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A Reliable Load Flow Method for Radial Distribution Systems

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Abstract

This paper presents a reliable load flow algorithm for radial distribution systems. The proposed algorithm exploits the particular topological structure of the radial distribution system. A system of algebraic equations which describe the steady state performance of the radial system is derived based on the relation between the nodal currents and the lateral currents. Different types of load models can be easily handled. The presented method is tested on several radial distribution systems. The results obtained reveal the robustness and effectiveness of the proposed algorithm for solving load flow problem in radial distribution system.

Key words: load flow, radial distribution network, ill-conditioned system.

طريقة موثوقة لحساب سريان الحمل في منظومات التوزيع الشعاعية

ملخص

يقدم هذا البحث خوارزمية موثوقة لحساب سريان الحمل في منظومات التوزيع الشعاعية. تستخدم الخوارزمية المقترحة بشكل فعال طبيعة تركيب منظومات التوزيع الشعاعية. تم اشتقاق منظومة من المعادلات الجبرية التي تصف سلوك المنظومة الشعاعية في الحالة المستقرة استنادا إلى العلاقة التي تربط تيارات العقد مع تيارات فروع المنظومة. يمكن تناول مختلف نماذج الحمل في الخوارزمية بسهولة. تم اختبار الطريقة المقترحة على عدد من منظومات التوزيع الشعاعية. ان النتائج المستحصلة تظهر كفاءة ومتانة الخوارزمية المقترحة لحل مسائل سريان الحمل في منظومات التوزيع الشعاعية.

1. Introduction

Load flow study has become the basic tool in power system design and operation. The commonly used methods to solve load flow problem in transmission systems are Newton-Raphson method and Fast decoupled method. These methods often fail to converge to a solution when applied to ill-conditioned system [1-2]. A modified version of the conventional load flow methods have been proposed by many researchers [3-6] to tackle the ill-conditioned problem. Utility distribution system and ship distribution system [7] are a typical form of ill-conditioned network. These networks are generally of radial structures with high R/X ratios. Application of Distribution Automated Systems and other distribution system optimization studies which requires a numerous run of load flow has motivating the researcher on the development of efficient and robust load flow method in radial distribution system.

Several algorithms have been developed to solve load flow for radial distribution systems [8-13]. Kersting[8] has presented a load flow technique based on the ladder network theory. Goswami et.al [2] proposed a direct method for getting load flow solution. The method was formulated as a problem of simple electric circuit with the load represented as constant impedance. The method is not applicable for distribution networks having nodes that are junctions of more than three branches. An algorithm presented in Ref.[9] for solving load flow problem using iterative solution of three fundamental equations representing real power, reactive power , and voltage magnitude. Some of the developed algorithms in Ref.[10-13] were ,generally, based on evaluating a an algebraic expression of the voltage at the receiving end voltage. The voltage magnitude and angle of the other nodes could be obtained from another set of recursive equations in a forward direction, starting from the node closer to the root node. These methods need a particular numbering scheme. Mekhamer et.al [14] used the same model equations of Ref. [9] but they used a simple iterative scheme instead of Newton-Raphson iterative technique. A forward sweep approach has been presented in Ref. [15]An approach to solve load flow in radial distribution system has been proposed by Aravindhababu et.al [16]. Their algorithm based on computing the voltage drop across all branches. The nodal voltages are calculated by using the branch to node matrix. They used a fixed point iterative technique.

In this paper, a reliable load flow algorithm for distribution system is presented. The method is based on solving a system of algebraic equations. The equations which describe the steady-state of the system are produced by exploiting the radial structure of the network. A conductance matrix is formed which describe the relation between the drawn nodal current at each bus and the lateral currents. The proposed method is applied on several test radial distribution systems. The results obtained are compared with other methods. The solution by the proposed methods is converged with less number of iteration regardless of the size of the system.

2. Mathematical Formulation

Most radial distribution systems are fed at one substation bus that is assumed to be the root bus (slack bus) rendering all the remaining nodes as load buses. Figure (1) shows a typical radial distribution system which has a main feeder and two laterals. The number of branches n_b and the number of nodes n are related by:

$$n_b = n - 1 \qquad \text{…………… (1)}$$

List of principal symbols used in the presented method is given in Appendix1. The branch and node nomenclature adopted in this paper is to number the nodes along the main feeder and each of the remaining laterals in an ascending scheme away from the root bus which would be assigned number 1. As shown in Fig. (1), the numbering of nodes in one lateral starts only after all the nodes in the preceding lateral has been numbered. Branch numbering scheme gives any branch connecting node ***i*** to node ***k*** a number ***k-1***. The above numbering process has been implemented in the load flow program of the proposed algorithm.

