ,rated primary voltages 11 kV, rated secondary voltage 416V, three-phase, wound core, oil-immersed, distribution transformer. Fig.(3) shows the active part of the three-phase, wound core, distribution transformer considered .The secondary winding comprises 19 layers (per phase) of copper sheet, while the primary consists of 914 turns (per phase) of insulated copper wire. In a typical rectangular wound core type transformer, the low voltage winding (secondary) is mounted about the vertical axis of a core leg, the high voltage winding (primary) is located around the outside of the low voltage winding and separated form it by the high-low space insulation. Fig.(4) illustrates the perspective view of LV and HV winding one-phase.

The transformer magnetic circuit is of shell type and is assembled from two small and two large irons wound cores, Fig.5 shows the small and large iron wound cores.

The main design parameters and the dimensions of this transformer under the study were taken from the design documents from the manufacturing company (Diyala Company of Electrical Industries) [19] as shown in Table (1).

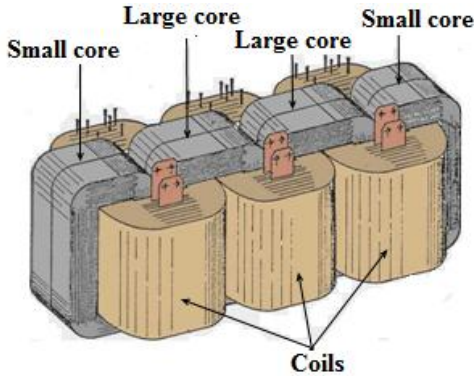**Table. (1) Design parameters of the Transformer**

Fig.3 Active part configuration of the Wound core distribution transformer

Rating	Capacity: 400 KVA Voltage :11000 ±5% / 416 V Current :21 / 555.14 A Frequency : 50 Hz Phase : 3-Phase
Core	Type : "Wound Core" Materials: M5 Nominal Flux Density:1.76 T Cross Section: 161.28×2 mm^2
HV-Coil	Winding Type : Cross Over Materials : Cu.Wire φ 2.5 mm No. of Turns : 914 Current Density : 2.46 A/mm^2
LV-Coil	Winding Type: Concentric Winding Materials : Cu. Strip(0.9×250)mm No. of Turn : 19 Current Density :2.47 A/mm^2
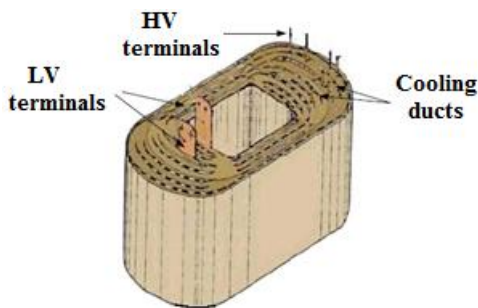
Fig.4   LV and HV winding of one phase
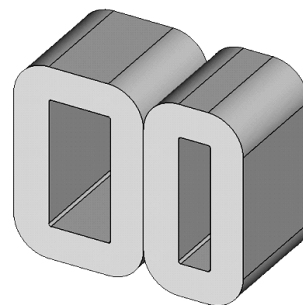
Fig.5 Small and Large Iron wound cores

6-Transformer Model Using FEM

To build the solid model, requires measuring the dimensions of the transformer accurately. The dimensions of this transformer under the study were taken from the design documents from the manufacturing company.

1-Building the Iron Core Model and Coil Model

The regions core and winding are represented by areas at 2D and by volumes at 3D then we copy these areas or volumes on x-axis to build half of the model at 2D or a quarter of model at 3D. Figs (6) and (7) show the iron core model and Coil Model.
In the iron core model, triangle elements are used in free mesh of the 2D iron-core model and hexahedral elements are used in mapped mesh in 3D iron-core models. The element type used for iron-core is PLANE53 in 2D model and BRICK97 in 3D model.
The non-linear characteristics between the magnetic flux density (*B*) and the magnetic field intensity (*H)* of the electrical steel used for the iron-core was input to ANSYS manually. In the coil Model, the element type PLANE 53 is suitable for the coil region in 2D model and the element type suitable for the coil region in 3D model is BRICK 97, because these elements have the capability of coupling with the external circuit.
The coil areas in 2D model are mapped meshed with quadratic elements and the coil volumes in 3D model are mapped meshed with hexahedral elements.

