δ-Fuzzy Ideal Of A Smarandache Near Ring

مثالية سكما الضبابية في الحلقة السمرندش القريبة

Showq Mohammed .E Dhuha Abdulameer Kadhim
Showqm.ibriheem@uokufa.edu.iq Dhuhaa.ebada@uokufa.edu.iq
AL-Kufa university College Of Education For Girls
Department of Mathematics

Abstract

In this paper ,we introduce the notions of δ-Fuzzy ideal of a Smarandache near ring related to the near filed M denoted by (S-δ FI) of a Smarandache near ring and discuss the problem concerning the intersection of a family of (S-δ FI)and we give some properties about it .

الخلاصة

قدمنا في هذا البحث مفهوم جديد وهو المثالية الضبابية δ في حلقة السمرندش القريبة بالنسبة للحقل القريب M والتي يرمزلها بالرمز (S-δ FI) وتم عرض بعض النتائج والبراهين الخاصة بها .

Key word

near ring, near field, Smarandache near ring, Smarandache ideal , fuzzy sub near field, Smarandache fuzzy ideal.

Introduction

Throughout this paper N will be a left Samarandache near ring . In 1977 the notion near ring denoted by G.Pilz [1],in 2002 The concept Samarandache near ring denoted by W.B.vasantha [2], In 2003 the notion Samarandache fuzzy ideal denoted by W.B.vasantha andasamy [3], in 2005 the concept $\bar{\lambda}^*$ introduced by S.Bharanri [4].

Definition (1.1) [1]

A left near ring is a set N together with two binary operations "+" and "." such that
(1) (N,+) is a group (not necessarily abelian),
(2) (N, .) is a semigroup ,
(3)$(n_1 + n_2) . n_3 = n_1 . n_3 + n_2 . n_3$, for all $n_1, n_2, n_3, \in N$;

Definition (1.2) [2]

Anon- empty set N is said to be a near field if on N is defined by two binary operations "+","." such that
(1) (N,+) is a group ,
(2) (N\{0}, .) is a group ,
(3) a.(b+c)=a.b+a.c for all a,b,c belong to N .

Definition (1. 3) [2]

The near ring (N,+,.) is said to be a smarandache near ring denoted by (S-near ring) if it has a proper subset M such that (M,+,.) is a near field .

Definition(1.4) [2]

Let N be S-near ring, a normal subgroup I of N is called a smarandache ideal (S-ideal) of N related to M if ,
(1) $\forall y,z \in M$ and $\forall i \in I, y.(z+i) - y.z \in I$, where M is the near field containd in N.
(2) $I.M \subseteq I$.

Definition (1.5)[1]

Let(N_1,+,.) and (N_2 ,+′,.′) be two S-near-rings , a function f : $N_1 \to N_2$ is called a Smarandache near-ring homomorphism (S-near-ring homomorphism) if for all m, n $\in M_1$ (M_1 is a proper subset of N_1 which is a near-field) we have

(1) f(m + n) = f(m) +′ f(n) .

(2) f(m. n) = f(m) .′ f(n) .

where f (m) and f (n) $\in M_2$ (M_2 is a proper subset of N_2 which is a near-field)

Definition(1.6) [4]

Let μ be a fuzzy ideal of a near ring N and

μ^* be a fuzzy set in N defined by $\mu^*(y) = \mu(y)+1-\mu(0)$,y $\in$ N .

Definition (1.7) [3]

Let μ is a fuzzy sub near ring of N and μ is a fuzzy ideal of N such that

(1) $\mu(y-z) \geq \min\{\mu(y), \mu(z)\}$

(2) $\mu(y.z^{-1}) \geq \min\{\mu(y), \mu(z)\}$, $\forall$ y,z $\in$ N. Then μ is a fuzzy sub near field of N.

Definition (1.8) [2]

Let N be a S-near ring. A non empty fuzzy subset μ of N is called a Smarandache fuzzy ideal (S-fuzzy ideal) related to the near field M of N if the following conditions are true

(i) N has a proper subset X such that

(1) $\mu(z - y) \geq \min\{\mu(z), \mu(y)\}$

(2) $\mu(z.y) \geq \min\{\mu(z), \mu(y)\}$ for all z, y $\in$ X . i.e. μ on X is a fuzzy ideal.

