

## ADAPTIVE CONTROL WITH A MULTI-INPUT POLE-ASSIGNMENT FOR THE DISTILLATION COLUMN

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### Abstract

*In this paper a multivariable adaptive controller is designed to achieve a suitable control action for a binary distillation column. The system is two input and two output, where the estimated model is obtained using a multivariable identification procedure for unknown order system. Then a pole- assignment control scheme is designed for multi-inputs to obtain the required output response. The results show that the bottom and distillate compositions are controlled accurately.*

**Indexing terms:** *Multivariable control theory, distillation columns, identification, and pole-assignment.*

### الخلاصة :

*تم تصميم مسيطر متكيف متعدد المتغيرات لتحقيق فعل السيطرة المناسب لأعمدة التقطير الثنائية. يتكون النظام من مدخلين وكذلك هناك مخرجين , حيث ان التركيبة الرياضية المتوقعة له يتم الحصول عليها من خلال استخدام تقنية تعريف النظم المتعددة لأنظمة ذو الدرجات غير المعروفة.*

*تم تصميم طريقة تحديد الاقطاب لعدة مداخل لغرض الحصول على الاستجابة المطلوبة لنظام السيطرة على اعمدة التقطير. بين البحث مدى التقارب في النتائج لمتغيري التركيب للقعر ( $X_B$ ) والتركيز ( $X_D$ ) بما هو مطلوب عند إشارات الدخل لكليهما.*

## 1- Introduction

Distillation is perhaps one of the most widely used unit operation in the process industry. The distillation column is probably very important process studied in the chemical engineering literature.

The column for which the model is presented in this paper separates a single multi-component liquid feed into two liquid products in a tray-type distillation column (binary type). The column is equipped with a reboiler and a total condenser. The model assumes that vapor holdups are negligible and that the effluent streams are in thermodynamics equilibrium. The column pressure is assumed to remain constant throughout the dynamic tests. The dynamics of the reboiler and condenser are neglected. We will assume the two components with constant relative volatility throughout the column and theoretical (100 percent efficient) trays. The foregoing assumptions can be relaxed if desired. More elaborate dynamic mathematical models of distillation columns are available in the literature [1]. The method developed by Howard [2], and later elaborated by luyben [3], utilizes a dynamic mathematical model of the lower and accessories. It is similar to the degree-of-freedom analysis. The method will be applied to the separation of a binary mixture into two products. The procedure involves writing out the system equations and then the number of variables.

In adaptive control schemes for unknown systems, the parameter estimation and state observation are very important part in control action. The method of Salman [4] has been adopted here for multivariable identification of unknown order systems together with the pole- assignment of multi-input systems[5].

## 2- Dynamic Model of a Binary Distillation Column

The model is based on unsteady state mass balances [3]. The assumption of constant molar overflow means that energy balances are not required. Referring to fig (1) and with the foregoing assumptions, the dynamic model can be expressed by set of differential and algebraic equations. The digital simulation of distillation column is fairly straightforward. The main complication is the larger of differential equation per tray (a total continuity equation and a light component continuity equation) and two algebraic equations per tray (a vapor-liquid phase equilibrium relationship and a liquid-hydraulic relationship).

$$dM_N / dt = L_{N+1} - L_N \quad (1)$$

$$d(M_N X_N) / dt = L_{N+1} X_{N+1} + V_{yN-1} - L_N X_N - V_{yN} \quad (2)$$

$$Y_N = \alpha X_N / (1 + (\alpha - 1) X_N) \quad (3)$$

$$L_N = L_{N+1} + (M_N - M_{N+1}) / \beta \quad (4)$$

We also need two equations representing the level controllers on the column based and reflux drums.

$$D = F_1 (M_D), \quad B = F_2 (M_B) \quad (5)$$

An N-stage distillation column is described by  $(4N+7)$  differential equations and many algebraic equations to compute the values of  $(4N+9)$  variables. Note that  $(F)$  and  $(X_F)$  are assumed to be given and whose values must be specified by designer. Therefore the system is under specified by two equations.

From a control viewpoint this means that there are only two variables that can be controlled. The two variables that be specified are  $(R)$  and  $(V)$ . They can be held constant or they can be changed by two controllers to hold some other two variables constant.