2-Building the Insulation Model

Different types of insulation are used in distribution transformer such as paper insulation, press board insulation, wood insulation, and oil. The positions of these insulations are distributed among the transformer parts. It is very difficult to represent these insulations by areas and volumes from assigning the key points and lines because these insulations have complex shapes and irregular areas or volumes. Therefore, the easiest and most favorite way of representation these insulations are to use overlap operation in ANSYS package. The insulation areas in 2D model are freely meshed with triangle elements and the insulation volumes in 3D model are freely meshed with tetrahedral elements. Figs (8) show the Insulation Model of 2D model and 3D model and Fig. (9) shows completely Model of 2D and 3D with mesh pattern.
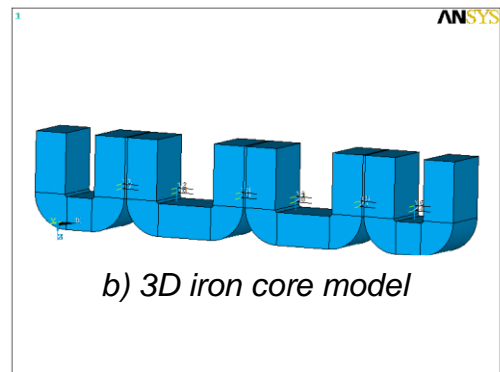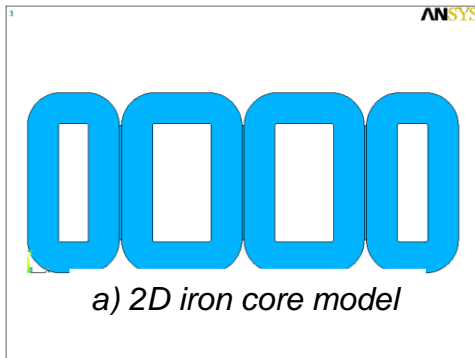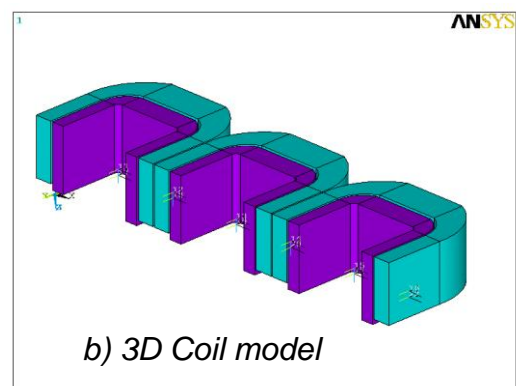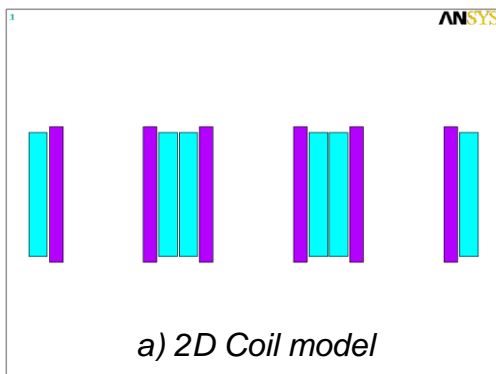
a) 2D iron core model

b) 3D iron core model

Fig.6   iron core model

a) 2D Coil model

b) 3D Coil model

Fig.7 Coil Model

a) 2D Insulation model

b) 3D Insulation model

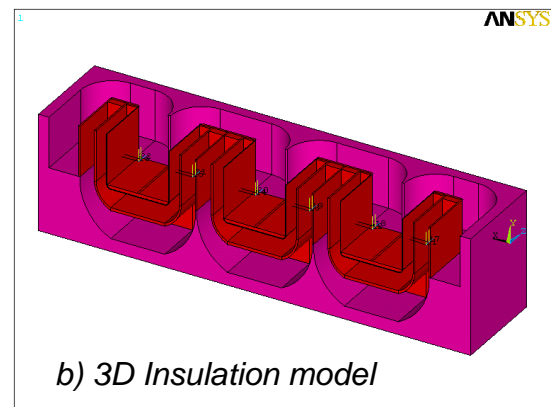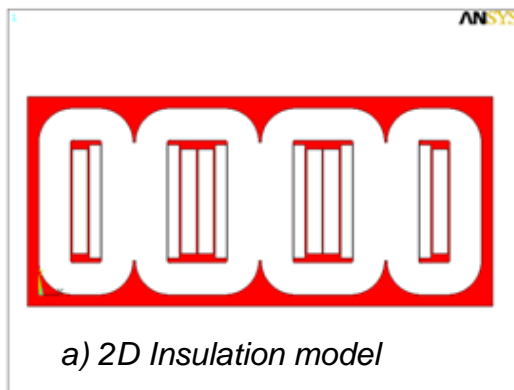*Fig.8 Insulation Model*

