Modeling Of Porosity Equation For Water Flow Through Packed Bed Of Monosize Spherical Packing

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Abstract

The porosity and the pore size have a great effect on the properties of packed beds. There is no doubt that any small change in the porosity of the packed bed leads to a big change in the pressure drop required for the fluid flow through the packed bed.

Modified equation for porosity had been attempted depending on the parameters affecting the porosity for water flow through packed bed for sphere particles of mono size packing system. The considered parameters affecting the porosity in the packed bed of sphere packing are the particle and bed diameters. Several types and kinds of packing materials with different sizes have been used in the packed bed such as Pea Gravel, Marbles, Glass Marbles, Black Marbles, Clear Marbles, Acrylic balls, Celite spheres and Glass spheres. The effects of different parameter on the porosity using the modified equation results have been studied in the present work, and The calculation results of the porosity modified equation have been compared with Furnas equation of porosity and with experimental results taken from documented literature data; the comparisons show a very good agreement between the porosity modified equation and experimental results.
Based and accurate semi-empirical equation for the pore size has been developed in this work by modifying Millington and Quirk equation. Model development and tests based on pore size analysis of experimental data. The effects of different parameter on pore size using the modified equation results have been studied in the present work, and the calculation results of the modified equation for the pore size have been compared with the experimental results taken from documented literature data; the comparisons show a very good agreement between the modified equation and experimental results.

Maxwell equation of tortuosity, which is one of the main parameters affecting the fluid flow through packed bed, was modified to satisfy the proposed equations of pore size, porosity.

Experimental data were used to get the best formula that gives the smallest average percentage error of 0.0001 %.

نمذجة جريان الموائع خلال العمود الحشوي لحشوات كروية الشكل

المستخلص

ان لقطر الفراغ والمسامية تاثير كبير جدا على خواص العمود الحشوي. لايوجد هناك شك بأن اقل تغيير في مسامية الحشوة للعمود الحشوي تؤدي الى تغير كبير في هبوط الضغط المجهز لجريان الموائع خلال العمود الحشوي. طورت معادلات لحساب المسامية بالاعتماد على العوامل المؤثرة على مسامية الحشوة اثناء جريان الماء خلال العمود الحشوي لحشوات الكروية الشكل الاحادية. ان العوامل المؤثرة على مسامية الحشوة في العمود الحشوي للحشوات الكروية وجدت انها قطر الحشوة وقطر العمود الحشوي. استخدمت انواع واشكال مختلفة من الحشوات وبحجوم مختلفة. ان تاثير العوامل المختلفة على مسامية العمود ةالحشوي تمت دراستها باستخدام المعادلة المطورة, وكذلك تمت مقارنة النتائج المستحصلة من المعادلات المطورة لحساب مسامية الحشوة مع معادلة فرناس لحساب المسامية ومع النتائج العملية المستحصلة من المصادر الموثقة، وقد اظهرت المقارنة تطابق جيد بين نتائجنا والنتائج العملية.
في هذا البحث تم ايجاد معادلات شبه عملية لحساب قطر الفراغ, حيث طورت معادلة ملنكتون وكورك بالأعتماد على تحليل النتائج العملية لحسابات قطر الفراغ. ان تاثير العوامل المختلفة على قطر الفراغ تمت دراستها باستخدام المعادلة المطورة, وكذلك مقارنة النتائج المستحصلة من معادلة ملنكتون وكورك المطورة لحساب قطر الفراغ للحشوات الكروية الشكل ذات حجم واحد مع النتائج العملية المستحصلة من المصادر الموثقة، وقد اظهرت المقارنة تطابق جيد بين معادلة ملنكتون وكورك المطورة والنتائج العملية.
معادلة ماكسويل التي تحسب المسار المتعرج خلال العمود الحشوي والتي تعتبراحدى العوامل الاساية المؤثرة على جريان المائع خلال العمود الحشوي, طورت هذه المعادلة لغرض تحقيق توافق مع معادلات قطر الفراغ والمسامية, حيث استخدمت التجارب العملية لإيجاد افضل صيغة للمعادلة وبأقل نسبة خطأ والتي هي 0.0001 %.

1. Introduction

There has been an increase in interest of the effect of porous media, because of their extensive practical applications in geophysics, thermal insulation in buildings, petroleum resources, packed-bed reactors and sensible heat-storage beds [1]. Porous materials are encountered everywhere in everyday life, in technology, and in nature. The most important structural characteristics of porous media include porosity, radial variations in void fraction, specific lateral surface area variations etc. All properties of porous media are influenced by the pore structure. Pore structure parameters represent average behavior of a sample containing many pores. The most important pore structure parameters are the porosity, the tortuosity, permeability and the pore size.

