Ibn Al Haitham Journal for Pure and Applied Science

Journal homepage: http://jih.uobaghdad.edu.iq/index.php/j/index

Employ Shrinkage Estimation Technique for the Reliability System in Stress-Strength Models: special case of Exponentiated Family Distribution

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Article history: Received 13 November 2019, Accepted 15 December 2019, Published in October 2020

Doi: 10.30526/33.4.2508

Abstract

A reliability system of the multi-component stress-strength model $R_{(s,k)}$ will be considered in the present paper ,when the stress and strength are independent and non-identically distribution have the Exponentiated Family Distribution(FED) with the unknown shape parameter α and known scale parameter λ equal to two and parameter θ equal to three. Different estimation methods of $R_{(s,k)}$ were introduced corresponding to Maximum likelihood and Shrinkage estimators. Comparisons among the suggested estimators were prepared depending on simulation established on mean squared error (MSE) criteria.

Keywords: Exponentiated Family Distribution, Reliability of multicomponent Stress–Strength models, Maximum likelihood estimator, Shrinkage estimator and mean squared error.

1. Introduction

A multicomponent system consuming k^{th} strength independently and non-identically random variables and everyone subjected to random stress was presented by Bhattacharyya and Johnson (1974)[1].The system reliability model, (s out of k) was denoted by $R_{(s,k)}$ when at least s ($1\leq s\leq k$) of components survive. In (2012), Pandit and Kantu studied the reliability estimation in multicomponent stress-strength(s-s) with the assumption that strengths and stresses followed the Pareto distribution [2] .Rao & Naidu in (2013) studied the multicomponent system reliability in (s-s) model for Exponentiated half Logistic distribution[3]. In (2014), Rao estimated the reliability system of the multicomponent (s-s) model for Generalized Rayleigh distribution through simulation [4].

In (2015), Kizilaslan and Nader, studied the multicomponent system assumed that the stress and strength of identical independent followed Weibull distribution using ML and Bayes methods [5]. In (2016) Rao et al, they estimated the multicomponent reliability in model when the stress and strength followed Exponentiated Weibull distribution [6]. In (2017), Abbas and Fatima estimated the system reliability of the multicomponent (s-s) model via Exponentiated

Weibull distribution, consuming; ML and MOM estimators and they accomplished results agreed that the shrinkage estimator was the best [7]. And in (2019), Abbas and Eman derived and estimated the formula of reliability for multicomponent in (s-s) model in case of Exponentiated Pareto distribution and they concluded that the shrinkage estimator was the best [8].

The aim of this paper is to estimate the reliability of multicomponent system in stress-strength model $R_{(s,k)}$ based on Family of Exponentiated distribution with unknown shape parameter α and known the parameters λ=2 and θ=3 and a=0 via different estimation methods like MLE, as well as some shrinkage methods and make comparisons among the proposed estimator methods using simulation, depending on mean squared error criteria by using special case of Family Exponentiated distribution .

The probability density function (p. d. f.) of a r.v. X following Exponentiated Family Distribution EFD has the form below [9]:

$$f(x;\alpha,\lambda)=\alpha\lambda\dot{g}(x;a,\underline{\theta})e^{-\lambda g(x;a,\underline{\theta})}\left(1-e^{-\lambda g(x;a,\underline{\theta})}\right)^{\alpha-1};\quad x>0, a\geq 0,\alpha,\lambda>0 \tag{1}$$

Here $g(x;a,\underline{\theta})$ is the function of x and may also be of the parameter $\underline{\theta}$ (may be vector valued) and a is constant. Furthermore, $g(x;\ a,\underline{\theta})$ refer to real valued, monotonically increasing function of X with $g(a;\ a,\underline{\theta})=0$, $g(\infty;\ a,\underline{\theta})=\infty$ and $g'(x;\ a,\ \underline{\theta})$ refer to the derived of $g(x;\ a,\ \underline{\theta})$ with respect to x [9].