a) 2D Transformer model
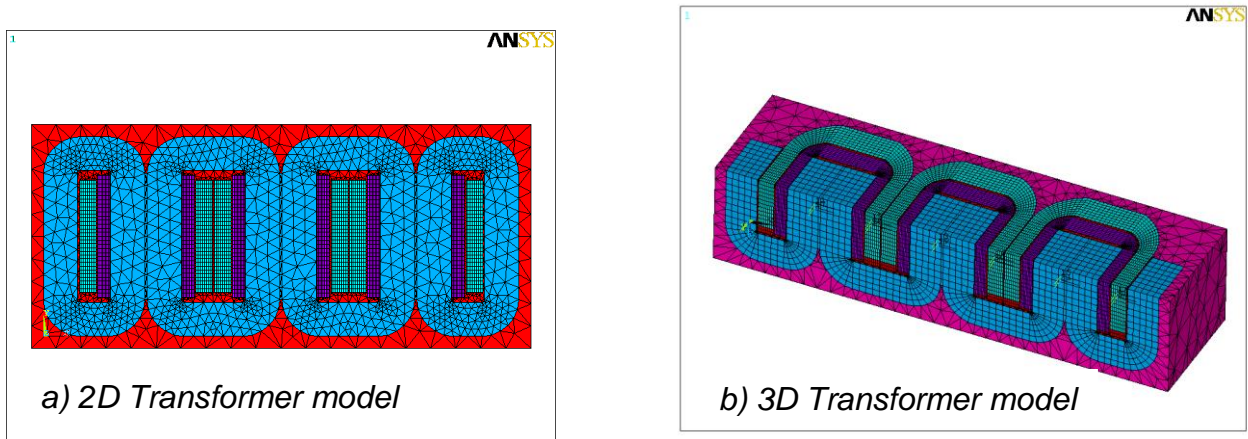

b) 3D Transformer model

Fig.9 Transformer Model with mesh pattern

7-Leakage reactance calculations by energy method

In this approach, use is made of an equivalent definition of inductance from the energy point of view[11].

$$L = \frac{2W_m}{I^2} \quad \text{-------} \quad (23)$$

Where Wm is energy in the magnetic field produced by a current I flowing in a closed path. The electromagnetic energy stored in the windings and the space between them can be used to calculate the inductance between the windings and the leakage inductance.

The magnetic field energy is obtained by leakage magnetic computing based on Maxwell equation, when numerical methods like Finite Element Method are used [11]; the magnetic field energy Wm of each part of magnetic field in a volume V is

$$W_m = \int_V \frac{1}{2} B.H \, dV = \frac{1}{2\mu} \int_V B^2 \, dV \quad \text{---------} \quad (24)$$

After computing magnetic field energy of each part of area, the whole magnetic field energy $W_m = \Sigma W_m(i)$ is obtained, and leakage inductance can be computed by equation (23).

$$L = \frac{2W_m}{I^2} = \frac{1}{I^2} \int_V B.H \, dV \quad \text{-------} \quad (25)$$

Solution of the field is generally obtained in terms of magnetic vector potential. The magnetic energy can be calculated from the product of current density and magnetic vector potential integrated over the volume and the inductance is obtained as

$$W_m = \frac{1}{2}\int_V A.J\,dV \qquad \text{-------- (26)}$$

$$L = \frac{1}{I^2}\int_V A.J\,dV \qquad \text{-------- (27)}$$

Where **A** is magnetic vector potential and **J** is current density vector.

In 2D magnetic field, the magnetic energy that stored in window space can be calculated by using two following formulas [9]

$$W_m = \frac{1}{2}\cdot t \iint B.H.dx\,dy \qquad \text{------- (28)}$$

$$W_m = \frac{1}{2}\cdot t \iint J.A.dx\,dy \qquad \text{-------- (29)}$$

To calculate the stored energy using equation (28), the integration should be done on the whole model, while if we use (29), the integration is only applied on the conductive parts of the current or on the windings. Once the magnetic energy is calculated, leakage reactance of transformer for each phase referred to primary can be calculated using following formula.

As it is known, the value of leakage reactance is equal to multiplication of inductance and angular frequency, therefore:

$$X_L = \omega L = 2\pi f.\frac{2W_m}{I^2} = \frac{4\pi f W_m}{I^2} \qquad \text{------- (30)}$$

$$X_L = \frac{4\pi f W_m}{I_{P1}^2 + I_{S1}^2} \qquad \text{-------- (31)}$$

Where:

X_L: is leakage reactance of transformer referred to primary side (per phase).

f   :is the supply frequency.

W: is the magnetic energy that calculated using (28) and (29) equations.

t: is the depth of defined model.

I_{P1} : is the instantaneous current of one phase of primary winding and

I_{S1} :is the instantaneous current of the same phase of secondary winding (referred to primary)

8-Results and discussion

In this paper the finite element model of a distribution transformer is built by using the FEM software (ANSYS) and the energy technique has been applied to the 400 kVA wound core distribution transformer .The distribution of flux and calculation of leakage reactance in the transformer winding were studied by using two types of analysis (static and transient) for the non-linear transformer models in the 2D models and 3D models.

The leakage reactance of (primary and secondary) can be calculated:

  a) Simulating the on-load behaviour of' the transformer (at working current ratings),

  b) Simulating the short-circuit behaviour of the transformer

8-1The Results of 2D Solution

1- No load condition

The validity of the models was firstly checked by computing the results of the finite element transient analysis in no load condition. In no load condition, the secondary coil in the transformer is open and the primary coil is connected to an alternating voltage source. The objective of transient analysis in no load case is to obtain the voltage waveform across the primary and secondary coils.

The magnitudes of the obtained voltages agree with that of the practical test and the design values as shown in Table (2). Fig.(10) and Fig. (11) show the voltage waveform of input and output voltage.

