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Performance of Quadrature OFDM System in Rayleigh Fading Channels

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Abstract

This paper uses the Divide-and-Conquer DFT/IDFT computation approach instead of direct computation approach in OFDM system introducing a proposed system called Quadrature OFDM (Q-OFDM). The use of this approach introduces a multi sub-channels each have a set of subcarriers orthogonal with each other in two orthogonal coordinates. This arrangement leads to decrease the probability that all subcarriers in a sub-channel see deep fading. Furthermore, this approach reduces the computational complexity and reducing peak-to-average power ratio problem which are common problems in common OFDM systems especially when large number of carriers is used. The performance of the Q-OFDM system has been tested and compared with conventional OFDM system in AWGN, flat fading and selective fading channels. The results show that the performance of Q-OFDM is superior that conventional OFDM especially for high values of SNR. A maximum gain of 5 dB gain in SNR has been obtained in both AWGN and flat fading channels in Q-OFDM system over the conventional one. While 3 dB maximum gain has been obtained for the case of selective fading channel.

الخلاصة

في هذا البحث تم استخدام طريقة التقسيم والتغلب لبناء محول فورير ومحول فورير العكسي في منظومة التقسيم الترددي المتعامد المتعدد مأأدى الى الحصول على منظومة جديدة تدعى منظومة التقسيم الترددي المتعامد المتعدد الرباعية. ان استخدام هذه الطريقة يؤدي الى تكوين عدة قنوات ثانوية كل منها يحوي على مجموعة من الحوامل المتعامدة مع بعضها البعض بمحاور ثنائية الابعاد. ان هذا الترتيب سوف يؤدي الى تقيل احتمالية ان تتعرض كل الحوامل الموجودة في القنوات الثانوية الى خفوت كبير. وعلاوة على ذلك فان هذه الطريقة سوف تقلل من كمية الحسابات اللازمة كما انها تقلل من مشكلة نسبة القيمة العظمى الى القيمة المتوسطة واللتان هما من المشاكل المهمة في منظومة التقسيم الترددي المتعامد المتعدد سيما عندما يكون عدد الحوامل المستخدم كبيرا. لقد تم فحص اداء المنظومة الجديدة المقترحة ومقارنة اداءها مع المنظومة التقليدية في ثلاث انواع مختلفة من القنوات هي قناة كاوس ذات الضوضاء البيضاء وقناة الخفوت الترددي المستوي وقناة الخفوت الترددي الانتقائية. لقد بينت النتائج التي تم الحصول عليها ان اداء تدعى منظومة التقسيم الترددي المتعامد المتعدد الرباعية أكثر رقيا من اداء المنظومة التقليدية خصوصا عند قيم عالية من نسبة الاشارة الى الضوضاء. لقد تم الحصول بواسطة المنظومة المقترحة على ربح اعظم مقدارة 5 ديسبل في نسبة الاشارة الى الضوضاء في حالتي قناة كاوس ذات الضوضاء البيضاء وقناة الخفوت الترددي المستوي مقارنة بالمنظومة التقليدية في حين تم الحصول على ربح اعظم مقدارة 3 ديسبل في نسبة الاشارة الى الضوضاء في حالة قناة الخفوت الترددي الانتقائية.

1. Introduction:

OFDM is becoming widely applied in wireless communications systems due to its high rate transmission capability with high bandwidth efficiency and its robustness with regard to multi-path fading and delay [1]. It has been used in digital audio broadcasting (DAB) systems, digital video broadcasting (DVB) systems, digital subscriber line (DSL) standards, and wireless LAN standards such as the American IEEE® Std. 802.11™ (Wi-Fi) and its European equivalent HIPRLAN/2. It has also been proposed for wireless broadband access standards such as IEEE Std. 802.16™ (WiMAX) and as the core technique for the fourth-generation (4G) wireless mobile communications [2].

Nevertheless, OFDM technique has certain drawbacks, such as the increased system complexity, which is associated with the generation of orthogonal subcarriers, and other problems, which might not occur in other modulation schemes. Such problems include the peak to average power ratio (PAPR)[3].where a high PAPR OFDM signal may cause poor power efficiency, in-band distortion, and undesired spectral spreading when it passes through a nonlinear power amplifier [4], and inter carrier interference (ICI)[5].Another weak point is that it is very sensitive to frequency and phase errors between the transmitter and receiver. The main sources of these errors are frequency stability problems; phase noise of the transmitter; and any frequency offset errors between the transmitter and receiver. This problem can be mostly overcome by synchronizing the clocks between the transmitter and receiver, by designing the system appropriately, or by reducing the number of carriers used [4].

To overcome these problems, Quadrature OFDM (Q-OFDM) systems is proposed. In particular, the proposed system can achieve the same guard-interval overhead and same bandwidth occupation to conventional OFDM systems while with reduced PAPR and improved sensitivity to carrier frequency offset (CFO) robustness and frequency diversity. Q-OFDM systems also promise low complexity in downlink receivers [6].

As it is known, the Discrete Fourier Transform (DFT) plays an important role in many applications of digital signal processing and communications. One of the key components in OFDM system is the FFT (the less complex version of DFT). There are more and more communication systems require higher points FFT and higher symbol rates. The requirement establishes challenges for low power and high speed FFT design with large points. In our target application, the IEEE 802.16-2004 (WiMAX) standard requires the OFDM symbol rates from 1.75MHz to 20MHz and the FFT up to 2048 points. There are in general two approaches in implementing the FFT/IFFT for digital signal processing and communications namely: the direct computation approach and the divide-and-conquer approach[7]. In OFDM literature, the most standards and designs use the direct computation approach to implement FFT/IFFT while the usage of the second approach has no longer be deeply discussed.