### **3- Dual composition control**

Recently the increasing demands of high product quality, minimal waste generation, and low energy consumption have called for better performance and tighter control of distillation columns. As Gould [6] emphasized 20 years ago, the two processes itself is the main focus of chemical process control. In the past few decades, however, chemical control engineers have paid more attention to column control systems than to distillation columns themselves. Although the non-linear and distributed character of distillation columns has long been recognized, distillation dynamics has been modeled with linearized and lumped approaches in conventional control system design to fit into the linear control theory [1]. Control systems implemented using such an approach have functioned acceptably for columns with products of low to moderate purity but have encountered difficulties recently, when applied to high purity columns, especially for control of compositions of both overhead and bottom products (dual-composition control) [7]. The column has two inventory-control loops and two composition -control loops. The composition control loops can interact among themselves, and thus this is a multivariable control problem. Several approaches for the solution of multivariable control problems are available. Perhaps the best known is based on the concept of relative gain [8] to find correct pairings of controlled and manipulated variables and concept and decoupling to achieve noninteracting feedback control [8]. Among other methods of multivariable control systems design are the method of inverse Nyquist arrays [9 and 10], the characteristic locus method [11], the pole-placement method, and optimal control method based on state-space models [12], model algorithmic control [13], dynamic matrix control [14], and singular value decomposition [15]. A survey of some of the other multivariable control methods found in Edgar [16].

In this work, the digital simulation of the column under study [3] was considered, so that the interactions between the loops (multivariable control problem) are not exit. Luyben [3] assumes that the control structure is adjust  $(R)$  and  $(V)$  to control  $(X_D)$  and  $(X_B)$  respectively.

### 4- Identification-Algorithm

The dynamic equation of the linearized model of the process under control are as follows:-

$$X(k+1) = A x(k) + B u(k) , \quad x(0)=X_0 \tag{6}$$

$$Y(k) = C x(k)$$

The method of Salman [4] can be used to construct the identifier for MIMO systems, the following should be complied:-

- 1- Divide the MIMO system into a q-number of MISO subsystems, where q is the number of outputs.
- 2- For subsystem number 1, implement the data vector which contains the input-output data as:-

$$O^T = [ \quad u^T(k-N+1) \quad \dots \quad u^T(k-1) \quad \quad y(k-N) \quad y(k-N+1) \dots \quad y(k-1) ] \tag{7}$$

Where  $u^T(.) = [ u_1 (.) \quad u_2 (.) \dots \dots \dots u_m (.) ]$

- 3- Perform the co-variance matrix as:-

$$s(k) = \begin{pmatrix} u^T(k-N) & u^T(k-N+1) & u^T(k-1) \dots & y(k-N) \dots & \dots & y(k-1) \\ u^T(k-N+1) & u^T(k-N+2) & u^T(k) & y(k-N+1) \dots & \dots & y(k) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u^T(k-2mN-N+1) \dots & \dots & u^T(k-2mN) & y(k-2mN-N+1) & \dots & y(k-2mN) \end{pmatrix} \tag{8}$$

- 4- construct the following equations:-

$$H(k+1) = i^2 H(k) + s(k) \tag{9}$$

$$v(k) = O y(k+1) \tag{10}$$

$$P(k+1) = i^2 P(k) + v(k) \tag{11}$$



And

$$C = \begin{pmatrix} 1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & \dots & \dots & 0 \\ 0 & 0 & \dots & 0 & 1 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & \dots & \dots & 0 \\ \vdots & \vdots & & \vdots & & & & \vdots & \vdots & & & & \vdots & & & \vdots \\ 0 & 0 & \dots & 0 & \dots & \dots & \dots & 1 & 0 & \dots & \dots & 0 & \dots & \dots & \dots & 0 \\ i=1 & & & i=2 & & & & i=q & & & & & & & & q(N \times q) \end{pmatrix}$$

Note that the order of the estimated model will be (Nxq) and that the final q-step gives a non-minimal representation. Hence the minimal representation to be estimated is not unique.

- 7- Estimate the correct order (n), which represents the controllable part. This is done by isolating controllable part from the uncontrollable part. Thus all the subsystems should be combined in such a way that the overall minimal realization of the system is obtained.

### 5- State Observer

In this section the construction of the state observer is introduced. The construction is done through collecting sets of input and output data arranged as follows:-

$$\begin{pmatrix} X_1(k+1) \\ X_2(k+1) \\ \vdots \\ X_N(k+1) \end{pmatrix} + \begin{pmatrix} u^T(k-N+1) & u^T(k-N+2) & \dots & u^T(k) & y(k-N+1) \dots y(k) \\ u^T(k-N+2) & u^T(k-N+3) \dots o^T & y(k-N+2) \dots o \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} b \\ -a \end{pmatrix} = \dots (13)$$

$u^T$        $o^T$        $o^T$        $y(k)$        $o$

Where (N) is the preassigned number and [ b -a]<sup>T</sup> are the identified parameters. Note that, this observer is suitable for the case of MISO systems. Then for the system of q-outputs there will be q-observers of equation (13).

