Al-khwarizmi
Engineering
Journal

Al-Khwarizmi Engineering Journal, Vol.2,No.2,pp 46-60 (2006)

Dynamic Analysis of Rotating Cantilever Plates

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(Received 20 February 2006; accepted 13 August 2006)

Abstract:-

A finite elements technique has been used to determine the natural frequencies of a cantilever plate mounted on the periphery of a rotating disc. The rotating cantilever plate has been idealized as an assemblage of three nodded triangular shell elements with six degrees of freedom at each node. In the analysis the initial stress effect (geometric stiffness) and other rotational effects except the carioles acceleration effect have been included.

The eigenvalues have been extracted by using simultaneous iteration technique. From the result of computations carried out for various values of the aspect ratio, the speed of rotation, disc radius and the setting angle. The numerical results have shown a good agreement compared with the available investigations using other methods.

Key Words: finite element, natural frequency, geometric stiffness, rotating plate.

1.Introduction

The natural frequencies of rotating turbomachinery blades are known to be significantly higher than those of the non- rotating blades. The design speed for turbomachine is generally established by drawing a Campbell diagram for each row of the blades [1]. The diagram essentially is a plot of the vibration of the first few natural frequencies of the blades with the speed of rotation with possible resonances. The design speed of the engine should be away from these possible resonances points. Obviously, the correct estimation of the natural frequencies of the blades at various speeds of rotation is important. Since the blades are generally idealized as cantilever beams, the vibrations of rotating cantilever beams have been studied in several investigations. Suther land [2] has used Myklestad type method by a suitable modification of the equations, relations the shear and the moments of consecutive conditions on the beam, to take into account the effect of the centrifugal forces. Schilhaus [3] has derived the equations of motion for the banding vibration and solved it by successive approximations to determine the effect of rotation on the fundamental frequency by considering the differential equation as Euler characteristic equation of a variational problem and minimizing energy function. Kissel [4] has also investigated the fundamental frequency of rotating cantilever beam. The effect of rotation on the fundamental frequency has been further by Carnegie [5]. the first three bending frequencies of a rotating beam have been obtained by Yntema [6] by using the Rayteigh energy method. Hsu Lo [7] and Bogdanoff [8] have shown that the effete of the non-liner terms arising form the corioles acceleration is negligible. The problem of torsional vibrations of rotating cantilever bars has been studied by Bogdanoff and Horner [9]. Dokainish and Rawtani [10] used a finite element technique to determine the modal characteristics of rotating cantilever plates. Asimilar Approach was taken by Ramanurti and Kielb [11] to determine the modal characteristics of a twisted rotating plate. They used a strain energy expression for a plate that employs steady- state in- plane stress components. The steady- state in-plane stress components were obtained either analytically from the partial differential equations of stretching motion or numerically from the equilibrium condition between the centrifugal inertia force and the steady-state in-plane stress. Then, the equations of motion were derived by using the strain energy expression in which the steady-state in-plane stress components previously obtained were employed. Because of the prohibitive complexities involved in this conventional modeling method, the procedure of deriving equations of motion was rarely described in detail in the Literature. Different from the conventional modeling method, which employs only cartesian deformations variable, a new medaling method for beams undergoing overall motion was introduced by Kane et al [12], and later extended by yoo et al [13]. This modeling method employs a non-Cartesian deformation variable that represents the stretch of the beam along its neutral axis to derive equations of motion directly. The use of the non- cartesian variable led to the accurate to capture of the stiffening effect. This modeling method was later successfully utilized to obtain modal characteristics of rotating beams By Yoo and Shin [14] similarly; a linear dynamic modeling method for plates undergoing overall motion was introduced by Yoo [15] and its accuracy was verified by Yoo and Chung [16]. A similar modeling method was also developed for another two–dimensional elastic body, a disk, by flower [17]. The key ingredient of the modeling method introduced in Ref [15] and [16] is the use of two in –plan stretch variables by which the exact in –plan strain energy can be expressed in a quadratic form. The use of the two stretch variables enables onto derive linearized equation of motion that include proper motion –induced stiffness variation terms.