In this paper, the divide and conquer approach is used for FFT/IFFT implementation in OFDM, to make use the multi-orthogonality property of this approach to enhance the performance the conventional OFDM system, introducing Q-OFDM system.

The performance of the Q-OFDM system in terms of signal to noise ratio versus bit error probability is analyzed in AWGN, flat fading and selective fading channels. The complexity analysis of both the conventional and Q-OFDM systems is also given.

2. OFDM based on Direct Computation of the IDFT:

OFDM theory states that the IFFT of magnitude N, applied on N samples, realizes an OFDM signal, where each samples is transmitted on one of the N orthogonal frequencies [8].The IFFT takes frequency domain spectrum $X(k)$ and converts it to time domain signal x(n) by successively multiplying it by a range of sinusoids as given by (1); For a complex-valued sequence x (n) of N points, the IDFT may be expressed as:

$$x(n) = \sum_{n=0}^{N-1} X_I(k)\,\sin\left(\frac{2\pi kn}{N}\right) - j\sum_{n=0}^{N-1} X_R(k)\,\cos\left(\frac{2\pi kn}{N}\right) \quad \ldots\ldots(1)$$

where:

$$X_R(k) = \sum_{n=0}^{N-1}\left[x_R(n)\cos\frac{2\pi kn}{N}\, x_I(n)\sin\frac{2\pi kn}{N}\right] \quad \ldots\ldots(2)$$

$$X_I(k) = \sum_{n=0}^{N-1}\left[x_R(n)\cos\frac{2\pi kn}{N}\, x_I(n)\sin\frac{2\pi kn}{N}\right] \quad \ldots\ldots(3)$$

The direct computation of (1) requires:

1. $2N^2$ evaluations of trigonometric functions:
2. N^2 complex multiplications.
3. N (N —1) complex additions.
4. A number of indexing and addressing operations.

These operations are typical of DFT computational algorithms. The operations in items 2 and 3 result in the DFT values $X_R(k)$ and $X_I(k)$. The indexing and addressing operations are necessary to fetch the data *x(n),* $0 < n < N$ -1, and the phase factors and to store the results. The variety of DFT algorithms optimize each of these computational processes in a different way [9].

Fig.1 shows the subchannel division in conventional OFDM system that uses DFT computation approach. The advantage of this conventional subchannel assignment scheme is that each user benefit from the large frequency diversity because of the randomness of the occupied subcarrier, and resource allocation can be optimized with little constraint to realize multi-user diversity. This scheme also enables efficient spectrum reuse between different cells. On the other hand, such systems suffer from problems including PAPR, CFO sensitivity and high complexity [6].

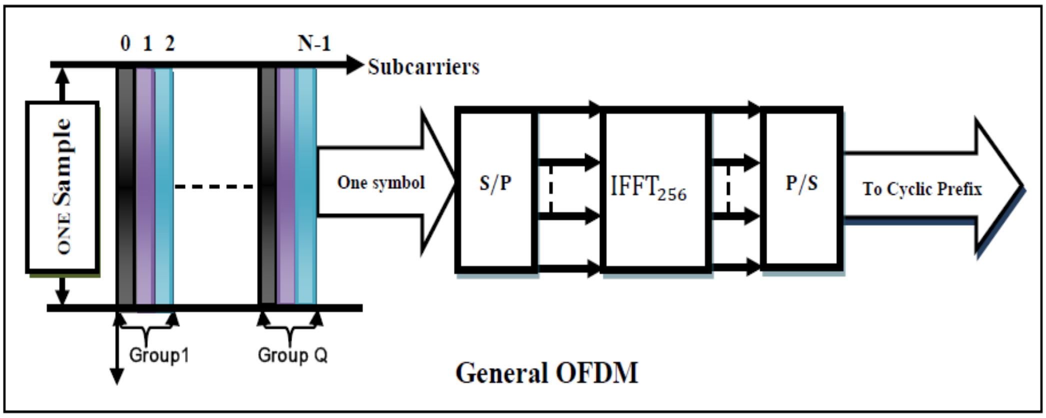


Fig.1. The subchannel division in conventional OFDM system that uses IDFT computation approach.

2. Proposed Q-OFDM based on Divide – and - Conquer DFT Computation Approach:

The proposed Q-OFDM systems are motivated by the judicious use of Divide-and-Conquer (D&C) approach in the computation of DFT. D&C approach is the basis of FFT and IFFT algorithms[6]. Unlike the subchaanel in conventional OFDMA systems, which are defined in frequency domain, subchannel in Q-OFDM system are defined over array of two dimensions in the intermediate domain. This array is P×Q, where N=PQ and both P and Q are powers of 2. There are in total Q subchannels, each with P subcarriers with the same index over P Q-point IDFTs. These channel coefficients have low correlation, and the probability that all subcarriers in a subchannel see deep fading is very low. Hence, the resource that each user occupies spread over two orthogonal coordinates, so the system is named Quadrature OFDM (Q-OFDM).

The process of the row-wise D&C approach is recalled in calculating an *N*-point inverse DFT (IDFT) of the signal $\tilde{\mathbf{x}} = \{\tilde{x}_i\}$, i = 0, 1, … , $N-1$. The samples in different layers with different accents are distinguished as: $\tilde{x}$ for a frequency-domain sample, $\hat{x}$ for an intermediate domain sample, and *x* for a time-domain sample. The overall process of Q-OFDM system is explained in Figures 2-6. Three process is distributed over three layers each with different domain representation. The first layer is in time domain where three operation steps S_1,S_2 and S_3 are performed for the input signal (Fig.2), the second layer is in intermediate domain with S_4 and S_5 (Figs. 4&5) and the third layer is in time domain with S_6 (Fig.6). The operation steps in different layers can be expressed as an algorithm of the following steps:

A) First layer (frequency domain):

$\mathbf{S_1}$: Store the signal $\tilde{\mathbf{x}} = \{\tilde{x}_i\}$, i = 0, 1, … , $N-1$ in row-wise as shown in Fig.2. Now the sequence $\tilde{\mathbf{x}}$(k), 0 < k < K— 1, can be stored in one-dimensional array indexed by k as illustrated in Fig. 3.