Many investigators described the porosity and found that the packing porosity depends upon the particle size, size distribution, particle shape, surface roughness, method of packing, and the size of the container relative to the particle diameter [2]; **Stanek** 1972[3] and **Szekely** 1973 [4] suggested a method to correct the effects of the variable porosity on flow through porous media by considering two distinct uniform void fractions. **Kubo et al.** in 1978 [5] reported photographic observations on flow patterns in porosity of porous media of equal sized spheres. **Standish** and **Borger** in 1979 [6], Standish and Mellor in 1980 [7], **Standish** and **Leyshon** in 1981[8], and **Standish** and **Collins** in 1983[9] also study experimentally the porosity and permeability of multi component mixtures of uniform and irregular shape particles, the study the porosity of multi component mixtures from the results of binary mixtures. **Ouchiyama** and **Tanka** in 1984 [10] proposed a mathematical model to calculate the porosity of particulate mixtures, especially those in the ternary system, from the knowledge of particle sizes involved and their proportion in the mixture. **Standish** and **Yu** in 1987 [11] studied the porosities of multi-size mixtures; they measured porosities for ternary systems of uniform and non-uniform mixtures of spherical particles. **Fuller** and **Thompson** in 1987 [12] studied the influence of distribution of the particle size upon the density of granular material. **Yu**, **Zou**, **Standish** and **Xu** in 1989 [13] presented a general discussion of the porosity and particle size distribution relation with the commonly used size distribution systems including the discrete binary, ternary and quaternary mixtures and the continuous Gaudin-Schuhmann, log-normal, Rosin-Rammler and Johnson's S_B size distributions. **Parkhouse and Kelly 1995**[14] gave an equation for the relationship between the porosity and the ratio of length to diameter of the bed using more than 7 cylinders, based on the statistical approach to the distribution of the pores in the stacks. **Zou** and **Yu** in 1996[15] proposed an empirical equation to quantify the relationship between the porosity and sphericity of cylindrical particles in dense random packing. This relation was based on the experiments of wood cylinder packing. **Moallemi** in 1989, **Yu** and **Standish** in 1991, **Summers** in 1994 and **Ismail** in 2000 [16] studied the local voidage for the mixtures of spheres packing (mono, binary and ternary) and found that the local voidage variations in the axial, radial and angular direction.

1-1 Variables Affecting Flow through Granular Bed

The variables affecting resistance to flow through packed bed can be classified into two basic categories [7], variables related to the fluid flowing through the bed such as viscosity, density, and rate of fluid flow, variables related to the nature of the bed are numerous and to be considered as shape and size of the particles, container walls effects, porosity of the bed, surface roughness of the particle, and orientation of particles.

For fluid flow through packed beds many particles of irregular shapes usually used. To treat this problem the particles are considered as spheres by introducing a factor called sphericity Φ which allows calculation of an equivalent diameter [17].

The sphericity of a particle is the ratio of the surface area of this sphere having the same volume as the particle to the actual surface area of the particle, as shown below:

$$\Phi = \frac{a_{sphere}}{a_{particale}} = \frac{6/d_p}{S_{particle}/V_{particle}} \qquad (1)$$

For a sphere, the surface area $S_p = \pi d_P^2$ and the volume is $V_p = \pi d_p^3/6$.

The specific surface area of a particle is used through most of the equations or formulas of fluid flow through packed bed, and it is defined as follows:

$$S = \frac{S_p}{V_p} \qquad (2)$$

Where S is specific surface area of a particle in m^{-1}, S_p is the surface area of a particle in m^2 and V_p is the volume of a particle in m^3. Therefore for spherical particle:

$$S = \frac{\pi\, d_p^{\,2}}{\pi\left(d_p^{\,3}/6\right)} = \frac{6}{d_p} \qquad (3)$$

where d_p is the particle diameter in m. For beds consisting of a mixture of different particle diameters, the effective particle diameter (dp_{eff}) can be used instead of d_p as [18]:

$$dp_{eff} = \frac{1}{\sum_{i=1}^{n} \frac{x_i}{d_{pi}}} \qquad (4)$$

where: x_i is the fractional weight of spherical particle.

d_p is the diameter of spherical particle.

1-2 Permeability

The permeability, *k*, is the measure of the flow conductance of the porous medium and it is defined by the Darcy's law, as shown below:

$$k = \left(\frac{\varepsilon}{\tau}\right)^2 \frac{\varepsilon\, dp_{eff}^2}{36(1-\varepsilon)^2 K_c} \qquad (5)$$

where ε is the porosity of the porous media, τ the tortuosity factor and K_c, is the kozeny's constant, and it is dependent of the porosity for packing [19]. One very well-known equation which relates the permeability to porous media properties was derived by Kozeny in 1927[20]. He viewed the porous bed as an assemblage of channels of various cross sections and expressed the permeability as:

$$k = c\frac{\varepsilon^3}{S^2\tau} \qquad (6)$$

where c is a proportionality parameter, which depends on the shape of the channels. The Kozeny equation has been largely applied and modified by other researchers. Carman introduced the specific surface exposed to the fluid S_B (S_B=(S-1)) and set the constant c to 1/5 which gave the best fit to his experimental results. The result is known as the Kozeny-Carman equation [21]:

$$K = c\frac{\varepsilon^3}{5S_b^2(1-\varepsilon)^2} \qquad (7)$$

A more recent modification of the Kozeny-Carman equation is due to Blake, who related permeability to the void fraction and primary particle size and introduced a correction factor derived from experimental results. In this case, the permeability k is written as:

$$K = \frac{d_p^2\varepsilon^3}{180(1-\varepsilon)^2} \qquad (8)$$

This equation is considered valid for media consisting of individual particles [22].

1-2 Tortuosity Factor

Tortuosity is defined by **Sheidegger** in 1974 [21] as the ratio of the average pore length (L_e) to the thickness of the medium (L):

$$\tau = \frac{L_e}{l} \tag{9}$$

Where L_e is the average length of porous medium and l is the bed thickness.

Figure 1shows the effect of tortuosity in a porous media between the average length and the bed thickness [23,24]. Tortuosity is not a physical constant and depends first of all on other porous media characteristics, like porosity, pore diameter, channel shape, etc. in general, in granular packing or beds the value of tortuosity lies in the range 1.1-1.7 [25]. It is difficult to determine tortuosity experimentally and in general, tortuosity is calculated by using the porosity and the effective diffusion coefficient or from the Kozeny coefficient [19]. The tortuosity may be expressed as a function of **kozeny's** coefficient K as [23]:

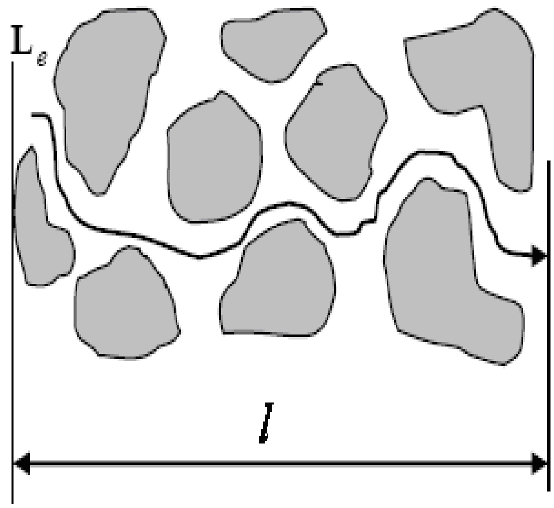

Figure 1 The effect of tortuosity in a porous media

$$\tau = \sqrt{\frac{K}{K_c}} \tag{10}$$

where: K_c is the kozeny's constant and K is the kozeny's coefficient.

Several empirical correlations, which suggested a relationship between tortuosity and porosity, such as Maxwell in 1873, Weissberg in 1963, Comiti and Renaud in 1989 and Boudreau in 1996 [25], as shown below:

$\tau = 1.5 - 0.5\varepsilon$ (Maxwell, 1873) (11)

$\tau = 1 - 0.5\ln(\varepsilon)$ (Weissberg, 1963) (12)

$\tau = 1 - 0.41\ln(\varepsilon)$ (Comiti and Renaud, 1989) (13)

$\tau = \sqrt{1 - \ln(\varepsilon^2)}$ (Boudreau, 1996) (14)

Archie in 1942 suggested most frequently relationship between tortuosity and porosity for a mixed bed of particles dependent on the methods applied for packing preparation, as:

$$\tau = \frac{1}{\varepsilon^n} \quad (15)$$

Where n is a numerical value, and depend on the properties of the packing bed. The value of n lies in the range from 0.4 for loose packing to 0.5 for dense packing [26]. The above equations all satisfy the condition τ =1 for ε =1, and this consistent with the physical situation observed [25]. Also tortuosity can be calculated from the effective diffusion coefficient D_e, which characterizes mass transfer in porous media, it can be written as [23]:

$$\tau = \frac{D_\circ}{D_e}\varepsilon \quad (16)$$

Sen in 1981 and Yun in 2005 showed that for an isotropic medium with spherical particles the tortuosity of porous and granular media decreases with increasing bed voidage [25].

1-3 Pore Size

The study of pore size is necessary to study the packing of a porous medium. Each void in the porous medium is connected to more than one other pore (through pore or interconnected), connected only to one other pore (blind pore or dead end), or not connected to any other pore (closed pore or isolated) and fluid flows through the interconnected pores [19]. Millington and Quirk in 1964 [27] derived a model for the determination of the pore size for gas flow though soil media by linking air permeability and gas diffusivity, combining Fick's law for diffusive transport with Poiseuille's law for convective fluid transport, and assuming soil pores to be uniform, tortuous, and nonjointed tubes of similar diameter. The pore size equation is shown below:

$$d_{pore} = 2\sqrt{\frac{8k}{D_{P,g}/D_{0,g}}} \tag{17}$$

Where $D_{P,g}$ is the gas diffusion coefficient in soil (m^3 soil air m^{-1} soil sec^{-1}), $D_{0,g}$ is the gas diffusion coefficient in free air (m^2 air sec^{-1}) [28].