(3) $\mu(z + y - z) \geq \mu(y)$

(4) $\mu(z.y) \geq \mu(y)$

(ii) X contains a proper subset M such that M is a near field under the operations of N and $\mu|_M$: $M \to [0, 1]$ is a fuzzy sub near field of M .

Definition (1.9) [3]

Let μ is a fuzzy subring of N such that μ^* is a field contains the identity of N and $\mu(y) = \mu(y^{-1})$ for all units y in N, then μ is called the fuzzy subfield of N.

Definition (1.10) [3]

Let $f:(N_1,+,.)\to(N_2,+',.')$ be a function .For a fuzzy set μ in N_2 we define $(f^{-1}(\mu))(x)=\mu(f(x))$ for every $x\in N_1$.For a fuzzy set λ in X , $f(\lambda)$ is defined by

$$(f(\lambda))(y)=\begin{cases}\sup\lambda(x) & \text{if } f(x)=y, y\in N_2\\ 0 & \text{otherwisey}\end{cases}$$

2. The main Results

In this section we study δ-fuzzy ideal of a Smarandache near ring.

Definition (2.1)

Let $\bar{\lambda}$ be a S-fuzzy ideal of S-near ring N related to the near filed M is called δ-Fuzzy ideal of N denoted by (S-δFI) if

(1)$\bar{\lambda}(x-y) \le \bar{\lambda}(x)$

(2)$\bar{\lambda}(xy) \ge \max\{\bar{\lambda}(x), \bar{\lambda}(y)\}, \forall x, y \in M$.

Example (2.2)

Consider the S- near ring $N = Z_6$ with addition and multiplication as defined by the following tables .

+6	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

.6	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

Let $\bar{\lambda}$ be a S-fuzzy ideal of S-near ring N related to the near filed M={0,2,4 } defined by

$$\bar{\lambda}(y) = \begin{cases} 1 & \text{if } y \in \{0,2,4\} \\ 0 & \text{if } y \in \{1,3,5\} \end{cases}$$

Then $\bar{\lambda}$ is (S-δFI) related to the near filed M={0,2,4 }of N.

Theorem (2.3)

Let $\overline{\lambda}$ be a fuzzy subset of an S-near ring N , then $\overline{\lambda}^*$ is a (S-δ FI) related to the near filed M of N if $\overline{\lambda}$ is a (S-δ FI) related to the near filed M of N

Proof

Let $\overline{\lambda}$ be (S-δ FI) related to the near filed M of N then $\overline{\lambda}$ is S-fuzzy ideal of N there exist X is a proper subset of N contain M as a near field and $\overline{\lambda}$ is fuzzy sub near filed of M , let $x, y \in N,$

(1) $\overline{\lambda}^*(x-y) = \overline{\lambda}\ (x-y) - \overline{\lambda}\ (0) + 1$

since $\overline{\lambda}$ is fuzzy ideal of N

$\geq \min\{\overline{\lambda}\ (x), \overline{\lambda}\ (y)\} - \overline{\lambda}\ (0) + 1$

$\geq \min\{\overline{\lambda}\ (x) - \overline{\lambda}\ (0) + 1, \overline{\lambda}\ (y) - \overline{\lambda}\ (0) + 1\}$

$= \min\{\overline{\lambda}^*(x), \overline{\lambda}^*(y)\}.$

(2) $\overline{\lambda}^*(xy^{-1}) = \overline{\lambda}\ (xy^{-1}) - \overline{\lambda}\ (0) + 1$

$\geq \min\{\overline{\lambda}\ (x), \overline{\lambda}\ (y^{-1})\} - \overline{\lambda}\ (0) + 1$

$= \min\{\overline{\lambda}\ (x), \overline{\lambda}\ (y)\} - \overline{\lambda}\ (0) + 1$

$= \min\{\overline{\lambda}\ (x) - \overline{\lambda}\ (0) + 1, \overline{\lambda}\ (y) - \overline{\lambda}\ (0) + 1\}$

$\geq \min\{\overline{\lambda}^*\ (x), \overline{\lambda}^*(y)\}$

From (1) and (2) $\Rightarrow \overline{\lambda}^*$ is a fuzzy sub near field of M that mean $\overline{\lambda}^*$ is S-fuzzy ideal related to the near filed M of N. to prove that $\overline{\lambda}^*$ is (S-δ FI) related to the near filed M of N, let $x, y \in M,$