Consequently, the cumulative distribution function (c.d. f.) of X will be:

$$F(x,\alpha,\lambda)=\left(1-e^{-\lambda g(x;a,\underline{\theta})}\right)^{\alpha}\qquad x>0 \tag{2}$$

The first who derived the reliability of a multicomponent system in stress-strength model $R_{(s,k)}$ was Bhattacharyya and Johnson (1974) as the following form,[1].

$R_{(s,k)}$ =P(at least s of the $X_1, X_2,\ldots, X_k$ exceed Y)

Where $X_1,X_2\ldots, X_k$ with common distribution function F(x) is subjected to the common random stress Y with distribution Function G(y)

$$R_{(s,k)}=\sum_{i=s}^{k}\binom{k}{i}\int_{o}^{\infty}(1-F_x(y))^{i}(F_x(y))^{k-i}dG(y)$$

When X~ EFD(α,λ,θ) and Y~ EFD (β,λ,θ) then:

$$R_{(s,k)}=\sum_{i=s}^{k}\binom{k}{i}\int_{o}^{\infty}(1-\left(1-e^{-\lambda g(y;a,\underline{\theta})}\right)^{\alpha})^{i}(\left(1-e^{-\lambda g(y;a,\underline{\theta})}\right)^{\alpha})^{k-i}\beta\lambda\dot{g}(y;a,\underline{\theta})\,e^{-\lambda g(y;a,\underline{\theta})}\,(1-e^{-\lambda g(y;a,\underline{\theta})})^{\beta-1}dy$$

$$R_{(s,k)}=\sum_{i=s}^{k}\binom{k}{i}\int_{o}^{\infty}(1-\left(1-e^{-\lambda g(y;a,\underline{\theta})}\right)^{\alpha})^{i}(\left(1-e^{-\lambda g(y;a,\underline{\theta})}\right)^{\alpha})^{k-i}\beta\lambda\dot{g}(y;a,\underline{\theta})e^{-\lambda g(y;a,\underline{\theta})}\,(1-e^{-\lambda g(y;a,\underline{\theta})})^{\beta-1}dy$$

Let $z=1-e^{-\lambda g(y;a,\underline{\theta})}$ then $dy=\frac{dz}{\lambda(1-z)\dot{g}(y;a,\underline{\theta})}$

$$R_{(s,k)}=\sum_{i=s}^{k}\binom{k}{i}\beta\lambda\int_{o}^{1}(1-z^{\alpha})^{i}(z)^{\alpha k-\alpha i+\beta-1}\,\dot{g}(y;a,\underline{\theta})(1-z)\frac{dz}{\lambda(1-z)\dot{g}(y;a,\underline{\theta})}$$

$=\sum_{i=s}^{k}\binom{k}{i}\beta\int_{o}^{1}(1-z^{\alpha})^{i}(z)^{\alpha k-\alpha i+\beta-1}\,dz$

Let $w=z^{\alpha}\rightarrow z=w^{\frac{1}{\alpha}}\rightarrow dz=\frac{1}{\alpha}w^{\frac{1}{\alpha}-1}\,dw$

And by simplification, $R_{(s,k)}$ became:

$$R_{(s,k)}=\frac{\beta}{\alpha}\sum_{i=s}^{k}\frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}\left(k+\frac{\beta}{\alpha}-j\right)\right]^{-1};\ k,i,j \text{ are integers} \quad (3)$$

2. Estimation methods of $R_{(s,k)}$

3. Maximum Likelihood Estimator (MLE):

Consider a random sample $x_1,x_2,\ldots,x_n$ of size n following EFD$(\alpha,2,3)$, $x_{(1)}<x_{(2)}<\ldots<x_{(n)}$, the order random sample of x and $y_1,y_2,\ldots,y_m$ is a random sample of size m follows EFD $(\beta,2,3)$, and $y_{(1)}<y_{(2)}<\ldots<y_{(m)}$ is the order random sample of y. Then the likelihood functions will be as follows: **[9]**