Moldrup (Moldrup et al., 2000b) found that gas diffusivity for porous media was best described by the following equation [29]:

$$\frac{D_{P,g}}{D_{0,g}} = \varepsilon^{1.5} \tag{18}$$

1-4 Porosity of the Bed

The porosity is the most important property of a porous medium and it affects most of the physical properties of the medium. The porosity is affected by many variables that may be classified into the categories of particle properties, container properties and packing method [30,31]. The porosity (ε) is defined as the ratio of the void volume to the total volume of the bed (the volume fraction occupied by the fluid phase) [32], i.e:

$$\varepsilon = \frac{Volume of\ voids in\ a\ bed}{total volume of\ the bed} \tag{19}$$

For spherical packing, geometric analysis predicts that the porosity will be constant with consistent packing methods, regardless of the diameter of the spheres [33]. The porosity has a great effect on the properties of porous media. **Leva** [32] found that a 1% decrease in the porosity of the bed produced about an 8% increase in the pressure drop, whilst **Carman** [34] reported a higher value, 10% increase in the pressure drop for every 1% decrease in porosity. Depending on the type of the porous medium, the porosity may vary from near zero to almost unity**. Kaviany in 1995** suggested that the normal range of average void fraction was from 0.36 to 0.43 [35]. Measurement of porosity is made by using several techniques, such as imbibitions, mercury injection and gas injection methods give an effective porosity value [30]. The porosity can be evaluated experimentally using the following equation [33]:

$$\varepsilon = 1 - \frac{\rho_b}{\rho_t} \tag{20}$$

Furnas [36] proposed equation for the porosity in packed column with sphere packing as function of particle and bed diameter, as shown below:

$$\varepsilon = 0.375 + 0.34\frac{d_p}{D_r} \qquad (21)$$

Where d_p is the particle diameter in m and D_r is the bed diameter in m.

Feng and **Yu** [37] formulated empirical equations in terms of dimensionless group for spherical particles with special reference to the effect of liquid addition. They show that the properties of both particle and liquid affect the packing behaviors significantly. Under given packing conditions, dry based porosity increases to a maximum and then keeps constant with the increase of liquid content. Particle size and surface tension are the main factors in the quantification of this porosity- liquid content relation.

The purpose of this work is to:

1.Study the effect of particles size and size distribution on the bed

2.Study the effect of bed porosity on the pressure drop.

3.Study the effect of bed porosity and permeability on the pore size.

4.Proposing equation for the porosity as a function of particle and bed diameter.

5.Proposing equation for the tortuosity as a function of bed porosity.

6.Derive a conceptually based and accurate semi-empirical equation model for pore diameter as a function of bed porosity and permeability.

2- Theory of the Model

This work deals with proposing equations for modeling fluid flow through packed bed. Many factors affecting the fluid flow through packed bed, such as particles diameter, porosity, pore diameter, tortuosity, and bed length. Semi-empirical equations have been proposed to evaluate the porosity of the bed, the pore diameter and the tortuosity factor using experimental data.

2-1 The Porosity Proposed Equation Model

The porosity has a great effect on the properties of packed beds. Several attempts were made to simulate the porosity in packed beds [4, 5, 7, 18, 38]. Semi-empirical equation was developed in the present work by modifying Furnas equation of porosity (equ. 21). The new forms of the suggested equations of porosity depend on particle diameter (d_p) and bed diameter (D_r). Experimental data were used to get the new forms of porosity. The proposed equations of porosity can be written as follows:

$$\varepsilon = i_1 + i_2 \left(\frac{d_p}{D_r} \right)^{i_3} \qquad (22)$$

Where i_1, i_2 and i_3 are constants and can be evaluated from experimental data taken from literatures for water flow through packed bed of sphere packing by using statistical fitting.