The branch currents are related to the nodal currents of all buses through the following connectivity matrix [16]:

$$\begin{bmatrix} i_{b1} \\ i_{b2} \\ i_{b3} \\ i_{b4} \\ i_{b5} \\ i_{b6} \\ i_{b7} \\ i_{b8} \\ i_{b9} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{n2} \\ I_{n3} \\ I_{n4} \\ I_{n5} \\ I_{n6} \\ I_{n7} \\ I_{n8} \\ I_{n9} \\ I_{n10} \end{bmatrix} \qquad \text{………….. (2)}$$

Or in compact form:

$$[i_b] = [C][I_n] \qquad \text{…………….. (3)}$$

Where the lower triangular of connectivity matrix are zeros

The nodal current drawn at bus ***i*** can be expressed in terms of the active and reactive loading power as:

$$I_{ni} = \frac{P_i - Q_i}{V_i^*}$$

$$i = 2, 3, 4, \ldots\ldots, n \qquad \text{……………. (4)}$$

By using primitive impedances of the branches, the voltage drop across all branches can be calculated as [17]:

$$\begin{bmatrix} v_{b1} \\ v_{b2} \\ v_{b3} \\ v_{b4} \\ v_{b5} \\ v_{b6} \\ v_{b7} \\ v_{b8} \\ v_{b9} \end{bmatrix} = \begin{bmatrix} z_{b1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & z_{b2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & z_{b3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & z_{b4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_{b5} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & z_{b6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & z_{b7} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & z_{b8} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & z_{b9} \end{bmatrix} \begin{bmatrix} i_{b1} \\ i_{b2} \\ i_{b3} \\ i_{b4} \\ i_{b5} \\ i_{b6} \\ i_{b7} \\ i_{b8} \\ i_{b9} \end{bmatrix} \qquad \text{............ (5)}$$

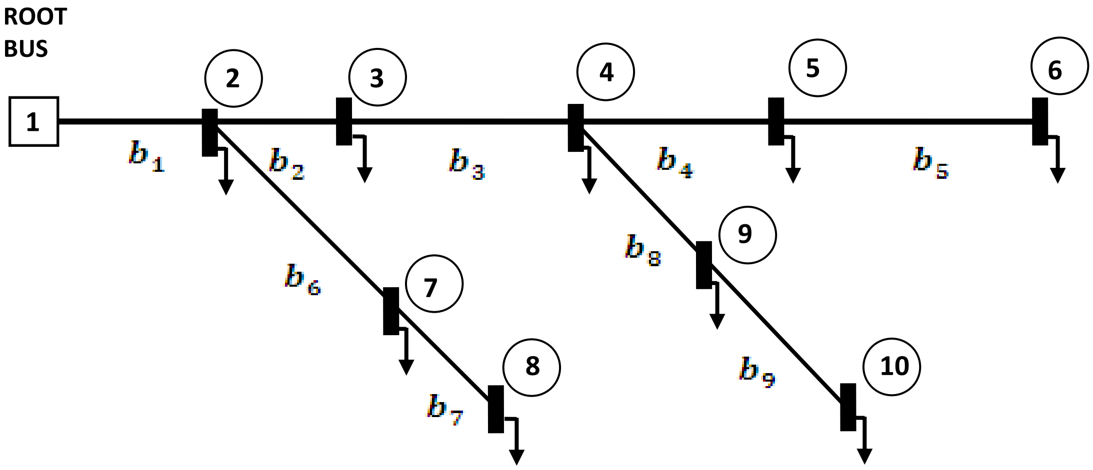

Figure.1 A typical radial distribution system with its branches being numbered

The complex bus voltage at load buses can be obtained from the following relation:

$$\begin{bmatrix} v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \end{bmatrix} = \begin{bmatrix} v_1 \\ v_1 \\ v_1 \\ v_1 \\ v_1 \\ v_1 \\ v_1 \\ v_1 \\ v_1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_{b1} \\ v_{b2} \\ v_{b3} \\ v_{b4} \\ v_{b5} \\ v_{b6} \\ v_{b7} \\ v_{b8} \\ v_{b9} \end{bmatrix} \qquad \text{.......... (6)}$$

In matrix form, the system of equations (6) can be written as:

$$[v] = [v_1] - [C]^T \, [v_b] \qquad \text{........... (7)}$$

Substituting (3) and (5) into Equation (7), yields:

$$[v] = [v_1] - [C]^T [z_b][C][I_n] \qquad \text{……… (8)}$$

For the nodal current given in (4), Equation (8) can be rewritten as:

$$[v] = [v_1] - [Z]\left[\frac{P-Q}{v^*}\right] \qquad \text{………….. (9)}$$

Where,

$$[Z] = [C]^T [z_b][C] \qquad \text{……… (10)}$$

For n bus radial system, the complex bus voltage at any bus i in terms of active power, reactive power, and complex voltage of other buses is given by the following nonlinear algebraic equation:

$$v_i = v_1 - \sum_{k=2}^{n} Z_{ik}\left\{\frac{P_k - jQ_k}{v_k^*}\right\}$$

i =2,3,4,…….., n ………..(11)

This can be resolved into the following two real equations, which are functions of the state variables $(|V|, \delta)$:

$$f_i(|v|,\delta) = |v_i|\cos\delta_i - |v_1|\cos\delta_1 + \sum_{k=1}^{n}\frac{|Z_{ik}||S_k|}{|V_k|}\{\cos(\gamma_{ik} - \theta_k + \delta_k)\} = 0$$

i =2,3,4,…….., n ………………………(12-a)

$$g_i(|v|,\delta) = |v_i|\sin\delta_i - |v_1|\sin\delta_1 + \sum_{k=1}^{n}\frac{|Z_{ik}||S_k|}{|v_k|}\{\sin(\gamma_{ik} - \theta_k + \delta_k)\} = 0$$

i =2,3,4,…….., n …………………… (12-b)

The system of equations (12) is solved iteratively by using Newton – Raphson. The linearised forms of these equations which are implemented in this paper are given in equation (13):

$$\begin{bmatrix} -f(|V|,\delta)^{cal} \\ -g(|V|,\delta)^{cal} \end{bmatrix} = \begin{bmatrix} \frac{\partial f(|V|,\delta)}{\partial\delta} & \frac{\partial f(|V|,\delta)}{\partial|V|} \\ \frac{\partial g(|V|,\delta)}{\partial\delta} & \frac{\partial g(|V|,\delta)}{\partial|V|} \end{bmatrix}\begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \qquad \text{……………….(13)}$$

The bus voltage angle and magnitude correction vector $[\Delta\delta \quad \Delta|V|]^T$ are found iteratively from equation (13). The process is repeated until the vector of the function $[-f(|V|,\delta)^{cal} \quad -g(|V|,\delta)^{cal}]^T$ fall within a specified tolerance.