Table. (2)

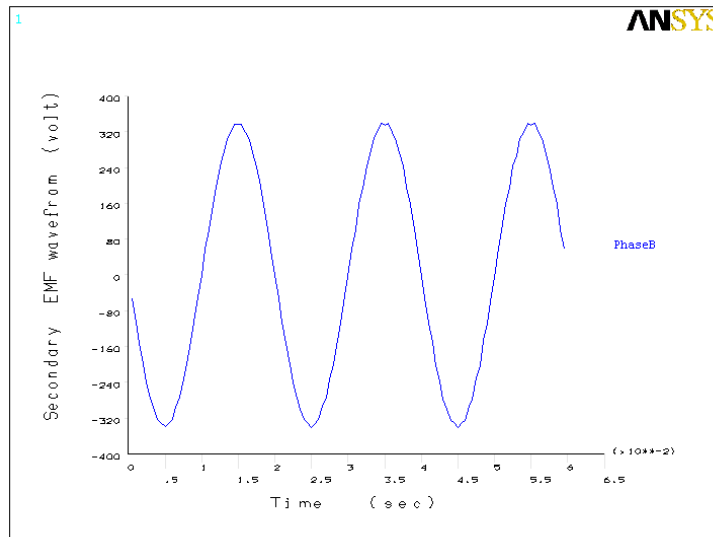|	Terminal voltage (peak value)	volt	
	Test value	Design value	FEM. Solution
Primary coil	16334.16	16334.166	16334.2
Secondary coil	339.41	339.66	339.55

Fig.11 Output voltage waveform

2- Magnetic Flux Distribution and Leakage reactance Results at short circuit condition

By using static analysis of ANSYS software, the magnetic flux distribution and calculation of the Leakage reactance of the defined model in different work conditions such as (full load), (symmetrical short circuit) are described below. Fig.(12) Shows the distribution of the equipotential lines of magnetic flux in windings area at short circuit condition with balanced ampere-turn in both side (HV coil and LV coil). As shown in Fig.(12) the axial component of leakage flux almost include whole area and the Radial component leakage flux only includes two areas on end winding that their direction is opposite to each other.

The computation results show that along Y direction (Height of coil), the axial flux density (B_Y) gradually decreases from the middle to the end of winding. Along X direction(coil width), the axial flux density linearly decreases from the outer side to the inner side of the LV winding and linearly decreases from the inner side to outer side of the HV winding as seen in Fig (13). This tendency of variation can be seen clearly from B_Y-X curve at the axis of translational symmetry in Fig.(14). Furthermore the radial flux density at the ends of winding is much higher than that in the middle of the winding as shown in Fig (15), and the field vector at two end area of primary and secondary windings are radial and at other area are axial as shown in Fig (16). The reason of differences between field vectors directions in primary and secondary windings is opposite current direction in two windings.
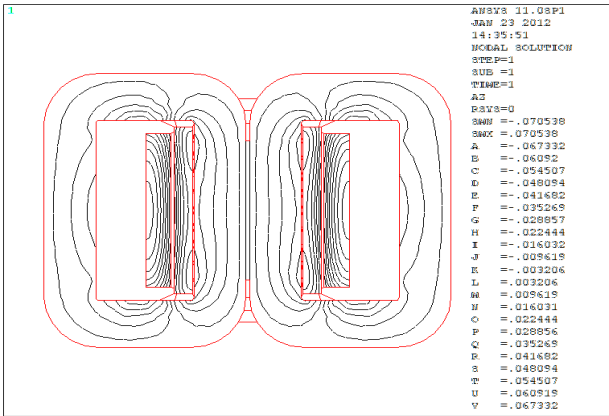
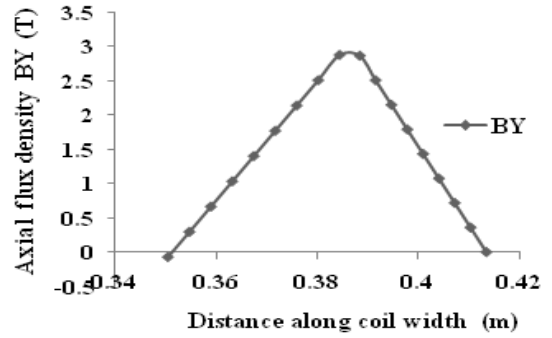**Fig. (12) b) Flux lines distribution of the HV and LV-Coil**

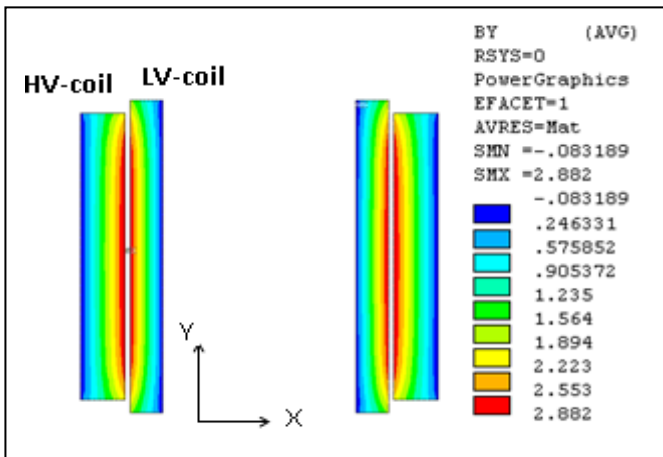**Fig.14) Local axial flux density distribution**