2-2 The Pore Size Proposed Equation Model

The pores are tortuous and interconnected, with a distribution of different sizes and shapes, it is convenient to assume that the pore diameter (d_{pore}) represents a cylindrical form of the hydraulic diameter (r_H) which is the cross sectional area perpendicular to fluid flow divided by the wetted perimeter, this can be represented as [39]:

$$r_H \;\; for\ pourous\ medium = \frac{volume\ open\ to\ flow}{total\ watted\ surface} \qquad (23)$$

$$r_H = \frac{volume\ of\ bed * \varepsilon}{No\ of\ spherical\ particles * surface\ area\ of\ one\ particle} \qquad (24)$$

But:

$$No.of\ particles = \frac{Volume\ of\ bed * (1-\varepsilon)}{Volume\ of\ one\ particle} \qquad (25)$$

Therefore:

$$r_H = \frac{volume\ of\ bed * \varepsilon}{volume\ of\ bed * (1-\varepsilon) * \frac{surface\ area}{volume}} \qquad (26)$$

i.e.

$$r_H = \frac{d_p}{6} \left(\frac{\varepsilon}{1-\varepsilon} \right) \qquad (27)$$

While the pore diameter (equivalent to the hydraulic diameter) is defined as four times the cross sectional area per wetted perimeter [40], therefore:

$$d_{pore} = 4r_H = \frac{2}{3}\frac{\varepsilon}{(1-\varepsilon)}d_p \tag{28}$$

Equation (28) can be taken as experimental value of pore diameter where the particle diameter and the porosity are determined experimentally.

Many theoretical equations for the pore diameter have been used before one of them is Millington and Quirk [27]. Millington and Quirk equation was developed to be used for pore size of sphere packing in water. In the modification, it was suggested that the pore diameter is a function of permeability and porosity of the packed bed, and can be written as follows:

$$d_{pore} = f(k,\varepsilon) \tag{29}$$

$$d_{pore} = c_1 \frac{k^{c_2}}{\varepsilon^{c_3}} \tag{30}$$

Where c_1, c_2 and c_3 are constants and can be evaluated from experimental data taken from literature by using statistical fitting.

2-3 The Tortuosity Factor Proposed Equation Model

One of the important parameter needed to represent in this work is the tortuosity of sphere packing which is one of the main parameters affecting the fluid flow through packed bed. There are many expressions that show the dependence of tortuosity in porosity of packed bed one of them Maxwell equation, equ.31 below [26]:

$$\tau = 1.5 - 0.5\varepsilon \tag{31}$$

Equation (31) can be modified to satisfy the proposed equations of pore size, porosity. Experimental data were used to get the best form as follows:

$$\tau = m_1 - m_2\varepsilon^{m_3} \tag{32}$$

Where m_1, m_2 and m_3 are constants that can be evaluated from experimental data taken from literature by statistical fitting. This formula gives the smallest average percentage error and gives the value of one when the porosity is one.

3- Results And Discussions

The present section deals with the results and discussions of the proposed semi-empirical equation for the porosity, the pore size and tortuosity. These results depend on values of porosities, bed diameters, particles diameters, velocities, bed length and other parameters taken from experimental work. This section also contains the discussions of the proposed equation results, and the comparisons between these results and experimental results taken from documented literatures, as well as comparisons were made between all these results and similar results taken from theoretical equations (Furnas equation, MQ equation).

Equations 22, 30 and 32 was fitted using experimental data obtained from documented literatures [41, 42,43,44,45,46,47,48], in order to calculate the different constants in it. This had been done for water flow through packed bed for mono size spherical packing system. Many types of packing were used in the present work such as Pea Gravel, Marbles, Glass Marbles, Black Marbles, Clear Marbles, Acrylic balls and Glass spheres. The diameters of the packing materials used in this model are from the range of (0.2-8.89) cm, the porosity is from the range of (0.3-0.76), the pore size is from the range of (0.087 cm to 4.86 cm), the permeability is from the range of (4.32 E-05 cm^2 to 0.16 cm^2), the bed diameters is from the range of (7.62 - 15.24) cm, velocity is from the range of (0.002 - 0.3) m/s, the pressure drop is from the range of (24.9 – 59097) Pa and the height of packing is from the range of (7.62 - 56) cm.

So the new model for the porosity for water flow through packed beds of mono size sphere packing was found to be as follows:

$$\varepsilon = 0.36 + 0.08 \left(\frac{d_p}{D_r} \right)^{0.5} \tag{33}$$

The correlation coefficient was 0.9662 and the average percentage error was found to be 0.00012% between experimental work and the proposed equation.

The new modified model for the pore size equation 30 was found to be as follows:

$$d_{pore} = 4.24 \ \frac{k^{0.52}}{\varepsilon^{-0.23}} \tag{34}$$

The average percentage error was found to be 0.03% between experimental work and the new model equation and with a correlation coefficient of 0.921.The tortousity model eq.32 for water flow through packed beds can be represented in the following equation:

$$\tau = 1.5312 - 0.5268\varepsilon^{0.8984} \qquad (35)$$

The above equation deviates from experimental data with a very small average percentage error of 0.0001 % and with a correlation coefficient of 0.9997.

