A MODEL FOR GROUNDWATER MOTION IN 3D RANDOM HYDROGEOLOGIC HETEROGENEOUS MEDIA +

نموذج حركة الماء الارضي في وسط ثلاثي الابعاد عشوائي هيدروجيولوجي غير متجانس

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Abstract:

A mathematical model for groundwater motion in three dimension hydrogeologic heterogeneous media at Al Hawija Area (4222km^{2}in extent) is developed. The theory of superposition is applied at the media and the results are verified using a theoretical background of Darcey's Law and Theis Equation for the confined and unconfined aquifers at the area. Under the highlight of the current conceptualization study, the resulted superimposed drawdown variation of theoretical and numerical solutions shows a good identity at the beginning of pumping up to 2500 days and acceptable identity in variation beyond these periods for the unconfined aquifer. Whereas the variation of drawdown with time for both theoretical and numerical solution shows an acceptable coincidence before one year and the two solutions begin to deviate with time beyond that under the effect the existing hydraulic boundaries for the confined aquifer.

المستخلص:

النموذج الرياضي لحركة المياه الجوفية الثلاثي الابعاد في وسط هيدروجيولوجي غير متجانس في منطقة الحويجة بمساحة (٤٢٢٢كم٢) قد طور. ان نظرية التراكب قد طبقت وحققت النتائج باستخدام الخلفية النظرية لقانون دارسي *ومعادلة ثايس* لكلا المكمنين المائيين المفتوح والمغلق في المنطقة. على ضوء الدراسة الحالية ان نتائج التغاير في الهبوط المتراكب النظري والعددي اظهر تطابقا جيدا في بداية الضخ حتى ٢٥٠٠ يوم وتطابقا مقبولا بعد ذلك للمكمن المائي المفتوح . في حين ان التغاير في الهبوط مع الزمن للنتائج النظرية والعددية اظهر تطابقا مقبولا لحد سنة ثم يبدأ الحلين بالانحراف بعد ذلك تحت تأثير الحدود الهيدروليكة الموجودة للمكمن المائي المحصور.

Introduction:

A long term background of superposition method is developed in various aspects of science. Sink and source in fluids and gases, heat, earthquake, contamination, sonic, ultrasonic, acoustic, elastic, optics, and electromagnatism, etc... Waves, cause superimposed effects in both homogenous and heterogeneous media.

In the current study, the simultaneous numerical and analytical solutions are adapted to treats the phenomena mathematically in heterogeneous media.

[1] generalized an earthquake-wave-motion model, which considers 3D random heterogeneous media, together with existing models for sources and realistic geological

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profiles for sedimentary basins and irregular topography. The model can be used not only to examine the influences of heterogeneous media, but also to explore the multiple interactions of source, site (irregularity and/or heterogeneity), and wave interference on spatial variations of ground motion.

[2] adapted the superposition theory in shear stress distribution of axial tension plates. Results are shown for one quarter of the plate cross section in the xy plane. The lines of symmetry are at the bottom and on the right hand side of the cross section. The uniform tension load is applied normal to the xy plane. The zoomed area of shear stress distribution in the close vicinity to the free edge is also shown. Results of the classical step-by-step procedure based on the homogenization theory are compared to the reference solution where the size of finite elements is of the same order of magnitude as that of material heterogeneity. It can be seen that a classical step-by-step procedure predicts accurate shear stress distribution except for the close vicinity to the free edge, where it significantly underestimates maximum stress values, and along the entire interface between the two dissimilar layers.

[3] reviewed recent results on the Eulerian Computation of high frequency waves in heterogeneous media to cover three recent methods: the moment, the level set, and the computational methods for interface problems in high frequency waves. These approaches are all based on high frequency asymptotic limits.

[4] investigated the performance of diffuse optical tomography to image highly heterogeneous media, such as breast tissue, as a function of background heterogeneity. To model the background heterogeneity, the functional information derived from Gadolinium-enhanced magnetic resonance images of the breast has been employed. Overall image quality and quantification accuracy worsens as the background heterogeneity is increased.

In this analysis a numerical solution based on a finite difference method is adapted and a theoretical solution depending on Darcey's law and Theis Equation is used for comparison purposes.