Fig.13 Axial flux density distribution of HV-coil and LV-coil
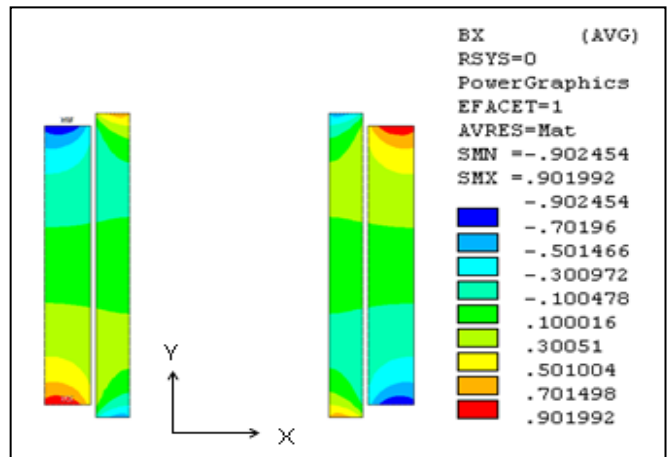

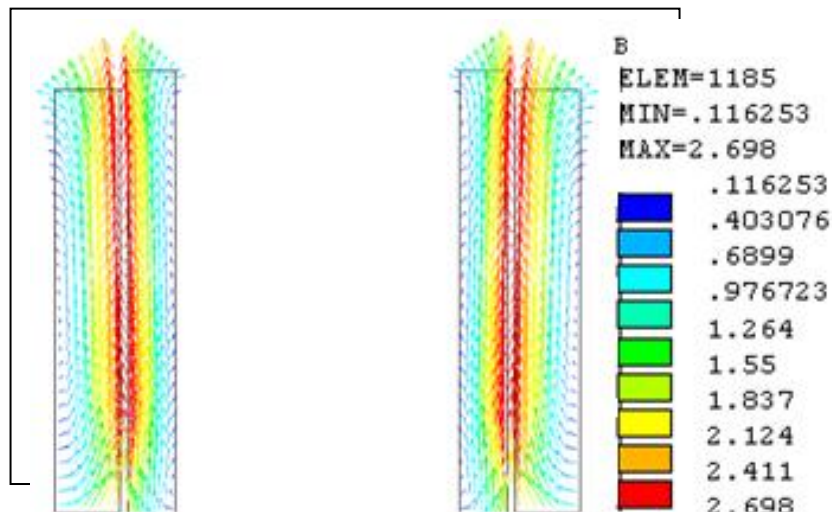**Fig.15 Radial flux density distribution**

Fig.16 Flux density vectors distribution on winding area

Figs. (17) and (18) show the distribution of radial and axial magnetic flux density along the height of the HV coil.
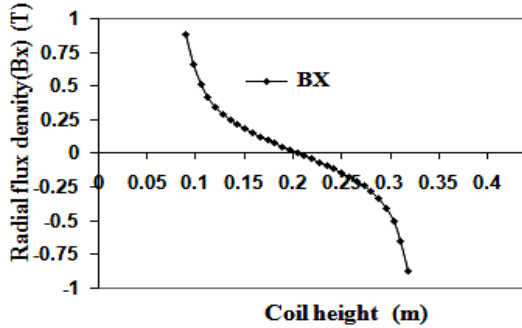

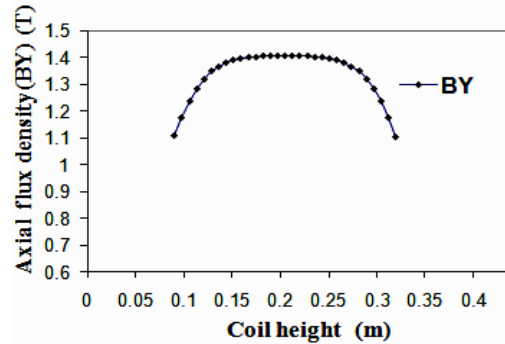**Fig.17 Radial flux distribution along center line of HV-coil**

Fig.18 Axial flux distribution along center line of HV-coil

After the solution of leakage magnetic Field using the finite element method, the leakage inductance can be calculated from the total magnetic energy. Equation (23) can describe the Relation of the leakage *inductance* and the leakage Magnetic field energy, and the magnetic energy can be calculated from Equations (28) or (29). As is known, the value of leakage reactance is equal to multiplication of inductance and angular frequency as Equations (31). Furthermore, the value of leakage reactance in the engineering applications, reactance can show the percentage of leakage reactance voltage and rated voltage.

Table.3 shows the values of Magnetic energy, leakage inductance and leakage reactance on primary and secondary windings.

Table.3. the values of Magnetic energy and leakage

Type windings	Magnetic energy (*J*)	Leakage inductance (*mH*)
Primary	8.0811	80.85
Secondary	4.2784	0.01815

The results obtained from energy method and classical method (conventional design) is compared with actual test results, as shown in Table.4

Table.4 Comparison of Energy method results of leakage reactance with test

Method	leakage reactance in percentage values ($\%IX_L$)	Error in respect to the test values (%)
Energy method	3.885	1.39%
Classical method	3.82	3.04%
Test results	3.94	------

The Leakage reactance value calculated with the use of the 2D FEM model was equal to 3.885%. The classical design methodology resulted equal to 3.82, while the test value of short-circuit reactance of the study transformer was found equal to 3.94. Therefore, the deviation between 2D FEM and the test value is equal to 1.39%, while the deviation of the classical method is equal to 3.04%.