3-1 Comparisons between Proposed Equation, Theoretical Equations and Experimental Results

Comparisons between the porosity obtained by using the modified equation (eq. 33), the experimental values of the porosity and the porosity obtained by using Furnas equation (eq.21), are shown in table 1

Table 1 The porosity results for water flow through packed bed

Type of packing	Dr (m)	d_p (m)	ε (Experiment)	ε (Present work)	ε Furnas
Black marbles[41]	0.0889	0.0191	0.47	0.4022	0.4479
Marbles[41]	0.1524	0.0127	0.4	0.388	0.4033
Marbles[42]	0.0889	0.0127	0.4	0.3953	0.4236
Pea gravel[43]	0.1524	0.0127	0.38	0.388	0.4033
Marbles[43]	0.1524	0.0127	0.38	0.388	0.4033
Pea gravel[43]	0.0889	0.0127	0.38	0.3953	0.4236
Pea gravel[43]	0.0889	0.0127	0.38	0.3953	0.4236
Pea gravel[43]	0.1524	0.0127	0.38	0.388	0.4033
Marbles[43]	0.0889	0.0127	0.38	0.3953	0.4236
Marbles[43]	0.1524	0.0127	0.38	0.388	0.4033
Marbles[44]	0.0889	0.0127	0.38	0.3953	0.4236
Pea gravel[44]	0.0826	0.0127	0.3	0.3964	0.4273
Pea gravel[44]	0.0826	0.0127	0.35	0.3964	0.4273
Pea gravel[44]	0.0825	0.0026	0.386	0.379	0.3857
Marbles[44]	0.0826	0.0127	0.38	0.3964	0.4273
Marbles[44]	0.0826	0.0021	0.38	0.3777	0.3838
Pea gravel[44]	0.0825	0.003	0.388	0.3802	0.3876
Acrylic ball [45]	0.08	0.0064	0.3571	0.3875	0.402
Acrylic ball [45]	0.08	0.0127	0.4028	0.3969	0.429
Acrylic ball [45]	0.08	0.0127	0.4028	0.3969	0.429
Glass marbles[45]	0.08	0.0127	0.406	0.3969	0.429
Glass marbles[45]	0.08	0.0127	0.406	0.3969	0.429
Acrylic ball [45]	0.08	0.0127	0.4054	0.3969	0.429
Glass marbles[45]	0.08	0.0127	0.406	0.3969	0.429
Marbles[45]	0.1524	0.0889	0.4207	0.4269	0.5733
Marbles[45]	0.08	0.0127	0.406	0.3969	0.429
Marbles[45]	0.0826	0.0127	0.4	0.3964	0.4273
Marbles[45]	0.08	0.0127	0.406	0.3969	0.429
Marbles[45]	0.1524	0.0127	0.376	0.388	0.4033
Acrylic ball [45]	0.08	0.0064	0.3571	0.3875	0.402
Marbles[45]	0.1524	0.0127	0.406	0.388	0.4033
Black marbles[46]	0.1461	0.019	0.41	0.3939	0.4192
Black marbles[46]	0.0889	0.019	0.4	0.4022	0.4477
Black marbles[46]	0.1461	0.019	0.41	0.3939	0.4192
Black marbles[46]	0.0889	0.019	0.4	0.4022	0.4477
Glass[47]	0.0762	0.0042	0.3793	0.3837	0.3937
Glass[47]	0.0762	0.0051	0.4051	0.3856	0.3978
Glass[48]	0.0762	0.0061	0.4156	0.3876	0.4022
Glass[48]	0.0762	0.0079	0.4265	0.3907	0.4102
Glass[48]	0.0762	0.0101	0.4321	0.3942	0.4201

Table 1 show a very good agreement between the porosity obtained by using the proposed equation and the experimental data, while results from Furnas equation for porosity was far away from the experimental data, this appears clear in the porosity of the marbles, where the experimental porosity was 0.4207 and the porosity obtained from the proposed equation was found to be 0.4269, while the porosity obtained from Furnas equation was 0.5733.

The results of the modified equation for the pore size (eq. (20)) are compared with the experimental values of the pore size as shown in table 2 below. The table also shows the experimental values of the porosity, permeability and particle diameter taken from literatures. The experimental values of the pore size were determined by using equation (18).

Table 2 The pore size results for water flow through packed bed

Type Of Packing	**ε**	**k** (m^2)	**d_p (m)** (Experiment)	**d_{pore} (m)** (Experiment)	**d_{pore} (m)** Modified Equation
Pea gravel[36]	0.38	4E-06	0.0127	0.00519	0.0056
Marbles[37]	0.3	2E-06	0.0127	0.00363	0.0038
Marbles[37]	0.35	4E-06	0.0127	0.00456	0.0048
Marbles[37]	0.4	5E-06	0.0127	0.00564	0.0061
Marbles[35]	0.4	5E-06	0.0127	0.00564	0.0060
Acrylic ball[38]	0.41	5E-06	0.0127	0.00577	0.0062
Glass marbles[38]	0.41	5E-06	0.0127	0.00579	0.0062
Glass marbles[38]	0.38	4E-06	0.0127	0.0051	0.0054
Black marbles[39]	0.41	6E-06	0.019	0.0088	0.0068
Celite spheres[39]	0.37	3E-06	0.0064	0.00251	0.0047
Celite spheres[39]	0.38	3E-06	0.0064	0.00262	0.0049

From table 2, it can be seen that the values of the pore size of the modified equation model used are comparable with the experimental values taken from literatures. It was observed that as the pore size increased the surface area increased creating additional flow resistance. The permeability increased and the inertia coefficient decreased with increasing pore size diameter.