Purpose of the Study:

The purpose of the current study is to develop the method of superposition in a hydrogeologic heterogeneous media using a numerical and theoretical background.

Description of the Problem:

It is suggested that a natural hydrogeologic regime of Al Hawija Basin with natural hydrogeologic properties is used as a pattern of the study in this analysis rather than assumed one. The covered area is about 4222km^2 as shown in Fig.(1). Actually the figure presents natural and essential boundaries that are perfect for the modeling process.

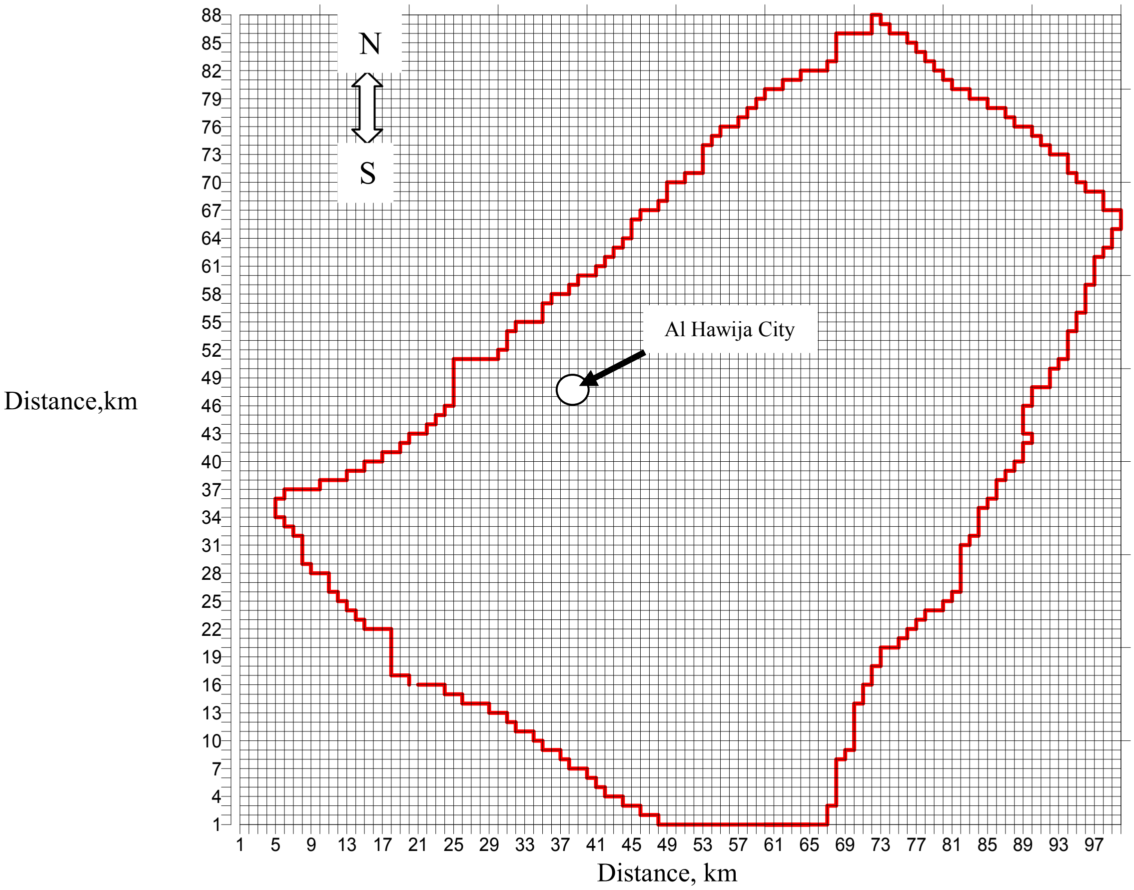

Fig.(1) The Boundary of the Modeled Area and Domain Descretization

Development of a Numerical Model:

Several programs have been written for aquifer simulation by a mathematical model, using a finite difference approach. It was preferred to use one of them instead of writing a new one. The program of [5] has been modified and used. It is chosen because of its flexibility for modification. The derivation of finite difference equation is based on the mass conservation principle and Darcy's Law. A detailed derivation of the finite differences equations and convergence test for errors study are found in the [5]. In general, the model is designed to remedy the instantaneous irregularity and heterogeneity of hydrogeologic media.
For more information about modeling technique, data evaluation, and model calibration, [6] presents a brief methodology

Simulation Technique:

Discretization of the Domain

The first step in modeling job is to discrete the domain by superimposing a mesh of finite difference grid over the map of the area. The size of mesh or in other words the number of rows and columns that is to be adapted depends on the required accuracy. However uniform spacing of 1mesh per 1km is specified in the present work because the area is relatively large as shown in Fig. (1).

Theoretical Background:

In transient groundwater flow regime, [7] derive the basic famous equation in unsteady flow at the vicinity of a pumping well which is written in the following simple form:

$$U = \frac{r^2 S}{4\pi t} \quad \ldots\ldots\ldots\ldots 1$$

$$s = \frac{Q}{4\pi T} w(u) \ldots\ldots\ldots\ldots 2$$

Where: r is the interested distance from the production well, S is storage coefficient, t is the time since pumping starts, s is the drawdown, Q is the production capacity, T is the average transmissivity at the vicinity of the production well, and w(u) is the well function.

So the drawdown in any point around the production well in the vicinity of the cone of depression can be obtained immediately if the previous parameters are known.

The Current Hypothetical Assumptions in Heterogeneous Aquifers:

a) Primary Assumptions

In this mathematical model, it is assumed that there are:

1- Many production wells with continuous pumping and constant productions of 50ℓ/s.

2- Equidistance of 1km apart (center to center) between each two consecutive wells in both X & Y coordinates.

b) Secondary Assumptions

To find the drawdown at the point of interest in the vicinity of the production wells shown in Fig.(2), a theoretical solution of Theis could be done for the purpose of verification with the numerical solution with some limitations; these are:

1- The resulting drawdown at the point of interest is produced by the cones of depression of production wells overlapped or superimposed over it.

2- The hydrogeologic parameters (transmissivity & specific yield or storage coefficient) of a certain mesh where the production well is located are specified for analytical solution of the drawdown.

3- If the point of interest is located at the center of a production well, the corresponding drawdown should be considered in the analytical solution of the drawdown.

Some Definitions:

The point of interest: It is a point at which the drawdown of groundwater scheme is required theoretically and numerically.

Theoretical Solution: It is an analytical solution by Laplas's Equation in 3Dgroundwater flow and Theis Equation.

Production Wells : Represent all wells have continuous discharge during certain pumping interval.

Effective Wells : All wells around the point of interest have cones of depression which are overlapped over.