In order to discuss the effect of the MMFs of the adjacent phase on the calculation leakage reactance at any windings area in another phase, Phase B is considered in two cases:

Case 1- One phase only is excited

Case 2- Three phases of the transformer are excited

The comparison between the two case shown in Fig.(19),which shows the distribution of magnetic flux density in phase(B) in each case. From this figure it is obvious that the magnitude of magnetic flux density in case(2) is greater than that of case(1) , because the flux density in each point in coil region is composed of the flux due to the current passing through phase B and other flux from the adjacent coils. This difference in the components of the flux density between the two cases will affect the magnitude of the leakage reactance in the coils, especially on the axial flux component.
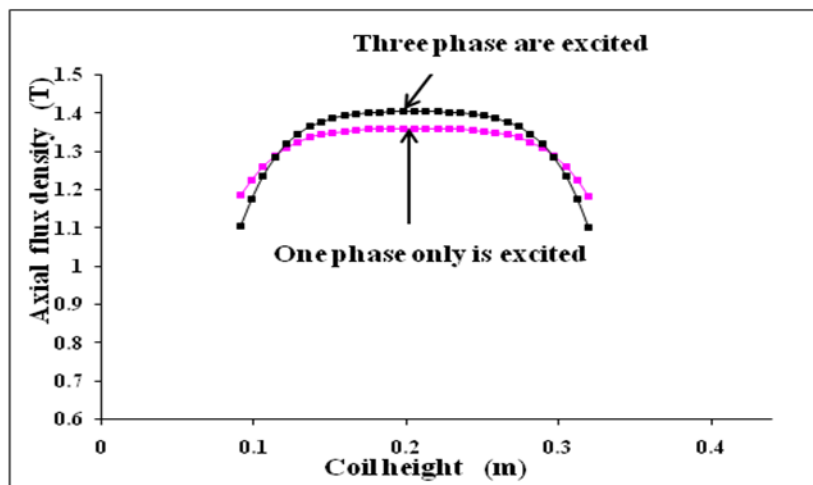

Fig.19 Axial flux distribution along center line of HV-coil in two

The analysis which is done in transformer model when three phases of the transformer are excited will permit to study the effects of MMF of the adjacent phase and also the effects of the high level saturation in the lateral limbs.

The obtained results show that the leakage reactance in the case of supplying the transformer with 3-phase voltage increase by (4.1%) over that when supplied with phase B only. as shown in Table (5).

Table (5) Comparison between (case1) and (case2) leakage reactance

Cases of excited	leakage reactance in percentage values (%IX$_L$)	Error in respect to the test values (%)
Case 1 Phase B only is excited	3.723	5.5 %
Case 2 Three phase are excited	3.885	1.39%
Test value	3.94	------

8-2 The Results of 3D Solution

The main objective of 3D Static Analysis is to study the distribution of the magnetic flux and calculation of the leakage reactance values especially in the portion of coil (End Region) out of the window of core which is not possible to calculate in 2D model.

To perform this type of electromagnetic analysis, the same procedure can be used as in a 2D static analysis but the 3D FE analysis of transformer model is complex and needs long time to be solved as compared with 2D analysis.

The distribution of the leakage flux in coil regions is complicated due to the difference in the direction of the current in the different regions around the coil. This leads to produce magnetic flux in the three dimensions (B_X, B_Y, B_Z). Fig. (20) Shows the distributions of radial and axial flux density in different Region of HV and LV coil.
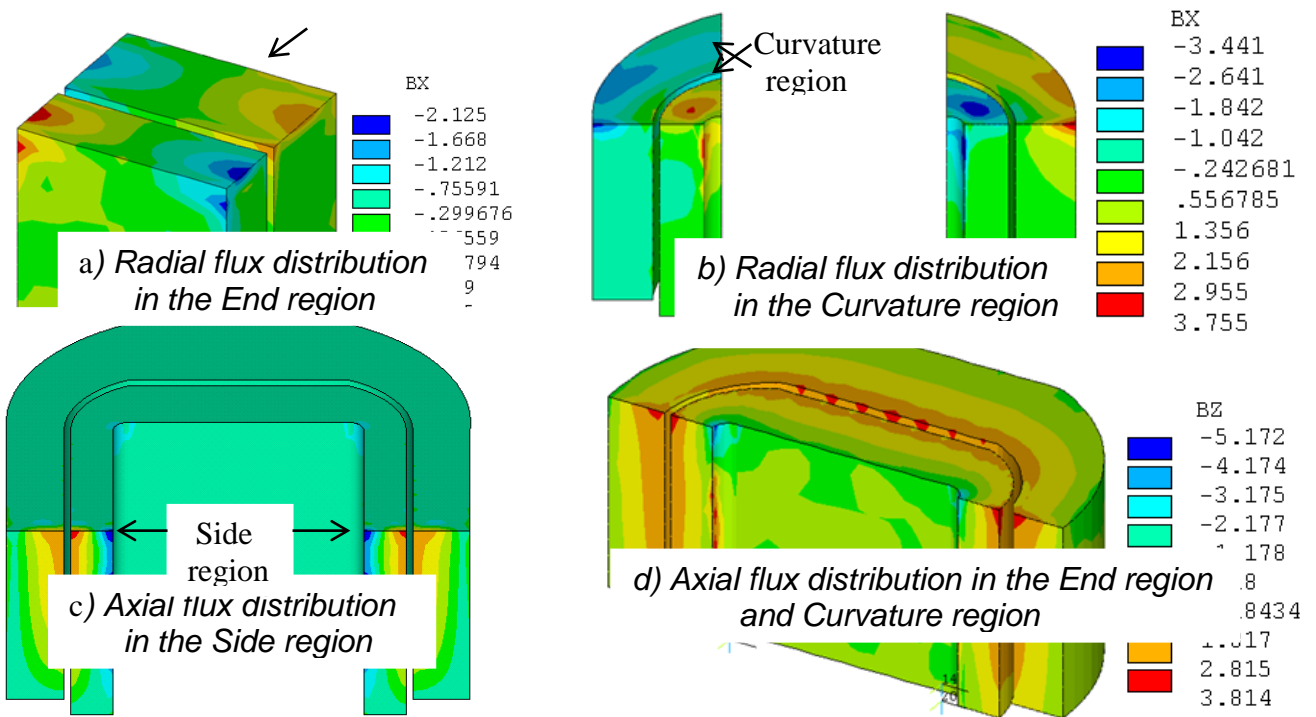
End
region