$$L=L(\alpha,\beta;x,y)=\prod_{i=1}^{n}f(x_i)\prod_{j=1}^{m}f(y_j)$$

$$=\prod_{i=1}^{n}\alpha\lambda\acute{g}(x_i;a,\underline{\theta})e^{-\lambda g(x_i;a,\underline{\theta})}\left(1-e^{-\lambda g(x_i;a,\underline{\theta})}\right)^{\alpha-1}\prod_{j=1}^{m}\beta\lambda\acute{g}(y_j;a,\underline{\theta})e^{-\lambda g(y_j;a,\underline{\theta})}\left(1-e^{-\lambda g(y_j;a,\underline{\theta})}\right)^{\beta-1}$$

$$=\alpha^{n}\lambda^{n+m}e^{-\lambda\sum_{i=1}^{n}g(x_i;a,\underline{\theta})}\prod_{i=1}^{n}\acute{g}(x_i;a,\underline{\theta})\prod_{i=1}^{n}(1-e^{-\lambda g(x_i;a,\underline{\theta})})^{\alpha-1}\beta^{m}e^{-\lambda\sum_{j=1}^{m}g(y_j;a,\underline{\theta})}\prod_{ij=1}^{m}\acute{g}(y_j;a,\underline{\theta})\prod_{j=1}^{m}(1-e^{-\lambda g(y_j;a,\underline{\theta})})^{\beta-1}$$

$$\ln(l)=nln\alpha+(n+m)\ln\lambda+\sum_{i=1}^{n}\acute{g}(x_i;a,\underline{\theta})-\lambda\sum_{i=1}^{n}g(x_i;a,\underline{\theta})+(\alpha-1)\sum_{i=1}^{n}\ln(1-e^{-\lambda g(x_i;a,\underline{\theta})})+mln\beta+\sum_{j=1}^{m}\acute{g}(y_j;a,\underline{\theta})-\lambda\sum_{j=1}^{m}g(y_j;a,\underline{\theta})+(\beta-1)\sum_{j=1}^{m}\ln(1-e^{-\lambda g(y_j;a,\underline{\theta})})$$

$$\frac{d\mathrm{Ln}(l)}{d\alpha}=\frac{n}{\alpha}+\sum_{i=1}^{n}ln\left(1-e^{-\lambda g(x_i;a,\underline{\theta})}\right)=0$$

$$\frac{d\mathrm{Ln}(l)}{d\beta}=\frac{m}{\beta}+\sum_{j=1}^{m}ln\left(1-e^{-\lambda g(y_j;a,\underline{\theta})}\right)=0$$

Thus, the maximum likelihood estimator of the parameters α and β will be as follows:

$$\widehat{\alpha}_{\mathrm{mle}}=\frac{-\mathrm{n}}{\sum_{\mathrm{i}=1}^{\mathrm{n}}\ln\left(1-e^{-\lambda g(x_i;a,\underline{\theta})}\right)} \quad (4)$$

$$\hat{\beta}_{mle}=\frac{-m}{\sum_{j=1}^{m}ln\left(1-e^{-\lambda g(y_j;a,\underline{\theta})}\right)} \quad (5)$$

By substituting $\widehat{\alpha}_{\mathrm{mle}}$, $\hat{\beta}_{mle}$ in equation (3), obtains the Maximum Likelihood estimator for $R_{(s,k)}$:

$$\hat{R}_{(s,k)_{mle}}=\frac{\hat{\beta}_{mle}}{\hat{\alpha}_{mle}}\sum_{i=s}^{k}\frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}\left(k+\frac{\hat{\beta}_{mle}}{\hat{\alpha}_{mle}}-j\right)\right]^{-1} \quad (6)$$

4. Shrinkage Estimation Method (Sh1):

Thompson in 1968 assumed the problem of shrink was a usual estimator $\hat{\alpha}$ of the parameter α to prior information α_0 using shrinkage weight factor $\emptyset(\hat{\alpha})$, such that $0 \leq \emptyset(\hat{\alpha}) \leq 1$. Thompson estimating α and he believed that α_0 was closed to the true value of α or α_0 that may be near the true value of α .Thus, the form of Thompson - Type shrinkage estimator of α say $\hat{\alpha}_{sh}$ will be[10].