The elements of the Jacobian matrix are given in Appendix 2.

3. The proposed algorithm

The basic step by step procedure of the proposed algorithm is as follows:

1) Form the connection matrix $[C]$ which relates the branch currents with the nodal currents.
2) Starting from gauss values (flat start) for the bus voltages, calculate the vector of the function$[-f(|V|,\delta)^{cal} \quad -g(|V|,\delta)^{cal}]^T$.
3) Calculate the elements of Jacobian matrix from equation(A2-3)to equation(A2-10) and solve the following set of equation for the state vector:

$$\begin{bmatrix}\Delta\delta \\ \Delta|V|\end{bmatrix} = \begin{bmatrix}\frac{\partial f(|V|,\delta)}{\partial\delta} & \frac{\partial f(|V|,\delta)}{\partial|V|} \\ \frac{\partial g(|V|,\delta)}{\partial\delta} & \frac{\partial g(|V|,\delta)}{\partial|V|}\end{bmatrix}^{-1} \begin{bmatrix}-f(|V|,\delta)^{cal} \\ -g(|V|,\delta)^{cal}\end{bmatrix}$$

Update the values of the state variables:

$$\delta^{(h+1)} = \delta^{(h)} + \Delta\delta$$
$$|v|^{(h+1)} = |v|^{(h)} + \Delta|v|$$

4) Check whether the elements of the vector of the function are within stipulated tolerance, or not. If so, the solution is converged. Calculate the branch power flow and branch voltage drop from the Equation (5):

If the elements of the vector of the function are out of the stipulated tolerance go to step 3 and continue the procedure.

4. Simulation and results

The proposed algorithm was implemented with MATLAB ver 5.6. The PC used in the studies presented has 2.09GHz processor with 400GB hard disk and 3.46GB RAM. To examine the robustness of the proposed algorithm, various tests have been conducted on several ill-conditioned radial distribution systems for different loading conditions. The single line diagram of the first system is shown in Fig.2. The system has a main feeder with 11 load buses fed from a root bus. The bus and line data [18] are given in Table (A3-1) and (A3-2) in

Appendix 3. The second test system is the 33bus system whose single line diagram is shown in Fig. (3)[19]. The system has one main feeder and three laterals. The bus and line data are given in Table (A3-3) and (A3-4) in Appendix 3. The third test system shown in Fig.(4) has 69-bus with one main feeder and four laterals. Its data are taken from [9] and given in Table(A3-5) and (A3-6) in Appendix 3

To compare the features of the proposed algorithm, the above systems have been solved for a base case load flow by applying Gauss-Siedel method, conventional newton-Raphson method, forward sweep approach[15], and the proposed method.

Figures 5, 6,and 7 show the bus voltage profile for the 12bus, 33bus and 69bus system, respectively. The results are matching that obtained by using other methods.

Table 1 shows the number of iterations and the execution time required for the solution. It can be seen that the proposed algorithm is efficient.

The branch voltage drop for the 12 bus test system is shown in Fig(8). The weakest branch is that connecting bus 3 with bus7.Fig. (9) Shows the branch voltage drop for the 33bus test system. Line number 5 seems to be the weakest branch. For the 69 bus system the weakest branch is that connecting bus 5 with bus 7 as shown from Fig.(10)

The heavy loading conditions are simulated by increasing the load at all buses with a loading factor. Table 2 shows the number of iteration required for these heavy loading conditions.