(1) $\overline{\lambda}^*(x-y) = \overline{\lambda}\ (x-y) - \overline{\lambda}\ (0) + 1$

$\leq \overline{\lambda}\ (x) - \overline{\lambda}\ (0) + 1 = \overline{\lambda}^*(x)$

(2) $\overline{\lambda}^*(xy\ ) = \overline{\lambda}\ (xy) - \overline{\lambda}\ (0) + 1$

$\geq \max\{\overline{\lambda}\ (x), \overline{\lambda}\ (y)\} - \overline{\lambda}\ (0) + 1$

$= \max\{\overline{\lambda}\ (x) - \overline{\lambda}\ (0) + 1, \overline{\lambda}\ (y) - \overline{\lambda}\ (0) + 1\}$

$= \max\{\overline{\lambda}^*(x), \overline{\lambda}^*(y)\}.$

From (1) and (2) $\Rightarrow \overline{\lambda}^*$ Is (S-δ FI) related to the near filed M of N.

Theorem (2.4)

Let $\{\overline{\lambda}_j\}_{j\in J}$ be a family of (S-δ FI) related to the near filed M of S-near ring N , then $\bigcap_{j\in J} \overline{\lambda}_j$ is (S-δ FI) related to the near filed M of N.

Proof

Since each $\overline{\lambda}_j$ is (S-δ FI) related to the near filed M of N when $j \in J$,then there exists a proper subset X_j of N such that $\overline{\lambda}_j$ is a fuzzy ideal on X_j ,M is a proper subset of X_j and $\overline{\lambda}_j$ is a fuzzy sub near field on M for all $j \in J$ $X = \bigcap_{j\in J} X_j$ is a proper subset of N contain M .

To prove that $\bigcap_{j\in J} \overline{\lambda}_j$ is S-fuzzy ideal

, let $x, y \in N$,

$$(1) \bigcap_{j\in J} \overline{\lambda}_i\ (x-y) = \inf_{j\in J} \overline{\lambda}_i\ (x-y)$$

since $\overline{\lambda}_j$ is fuzzy ideal of N

$$\geq \inf_{j\in J} \{\min\{\overline{\lambda}_j\ (x), \overline{\lambda}_j\ (y)\}\}$$

$$\geq \min\{\inf_{j\in J} \overline{\lambda}_j(x), \inf_{j\in J} \overline{\lambda}_j\ (y)\}.$$

$$\geq \min\{\bigcap_{j\in J} \overline{\lambda}_j(x), \bigcap_{j\in J} \overline{\lambda}_j\ (y)\}.$$

$$(2) \bigcap_{j\in J} \overline{\lambda}_j(xy^{-1}) = \inf_{j\in J} \overline{\lambda}_j(xy^{-1})$$

$$\geq \inf_{j\in J} \{\min\{\overline{\lambda}_j\ (x), \overline{\lambda}_j\ (y^{-1})\}\}$$

$$\geq \inf_{j\in J} \{\min\{\overline{\lambda}_j\ (x), \overline{\lambda}_j\ (y)\}\}$$

$$\geq \min\{\inf_{j\in J} \overline{\lambda}_j\ (x), \inf_{j\in J} \overline{\lambda}_j\ (y)\}$$

$$\geq \min\{\bigcap_{j\in J} \overline{\lambda}_j(x), \bigcap_{j\in J} \overline{\lambda}_j(y)\}$$

From (1) and (2) $\Rightarrow \bigcap_{j\in J} \overline{\lambda}_j$ is a fuzzy sub near field of M that mean $\bigcap_{j\in J} \overline{\lambda}_j$ is S-fuzzy ideal related to the near filed M of N. To prove that $\bigcap_{j\in J} \overline{\lambda}_j$ is (S-δ FI) related to the near filed M of N let

$x, y \in M$,

$$(1) \bigcap_{j\in J} \overline{\lambda}_i\ (x-y) = \inf_{j\in J} \overline{\lambda}_i\ (x-y)$$

$$\leq \inf_{j\in J} \overline{\lambda}_i\ (x) = \bigcap_{j\in J} \overline{\lambda}_i\ (x)$$

$$(2)\bigcap_{j\in J}\overline{\lambda}_i\ (xy\ ) = \inf_{j\in J}\ \overline{\lambda}_i(xy)$$

$$\geq \inf_{j\in J}\{\max\{\overline{\lambda}_i(x), \overline{\lambda}_i\ (y)\}\}$$

$$= \max\{\inf_{j\in J}\ \overline{\lambda}_i(x), \inf_{j\in J}\ \overline{\lambda}_i\ (y)\}$$

$$= \max\{\bigcap_{j\in J}\overline{\lambda}_i\ (x), \bigcap_{j\in J}\overline{\lambda}_i(y)\}.$$

From (1) and (2) $\Rightarrow \bigcap_{j\in J}\overline{\lambda}_i$ Is (S- δ FI) related to the near filed M of N.