$\mathbf{S_2}$: Factor the input signal $\tilde{\mathbf{x}}$ row-wise into N = P × Q matrix. Where $\tilde{\mathbf{x}} = \{ \tilde{x}_{p,q} \}$, p = 0, 1, . . . , P − 1, q = 0, 1, . . . , Q − 1 with mapping $\tilde{x}_{p,q} = \tilde{x}_{pQ+q}$ and N = P×Q.

$\mathbf{S_3}$: Per-Column of $\tilde{\mathbf{x}}$ calculate P-Point (IFFT) and result new matrix $\hat{\mathbf{x}}$ with *q*-th column $\hat{\mathbf{x}}_q = (\hat{x}_{0,q} , \hat{x}_{1,q} , \dots , \hat{x}_{P-1,q})^T$ where $\hat{x}_{l,q} = \frac{1}{\sqrt{P}} \sum_{p=0}^{P-1} \tilde{x}_{p,q} W_P^{pl}$ and $W_P^{pl} = \exp(-j2\pi pl/P)$;

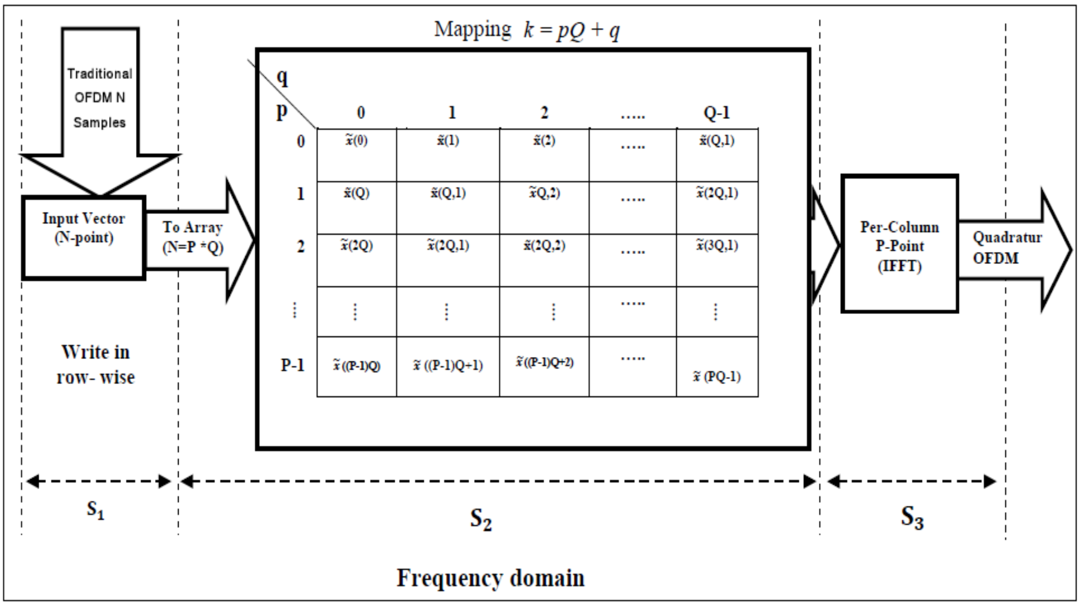


Fig.2. First part of Divide and Conquer IDFT algorithm (frequency domain)

k →	0	1	2	…………	K-1
	x̃(0)	x̃(1)	x̃ (2)	…………	x̃(K-1)

Fig.3. One dimensional data array for storing the sequence x(k). (0 < k < K-1).

B) Second layer (Intermediate domain):

$\mathbf{S_4}$: Multiply $\hat{\mathbf{x}}$ by a phase weighting matrix and generate a new matrix $\hat{\mathbf{V}} = \{\hat{v}_{l,q}\}$ with (*l*, *q*)-the element $\hat{v}_{l,q} = W_N^{-lq} \hat{x}_{l,q}$ as shown in Figs.4 and 5.

$\mathbf{S_5}$: Per-Row of $\hat{\mathbf{V}}$ calculate *Q*-Point (IFFT)

C) Third layer (Time domain)