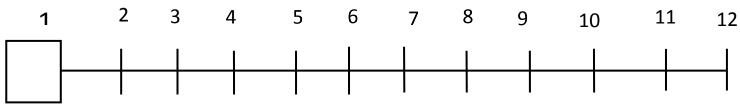

Fig.(2) Single line diagram of the 12 bus test system

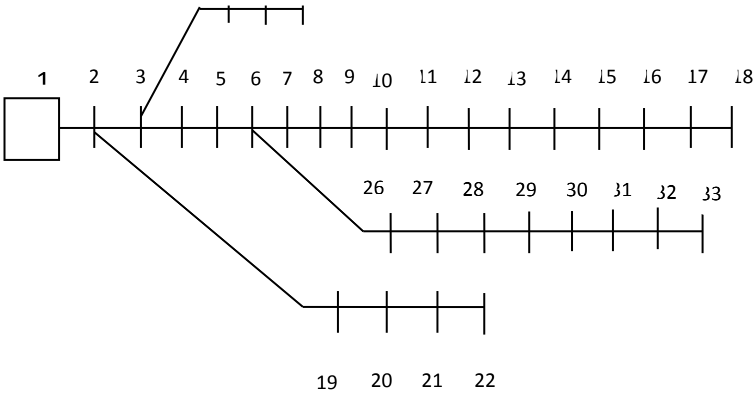

Fig.(3) Single line diagram of the 33 bus test system

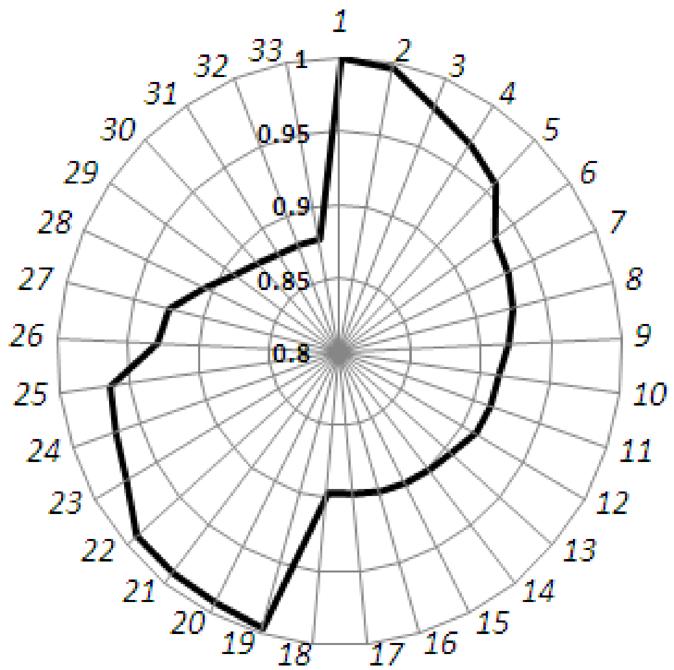

Fig.6 Bus voltage profile of the 33 bus test system for base case loading condition

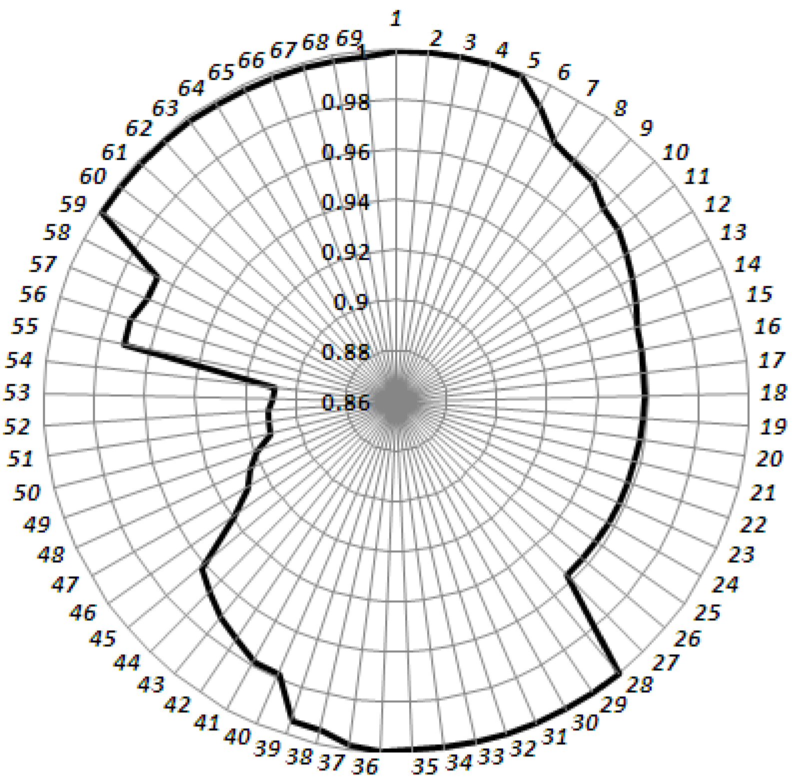

Fig.7 Bus voltage profile of the 69 bus test system for base case loading condition

Table 1 The number of iterations and execution time required to converge for different systems by different methods

Test system	Gauss_Siedle method		Conventional Newton_Raphson		Method given In [15]		Proposed algorithm	
	No. of Iterations	Execution time(s)	No. of Iterations	Execution time(s)	No. of Iterations	Execution time(s)	No. of Iterations	Execution time(s)
12 bus	67	0.109	4	0.093	N/A	N/A	3	0.078
33 bus	N/A	N/A	4	0.125	4	0.14	3	0.092
69 bus	4412	6.265	4	0.219	4	0.33	3	0.125