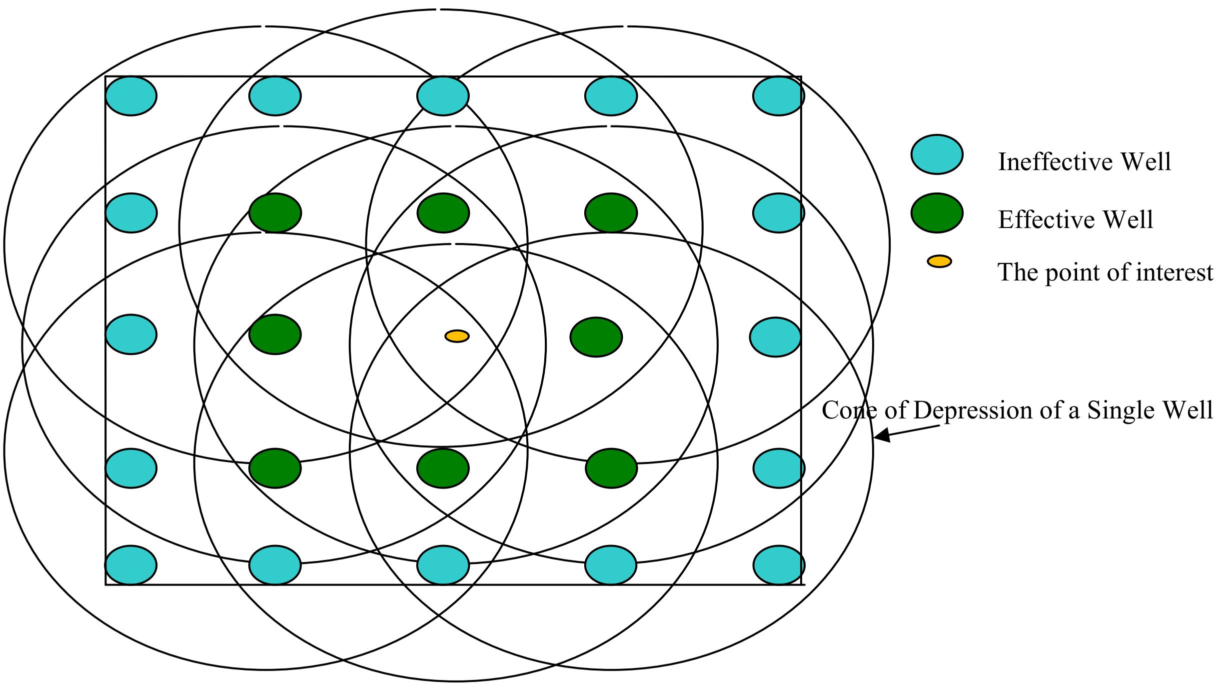

Fig (2) shows the Cones of Depression of the Effective Wells Overlapped over the Point of Interest

Methodology of the Work:

The purpose of this analysis begins with a selection of a specimen model; this may or may not be a true simulated area. Al Hawija Basin is selected to be as an interesting modeled area for the availability of the necessary data. In general, the model information is overset as follows:

Input Data Files

[6] indicated the requested data files to be prepared and lunched in the modeling process.

a) Boundary Conditions

The hydraulic heads and water levels at the all of natural rivers, streams, bonds and reservoirs are measured by the author (2005) and fixed in the model as boundary conditions in the model.

b) Recharge (or Constant Head) Boundary Condition

A recharge boundary is defined as a boundary along which groundwater head is fixed. This type of boundaries is handled in the simulation by setting the storage factor of the nodes along the position of the recharge boundary equal to extremely large values. A typical value of 10^{30} m^2 storage factor is use in this simulation**.** A natural recharge value of 1.29 cm/year (Iraqi Metrological Organization, 2003) is assigned for each nodal point inside the modeled area.

c) Calibration of the Model

[8] presents a brief explanation on how the model should be calibrated and adjusted before any environmental application.

Simulation of the Unconfined Aquifer:

a) Properties of the Unconfined Aquifer (Aquifer Coefficients)

In the absence of hydrogeologic data in an area, many methods may be used to estimate aquifer coefficients like, geoelectric methods outlined by [9]. These methods appear to be capable of providing a relatively rapid mean of assessing the variation in aquifer properties such as the specific yield, [9].

Specific yield and transmissivity are unknown parameters; since no pumping tests have been conducted in the study area for the unconfined aquifer, it becomes necessary to assume approximate and reasonable values for the aquifer coefficients, and then carry out any necessary adjustments on the bases of the results of the model calibration. [10] presented a value of (0.2) for the specific yield of rocks similar to those characteristics of the unconfined aquifer (fine-grained, silt, clay, and sandstone). This value is specified for each nodal point in the model.

The hydraulic conductivity values are assigned for each nodal point within the model area. These values have been estimated on the bases of the data of existing production wells. The assumed values were extrapolated over all the area of the model. The approximated values are adjusted through the steady calibration of the model.

b) Recharge of the Unconfined Aquifer

A recharge value of 1.29 cm/year, which is equal to 3.2% of the total annual rainfall, is used for the infiltration rate as remembered before. This value was specified for each nodal point within the modeled area (4222km^2). Several attempts have been conducted for the calibration of the recharge values through the calibration of the model.

c) Bottom Levels

The bottom level of the unconfined aquifer is determined, from the stratigraphy (lithological well logs) of 56 existing wells of table (1). These levels are extrapolated over the nodal points of the modeled area.