$$\hat{\alpha}_{sh} = \emptyset(\hat{\alpha})\hat{\alpha} + (1 - \emptyset(\hat{\alpha}))\alpha_0 \tag{7}$$

In this paper, we apply the unbiased estimator $\hat{\alpha}_{ub}$ as a usual estimator and $\alpha_0 \approx \alpha$ as a prior estimation of α in equation (7), as we mentioned, $\emptyset(\hat{\alpha})$ denotes the shrinkage weight factor such that $0 \leq \emptyset(\hat{\alpha}) \leq 1$, which may be a function of $\hat{\alpha}_{ub}$; a function of sample size (n,m) or may be constant (ad hoc basis) .

Note that: [9]

$$\hat{\alpha}_{ub} = \frac{n-1}{n}\hat{\alpha}_{mle}, \quad \text{Var}(\hat{\alpha}_{ub}) = \frac{\alpha^2}{n-2}$$

And, $\hat{\beta}_{ub} = \frac{m-1}{m}\hat{\beta}_{mle}$, $Var(\hat{\beta}_{ub}) = \frac{\beta^2}{m-2}$

4.1. The Shrinkage Weight Function (sh1)

The shrinkage weight factor as a function of sample sizes n and m respectively will be considered in this subsection and take the form below:

i.e. $\emptyset_1(\hat{\alpha})$ =Beta (n,m) and $\emptyset_2(\hat{\beta}) = \text{Beta}(m, n)$

Where n, and m are sample sizes, therefore the shrinkage estimator using shrinkage weight function of α and β which is defined in equation (7), will take the following formula:

$$\hat{\alpha}_{sh} = \emptyset_1(\hat{\alpha})\hat{\alpha}_{ub} + (1 - \emptyset_1(\hat{\alpha}))\alpha_0 \tag{8}$$

$$\hat{\beta}_{sh} = \emptyset_2(\hat{\beta})\hat{\beta}_{ub} + (1 - \emptyset_2(\hat{\beta}))\beta_0 \tag{9}$$

Also, as Thompson mentioned, α_0 and β_0 are closed to the real value of α *and* β respectively.Then, the shrinkage estimation $\hat{R}_{(s,k)}$ in equation (3) using shrinkage weight function will be:-

$$\hat{R}_{(s,k)_{sh1}} = \frac{\hat{\beta}_{sh1}}{\hat{\alpha}_{sh1}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} \left(k + \frac{\hat{\beta}_{sh1}}{\hat{\alpha}_{sh1}} - j\right) \right]^{-1} \tag{10}$$

4.2. Constant Shrinkage Weight Factor (Sh2):

We suggest in this subsection constant shrinkage weight factor $\emptyset_1(\hat{\alpha}) = 0.05$, and $\emptyset_2(\hat{\beta}) = 0.05$. Therefore, the shrinkage estimator through specific constant weight factor will be as follows:

$$\hat{\alpha}_{sh2} = (0.05)\,\hat{\alpha}_{ub} + (0.95)\alpha_0 \tag{11}$$

$$\hat{\beta}_{sh2} = (0.05)\hat{\beta}_{ub} + (0.95)\beta_0 \tag{12}$$

Substitute equation (11) and (12) in equation (3) to obtain the shrinkage estimation of $R_{(s,k)}$ using the above constant shrinkage weight factor as below:

$$\hat{R}_{(s,k)_{sh2}} = \frac{\hat{\beta}_{sh2}}{\hat{\alpha}_{sh2}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} (k + \frac{\hat{\beta}_{sh2}}{\hat{\alpha}_{sh2}} - j) \right]^{-1} \quad (13)$$