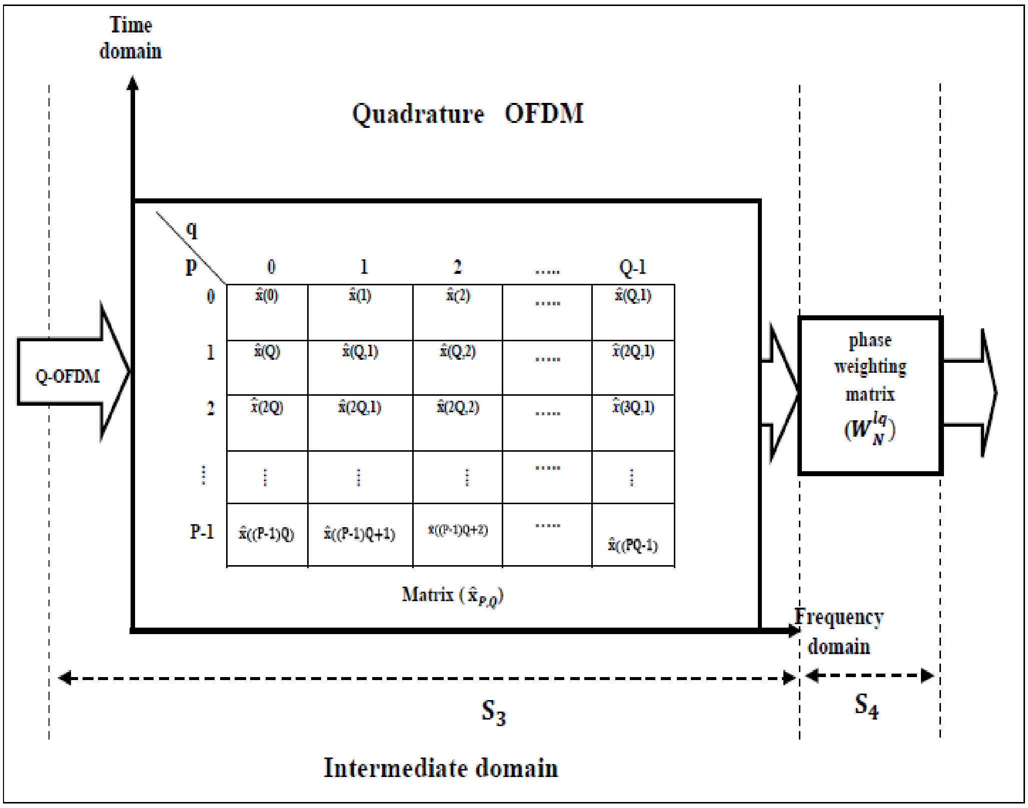


Fig.4. Second part of D&C IDFT algorithm (Intermediate domain).

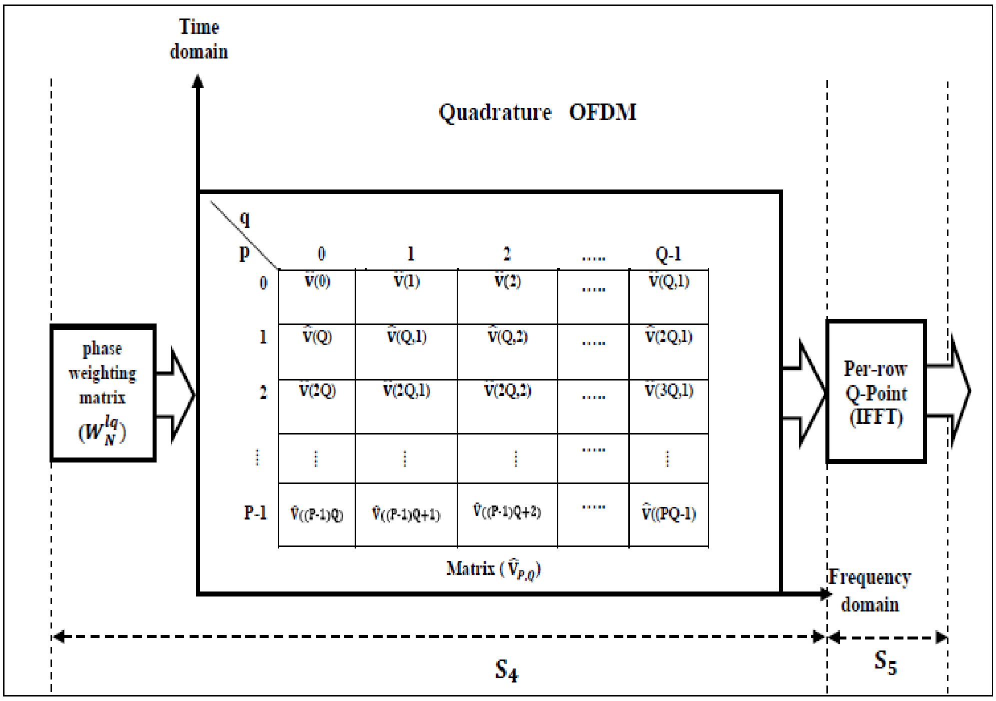


Fig.5. Second part of D&C IDFT algorithm (Intermediate domain).

$\mathbf{S_6}$ **:** Read out the resulting P × Q matrix $\mathbf{X} = \{x_{l,m}\}$, $l = 0, 1, \ldots, P-1$, $m = 0, 1, \ldots, Q-1$ column-wise, and the resulting vector $\mathbf{X}$ is the IDFT of $\tilde{\mathbf{x}}$, and $\mathbf{X} = \{x_i\}$, $i = 0, 1, \ldots, P-1$ with $x_{l+mP} = x_{l,m}$ as shown in Fig.6.