Table (A-1) Existing Production Wells Data

Well. No	Type	Longitude ° ′ ″	Latitude ° ′ ″	P.L or WL M.a.s.l	TDS (PPM)	DrawDown (m)	Q M^3/day	B.L m.a.s.l	B (m)	K M/day
1	C	43 59 22	34 59 19	164		27	1765			
2	C	43 57 30	34 57 45	160		4	786			
3	C	43 55 31	34 58 59	162	3500	10	842			
4	C	43 55 00	34 58 04	158	1251	10	907			
5	C	43 51 50	34 58 45	170			1627			

6	C	43 54 17	35 01 00	172	2019		1166			
7	C	43 58 45	35 12 45	187.5	4151		3888			
8	C	44 17 52	35 25 05	238	1800	8	1166			
9	C	43 50 45	35 03 29	166	4893	10	1296			
10	U	43 50 00	35 05 00	165	3657		173	90	75	7.414
11	U	43 46 27	35 05 30	168			778	68	100	
12	C	43 42 12	35 01 36	172	3533	66	648			
13	C	43 40 45	35 06 40	161		21	972			
14	C	43 37 15	35 06 12	164			140			
15	C	43 36 12	35 06 30	173	3446		583			
16	C	43 35 30	35 06 10	174	2094	28	583			
17	U	43 39 00	35 09 00	142	2487		198	68	74	4.7
18	U	43 57 28	35 08 00	166		1	1102			8.5
19	C	44 01 14	35 02 13	160	2980		196			
20	C	44 07 52	35 25 15		1166		1800			
21	C	44 00 00	35 01 00	161	4852					
22	C	44 01 10	35 04 00	172			812			
23	U	44 03 30	35 05 15	175	4500		105			
24	U	44 14 15	35 05 59	173	2693		1296	131	42	20.7
25	C	44 00 53	35 07 13	160	1427					
26	C	44 03 18	35 08 52	170						
27	C	44 05 54	35 08 54	175	6713					
28	C	44 19 28	35 11 58	213	2020		648			
29	C	44 02 00	35 13 00	190			151			
30	C	44 02 10	35 13 30	191.5	5600		2263			
31	U	44 01 51	35 14 23	179			709			5.8
32	C	44 04 23	35 14 15	183	6660					
33	C	44 08 00	35 14 00	173	4820		605			
34	U	44 14 58	35 20 55	238	420		198			2.83
35	C	44 08 42	35 20 23	225	2100		845			
36	C	44 11 20	35 18 35	195	6222		486			
37	C	44 11 00	35 18 00	185	1570					
38	C	44 19 50	35 18 11	214			846			
39	C	44 08 30	35 17 30	190	7670					
40	C	44 13 06	35 17 59	188	7437					
41	C	44 02 36	35 21 42	186	3236		130			
42	C	44 00 56	35 20 52	177	2662		907			
43	C	44 00 26	35 20 15	180	1172		1166			
44	U	44 07 00	35 19 00	183	2100		845	124	59	2.934
45	C	44 05 50	35 19 04	204			842			
46	C	44 05 00	35 20 00	209	4600		3149			
47	C	44 04 10	35 19 14	184	4500		786			
48	C	44 03 45	35 18 25	183	6180		1620			
49	U	44 02 00	35 18 00	184	5400		227	138	40	6.33
50	U	44 06 05	35 18 00	180	5399		778	145	35	1.932
51	C	44 05 50	35 16 00	181	5432		3936			
52	U	44 03 30	35 15 45	181			875	144	37	5.295
53	C	43 58 20	35 13 00	183			648			
54	U	43 57 30	35 19 00	185	3595		907			2.733
55	C	43 57 12	35 19 30	180			1296			
56	C	43 54 00	35 19 03	182	5601		464			

After [11]

d) Model Calibration

The flowchart of Fig.(3) shows the step by step program operation of the mathematical model. After the program has been run for long period, the predicted static water levels are obtained and show a relatively acceptable similarity with the measured water levels which are shown in Fig (4).