Theorem (2.5)

let $f:(N_1,+,.)\to(N_2,+',.')$ be epimomorphism and let $\overline{\lambda}$ is (S- δ FI)) related to the near filed M_2 of a S-near ring N_2 if and only if $f^{-1}(\overline{\lambda})$ is (S- δ FI)) related to the near filed M_1 of a S-near ring $N_{1,}$when $f^{-1}(M_2) = M_1$.

Proof

Let $\overline{\lambda}$ is a is (S- δ FI) related to the near filed M_2 of N_2, let $x, y \in N$,

$$(1)\ f^{-1}(\overline{\lambda})(x-y) = \overline{\lambda}(f(x-y))$$

$$\overline{\lambda}(f(x) - f(y)) = \min\{\overline{\lambda}(f(x), \overline{\lambda}(f(y)\}$$

$$= \min\{f^{-1}(\overline{\lambda})(x), f^{-1}(\overline{\lambda})(y)\}.$$

$$(2)\ f^{-1}(\overline{\lambda})(xy^{-1}) = \overline{\lambda}(f(xy^{-1}))$$

$$\overline{\lambda}(f(x).f(y^{-1})) \geq \min\{\overline{\lambda}(f(x), \overline{\lambda}(f(y)\}$$

$$= \min\{f^{-1}(\overline{\lambda}(x)), f^{-1}(\overline{\lambda}(y)\}.$$

Frome (1),(2) we have $f^{-1}(\overline{\lambda})$ is S-near field related to the near filed M_1 of a S-near ring N_1 , $f^{-1}(\overline{\lambda})$ is S-fuzzy ideal of N_1.

To proof that $f^{-1}(\overline{\lambda})$ is (S- δ FI) related to the near field M_1 of N_1 $x, y \in M_2$,

$$(1)\ f^{-1}(\overline{\lambda})(x-y) = \overline{\lambda}(f(x-y))$$

$$\leq \overline{\lambda}(f(x)) = f^{-1}(\overline{\lambda})(x)$$

$$(2)\ f^{-1}(\overline{\lambda})(xy) = \overline{\lambda}(f(xy))$$

$$\overline{\lambda}(f(x).f(y)) \geq \max\{\overline{\lambda}(f(x), \overline{\lambda}(f(y)\}$$

$$\geq \max\{f^{-1}(\overline{\lambda}(x)), f^{-1}(\overline{\lambda}(y)\}.$$

From (1) and (2) $f^{-1}(\overline{\lambda})$ is (S- δ FI) related to the near field M_1 of N_1

$\leftarrow$ suppose $f^{-1}(\overline{\lambda})$ is (S- δ FI) related to the near field M_1 of N_1,let $f(x), f(y) \in N_2$,

(1) $\overline{\lambda}(f(x) - f(y)) = \overline{\lambda}(f(x-y)) = f^{-1}(\overline{\lambda}(x-y))$

Since $f^{-1}(\overline{\lambda})$ is (S-δ FI) related to the near field M_1 of N_1

$\geq f^{-1}\{\min\{\overline{\lambda}(x),\overline{\lambda}(y)\}$

$=\min\{f^{-1}(\overline{\lambda})(x), f^{-1}(\overline{\lambda})(y)\}=\min\{\overline{\lambda}(f\ (x)),\overline{\lambda}(f(y))\}.$

$(2)\overline{\lambda}(f(xy^{-1}))= f^{-1}(\overline{\lambda})(xy^{-1})= f^{-1}\{\min\{\overline{\lambda}(x),\overline{\lambda}(y)\}$

$=\min\{f^{-1}(\overline{\lambda}(x)), f^{-1}(\overline{\lambda}(y)\}=\min\{\overline{\lambda}(f(x)),\overline{\lambda}(f(y))\}.$

Frome (1),(2) we have $f(\overline{\lambda})$ is S-near field related to the near filed M_2 of a S-near ring N_2 , $f(\overline{\lambda})$ is S-fuzzy ideal of N_2.