Where k, i, j are integers

4.3. Modified Thompson type Shrinkage Weight Factor (th):

In this paper, the shrinkage weight factor which was considered by Thompson will be modified to give the following formulas:

$$\gamma(\hat{\alpha}) = \frac{(\hat{\alpha}_{ub}-\alpha_0)^2}{(\hat{\alpha}_{ub}-\alpha_0)^2 + Var(\hat{\alpha}_{ub})} *0.005 \quad (14)$$

$$\gamma(\hat{\beta}) = \frac{(\hat{\beta}_{ub}-\beta_0)^2}{(\hat{\beta}_{ub}-\beta_0)^2 + Var(\hat{\beta}_{ub})} * 0.005 \quad (15)$$

Therefore, the shrinkage estimator of α and β using modified shrinkage weight factor are respectively as bellow:

$$\hat{\alpha}_{th} = \gamma(\hat{\alpha})\hat{\alpha}_{ub} + (1-\gamma(\hat{\alpha}))\alpha_0 \quad (16)$$

$$\hat{\beta}_{th} = \gamma(\hat{\beta})\hat{\beta}_{ub} + \left(1-\gamma(\hat{\beta})\right)\beta_0 \quad (17)$$

Then the Shrinkage estimation of $R_{(s,k)}$ in equation (3) based on Modified Thompson type shrinkage weight factor will be:

$$\hat{R}_{(s,k)_{th}} = \frac{\hat{\beta}_{th}}{\hat{\alpha}_{th}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} (k + \frac{\hat{\beta}_{th}}{\hat{\alpha}_{th}} - j) \right]^{-1} \quad (18)$$

Now, we take some special case for the Exponentiated Family distribution, by the following table:

Table1. special case for the Exponentiated Family distribution by different values the function $g(x; a, \underline{\theta})$

Name	Exponentiated Exponential distribution	Exponentiated Lomax distribution	Exponentiated Weibull distribution	Exponentiated Pareto distribution	Exponentiated Rayleigh distribution
Value $g(x; a, \underline{\theta})$	$g(x; a, \underline{\theta}) = x$	$g(x; a, \underline{\theta}) = \ln(1 + \theta x)$	$g(x; a, \underline{\theta}) = x^{\theta}$	$g(x; a, \underline{\theta}) = \ln(1 + x)$	$g(x; a, \underline{\theta}) = x^2$

5. Simulation Experiments

In this section, numerical results were studied to compare the performance of the different estimators of reliability using different sample size = (10, 30, 50 and 100), based on 1000 replication via MSE criteria. For this purpose, Monte Carlo simulation was employed by generating the random sample from the continuous uniform distribution defined on the interval

(0,1) as $u_1, u_2, \ldots, u_n$; $v_1, v_2, \ldots, v_m$. Transform uniform random samples to follow some distributions special case of the Family of Exponentiated distribution using (c. d. f.),[11].

The following steps compute the real value of $R_{(s,k)}$ in equation (3) and the value of estimation methods of all the proposal methods $\hat{R}_{(s,k)_{mle}}$, $\hat{R}_{(s,k)_{sh1}}$, $\hat{R}_{(s,k)_{sh2}}$ and $\hat{R}_{(s,k)_{th}}$ in equations (6), (10), (13) and (18) respectively.

Based on (L=1000) Replication, we calculate the MSE for all proposed estimation methods of $\hat{R}_{(s,k)}$ as follows:

$$\text{MSE} = \frac{1}{L}\sum_{i=1}^{L}(\hat{R}_{(s,k)_i} - R_{(s,k)})^2$$

Suppose the reliability system of multicomponent stress- strength model for some distributions as the following:

Table2. The symbol of the reliability system of multicomponent stress- strength model for special case of Exponentiated Family distribution.

Distributions	Exponentiated Exponential distribution	Exponentiated Lomax distribution	Exponentiated Weibull distribution	Exponentiated Pareto distribution	Exponentiated Rayleigh distribution
Symbol	$R1_{(s,k)}$	$R2_{(s,k)}$	$R3_{(s,k)}$	$R4_{(s,k)}$	$R5_{(s,k)}$

Now, we use some special case of Exponentiated Family distribution and applied the Simulation to find the value of the reliability and MSE to compare with the other methods.