Fig.20 leakage flux distribution in the different regions around

From Fig (21) - Fig.(22), it can see that the behavior of flux density are the same as in 2D analysis, but simple different about the values, due to the effect of Z-component of flux in 3D analysis. The magnetic energy can be calculated from the product of current density and vector potential integrated over the volume. The results obtained for the Leakage reactance are given in Table (5). These results show that the Leakage reactance on the coil region (Side Region) which is situated in the window of the core are approximately the same as that for 2D model, and they are expected to be more accurate than that of 2D because they include the z-component of the flux.
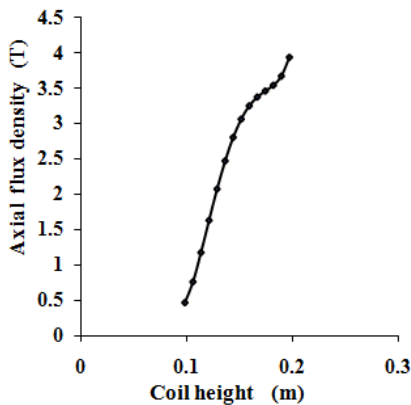

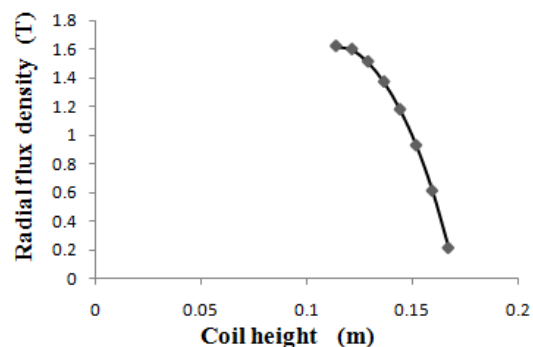**Fig.21 Axial flux distribution along Center line of HV-coil**

Fig.22 Radial flux distribution along Center line of HV-coil

Table.5 Comparison of 3D and 2D results of leakage reactance with test

Method	leakage reactance in percentage values ($\%IX_L$)	Error in respect to the test values (%)
Energy method 2D Results	3.885	1.39%
Energy method 3D Results	3.9399	0.02%
Classical method	3.82	3.04%
Test results	3.94	------

The short-circuit leakage reactance value calculated with the use of the 3D FEM model was equal to 3.939 and the Test value is equal to 3.94, Therefore, The results are agrees with test value, and the deviation between 3D FEM and the test value is equal to 0.02% while the deviation of the classical method is equal to 3.04%. This difference demonstrates the ability of 3D FEM to accurately predict the Leakage reactance, due to the better representation of the real transformer geometry.

9-CONCLUSIONS

In the present paper, the classical design methodology and energy method in 2D and 3D FEM models have been applied for the calculation of the leakage reactance of three-phase, wound core, distribution transformers. The results of the method were compared to the ones of the actual test values,

The obtained results have shown that:

The energy method is the more accurate than classical design method because of the results that obtained from ANSYS in 3D FEM model agrees with test values, the deviation equal to 0.02% as compared with nearly 3.04 % for the results based on classical design.

The 3D FEM model provides higher accuracy in the prediction of the leakage reactance than 2D FEM, with respect to the actual test values, due to the better representation of the transformer geometry. Also 3D FEM analysis is necessary to compute the magnetic flux (out of the window) in Curvature and End regions that are not possible with 2D analysis

In order to get accurate results on the calculation leakage reactance, the FEM analysis must be done in transformer model when three phases of the transformer are excited to take into account simultaneously both the MMF of the adjacent phase and the effect of saturation in some branches of limb core.