To proof that $f(\overline{\lambda})$ is (S-δ FI) related to the near field M_2 of N_2 $x, y\in M_2$,

(1) $f(\overline{\lambda}(x-y))\leq f(\overline{\lambda}(x))$

(2) $f(\overline{\lambda}(xy))\geq f\{\max\{\overline{\lambda}(x),\overline{\lambda}(y)\}$

$=\max\{f(\overline{\lambda}(x)), f(\overline{\lambda}(y))\}$

From (1) and (2) $f\ (\overline{\lambda})$ is (S-δ FI) related to the near field M_2 of N_2.

Proposition (2.6)

If $\overline{\lambda}$ is a (S-δ FI) related to the near filed M of N then $\overline{\lambda}^c$ is a (S-δ FI) related to the near filed M of N.

<u>Proof</u>

To prove that $\overline{\lambda}^c$ is S-fuzzy ideal

, let $x, y\in N$,

(1) $\overline{\lambda}^c(x-y)=1-\overline{\lambda}(x-y)$

$\geq 1-\min\{\overline{\lambda}(x),\overline{\lambda}(y)\}$

$\geq \min\{1-\overline{\lambda}(x),1-\overline{\lambda}(y)\}$

$\geq \min\{\overline{\lambda}^c(x),\overline{\lambda}^c(y)\}$

(2) $\overline{\lambda}^c(xy^{-1})=1-\overline{\lambda}(xy^{-1})$

$\geq 1-\min\{\overline{\lambda}(x),\overline{\lambda}(y^{-1})\}$

$\geq 1-\min\{\overline{\lambda}(x),\overline{\lambda}(y\ )\}$

$\geq \min\{1-\overline{\lambda}(x),1-\overline{\lambda}(y)\}$

$\geq \min\{\overline{\lambda}^c(x),\overline{\lambda}^c(y)\}$

From (1) and (2) $\Rightarrow \overline{\lambda}^c$ is a fuzzy sub near field of M that mean $\overline{\lambda}^c$ is S-fuzzy ideal related to the near filed M of N. to prove that $\overline{\lambda}^c$ is (S-δ FI) related to the near filed M of N $x, y\in M$,

(1) $\overline{\lambda}^c(x-y)=1-\overline{\lambda}(x-y)$

$\leq 1-\overline{\lambda}(x)=\overline{\lambda}^c(x).$

(2) $\overline{\lambda}^c(xy)=1-\overline{\lambda}(xy\ )$

$\geq 1-\max\{\overline{\lambda}(x),\overline{\lambda}(y)\}$

$\geq \max\{1-\overline{\lambda}(x),1-\overline{\lambda}(y)\}$

$\geq \max\{\overline{\lambda}^c(x),\overline{\lambda}^c(y)\}.$

From (1)and (2) $\overline{\lambda}^c$ is a (S-δ FI) related to the near filed M of N.

Proposition (2.7)

Let A,B be any two (S-δ FI) related to the near filed M of N ,if $A \subset B \vee B \subset A$ then $A \cup B$ is (S-δ FI) related to the near filed M of N is a (S-δ FI) related to the near filed M of N.

Proof

Let A,B be any two (S-δ FI) related to the near filed M of N, then A^c, B^c are two (S-δ FI) related to the near filed M of N, by using (2.6) $A^c \bigcap B^c$ is (S-δ FI) related to the near filed M of N.

, $A^c \cap B^c = (A \cup B)^c$ by using (2.4) $((A \cup B)^c)^c = (A \cup B)$ is (S-δ FI) related to the near filed M of N

Reference

[1] G.Pilz,"Near ring" ,North Hollanda Publ.and co.,1977.

[2] W.B.vasantha andasamy,"Samarandache near ring "United States of America" ,pp.20-30,pp.69-71,158,2002.

[3] W.B.vasantha andasamy,"Samarandache fuzzy a lgebra "United States of America" ,pp.9-10,pp.47-58,292,2003.

[4] S.Bharanri," On Fuzzy Costs Of Gamma Near ring ",Turk.J.math ,pp.11-22,2005.