Table 3. Values of the $R1_{(s,k)}$ for Exponentiated Exponential Distribution when$R1_{(s,k)} = 0.48485$, s=2, k=4 and λ=2

(n,m)	$\hat{R}1_{(s,k)MLE}$	$\hat{R}1_{(s,k)sh1}$	$\hat{R}1_{(s,k)sh2}$	$\hat{R}1_{(s,k)Th}$
(10,10)	0.48236	0.48489	0.48495	0.48491
(30,30)	0.48504	0.48489	0.48494	0.48490
(50,50)	0.48455	0.48489	0.48491	0.48489
(100,100)	0.48634	0.484897	0.48498	0.48490

Table 4. Values of MSE the $R1_{(s,k)}$ for Exponentiated Exponential Distribution when $R1_{(s,k)} = 0.48485$, s=2, k=4 and λ=2

(n,m)	$\hat{R}1_{(s,k)MLE}$	$\hat{R}1_{(s,k)sh1}$	$\hat{R}1_{(s,k)sh2}$	$\hat{R}1_{(s,k)Th}$
(10,10)	0.01521	0.239E-8	0.494E-4	0.202E-6
(30,30)	0.00576	0.239E-8	0.159E-4	0.777E-7
(50,50)	0.00349	0.239E-8	0.916E-5	0.442E-7
(100,100)	0.00165	0.239E-8	0.421E-5	0.217E-7

Table 5. Values of the $R2_{(s,k)}$ for Exponentiated Lomax Distribution when $R2_{(s,k)} = 0.48485$, s=2, k=4, λ=2 and θ=3.

(n,m)	$\hat{R}2_{(s,k)MLE}$	$\hat{R}2_{(s,k)sh1}$	$\hat{R}2_{(s,k)sh2}$	$\hat{R}2_{(s,k)Th}$
(10,10)	0.74265	0.48489	0.50888	0.48744
(30,30)	0.78786	0.48489	0.50176	0.48651
(50,50)	0.78828	0.48489	0.49980	0.48633
(100,100)	0.78964	0.48489	0.49862	0.48622

Table 6. Values of the MSE $R2_{(s,k)}$ for Exponentiated Lomax Distribution when $R2_{(s,k)} = 0.48485$, s=2, k=4 ,λ=2 and θ=3.

(n,m)	$\hat{R}2_{(s,k)MLE}$	$\hat{R}2_{(s,k)sh1}$	$\hat{R}2_{(s,k)sh2}$	$\hat{R}2_{(s,k)Th}$
(10,10)	0.11060	0.247E-8	0.00242	0,549E-4
(30,30)	0.10323	0.239E-8	0.00048	0.506E-5
(50,50)	0.09969	0.239E-8	0.00031	0. 301E-5
(100,100)	0.09657	0.239E-8	0.00022	0.219E-5

Table 7. Values of the $R3_{(s,k)}$ for Exponentiated Weibull Distribution when $R3_{(s,k)} = 0.48485$, s=2, k=4 ,λ=2 and θ=3.

(n,m)	$\hat{R}3_{(s,k)MLE}$	$\hat{R}3_{(s,k)sh1}$	$\hat{R}3_{(s,k)sh2}$	$\hat{R}3_{(s,k)Th}$
(10,10)	0.45181	0.48489	0.48437	0.48484
(30,30)	0.45505	0.48489	0.48451	0.48486
(50,50)	0.45617	0.48489	0.48454	0.48486
(100,100)	0.46025	0.48489	0.48459	0.48487

Table 8. Values of the MSE $R3_{(s,k)}$ for Exponentiated Weibull Distribution when $R3_{(s,k)} = 0.48485$, s=2, k=4 , λ=2 and θ=3.