2.Theoretical Back Ground

The formulation follows a pattern similar to that in Ref. [11]. Two Cartesian co-ordinate systems are used, an absolute fixed system $R_0\,(X_0 Y_0 Z_0)$ and a local system $R_1\,(X\,Y\,Z\,)$ (see Fig. 1) attached to the rotating disc.
The potential strain energy U and kinetic T are, respectively,

$$U = \frac{1}{2}\int_{vol} \varepsilon^t\, \sigma\, d(vol) \qquad \ldots(1)$$

$$T = \frac{1}{2}\int_{vol} \rho \bar{V}^2\, d(vol) \qquad \ldots(2)$$

For plate bending problems, according to Zienkiewicz [15], the strains are given by

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_p \\ \varepsilon_f \end{Bmatrix} + \begin{Bmatrix} \varepsilon_g \\ 0 \end{Bmatrix} \qquad ...(3)$$

ε_p and ε_f are strains due to in-plane and bending displacements respectively and ε_g is the effect of bending displacements on mid-surface strains.

The stresses are given by,

$$\{\sigma\} = \begin{Bmatrix} \sigma_p \\ \sigma_f \end{Bmatrix} \qquad ...(4)$$

Where $\{\sigma_p\}$ and $\{\sigma_f\}$ are in – plane stress resultants and bending and twisting moments, respectively.

With the definitions for stresses and strains U is given by,

$$U = P_1 + P_2 + P_3 \qquad ...(5)$$

Where P_1, P_2 plane stress, bending strain energy and P_3 supplementary strain energy due to the effect of bending displacement on mid-surface strains, more expressions for P_1, P_2 and P_3 are standard [16].

At rest the co-ordinates of a typical Point M on the mid-surface are $(x, y, 0)$. Due to the displacement

$$\{d\} = [u, v, w] \qquad ...(6)$$

The instantaneous co-ordinates of M are $(x+u, y+v, z+w)$, and then

$$\overline{OM}^{R_1} = \overline{OI}^{R_1} + \overline{IM}^{R_1} = \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix} \begin{Bmatrix} x+u \\ y+v \\ w \end{Bmatrix} = \begin{Bmatrix} x_i + x + u \\ y_i + y + v \\ z_i + w \end{Bmatrix} \qquad ...(7)$$

The angular velocity in the R_1 system is

$$\overrightarrow{\Omega}^{R_1} = [\Omega_1 \ \Omega_2 \ \Omega_3]^T \qquad ...(8)$$

And the absolute velocity of the point M is given by

$$\vec{V} = \frac{\partial \overline{OM}^{R_1}}{\partial t_{R_0}} = \begin{Bmatrix} u^* \\ v^* \\ w^* \end{Bmatrix} + \begin{Bmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{Bmatrix} \times \begin{Bmatrix} x_i + x + u \\ y_i + y + v \\ z_i + w \end{Bmatrix} = \begin{bmatrix} u^* + \Omega_1 (z_i + w) - \Omega_3 (y_i + y + v) \\ v^* + \Omega_3 (x_i + x + u) - \Omega_1 (z_i + w) \\ w^* + \Omega_1 (y_i + y + v) - \Omega_2 (x_i + x + u) \end{bmatrix} \qquad ...(9)$$

Computing $\vec{V}^2 (i.e., \vec{V}^t \vec{V})$, canceling the terms like those proportional to z_i^2 which give no contribution when Lagrange's equations are applied, and substituting the result in

Eq.(2) one can write the kinetic energy as [16],

$$T=\frac{1}{2}\int_{vol}\rho\begin{Bmatrix}u^{\bullet}\\v^{\bullet}\\w^{\bullet}\end{Bmatrix}^{t}\begin{Bmatrix}u^{\bullet}\\v^{\bullet}\\w^{\bullet}\end{Bmatrix}d(vol)+\frac{1}{2}\int_{vol}\rho\begin{Bmatrix}u^{\bullet}\\v^{\bullet}\\w^{\bullet}\end{Bmatrix}^{t}[A_1]\begin{Bmatrix}u\\v\\w\end{Bmatrix}d(vol)+$$

$$\frac{1}{2}\int_{vol}\rho\begin{Bmatrix}u^{\bullet}\\v^{\bullet}\\w^{\bullet}\end{Bmatrix}^{t}[A_2]\begin{Bmatrix}u^{\bullet}\\v^{\bullet}\\w^{\bullet}\end{Bmatrix}d(vol)+\frac{1}{2}\int_{vol}\rho\begin{Bmatrix}x_i+x\\y_i+y\\z_i\end{Bmatrix}^{t}[A_2]\begin{Bmatrix}u\\v\\w\end{Bmatrix}d(vol) \quad \ldots(10)$$