The paper describes a simple technique for modeling distribution transformer "wound core" in 2D and 3D model

These accurate results provide significant economic gains to the transformer manufacturer like (Diyala Company of Electrical Industries) through reduction of the industrial cycle and the production cost before manufacturing

10-REFERENCES

[1]A.M. Kashtiban "**Finite Element Calculation of Winding Type Effect on Leakage Flux in Single Phase Shell Type Transformers**" Proceedings of the 5th WSEAS International Conference on Applications of Electrical Engineering, Prague, Czech Republic, March 12-14, 2006 (pp39-43)

[2]Marina A. Tsili "**Advanced design methodology for single and dual voltage wound core power transformers based on a particular finite element model**" Electric Power Systems Research 76 (2006) 729–741

[3]A. Naderian Jahromi "**A Fast Method for Calculation of Transformers Leakage Reactance using Energy Technique**" IJE Transactions B: ApplicationsVol. 16, No. 1, April 2003

[4] William M. Flanagan "**Handbook of Transformer Design and Applications**"
    Second edition, McGraw-Hill,1992

[5]A. D. Yamkov, "**Transformer Design** " Mir Publisher, Moscow, 1975

[6]V.N. Mittit "**Design of Electrical Machines**" fourth edition 1996 ,Published by N. C. Jain for Standard publishers distributors ,delhi

[7]A.G. kladas, M. P. Papadopulos and J. A. Tegopoulos " **Leakage flux and force calculation on power transformer winding under short-circuit :2D and 3D models based on the finite element method compared to measurement**", IEEE Trans. On, Magnetics. , vol. 30, No. 5 September 1994

[8] Simon C. Bell and Pat S. Bodger "**Power Transformer Deign using Magnetic circuit theory and Finite Element Analysis-A comparison of Techniques**" Presented in AUPEC 2007, Perth, Western Australia, 9-12 December, 2007

[9] S. Jamali Arand1, K. Abbaszadeh2 "**The Study of Magnetic Flux Shunts Effects on the Leakage Reactance of Transformers via FEM**" Majlesi Journal of Electrical Engineering Vol. 4, No. 3, September 2010

[10]Robert M.Del Vecchio "**Transformer Design Principles with Applications to Core-Form Power Transformers**", CRC PRESS London 2002, Web site at www.crcpress.com

[11]S.V.Kulkarni and S.A.Khaparde " **Transformer Engineering Design and Practice**", Marcel Dekker, Inc, New York • Basel 2005

[12]T. Yun-Qiu & other ," **Numerical calculation of short circuit electromagnetic forces on the transformer winding** ", IEEE Trans. On Magnetics. vol.26, No.2, March 1990.

[13] O. W. Andersen, "**Transformer leakage flux program based on the finite element method**," *IEEE Trans. Power App. Sys.*, vol. PAS-92, no. 2, pp.682–689, Mar. 1973.

[14]M.A. TSILI1, A G. KLADAS1" **3D Finite Element and Mixed Finite-Boundary Element Method for the Magnetic Field Analysis of Power Transformers**" Wseas Transactions ON Circuits and Systems Issue 5,Volume 3, 2004

[15]Pavlos S. Georgilakis "**Spotlight on Modern Transformer Design**", Springer,2009

[16]S.Jamali, M. Ardebili, and K. Abbaszadeh "**Calculation of short-circuit reactance and electromagnetic forces in three-phase transformer by finite element method**" - *Proceedings of the Eighth International Conference on Publication Electrical Machines and Systems, 2005. (ICEMS 2005),* pp. 1725- 1730 - Vol. 3.

[17]GUEMES-ALONSO, J.A. - "A **new method for calculating of leakage reactances and iron losses in transformers**" - *Proceedings of the Fifth International Conference on Electrical Machines and Systems, 2001. (ICEMS 2001), Vol: 1, 18-20 Aug. 2001, pgs: 178-181*

[18]KASHTIBAN, A.M.; VAHEDI, A.; HALVAEI, A.; - "**Investigation of winding type effect on leakage flux of single phase shell type transformer using FEM**" - *Proceedings of the Eighth International Conference on Electrical Machines and Systems, 2005. (ICEMS 2005), Vol: 3, Sept. 2005, pgs:1755 - 1758.*

[19] "**Design Calculation Sheet of 400 KVA distribution transformer** "Mitsubishi Electric Corporation, 1983.

[20] Martin J. Heathcote, CEng, FIEE "**The J&P Transformer** ", Twelfth edition, Newnes, Reed Educational and Professional Publishing Ltd 1998

[21] Marina A. TSILI ,Antonios G. KLADAS "**Wound Core Power Transformer Design: Classical Methodology and Advanced Magnetic Field Analysis Techniques**" Advanced Research Workshop on Modern Transformers. 28 -30 October 2004. Vigo – Spain

[22]"**Design standard of distribution transformer**" Mitsubishi Electric Corporation, 1983

[23] J. D. Lavers "**Electromagnetic Field Computation in power Engineering**" IEEE Trans. On, Magnetics, vol. 29, No. 6 ,November 1993

[24] Documentation for ANSYS11

[25] P.Silvester,A. Kourad, "**Analysis of transformer leakage phenomena by high order finite element**", IEEE Trans. PAS., vol. 92,pp.,1843-1855, 1973

[26]Jan Sykulski, "**Field Simulation an Aid to Machine Design: The State of the Art**", IEEE 2006,University of Southampton, United Kingdom