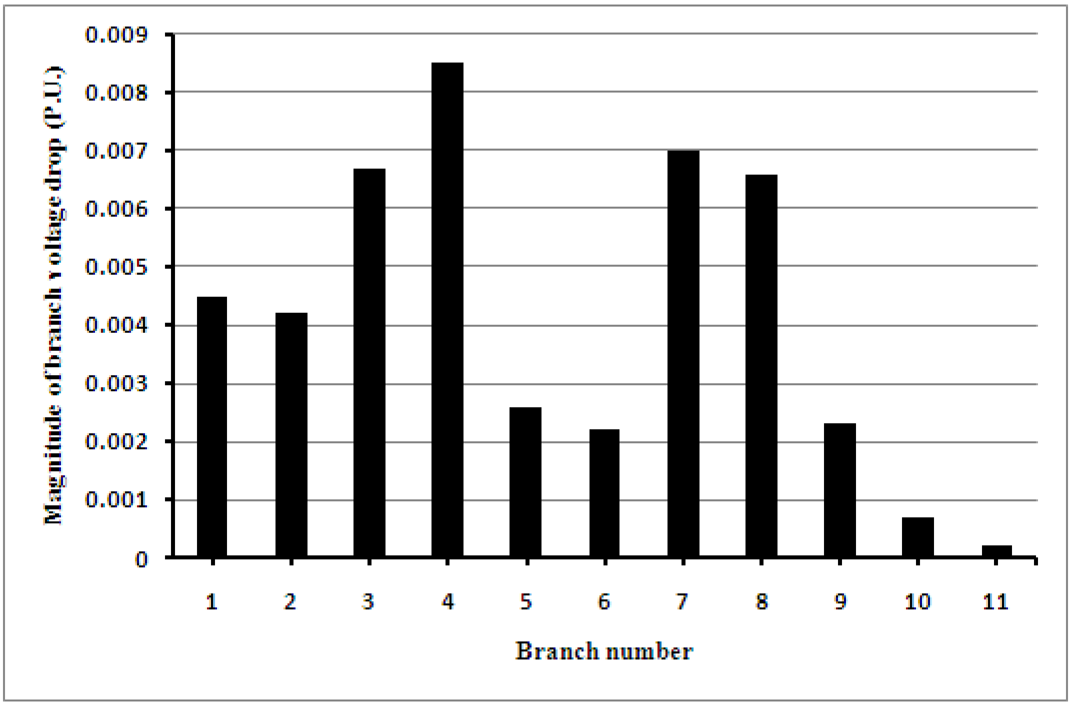

Fig.8 Magnitude of branch voltage drop of the 12 bus test system for base case loading condition

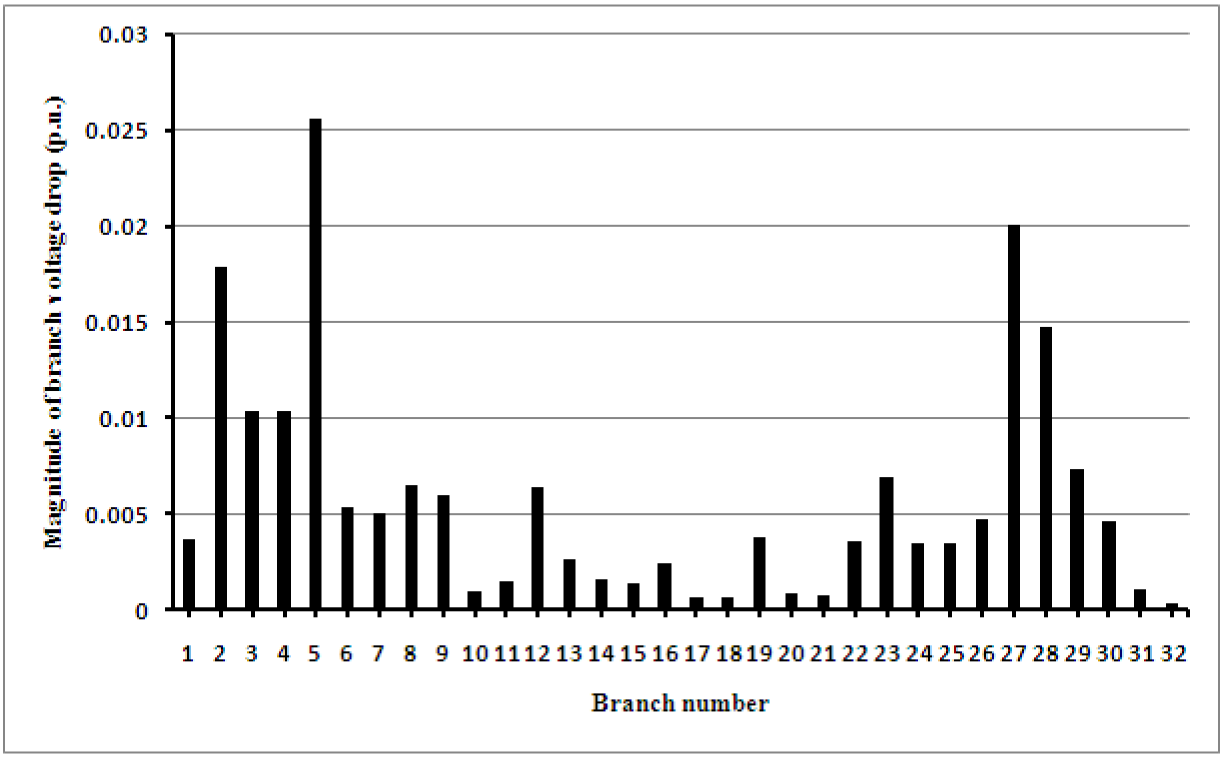

Fig.9 Magnitude of branch voltage drop of the 33 bus test system for base case loading condition