## 6- Pole-Assignment

Pole assignment technique can be viewed as an extension to the classical root locus method. The purpose is to arrange the feedback loop gain so that all the poles of the closed-loop system are assumed to have prescribed values.

The open-loop state equation of system (6) is:-

$$X(k+1) = A X(k) + B U(k)$$

If the control law is expressed as a linear negative feedback as:-

$$U(k) = -K X(k) + r(k) \quad (14)$$

Where (K) is the feedback gain matrix of dimension (n \* m), U(k) is m-dimensional vector of the input signal, X is n-dimensional vector of the states and r(k) is m-dimensional vector of the reference signal, then the closed-loop system has the state equation represented by:-

$$\begin{aligned} X(k) &= A X(k) - B K X(k) - B r(k) \\ &= (A - B K) X(k) - B r(k) \end{aligned} \quad (15)$$

Equation (15) has a t-set of specified poles. Where (t) is assumed to be real explicit eigenvalues.

The procedure of Salman[5] can be used to assign the poles at the desired locations using the feedback gain matrix , which has the following form:-

$$K = \begin{bmatrix} K_1 & K_2 & \dots & K_n \\ I_{(m-1)(m-1)} & : & O_{(m-1)(n-m+1)} \end{bmatrix} \quad \dots \quad (16)$$

Multiplying this gain matrix (K) by the estimated state equation (14) will produce the control law ( U ) which enables the assigning of all the poles in the prespecified locations. The pole here refers to that of the closed-loop transfer function, which also the same as the roots of the characteristics equation.

## 7- Numerical example

The data obtained from Luyben[3] is used to construct the dynamic model. The method of Salman[4] is used to obtain the state-space estimated model. The preassigned number is two. The following second order system with two inputs is obtained:-

$$A = \begin{bmatrix} 0.21564 & 1 \\ 0.57162 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0.00345 & 0.00184 \\ -0.00208 & -0.00165 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Fig(2) shows the block diagram of the complete system with the adaptive controller. When the location values of ( $X_B=0.98$ ) and ( $X_D=0.02$ ) are desired, the feedback gain matrix( $K_1$ ) and( $K_2$ ) can be calculated, from equ.(16).

The performance of a pole-assignment adaptive control algorithm is then as shown in fig(3) and fig(4). It can be seen that algorithm can identify the unknown process and also the variation of the process, and automatically adjust its controller settings to achieve perfect tracking. In addition, the parameter convergence is observed in this case to be quite rapid.

## 8- Conclusions

An adaptive controller is proposed to control binary distillation column. The method depends on the identification of a real data of the binary distillation column using multivariable identification procedure with a state observer.

The selections of the roots for a pole assignment scheme are decided by the steady state and the transient responses of the desired compositions.

A good feature of the proposed algorithm is that the order of the system is not necessary to be known, as instead, a preassigned number is used to obtain the actual system order. The preassigned number should be higher than the actual order of the system.

The results show that the bottom and distillate compositions are controlled smoothly and the output responses are not started immediately due to the time delay of the system.

## Notation

B : bottom product flow rate	XD: distillate composition
D : distillate product flow rate	Xn, Yn: tray compositions
M <sub>B</sub> : base hold up	Ln : tray liquid flows
M <sub>D</sub> : reflux drum hold up	Mn: tray liquid holdups
R : reflux drum flow rate	Mc: condenser holdups
V : vapor boil up in reboiler	Ms: steam holdups
F : feed flow rate	$\alpha$ : Relative volatility
X <sub>F</sub> :feed composition	$\beta$ :Hydraulic time constant (3 to 6 second per tray)
X <sub>B</sub> : bottom product composition	