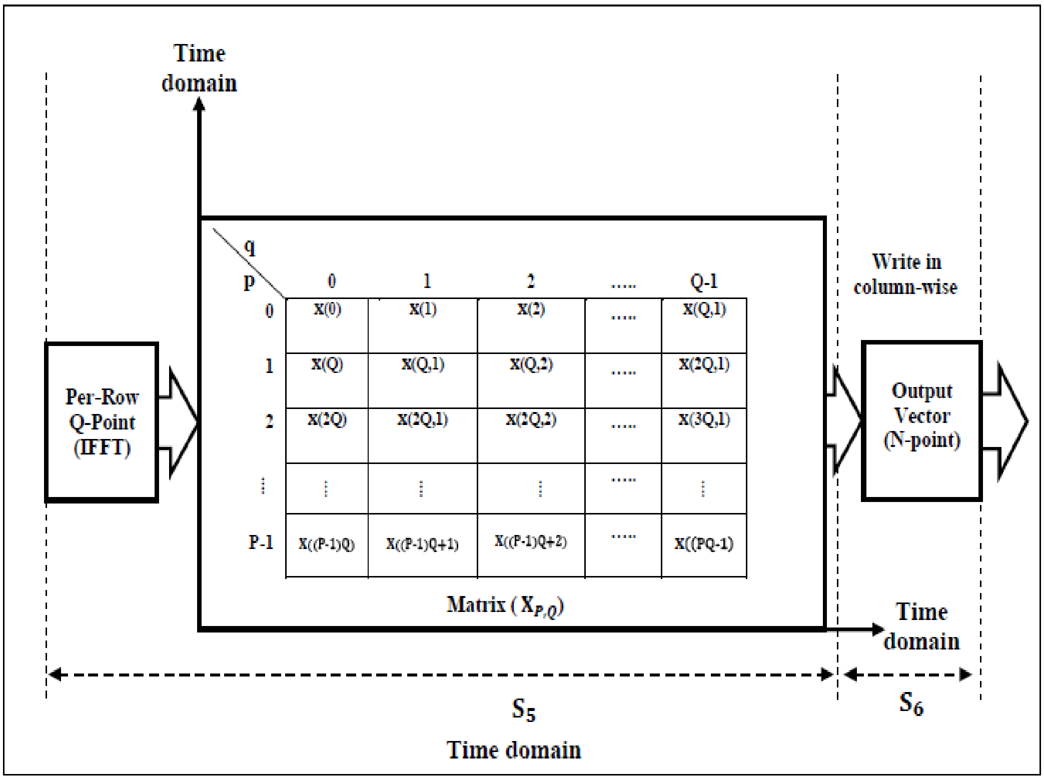


Fig.6. Third part of Divide and Conquer IDFT algorithm (time domain).

3. Complexity Analysis:

On the surface it may appear that the computational procedure outlined above is more complex than the direct computation of the DFT. However, let us evaluate the computational complexity of the equation that describes the DFT computation using divide and conquer algorithm which is given by:

$$X(p, q) = \sum_{l=0}^{P-1} \left\{ W_N^{lq} \left[\sum_{m=0}^{Q-1} x(l,m)\, W_Q^{mq} \right] \right\} W_P^{lp} \qquad \dots\dots (4)$$

The first step involves the computation of *P* DFTs, each of M points. Hence this step requires PQ^2 complex multiplications and $PQ(Q — 1)$ complex additions. The second step requires *PQ* complex multiplications. Finally, the third step in the computation requires QP^2 complex multiplications and $PQ(P —1)$ complex additions. Therefore, the computational complexity is:

Complex multiplications: $N(Q + P + 1)$
Complex additions: $N(Q + P - 2)$
where $N = PQ$. Thus the number of multiplications has been reduced from N^2 to $N(Q + P + 1)$ and the number of additions has been reduced from $N(N —1)$ to $N(Q + P - 2)$.

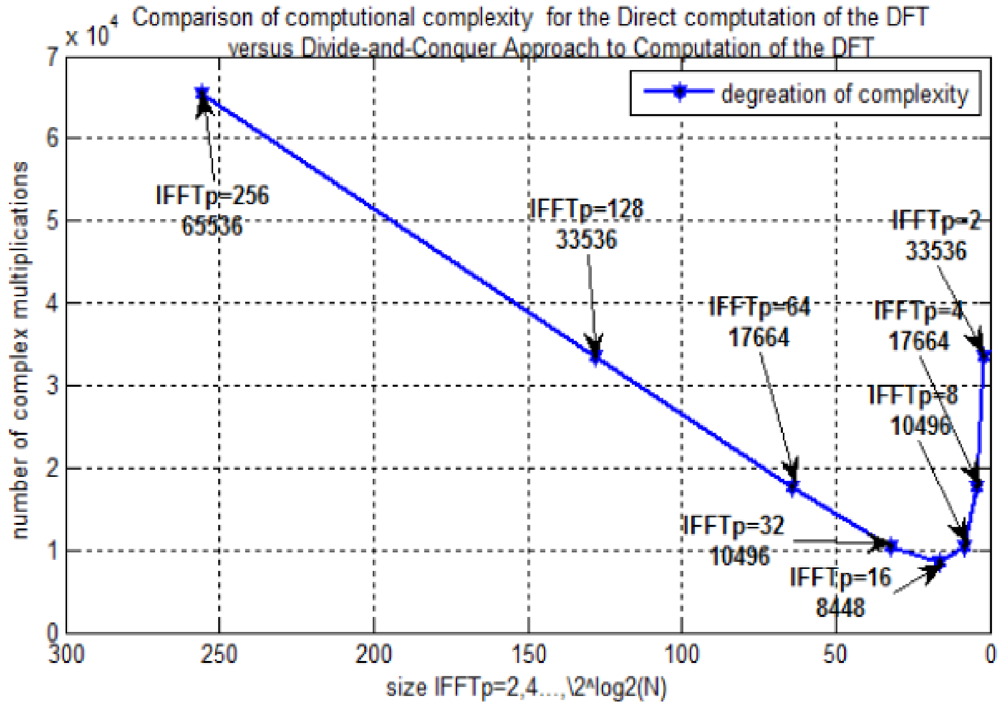


Fig.7. Comparison of computational complexity of Direct computation Approach of DFT with D&C one.

For example, suppose that $N = 1024$ and we select $P = 2$ and $Q = 512$. Then, instead of having to perform 1,048,576 complex multiplications via direct computation of the DFT, this approach leads to 527,360 complex multiplications. This represents a reduction by approximately a factor of 2. The number of additions is also reduced by about a factor of 2. If FFT is used instead of DFT, the number of complex multiplications would be $N\ log_2 N$ for direct computation approach while it becomes $N\ log_2 Q + N/2\ log_2 P$ for divide and conquer approach[6]. For the similar assumption above, Instead of having 10,240 complex multiplications via direct computation, the divide and conquer approach leads to 8,704 complex multiplications which represents a reduction of a factor 0.15.