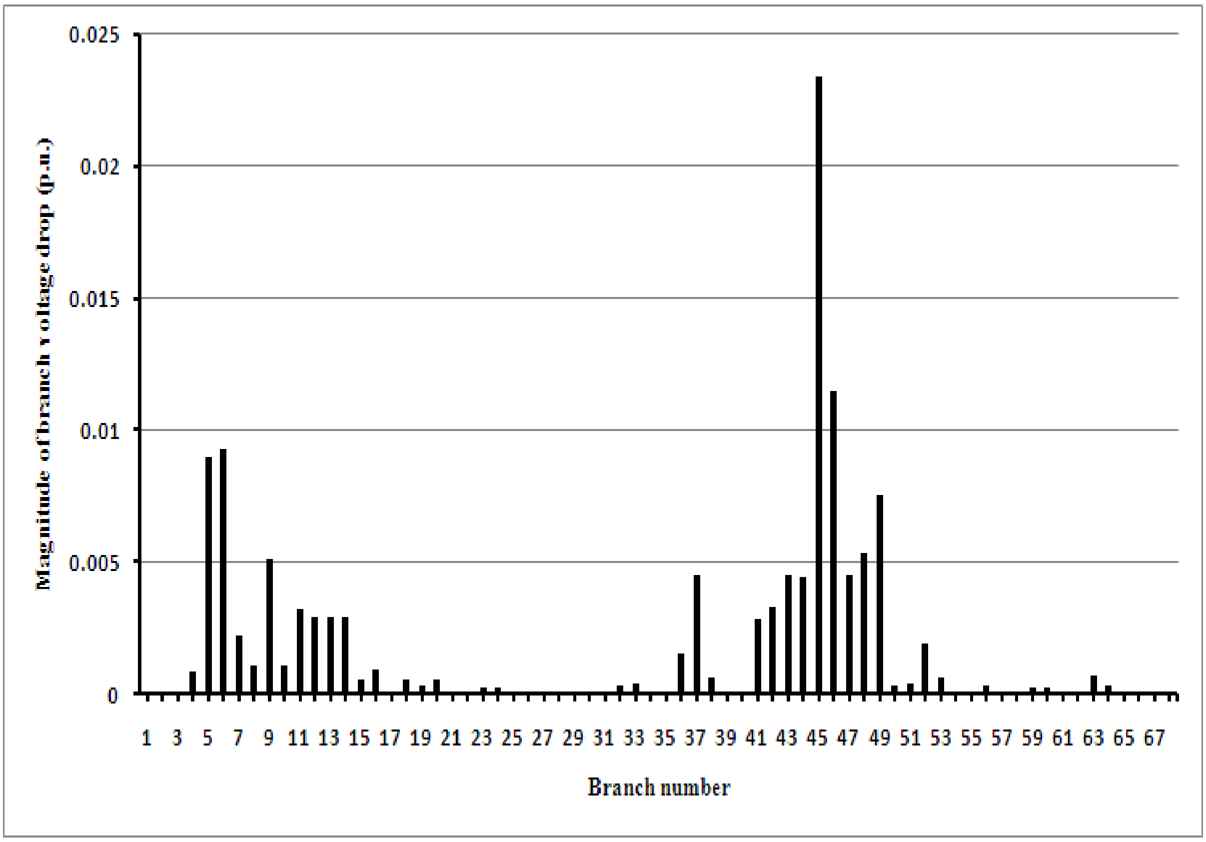

Fig.10 Magnitude of branch voltage drop of the 69 bus test system for base case loading condition

Table 2 The number of iterations required for convergence for different loading conditions

Test System	System loading status (kW+jkVAR)	Number of iterations required for convergence		
		Gauss-Siedle	Newton-Raphson	Proposed algorithm
12 bus	435+j405	67	4	3
	3058+j2847	4952	11	7
	3059+j2848	Div	Div	7
33 bus	4715+j2300	NA	4	3
	12273+j5987	NA	11	7
	12274+j5989	NA	Div	7
69 bus	3802+j2668	4412	4	3
	12216+j8571	13917	9	6
	12220+j8575	Div	Div	7

5. Conclusion

A new contribution to solve radial distribution load flow is proposed in this paper. The robustness of the proposed method is tested on several ill-conditioned radial systems. The method has a lower execution time. It exhibits a constant number of iterations. The algorithm presented is simply determining the weakest branch for any loading scenario. The proposed method provides a robust work horse for other distribution system studies

6. References

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7. Appendixes

Appendix 1 List of Symbols

$[v]$	Vector of complex bus voltages.
$v_i = \|v_i\| \llcorner \delta_i$	Complex voltage at bus *i*
$[z_b]$	Vector of primitive branch impedance.
$[i_b]$	Vector of branch currents
$[I_n]$	Vector of bus injected currents(load currents)
$S_i = P_i + Q_i$	
$S_i = \|S_i\| \llcorner \theta_i$	Complex power drawn at bus *i*
$Z_{ik} = \|Z_{ik}\| \llcorner \gamma_{ik}$	the *ik* element of the Impedance matrix Z

Appendix 2 Derivation of the elements of the Jacobian matrix

For n bus radial distribution system, the real equations which describe the performance in the steady-state as given by equation(12-a and 12-b):

$$f_i(|v|,\delta) = |v_i|\cos\delta_i - |v_1|\cos\delta_1 + \sum_{k=1}^{n}\frac{|Z_{ik}||S_k|}{|V_k|}\{\cos(\gamma_{ik}-\theta_k+\delta_k)\} = 0$$

i =2,3,4,…….., n ……..…………………………….(12-a)

$$g_i(|v|,\delta) = |v_i|\sin\delta_i - |v_1|\sin\delta_1 + \sum_{k=1}^{n}\frac{|Z_{ik}||S_k|}{|v_k|}\{\sin(\gamma_{ik}-\theta_k+\delta_k)\} = 0$$

i =2,3,4,…….., n …..………………………………… (12-b)

Expanding equation (12-a)&(12-b) using Taylor series and neglecting higher order terms yields:

$$\frac{\partial f_i(|V|,\delta)}{\partial\delta}\Delta\delta + \frac{\partial f_i(|V|,\delta)}{\partial|V|}\Delta V = -f_i(|V|,\delta)$$ …………………………….(A2-1)

$$\frac{\partial g_i(|V|,\delta)}{\partial\delta}\Delta\delta + \frac{\partial g_i(|V|,\delta)}{\partial|V|}\Delta V = -g_i(|V|,\delta)$$ …………………………….(A2-2)