(n,m)	$\hat{R}3_{(s,k)MLE}$	$\hat{R}3_{(s,k)sh1}$	$\hat{R}3_{(s,k)sh2}$	$\hat{R}3_{(s,k)Th}$
(10,10)	0.02444	0.239E-8	0.797E-5	0.847E-7
(30,30)	0.00955	0.239E-8	0.164E-5	0.153E-7
(50,50)	0.00596	0.239E-8	0.927E-6	0.820E-8
(100,100)	0.00317	0.239E-8	0.448E-6	0.404E-8

Table 9. Values of the $R4_{(s,k)}$ for Exponentiated Pareto Distribution when $R4_{(s,k)} = 0.48485$, s=2, k=4, and λ=2.

(n,m)	$\hat{R}4_{(s,k)MLE}$	$\hat{R}4_{(s,k)sh1}$	$\hat{R}4_{(s,k)sh2}$	$\hat{R}4_{(s,k)Th}$
(10,10)	0.74995	0.48489	0.51295	0.48795
(30,30)	0.78227	0.48489	0.50203	0.48654
(50,50)	0.78877	0.48489	0.49994	0.48634
(100,100)	0.78742	0.48489	0.49821	0.48618

Table 10. Values of the MSE $R4_{(s,k)}$ for Exponentiated Pareto Distribution when $R4_{(s,k)} = 0.48485$, s=2, k=4 and λ=2

(n,m)	$\hat{R}4_{(s,k)MLE}$	$\hat{R}4_{(s,k)sh1}$	$\hat{R}4_{(s,k)sh2}$	$\hat{R}4_{(s,k)Th}$
(10,10)	0.11724	0.248E-8	0.00375	0.105E-3
(30,30)	0.10319	0.239E-8	0.00050	0.507E-5
(50,50)	0.09941	0.239E-8	0.00031	0.305E-5
(100,100)	0.09519	0.239E-8	0.00021	0. 203E-5

Table 11. Values of the $R5_{(s,k)}$ for Exponentiated Rayleigh Distribution when $R5_{(s,k)} = 0.48485$, s=2, k=4 and λ=2.

(n,m)	$\hat{R}5_{(s,k)MLE}$	$\hat{R}5_{(s,k)sh1}$	$\hat{R}5_{(s,k)sh2}$	$\hat{R}5_{(s,k)Th}$
(10,10)	0.45555	0.48489	0.48443	0.48485
(30,30)	0.45467	0.48489	0.48450	0.48486
(50,50)	0.45551	0.48489	0.48453	0.48486
(100,100)	0.45835	0.48489	0.48458	0.48487

Table 12. Values of the $R5_{(s,k)}$ for Exponentiated Rayleigh Distribution when $R5_{(s,k)} = 0.48485$, s=2, k=4 and λ=2.

(n,m)	$\hat{R}5_{(s,k)MLE}$	$\hat{R}5_{(s,k)sh1}$	$\hat{R}5_{(s,k)sh2}$	$\hat{R}5_{(s,k)Th}$
(10,10)	0.02510	0.239E-8	0.886E-5	0.917E-7
(30,30)	0.00952	0.239E-8	0.163E-5	0.151E-7
(50,50)	0.00611	0.239E-8	0.956E-6	0.839E-8
(100,100)	0.0031	0.239E-8	0.441E-6	0.379E-8

6. Discussion Simulation results

From the tables above, for n= (10,30,50,100) besides m=(10,30,50,100) we conclude that the minimum (MSE) for the estimator of system reliability $R_{i(s,k)}$, (i=1,2,3,4,5),held for shrinkage estimator by using shrinkage weight function .This indicates that the shrinkage reliability estimator ($\hat{R}_{(s,k)_{sh1}}$) is the best for any n and m and follows by shrinkage estimator using modified Thomson weight factor.

7. Conclusion

The simulation results exhibited that the shrinkage estimator method is suitable for estimation the parameters and system reliability for the Exponentiated Family distribution especially when shrinkage weight factor as a linear combination between unbiased estimator, and prior estimate. The result estimator $\hat{R}_{(sh1)}$ perform well and will be the best estimator than the other in the sense of MSE.

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