However, the reduction factor depends on the choose of values of P and Q. If N=256 (the case considered in simulations), and P take the values in the set {2,4,8,16,32,64,128} as shown in Fig.7. It can be easily seen that the case P=16 (Q=16) gives the minimum reduction in the number of complex multiplications (8448 as compared with 65536 when P=256 (direct computation case)) . This means that as the values of P and Q become close from each other a maximum reduction in complex multiplication is obtained i.e the highest possible data rate can be obtained.

5. Simulation Results:

The performance of conventional OFDM system based on IFFT of size N=256(IFFTp where P=256 and Q=1) and QOFDM based on IFFT of size N=P×Q where P=128,64,32,16,8,4 and 2 have been tested and compared using MATLAB package version 7.8 for three types of channels.

The channel types considered are: additive white Gaussian noise (AWGN), flat fading and frequency selective fading channels. Modulation type is taken to be BPSK. System parameters used through the simulation are listed in Table 1. Noting that a bit rate of 1.92 Mbps is used.

Table 1: Simulation parameters

Modulation type	BPSK
Channel bandwidth (BW)	5MHz
Cyclic prefix factor(G)	1/8
Number of subcarrier, IFFTp=	256
Number of subcarrier, IFFTp=	128,64,32,16,8,4,2
Number of bits sent	5×10^6
Sampling factor(n)	144/125
Subcarrier spacing(Δf)	22.5kHz
Useful sample time(T_b)	44.444 µsec
Cyclic prefix time(T_g)	5.5556 µsec
OFDM symbol time(T_s)	50 µsec
Normalized maximum Doppler Shift	0.5 Hz
Path delay	[0 0.4 0.9]µsec
Path gain	[0 -5 -10]dB
Channel model	AWGN
	Flat fading + AWGN
	Frequency selective fading + AWGN

5-1 AWGN Channel Results:

The simulation results in terms of error probability versus SNR for the Q-OFDM system in AWGN channel is shown in Fig.8. The figure shows the performance of the IFFTp=256-based- OFDM (IFFTp=256) and the others IFFTp=128,64,…,2-based-QOFDM (IFFTp=128,…,2) systems for uncoded BPSK modulation type in AWGN channel. Some effective numerical values of Fig.8 is given in table 2. It is clearly shown in Figure that the performance of Q-OFDM system is better than the conventional OFDM system when signal to noise ratio (SNR) is above a threshold depending on the channel condition, This improvement starts at SNR values exceed 11dB.

Some values of maximum gains in SNR in dB can be recognized in the bottom of Table 2. The maximum gain obtained in our simulations is 5 dB when p=4 at Pe=4e-06. From other hand, the maximum reduction in Pe for specific values of SNR can be read in the table by those underlined values. For example at SNR=20 dB reduces from 3e-05 in IFFTp=256 based OFDM system to 4e-06 in IFFTp=4 based OFDM system.

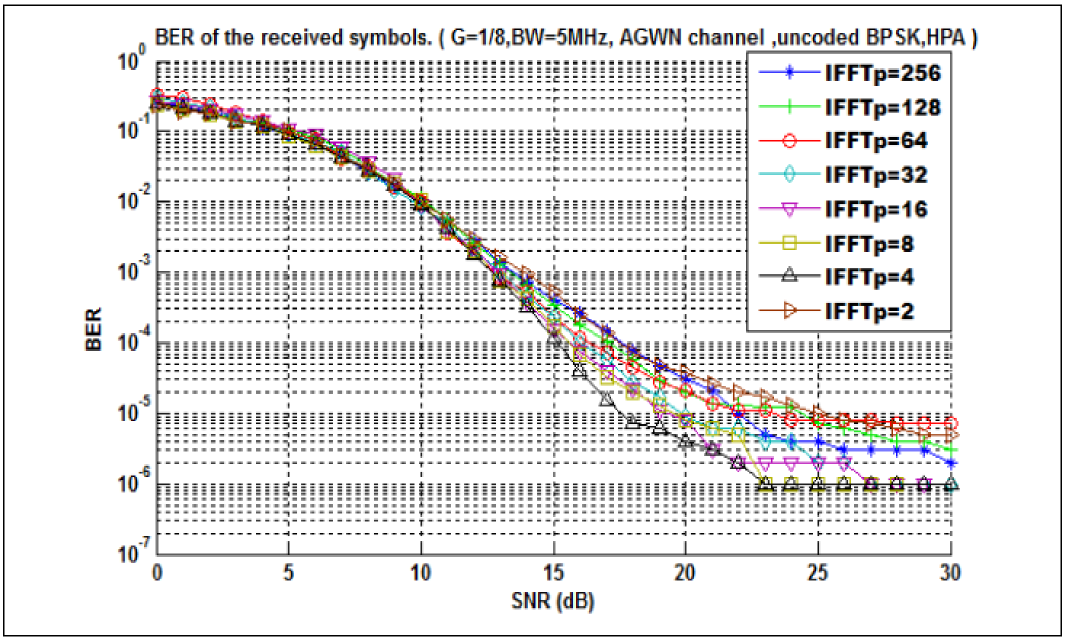


Fig.8. Performance of OFDM and Q-OFDM of different of sub-channels in AWGN channel.

5-2 Flat fading Results:

The simulation results of Q-OFDM system in flat fading channel is shown in Fig.9. The figure shows the performance of the IFFTp=256-based- OFDM (IFFTp=256) and the others IFFTp=128,64,…,2-based-QOFDM (IFFTp=128,…,2) systems for uncoded BPSK modulation type in flat fading channel.