$$[A_1]=\begin{bmatrix}0 & -2\Omega_3 & 2\Omega_2\\ 2\Omega_3 & 0 & -2\Omega_1\\ -2\Omega_2 & 2\Omega_1 & 0\end{bmatrix} \quad \ldots(11)$$

$$[A_2]=\begin{bmatrix}\Omega_2^2+\Omega_3^2 & -\Omega_1\Omega_2 & -\Omega_1\Omega_3\\ -\Omega_1\Omega_2 & \Omega_1^2+\Omega_3^2 & -\Omega_3\Omega_2\\ -\Omega_1\Omega_3 & -\Omega_3\Omega_2 & \Omega_2^2+\Omega_2^2\end{bmatrix} \quad \ldots(12)$$

2.1.Derivation of The Stiffness Matrix

The polynomials for the displacements u and v are linear in L_1, L_2 *and* L_3 while for the displacements w the polynomial assumed is cubic [15].

The in – plane nodal displacements are defined by,

$$\{q_1\}=\lfloor u_1 \quad v_1 \quad u_2 \quad v_2 \quad u_3 \quad v_3\rfloor^{t} \quad \ldots(13)$$

And the bending nodal displacements are defined by,

$$\{q_2\}=\lfloor w_1,\theta_{x_1},\theta_{y_1},w_2,\theta_{x_2},\theta_{y_2},w_3,\theta_{x_3},\theta_{y_3}\rfloor^{t} \quad \ldots(14)$$

$$\theta_{x_i}=-\left(\partial w/\partial y\right)_i,\ \theta_{y_i}=-\left(\partial w/\partial x\right)_i \quad \ldots(15)$$

After standard finite elements procedure one arrives at

$$\begin{Bmatrix}u\\v\end{Bmatrix}=[N_1]\{q_1\},[N_2]=\begin{bmatrix}L_1 & 0 & L_2 & 0 & L_3 & 0\\ 0 & L_1 & 0 & L_2 & 0 & L_3\end{bmatrix} \quad \ldots(16, 17)$$

$$\{w\}=[N_2]\{q_2\},[N_2]=[N_{b_1} \ N_{b_2} \ldots N_{b_9}]^{t} \quad \ldots(18, 19)$$

$$N_{b_1}=L_1+L_1^2L_2+L_1^2L_3-L_1L_2^2-L_1L_3^2 \quad \ldots(20)$$

$$N_{b_2}=b_3(L_1^2L_2+\frac{1}{2}L_1L_2L_3)-b_2(L_3L_1^2+\frac{1}{2}L_1L_2L_3) \quad \ldots(21)$$

$$N_{x_1} = a_3(L_1^2 L_2 + \frac{1}{2} L_1 L_2 L_3) - a_2 (L_3 L_1^2 + \frac{1}{2} L_1 L_2 L_3) \quad ...(22)$$

The other shape functions for nodes 2 or 3 can be written down by a cyclic permutation of the suffixes 1, 2, 3.

L_1, L_2, L_3 and the area co-ordinates, and a_i and b_i, are defined in Fig.2.

Once one knows the expression for the strain and the shape functions, then k_p the in-plane stiffness matrix, and k_f, the bending stiffness matrix, can be easily derived. The integration is performed by using numerical three point integration [17] over the triangular area.

2.2.Derivation Of Geometric Stiffness Matrix

Owing to the presence of the in-plane stresses σ_x^0, σ_y^0 and τ_{xy}^0 in the middle surface caused by rotation, the additional strain energy stored in the element is given by P_3. This additional strain energy results in an increase in the stiffness of the elements by an amount,

$$\{k_g\} = \iint_A [G]^t \begin{bmatrix} \sigma_x^0 & \tau_{xy}^0 \\ \tau_{xy}^0 & \sigma_y^0 \end{bmatrix} [G]\, h\, dA \quad ...(23)$$

Where $[G]$ is defined by

$$\begin{Bmatrix} \dfrac{\partial w}{ax} \\ \dfrac{\partial w}{ax} \end{Bmatrix} = [G]\{q_2\} \quad ...(24)$$

For details see Ref. [15].