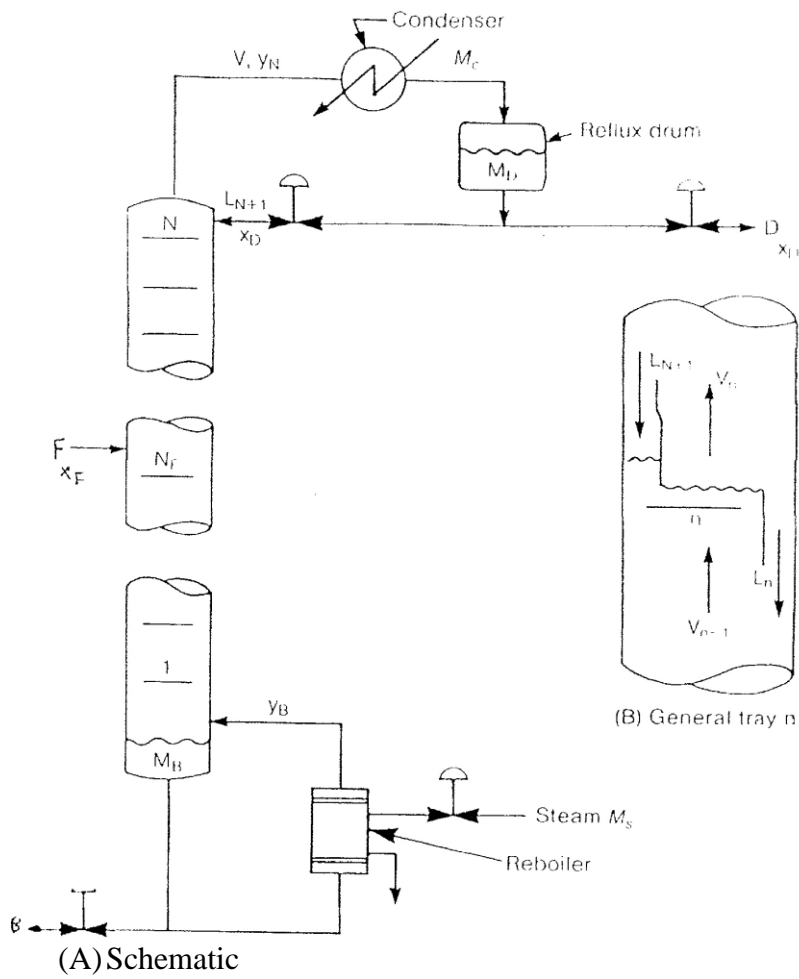


Fig. (1) Modeling of a distillation column

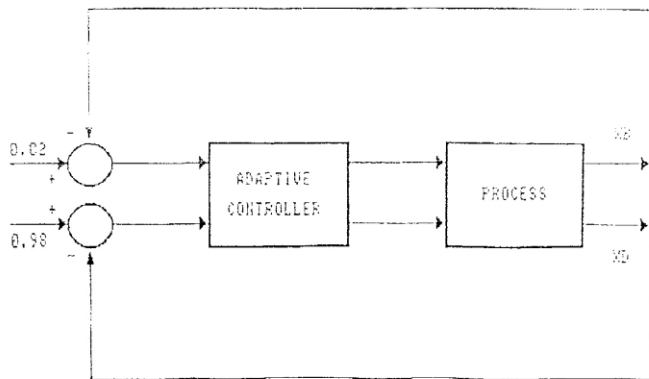


Fig. (2) Distillation column with adaptive controller

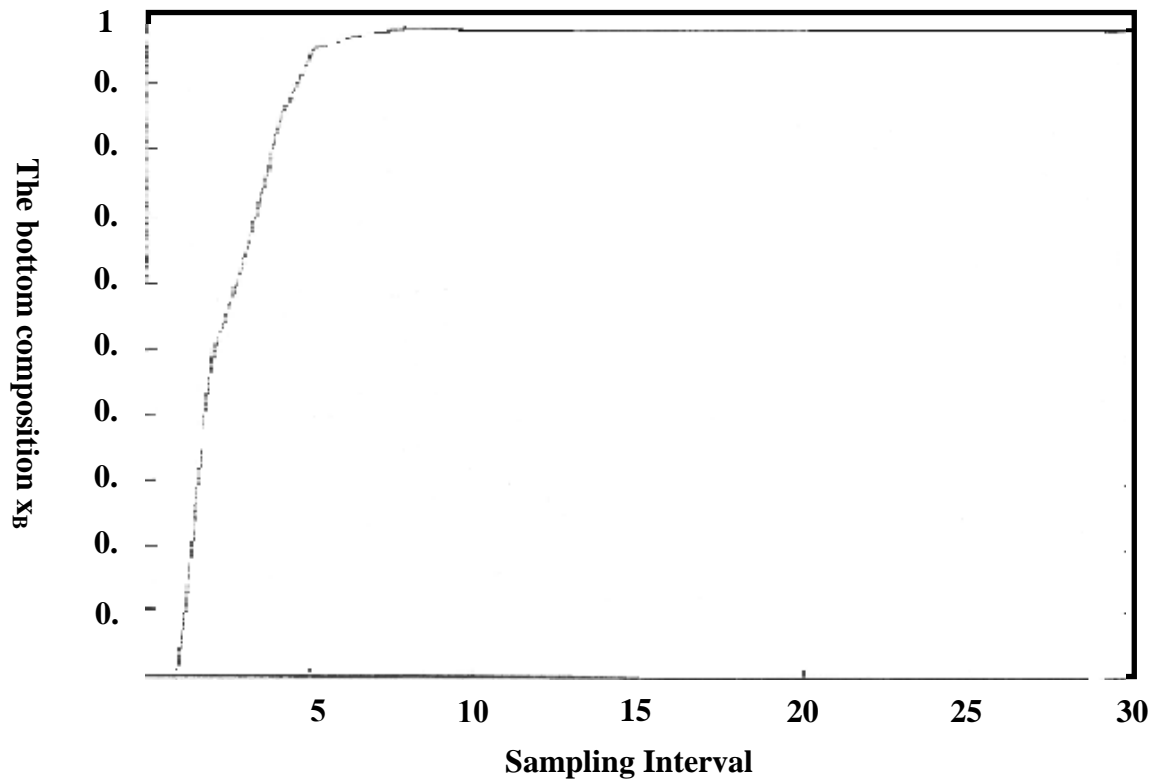