Some effective numerical values of Fig.9 is given in table 3. It is clearly shown in Figure that the performance of Q-OFDM system is better than the conventional OFDM system when signal to noise ratio (SNR) is above a threshold depending on the channel condition, this improvement starts at SNR values exceed 24dB. Some values of maximum gains in SNR in dB can be recognized in the bottom of Table 3. The maximum gain obtained in our simulations is 5 dB when p=4 & 8 at Pe=4.6e-06. From

Table 2.Numerical results of Fig.8

(IFFTp=) / SNR(dB)	256	128	64	32	16	8	4	2
20	3e-005	2e-005	2.1e-005	9e-006	8e-006	8e-006	4e-006	3.8e-005
21	2.1e-005	1.4e-005	1.4e-005	6e-006	3e-006	6e-006	3e-006	2.6e-005
22	1e-005	1.3e-005	1.1e-005	6e-006	2e-006	5e-006	2e-006	2e-005
23	5e-006	1.2e-005	1.1e-005	4e-006	2e-006	1e-006	1e-006	1.7e-005
24	4e-006	1.2e-005	8e-006	4e-006	2e-006	1e-006	1e-006	1.3e-005
25	4e-006	7e-006	8e-006	2e-006	2e-006	1e-006	1e-006	1e-005
MaxGain in (dB)			1dB At (2.1e-005)	2dB At (9e-006)	4.5dB At (4e-006)	1.5dB At (6e-006)	5dB At (4e-006)	

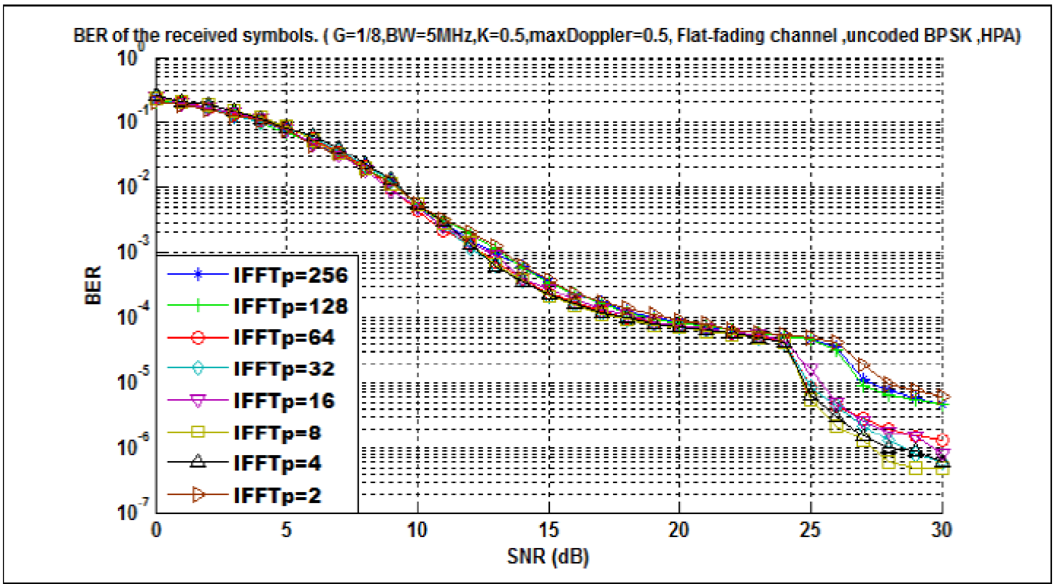


Fig.9. Performance of OFDM and Q-OFDM of different number of sub-channels in flat fading channel.

Other hand, the maximum reduction in Pe for specific values of SNR can be read in the table by those underlined values. For example at SNR=26 dB reduces from 3.6e-05 in IFFTp=256 based OFDM system to 4.5e-06 in IFFTp=32 and 64 based OFDM system.

Table 3.Numerical results of Fig.9

(IFFTp=) / SNR(dB)	256	128	64	32	16	8	4	2
25	4.697e-005	4.607e-005	8.2e-006	8.3e-006	1.563e-005	5.6e-006	6.5e-006	4.881e-005
26	3.602e-005	3.274e-005	4.5e-006	4.5e-006	5e-006	2.2e-006	3e-006	4.219e-005
27	1.12e-005	9.1e-006	2.9e-006	2.2e-006	2.6e-006	1.3e-006	1.5e-006	1.915e-005
28	7.4e-006	6.5e-006	2e-006	1.3e-006	1.8e-006	6e-007	1e-006	9.2e-006
29	5.7e-006	5.5e-006	1.5e-006	8e-007	1.5e-006	5e-007	9e-007	7.5e-006
30	4.6e-006	4.7e-006	1.3e-006	6e-007	8e-007	5e-007	6e-007	6.2e-006
MaxGain in (dB)			4dB At 4.5e-006	4dB At 4.5e-006	4dB At 5e-006	5dB At 4.6e-006	5dB At 4.6e-006	

5-3 Selective Fading Results:

The simulation results of the Q-OFDM system in selective fading channel is shown in Fig.10. The figure shows the performance of the IFFTp=256-based- OFDM (IFFTp=256) and the others IFFTp=128,64,…,2-based-QOFDM (IFFTp=128,…,2) systems for uncoded BPSK modulation type in selective fading channel with maximum normalized Doppler shift of 0.5. Some effective numerical values of Fig.10 is given in table 4. It is clearly shown in Figure that the performance of Q-OFDM system is better than the conventional OFDM system when signal to noise ratio (SNR) is above a threshold depending on the channel condition, this improvement starts at SNR values exceed 23dB.