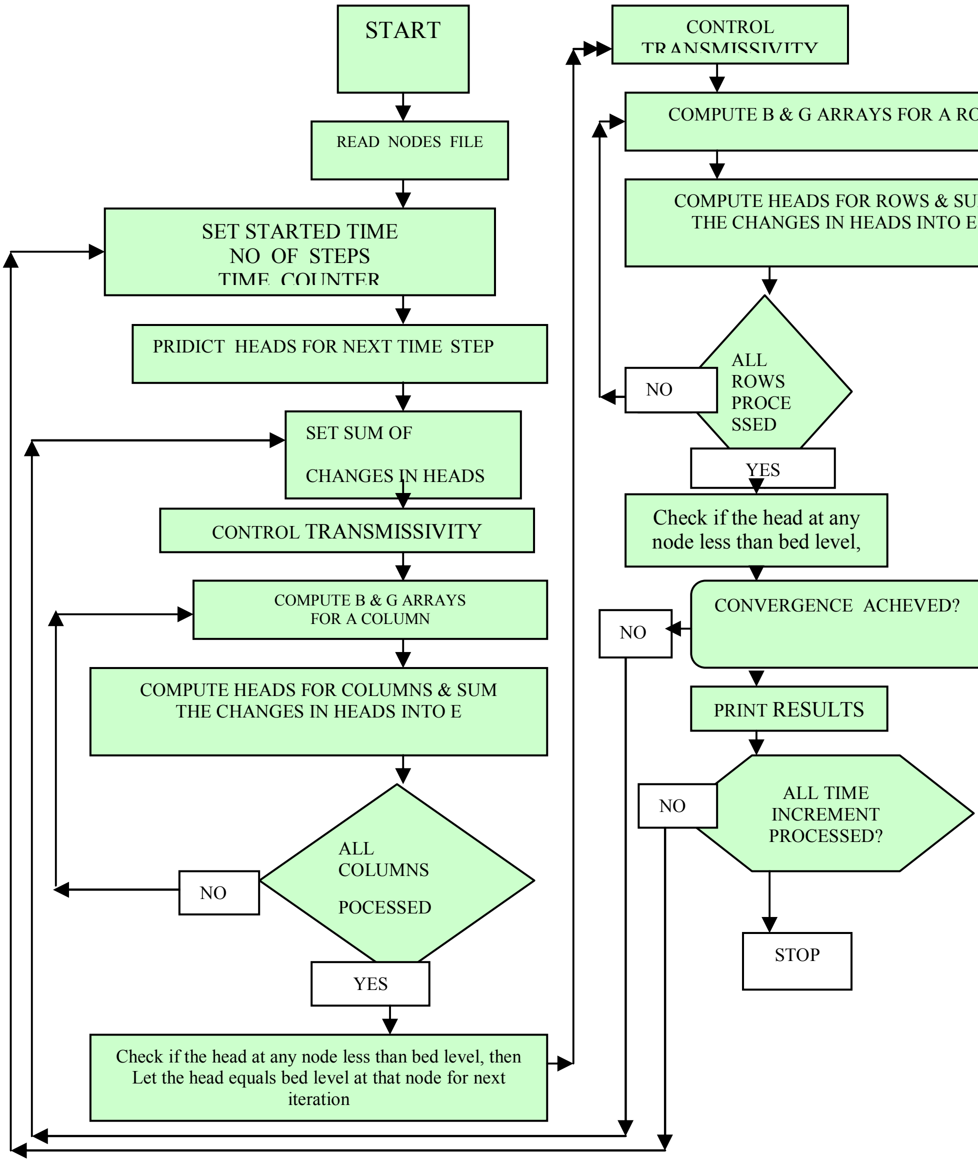

Fig.(3) Modified Simulation Flowchart of Water Table Aquifer, after [5]

e) Prediction of Aquifer Response to Pumping Effects

The ... ial head values for computatic ... nping rate of 4320 m^3/day (50ℓ/s) from a hypothetical well is introduced in a representative location of Fig (5). The superposition effects of all wells around the vicinity of the pumping well were considered according to the concept of Fig.(2) in the modeling process theoretically and numerically. The program was run for a simulated period of 10000 days. The resulting drawdown contour map is also shown in Fig. (5).