It is easy to show that

$$U = \frac{1}{2} q\,'[k + k_f]q + \frac{1}{2} q'\, k_G\, q \quad ...(25)$$

$$\{q\} = [q_1, q_2]' \quad ...(26)$$

Therefore

$$\partial U / \partial q = [k + k_f]q + k_G\, q \quad ... (27)$$

2.3.Derivation of Mass, Carioles and Supplementary Matrices, And The Load Vector

By using Eq. (16 and 18) in Eq. (10) and then applying Lagrange's equations one obtain,

$$\left(d/dt\right)\left(\partial T/\partial q^{\cdot}\right) - \left(\partial T/\partial q\right) = m_E q^{\cdot\cdot} + cq^{\cdot} + k_R q - F$$

...(28)

With $[N]$ defined by,

$$\begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix} = [N]$$

The element mass matrix is,

$$[m_E] = \rho \int_{vol} [N]^t [N]\, d\,(vol) \quad ...(29)$$

The element coriolis matrix is

$$[C] = \rho \int_{vol} [N]^t [A_1][N]\, d\,(vol) \quad ...(30)$$

The element supplementary or rotational stiffness matrix is,

$$[k_R] = \rho \int_{vol} [N]^t [A_2][N]\, d\,(vol) \quad ...(31).$$

And the force vector is,

$$\{F\} = \rho \int_{vol} [N]^t [A_2] \begin{Bmatrix} x_i + x \\ y_i + y \\ z \end{Bmatrix} d\,(vol) \quad ...(32)$$

2.4.Final Matrices After Assembly

Adding expressions (27) and (28) and equating the result to zero gives the final differential equation of the structure after assembly in from,

$$M_E\ddot{q} + C\dot{q} + [K_E + K_G + K_R]q = \vec{F}(\Omega^2) \quad \ldots(33)$$

The matrix K_G depends on the initial stress distribution. Initially the stresses are taken as zero and the equation.

$$[K_E + K_R]q = \vec{F}(\Omega^2) \quad \ldots(34)$$

Is solved for the initial stress distribution σ_0. Then the solution of,

$$(K_E + K_G(\sigma_0) + K_R)q = \vec{F}(\Omega^2) \ldots(35)$$

Gives a new stress distribution σ. The stress values were found to converge within two iterations.
Finally the frequencies and eigenvectors are found for the deformed configuration. The equation of motion of the structure, with the carioles matrix neglected, is

$$M_E\ddot{q} + [K_E + K_G(\sigma) + K_R]q = 0 \ldots(36)$$

Assuming harmonic vibrations,
$q = q_o\, e^{i\omega t}$, One has

$$[K_E + K_G(\sigma) + K_R - \omega^2 M_E]q_o = 0 \ldots(37)$$

In which M_E and $K_E + K_G + K_R$ are symmetric and positive definite matrices Eq. (37) is standard eigenvalue problem and is solved for the eigenvalues and eigenvectors by using a simultaneous iteration technique [18 and 19].
The tapered and skewed plate can also be modeled by triangular shell elements, the variation in thickness being accounted for by defining the thickness of the element at the three nodes. For formulating all the matrices the element thickness can by taken as the mean of the nodal thicknesses.

3.Check Problem

To check the suitability of the present triangular shell elements for modeling the rotating plate structures, the problem to determining the natural frequencies for rotating and stationary cantilever blade solved by Garngie [24]. The current results that was exhibited in Table. (1) measure with the case study in Ref. [24]. Table. (1) shows the fundamental natural frequencies for rotating and stationary blade. In this table the maximum error not exceeds (1.7%). The data for the verification case are:

$E = 217\,\text{Gpa},\ a = 328\,mm,\ \rho = 7850\,Kg/m^3$

$b = 28\,mm,\ \upsilon = 0.3,\ r = 150\,mm$

$\Omega = 100\pi\ rad/sec,\ t = 3\,mm$

4.Case Study

The aim of this study is to fill the gap by furnishing the information about the behaviour of rotating plate regarding vibration behaviour having different skew angle, speed of rotation, disc radius and thickness subjected to centrifugal loading. Analysis has been done on flat plate $\alpha = 0$. The mesh sizes (5*5), (6*4) and (9*4) were chosen for the analysis for the cases in which the aspect ratios were (1, 2 and 3) respectively (See Fig. 3). In all computations the material of the plate has been assumed to be homogeneous and isotropic. The material properties are:

$E = 210\,\text{Gpa},\ a = 30\,mm,\ t = 3\,mm$

$b = 30\,mm,\ \rho = 7850\,Kg/m^3,\ \upsilon = 0.3$

5.Results and Discussions

The natural frequencies are computed for plates of different non-dimensional speed $(\bar{\Omega}\ from\ 0\ to\ 1)$, non-dimensional-radius of disc $(\bar{r}\ from\ 0\ to\ 3)$, setting angle $(\theta\ from\ 0^{\circ}\ to\ 90^{\circ})$, thickness of the plate $(t\ from\ 2mm\ to\ 5mm)$ and aspect ratio $\left(a/b = 1, 2\ and\ 3\right)$.