Some values of maximum gains in SNR in dB can be recognized in the bottom of Table 4. The maximum gain obtained in our simulations is 3 dB when p=4 at Pe=1.8e-05. From other hand, the maximum reduction in Pe for specific values of SNR can be read in the table by those underlined values. For example at SNR=25 dB reduces from 5.626e-04 in IFFTp=256 based OFDM system to 7.7e-05 in IFFTp=128 based OFDM system.

6. Conclusions:

The Divide-and-Conquer computation approach of DFT/IDFT was used in conventional OFDM system instead of direct computation approach, introducing Quadrature OFDM system. The usage of this approach reduces the effect of PAPR problem, reduces the error probability as well as increases the data rate by reducing the number of computations. With Divide-and Conquer approach new sub-channel assignment is defined. These sub-channel coefficients have low correlation, and the probability that all subcarriers in a sub-

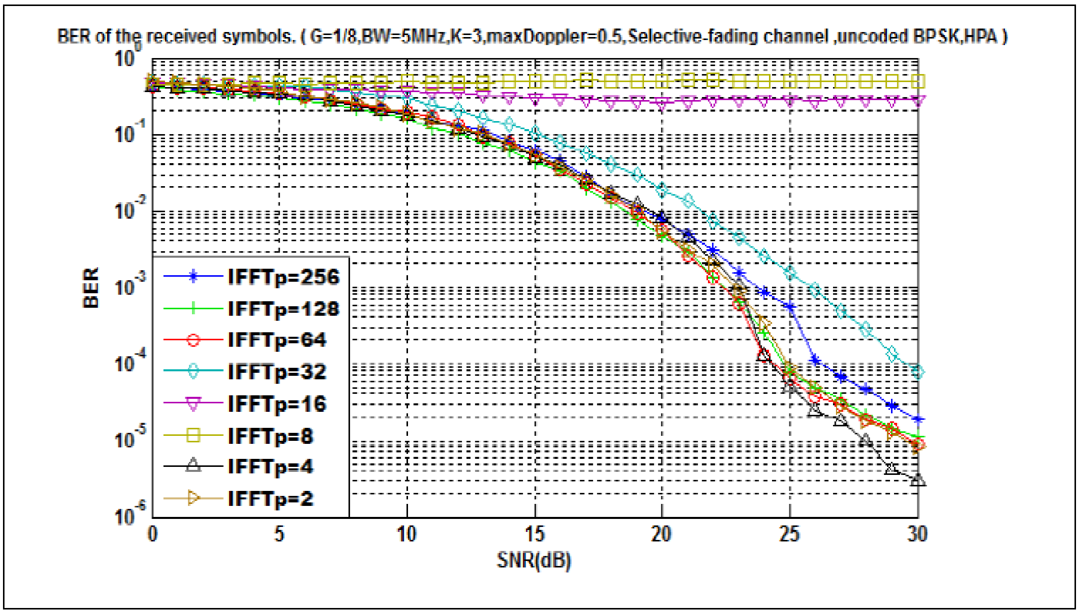


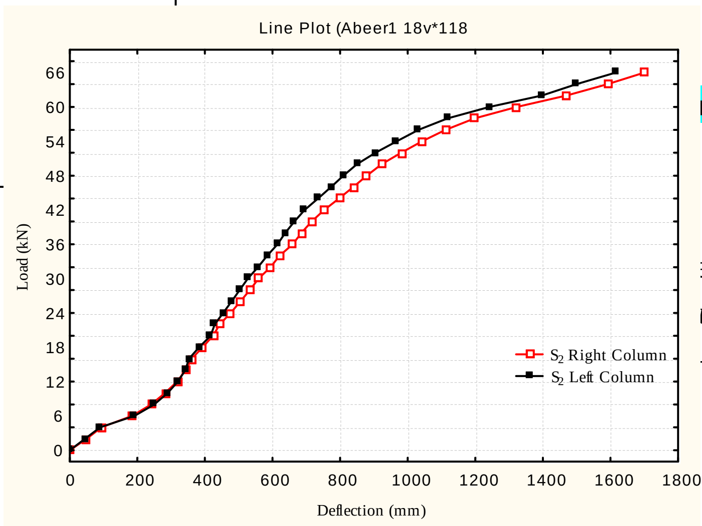
Fig.10 Performance of OFDM and Q-OFDM of different number of sub channels in selective fading channel.

Table 4.Numerical results of Fig.10

SNR(dB) \ (IFFTp=)	256	128	64	32	16	8	4	2
25	5.626e-004	7.7e-005	6.4e-005	1.557e-003	2.772e-001	0.5	5.1e-005	8.8e-005
26	1.134e-004	5e-005	8.3e-005	9.435e-004	2.745e-001	0.5	2.4e-005	4.9e-005
27	6.7e-005	3.4e-005	3e-005	4.892e-004	2.826e-001	0.5	1.8e-005	2.8e-005
28	4.6e-005	2.1e-005	1.9e-005	2.854e-004	2.826e-001	0.5	1e-005	1.8e-005
29	2.8e-005	1.4e-005	1.4e-005	1.326e-004	2.853e-001	0.5	4e-006	1.3e-005
30	1.9e-005	1.1e-005	9e-006	7.8e-005	2.853e-001	0.5	3e-006	8e-006
MaxGain in (dB)		2dB At 7.7e-005	2dB At 1.9e-005				3dB At 1.8e-005	2dB At 2.8e-005

Channel see deep fading is very low reducing the error probability. Hence, the resource that each user occupies spread over two orthogonal coordinates. The selection of number of sub-channels sets the improvement introduced by the Q-OFDM.

This improvement should make a compromising between the SNR gain obtained and the computation complexity required.



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