3-2 Studying The Effect Of Different Parameters On Porosity Modified Equation

The porosity is affected by many variables. The main two are particle diameter and bed diameter. A certain range for each parameter was taken in this study according to the available experimental data from literatures.

3-2-1 Effect of particle diameter on porosity

Figure 2 indicates that any increase in the particle diameter causes increase in the bed porosity for the same bed diameter range. For example at bed diameter 0.0762 m, the particle diameter was 0.0195 m and porosity was 0.4, but when the particle diameter was increased to 0.0889 m the porosity increased to 0.446 for the same bed diameter.

3-2-2 Effect of Bed Diameter on Porosity

Figure 3 shows that when the bed diameter was increased the porosity decreased for the same particle diameter. For example when the bed diameter was 0.0914m, the particle diameter was 0.0889m, and the porosity was found to be 0.4388. When the bed diameter increased to 0.1524m the porosity was decreased to 0.4211 for the same particle diameter.

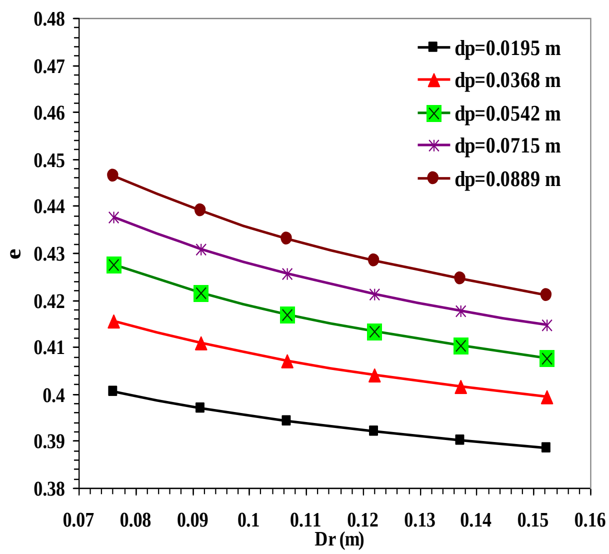

Figure 2 Porosity versus bed diameter

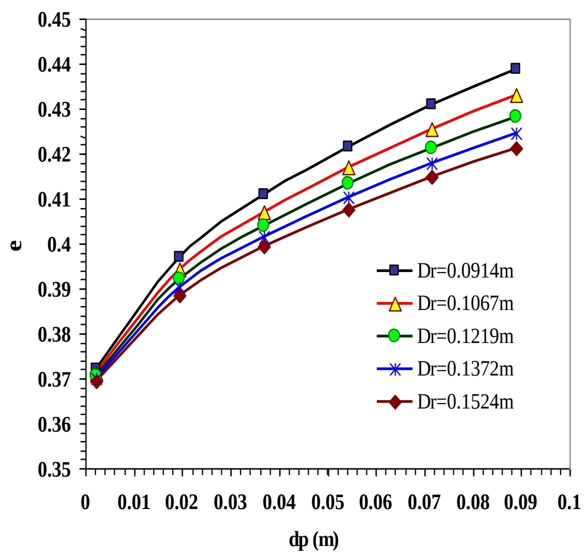

Figure 3 Porosity versus particle diameter

3-3 Studying the Effect of Different Parameters the Pore Size Modified Equation

This section shows the effect of different parameter on pore size using the modified equation results; a certain range for each parameter was taken in this study according to the available experimental data from literatures.

3-3-1 Effect of Porosity on Pore Size

Figure 4 shows that when the porosity increases the pore diameter increases, this is because when the void fraction between particles become larger leads to a large space between particles. It is also evident that as the pore density increases or the porosity increases the inertia coefficient increases which is consistent with the physical notion that the more tortuous the passage of fluid flowing through the sample the greater the inertia forces inside the porous medium increasing this way the value of this coefficient.

3-3-2 Effect of Permeability on Pore Size

Figure 5 indicates that an increase in the permeability causes an increase in the pore diameter. The permeability increased and the inertia coefficient decreased with increasing pore size diameter but did not show any clear relation with the porosity. They mentioned that there was strong influence on the drag force exerted on the fluid by which permeability increased with larger pores in the sample which also contributed to the effect of pressure drop.