The variation of the first three non-dimensional frequency of vibration (β) with $(\bar{\Omega})$ were shown in Figs. (4, 5, 6 and 7) for the plates having $(\bar{r})$ of (0, 1, 2 and 3) respectively and $(\theta = 0^{\circ}, t = 3\,mm)$.

The second set of results will initiate the tendency of change in the (β) with $(\bar{\Omega})$ for different setting angle $(\theta = 0^{\circ}, 45^{\circ}\ and\ 90^{\circ})$ were shown in Figs. (8, 9 and 10) for the plates having aspect ratios of $\left(a/b = 1, 2\ and\ 3\right)$ respectively and $(\bar{r} = 1)$.

Figs. (11 and 12) show the variation of the fundamental natural frequency with the variation of $(\bar{\Omega})$ for different thickness $(t = 2, 3, 4\ and\ 5mm)$ for the plates having aspect ratios of $\left(a/b = 1\ and\ 2\right)$ respectively, and $(\theta = 0^{\circ}, \bar{r} = 1)$.

All the natural frequencies are observed to increase with the speed of rotation and with the increasing radius of the disc. An increase in the setting angle causes a decrease in the natural frequencies. It can be recognized that when the thickness increase the natural frequencies increases.

Also it can be observed the speed of rotation and disc radius are effect of large part of results, which the natural frequencies are proportional to the speed of rotation and disc radius.

6.Conclusion

The finite element method has been applied to study the influence of various parameters such as skew angle, aspect ratio, speed of rotation, thickness and disc radius on the natural frequencies of a rotating cantilever plate.

The conclusions obtained from the present analysis can be summarized as follows:

1. The frequencies of all modes of vibration are independent of skew angle and disc radius when the cantilever plate is stationary.
2. The maximum effect of skew angle occurs when skew angle is $(\theta = 0^{\circ})$.
3. The plate thickness is very effective on the natural frequencies for both cases (stationary and rotating plate).
4. The speed of rotation and disc radius are very effective parameters on the natural frequencies for rotating cantilever plate.

Table. 1 Values of fundamental natural frequency (Hz) for stationary and rotating cantilever plate.

	Present Work	Ref. [24]	Error %
$(\Omega = 0)$	23.9	23.6	1.2
$(\Omega = 100\pi\, rad/\sec)$	52.9	52.0	1.7

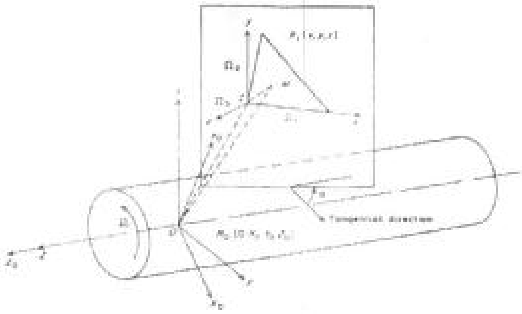


Fig.1 Cartesian co-ordinate system.

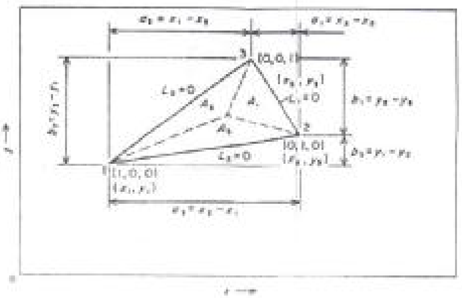
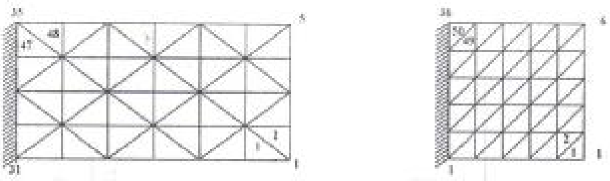
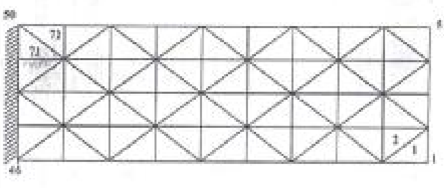


Fig. 2 Area co-ordinates.