1- The elements of sub matrix $\left[\frac{\partial f(|V|,\delta)}{\partial\delta}\right]$ are:

-the diagonal elements

$$\frac{\partial f_i(|V|,\delta)}{\partial\delta_i} = -|V_i|\,sin\,\delta_i - \frac{|Z_{ii}||S_i|}{|V_i|}sin\,(\gamma_{ii}-\theta_i+\delta_i)$$ ………………………..(A2-3)

-the off diagonal elements

$$\frac{\partial f_i(|V|,\delta)}{\partial\delta_k} = -\frac{|Z_{ik}||S_k|}{|V_k|}sin\,(\gamma_{ik}-\theta_k+\delta_k)$$ ……………………..….(A2-4)

2- The elements of sub matrix $\left[\frac{\partial f(|V|,\delta)}{\partial|V|}\right]$ are:

-the diagonal elements

$$\frac{\partial f_i(|V|,\delta)}{\partial|V_i|} = cos\delta_i - \frac{|Z_{ii}||S_i|}{|V_i|^2}cos\,(\gamma_{ii}-\theta_i+\delta_i)$$ ………………………...(A2-5)

-the off diagonal elements

$$\frac{\partial f_i(|V|,\delta)}{\partial|V_k|} = -\frac{|Z_{ik}||S_k|}{|V_k|^2}cos\,(\gamma_{ik}-\theta_k+\delta_k)$$ ……………………..….(A2-6)

3- The elements of sub matrix $\left[\frac{\partial g(|V|,\delta)}{\partial\delta}\right]$ are:

-the diagonal elements

$$\frac{\partial g_i(|V|,\delta)}{\partial\delta_i} = |V_i|\,cos\,\delta_i + \frac{|Z_{ii}||S_i|}{|V_i|}cos\,(\gamma_{ii}-\theta_i+\delta_i)$$ ………………….…. (A2-7)

-the off diagonal elements

$$\frac{\partial g_i(|V|,\delta)}{\partial \delta_k} = \frac{|Z_{ik}||S_k|}{|V_k|} cos\,(\gamma_{ik} - \theta_k + \delta_k) \quad \text{(A2-8)}$$

4- The elements of sub matrix $\left[\frac{\partial g(|V|,\delta)}{\partial |V|}\right]$ are:

-the diagonal elements

$$\frac{\partial g_i(|V|,\delta)}{\partial |V_i|} = sin\delta_i - \frac{|Z_{ii}||S_i|}{|V_i|^2} sin\,(\gamma_{ii} - \theta_i + \delta_i) \quad \text{(A2-9)}$$

-the off diagonal elements

$$\frac{\partial g_i(|V|,\delta)}{\partial |V_k|} = -\frac{|Z_{ik}||S_k|}{|V_k|^2} sin\,(\gamma_{ik} - \theta_k + \delta_k) \quad \text{(A2-10)}$$

Appendix 3 The bus data and line data of test distribution systems

Table A3-1 Bus data of the 12 bus test system

Bus No.	P_D (kW)	Q_D (kVAr)	Bus No.	P_D (kW)	Q_D (kVAr)
2	60	60	8	45	45
3	40	30	9	40	40
4	55	55	10	35	30
5	30	30	11	40	30
6	20	15	12	15	15
7	55	55			

Table A3-2 Line data of the 12 bus test system

Line No.	SE	RE	Resistance (Ω)	Reactance (Ω)	Line No.	SE	RE	Resistance (Ω)	Reactance (Ω)
1	1	2	1.093	0.455	7	7	8	4.403	1.215
2	2	3	1.184	0.494	8	8	9	5.642	1.597
3	3	4	2.095	0.873	9	9	10	2.890	0.818
4	4	5	3.188	1.329	10	10	11	1.514	0.428
5	5	6	1.093	0.455	11	11	12	1.238	0.351
6	6	7	1.002	0.417					

Table A3-3 Bus data of the 33 bus test system

Bus No.	P_D (kW)	Q_D (kVAr)	Bus No.	P_D (kW)	Q_D (kVAr)
2	100	60	18	90	40
3	90	40	19	90	40
4	120	80	20	90	40
5	60	30	21	90	40
6	60	20	22	90	40
7	200	100	23	90	50
8	200	100	24	420	200
9	60	20	25	420	200
10	60	20	26	60	25
11	45	30	27	60	25
12	60	35	28	60	20
13	60	35	29	120	70
14	120	80	30	200	600
15	60	10	31	150	70
16	60	20	32	210	100
17	60	20	33	60	40

Table A3-4 Line data of the 33 bus test system

Line No.	SE	RE	Resistance (Ω)	Reactance (Ω)	Line No.	SE	RE	Resistance (Ω)	Reactance (Ω)
1	1	2	0.0922	0.0470	17	17	18	0.7320	0.5740
2	2	3	0.4930	0.2511	18	2	19	0.1640	0.1565
3	3	4	0.3660	0.1864	19	19	20	1.5042	1.3554
4	4	5	0.3811	0.1941	20	20	21	0.4095	0.4784
5	5	6	0.8190	0.7070	21	21	22	0.7089	0.9373
6	6	7	0.1872	0.6188	22	3	23	0.4512	0.3083
7	7	8	0.7114	0.2351	23	23	24	0.8980	0.7091
8	8	9	1.0300	0.7400	24	24	25	0.8960	0.7011
9	9	10	1.0440	0.7400	25	6	26	0.2030	0.1034
10	10	11	0.1966	0.0650	26	26	27	0.2842	0.1447
11	11	12	0.3744	0.1238	27	27	28	1.0590	0.9337
12	12	13	1.4680	1.1550	28	28	29	0.8042	0.7006
13	13	14	0.5416	0.7129	29	29	30	0.5075	0.2585
14	14	15	0.5910	0.5260	30	30	31	0.9744	0.9630
15	15	16	0.7463	0.5450	31	31	32	0.3105	0.3619
16	16	17	1.2890	1.7210	32	32	33	0.3410	0.5302