Fig. (3) dynamic behavior of the bottom composition

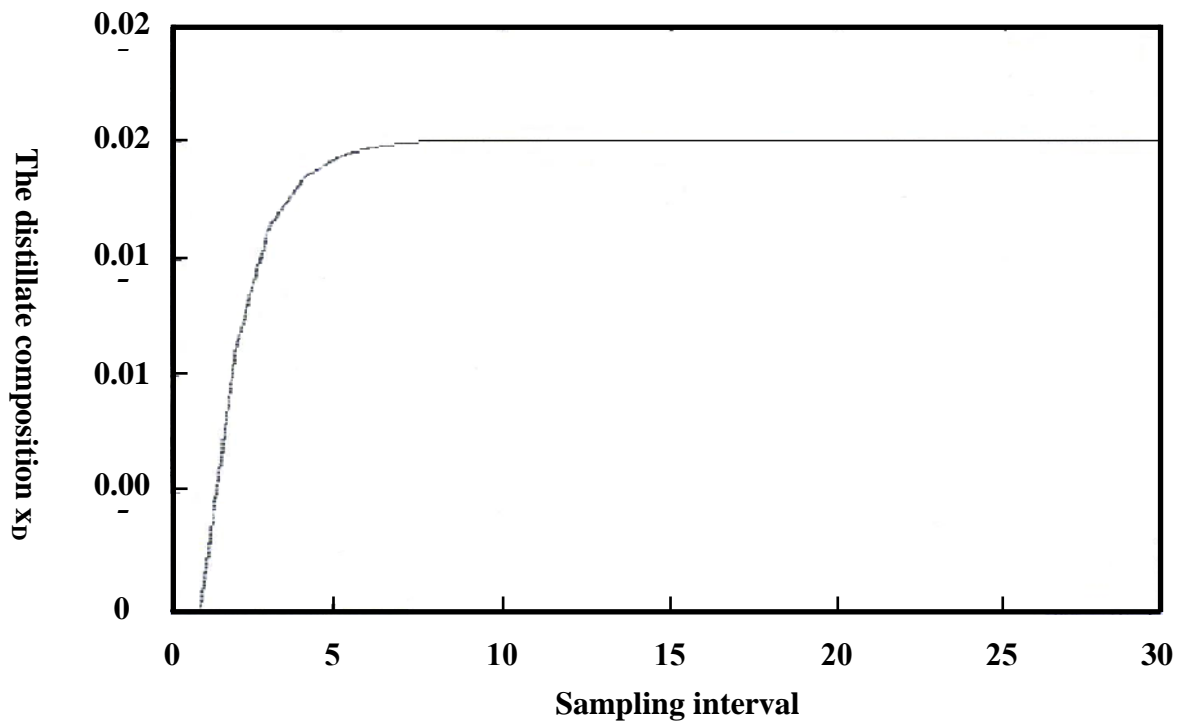


Fig. (4) dynamic behavior of the distillate composition

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