	(9*4)	(6*4)	(5*5)
No. of elements	72	50	48
No. of nodes	50	35	36

Fig. 3 (9*4), (6*4) and (5*5) mesh used in the shell analysis.

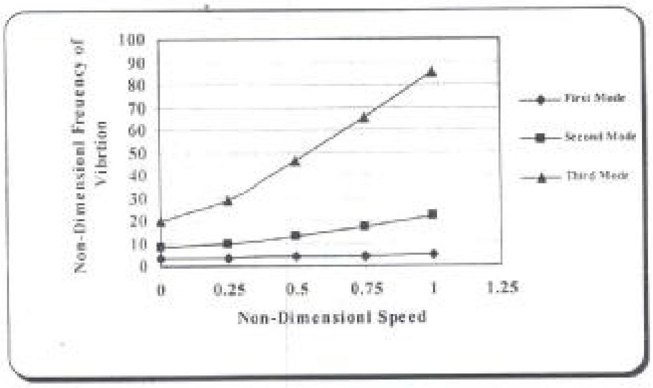


Fig. 4 Variation of non-dimensional frequency of vibration with non-dimensional speed (a/b=1, $\theta = 0, \bar{r} = 0$)

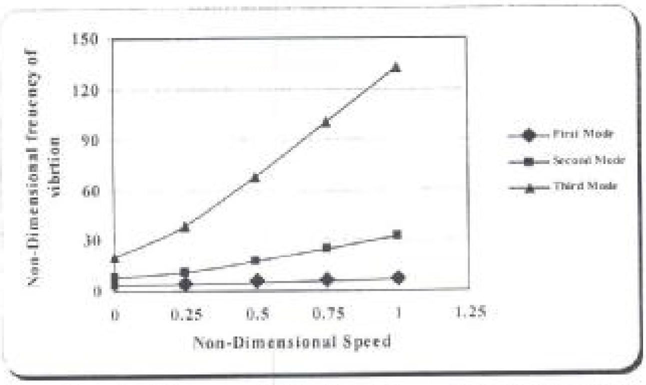


Fig. 5 Variation of non-dimensional frequency of vibration with non-dimensional speed (a/b=1, $\theta = 0, \bar{r} = 1$)

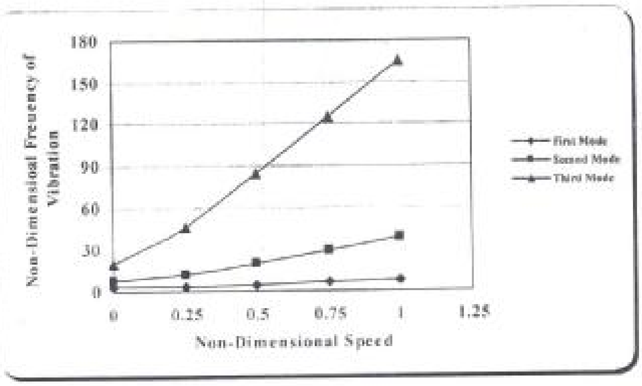


Fig. 6 Variation of non-dimensional frequency of vibration with non-dimensional speed (a/b=1, $\theta = 0, \bar{r} = 2$)

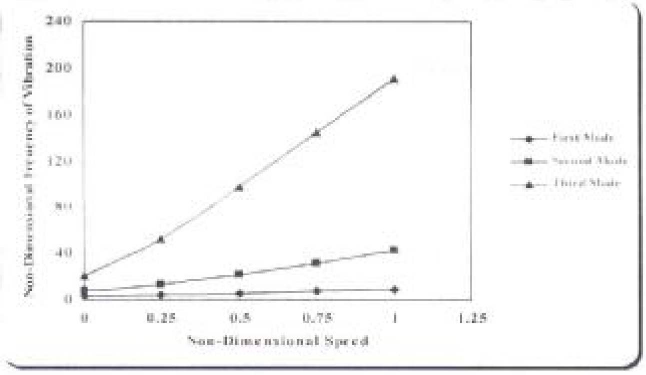


Fig. 7 Variation of non-dimensional frequency of vibration with non-dimensional speed (a/b=1, $\theta = 0, \bar{r} = 3$)