Table A3-5 Bus data of the 69 bus test system

Bus No.	P_D (kW)	Q_D (kVAr)	Bus No.	P_D (kW)	Q_D (kVAr)
1	0.0	0.0	36	0.0	0.0
2	0.0	0.0	37	79.0	56.0
3	0.0	0.0	38	384.7	274.5
4	0.0	0.0	39	384.7	274.5
5	0.0	0.0	40	40.5	28.3
6	2.60	2.2	41	3.6	2.7
7	40.4	30.0	42	4.35	3.5
8	75.0	54.0	43	26.4	19.0
9	30.0	22.0	44	24.0	17.2
10	28.0	19.0	45	0.0	0.0
11	145.0	104.0	46	0.0	0.0
12	145.0	104.0	47	0.0	0.0
13	8.0	5.5	48	100.0	72.0
14	8.0	5.5	49	0.0	0.0
15	0.0	0.0	50	1244.0	888.0
16	45.5	30.0	51	32.0	23.0
17	60.0	35.0	52	0.0	0.0
18	60.0	35.0	53	227.0	162.0
19	0.0	0.0	54	59.0	42.0
20	1.0	0.6	55	18.0	13.0
21	114.0	81.0	56	18.0	13.0
22	5.3	3.5	57	28.0	20.0
23	0.0	0.0	58	26.0	20.0
24	28.0	20.0	59	26.0	18.55
25	0.0	0.0	60	26.0	18.55
26	14.0	10.0	61	0.0	0.0
27	14.0	10.0	62	24.0	17.0
28	26.0	18.6	63	24.0	17.0
29	26.0	18.6	64	1.2	1.0
30	0.0	0.0	65	0.0	0.0
31	0.0	0.0	66	6.0	4.3
32	0.0	0.0	67	0.0	0.0
33	14.0	10.0	68	39.22	26.3
34	19.5	14.0	69	39.22	26.3
35	6.0	4.0			

Table A3-6 Line data of the 69 bus test system

Line No.	SE	RE	Resistance (Ω)	Reactance (Ω)	Line No.	SE	RE	Resistance (Ω)	Reactance (Ω)
1	1	2	0.0005	0.0012	36	36	37	0.0851	0.2083
2	2	3	0.0005	0.0012	37	37	38	0.2898	0.7091
3	3	4	0.0015	0.0036	38	38	39	0.0822	0.2011
4	4	5	0.0251	0.0294	39	8	40	0.0928	0.0473
5	5	6	0.3660	0.1864	40	40	41	0.3319	0.1114
6	6	7	0.3811	0.1941	41	9	42	0.1740	0.0886
7	7	8	0.0922	0.0470	42	42	43	0.2030	0.1034
8	8	9	0.0493	0.0251	43	43	44	0.2842	0.1447
9	9	10	0.8190	0.2707	44	44	45	0.2813	0.1433
10	10	11	0.1872	0.0619	45	45	46	1.5900	0.5337
11	11	12	0.7114	0.2351	46	46	47	0.7837	0.2630
12	12	13	1.0300	0.3400	47	47	48	0.3042	0.1006
13	13	14	1.0440	0.3450	48	48	49	0.3861	0.1172
14	14	15	1.0580	0.3496	49	49	50	0.5075	0.2585
15	15	16	0.1966	0.650	50	50	51	0.0974	0.0496
16	16	17	0.3744	0.1238	51	51	52	0.1450	0.0738
17	17	18	0.0047	0.0016	52	52	53	0.7105	0.3619
18	18	19	0.3276	0.1083	53	53	54	1.0410	0.5302
19	19	20	0.2106	0.0696	54	11	55	0.2012	0.0611
20	20	21	0.3416	0.1129	55	55	56	0.0047	0.0014
21	21	22	0.0140	0.0046	56	12	57	0.7394	0.2444
22	22	23	0.1591	0.0526	57	57	58	0.0047	0.0016
23	23	24	0.3463	0.1145	58	3	59	0.0044	0.0108
24	24	25	0.7488	0.2475	59	59	60	0.0640	0.1565
25	25	26	0.3089	0.1021	60	60	61	0.1053	0.1230
26	26	27	0.1732	0.0572	61	61	62	0.0304	0.0355
27	3	28	0.0044	0.0108	62	62	63	0.0018	0.0021
28	28	29	0.0640	0.1565	63	63	64	0.7283	0.8509
29	29	30	0.3978	0.1315	64	64	65	0.3100	0.3623
30	30	31	0.0702	0.0232	65	65	66	0.0410	0.0478
31	31	32	0.3510	0.1160	66	66	67	0.0092	0.0116
32	32	33	0.8390	0.2816	67	67	68	0.1089	0.1373
33	33	34	1.7080	0.5646	68	68	69	0.0009	0.0012
34	34	35	1.474	0.4873					
35	4	36	0.0034	0.0084					