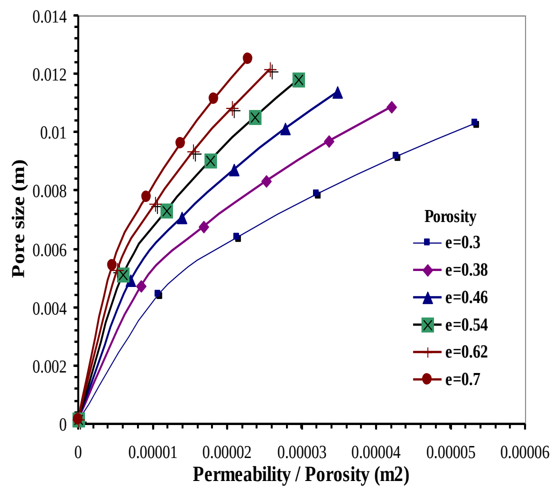

Figure 4 Pore size versus the ratio of permeability to porosity

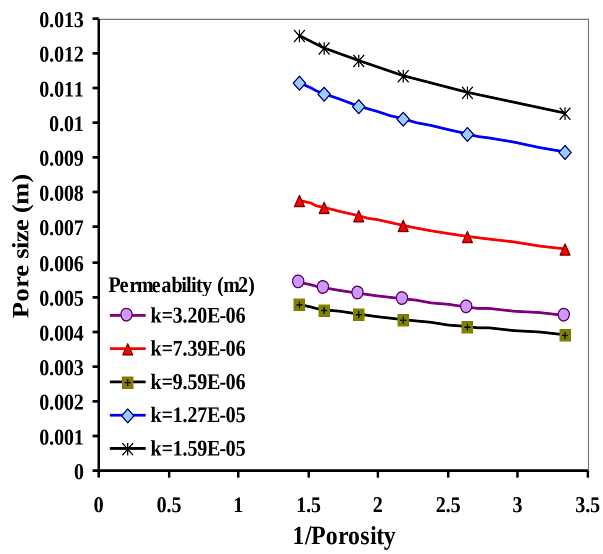

Figure 5 Pore size versus the porosity inverse

4- Conclusions

1. The porosity proposed equation results deviate's from experimental results with a very small average percentage error, and they are almost identical, while Furnas equation of porosity was far from the experimental data.
2. The porosity is affected by many variables. The main two are particle diameter and bed diameter, any increase in the particle diameter causes increase in the bed porosity for the same bed diameter range, and whenever the bed diameter was increased the porosity decreased for the same particle diameter.
3. The particle size and size distribution highly affect the bed porosity. For mono size packing, the lower the particle size, the lower is the bed porosity. The porosity value of the multi- size systems are generally less than those of mono size systems, because the particles of smaller sizes tend to fill the void spaces between the larger sizes particles.
4. New and interesting pore size equation has been obtained for water flow through porous media of mono size packing. The pore size modified equation had successfully described the effects of particles diameter, bed diameter and permeability on the fluid flow through porous media, compared with the experimental results.
5. It was found that an increase in pore diameter causes a decrease in pressure drop, this is due to the fact that when the pore diameter increase's the void between particles increases, and this leads to a decrease in the resistance to fluid flow.
6. It was revealed that as the pore diameter increased in the porous media the value of the permeability increased with decreasing pressure drop. Also as the pore size decreased, the surface area increased resulting in higher mechanical energy dissipation.

5- Notations

Symbols		***Notations***
A	=	The bed cross sectional area (m)
D_r	=	Diameter of the bed (m)
$D_{P,g}$	=	The gas diffusion coefficient in soil (m^3 soil air m^{-1} soil sec^{-1})
$D_{0,g}$	=	The gas diffusion coefficient in air (m^2 air sec^{-1})
d	=	The diameter of glass bed (m)
d_p	=	Diameter of the particle (m)
dp_{eff}	=	Effective particles diameter (m)
dpore	=	Effective pore diameter (m)

e	=	Porosity of the bed.
K	=	Kozeny's coefficient.
K_C	=	Kozeny's constant.
k	=	Permeability coefficient for the bed (m^2).
L	=	The height of packing in the bed (m)
L_e	=	Average length of porous medium (m).
l	=	Thickness of the bed (m)
r_H	=	The hydraulic diameter
S	=	Specific surface area of the particles (m^2/m^3)
S_B	=	Specific surface area of the bed (m^2/m^3)
S_c	=	Surface of the container per unit volume of bed (m^{-1})
u	=	Superficial velocity (m/s)

Greek Symbols

θ	=	The angle which the normal to the solid-liquid interface makes with the stream direction
ε	=	Porosity of the bed.
μ	=	Fluid viscosity (kg/m.s).
Φ	=	Sphericity.
ρ	=	Density of fluid (kg/m^3).
ρ_b	=	Bulk density (g/cm^3).
$\O_s$	=	The sphericity shape factor
δ	=	The orientation factor
τ	=	Tortuosity factor.

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