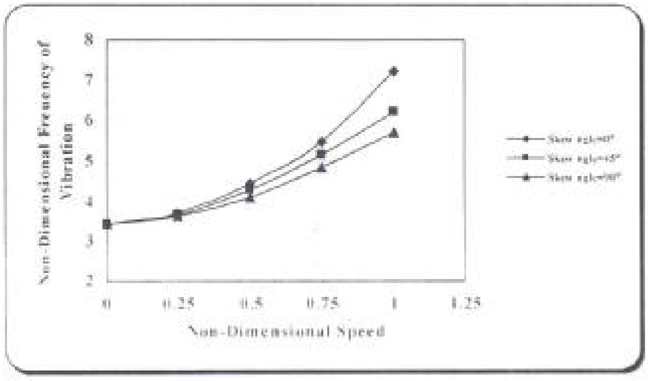


Fig. 8 Variation of non-dimensional frequency of vibration with non-dimensional speed ($a/b = 1, \bar{r} = 1$)

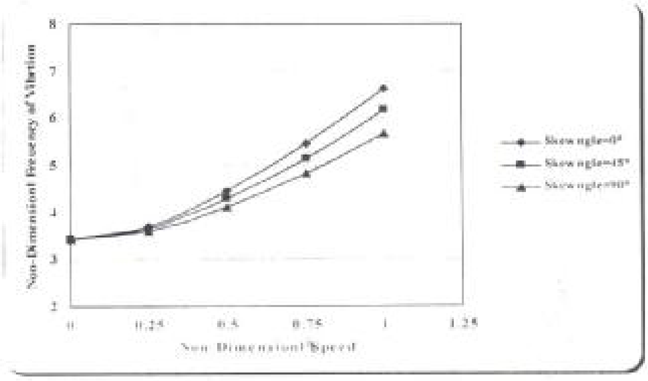


Fig. 9 Variation of non-dimensional frequency of vibration with non-dimensional speed ($a/b = 2, \bar{r} = 1$)

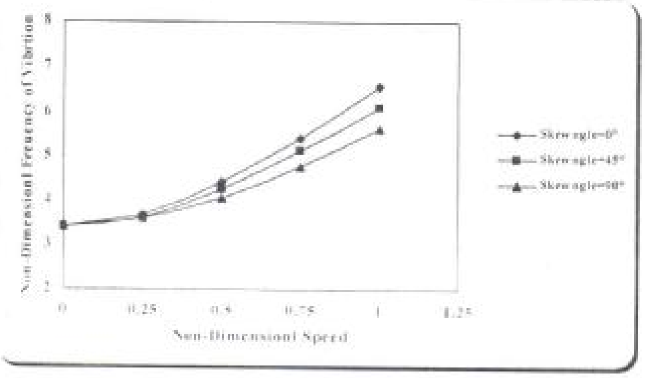


Fig. 10 Variation of non-dimensional frequency of vibration with non-dimensional speed ($a/b = 3, \bar{r} = 1$)

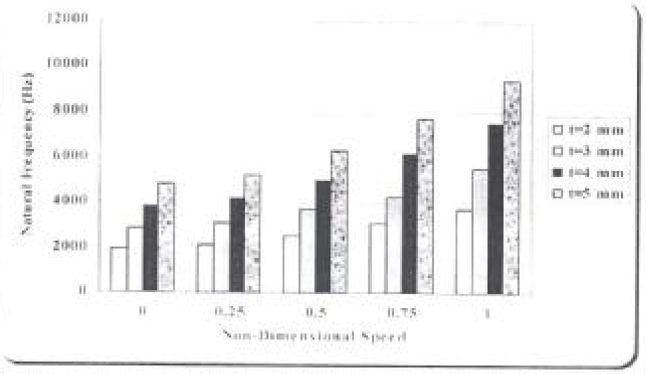


Fig. 11 Variation of fundamental natural frequency for rotating cantilever plate (Hz) with non-dimensional speed ($a/b = 1, \theta = 0, r = 1$)

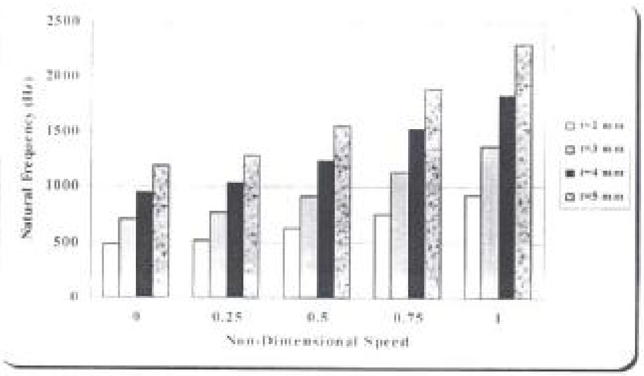


Fig. 12 Variation of fundamental natural frequency for rotating cantilever plate (Hz) with non-dimensional speed ($a/b = 1, \theta = 0, r = 1$)

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8.Nomenclature

c	Coriolis matrix of the element
b	Width of the rotating plate m
a	Length of the rotating plate m
$\vec{d}$	Displacement of a typical point in the mid-surface
h	Thickness of a typical of an element
h_1, h_2, h_3	Thickness at the nodes 1, 2, 3
t	Thickness of the rotating plate
k_p, k_f, k_G	In-plane, bending and geometric stiffness matrices of an element
k_R	Additional stiffness of an element
m_E	Mass matrix of an element
q	Nodal displacements of the structure
u, v	In-plane components of the displacement $\vec{d}$
u_i, v_i	In-plane nodal displacements of node i
w	Components of the displacement $\vec{d}$ normal to the mid-surface
w_i	Bending nodal displacements of node i
x, y	Components of $\overline{IM}$ in R_i
x_1, t, z_i	Components of $\overline{OI}$ in R_o
A	Area of the triangular element
C	Coriolis matrix of the structure
D	Flexural rigidity of the plate, $= Et^3/12(1-\upsilon^2)$
E	Young's modulus
F	Nodal centrifugal force vector for the element
$\vec{F}(\Omega^2)$	Nodal centrifugal force vector for the structure
K_E, K_G	Elastic and geometric stiffness of the structure
K_R	Additional stiffness of the structure
L_1, L_2, L_3	Area co-ordinates of the triangle
M_E	Mass matrix of an structure
$N, N_1, N_2, N_{k1} - N_{ml}$	Shape function
P_1, P_2	Plane stress, bending strain energy
P_3	Supplementary strain energy due to the effect of bending displacement on mid-surface strains

$R(OXYZ)$	Global Cartesian co-ordinate system attached to the rotating disc
$R_0\,(OX_0Y_0Z_0)$	Absolute fixed Cartesian co-ordinate system
$R_1\,(IX_0Y_0Z_0)$	Local Cartesian co-ordinate system
T	Kinetic energy
U	Total potential energy
$\overline{V}$	Absolute velocity of M
β	Non-Dimensional frequency of vibration, $=\omega a^2\sqrt{\rho t/D}$
ε	Strains
$\varepsilon_p, \varepsilon_f$	Strain due to in-plane and bending displacements
ε_g	Effect of bending displacements on mid-surface strains
θ	Skew angle, setting angle
υ	Poisson's ratio
ρ	Mass density Kg/m^3
σ	Stress N/m^2
σ_p	In-plane stress resultant
σ_f	Bending and twisting moment
$\sigma_x^0, \sigma_y^0, \tau_{xy}^0$	Initial in-plane stress
ω	Frequency in rotation $(rad/\sec)$
ω_o	Frequency at rest $(rad/\sec)$
$\overline{\Omega}$	Speed of rotation $(rad/\sec)$
$\Omega_1, \Omega_2, \Omega_3$	Components of $\overline{\Omega}$ in R_1
$\overline{\Omega}$	Non-dimensional speed, $=\Omega/\omega_n$
$\overline{r}$	r/a

التحليل الديناميكي لصفيحة دوارة مثبته من جهة واحدة

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الخلاصة:
في هذا البحث تم أستخدام تقنية العناصر المحددة لحساب الترددات الطبيعية لصفيحة مثبته من جهة واحدة بقرص دوار. تم تمثيل هذه الصفيحة كقشرة دوارة، طور برنامج حاسبة باستعمال العنصر المثلث كعنصر تجزئة لهيكل الصفيحة، يحتوي هذا العنصر على ثلاث عقد لكل عقدة ست درجات من الحرية. في هذا التحليل تمت دراسة تأثير الاجهاد الابتدائي (الجساة الهندسية) و التأثير الدوراني بدون الاخذ بنظر الاعتبار وجود تأثير تعجيل كيريولس (carioles acceleration).
تم حساب الترددات الطبيعية بأستخدام تقنية التكرار المتزامن، تم أيضا في هذا البحث دراسة عدة متغيرات مثل سرعة الدوران و قطر القرص و زاوية التثبيت و سمك الصفيحة و نسبة الطول الى العرض.بينت النتائج العددية تطابقا مقبولا مع نتائج باحثين اخرين بأستخدام طرق اخرى.