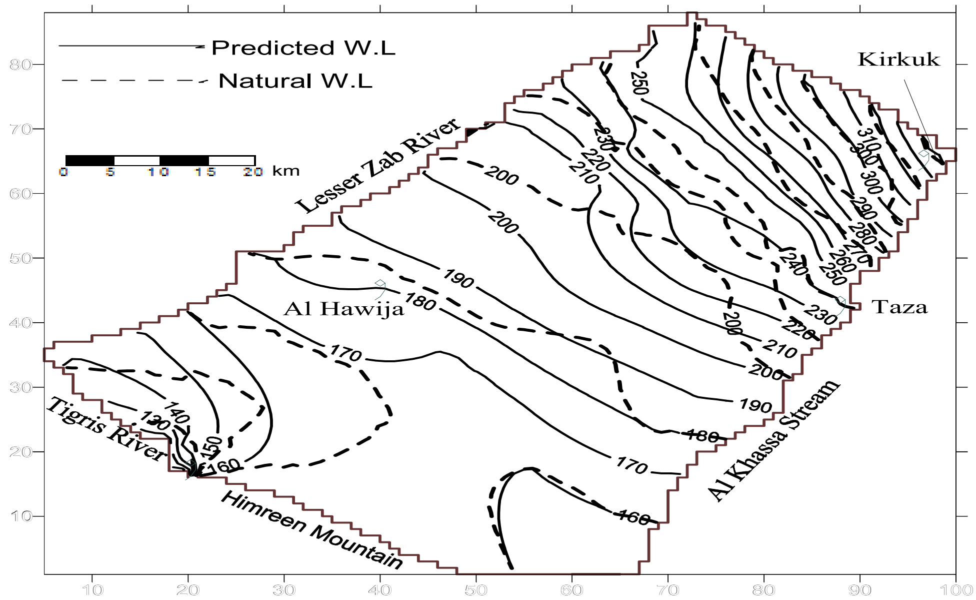

Fig.(4) Comparison between the measured & simulated water levels of the Unconfined Aquifer, m.a.s.l

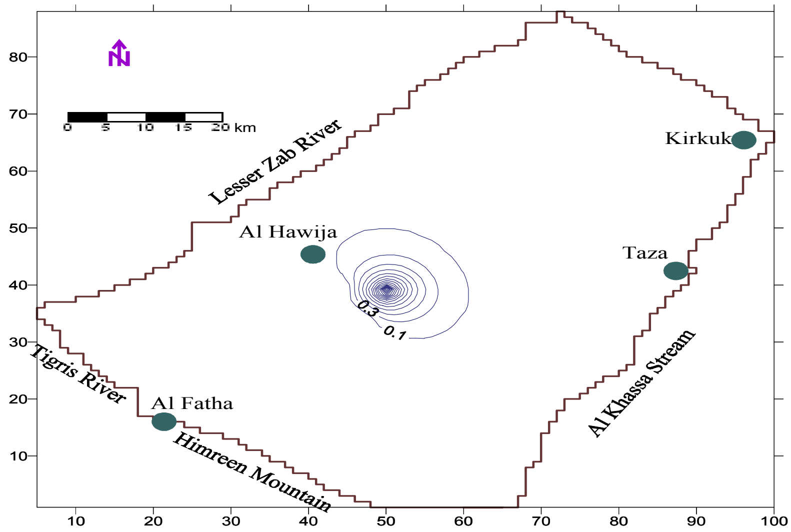

Fig.(5) Drawdown Values(in meters) Contour Map as a Result of Pumping 50 L/s Production Rate for a Period of 27 Years for a Signifying Well Location

f) Discussion and Comparisons

The predicted drawdown which is shown in the contour map of Fig (5) and the comparison of the distance-drawdown curves between Theis and the numerical solutions of Fig (6) was found to be 4.08m in the pumping well site after 10000days of continuous withdrawal rate of 4320 m^3/day (50 ℓ/s). Figs. (7, 8, and 9) show the comparison between the theoretical and predicted time-drawdown curves for a period of 10000days for the same abstraction well with the same discharging rate. The theoretical and predicted drawdown values seem closely coincident. Few differences between the predicted and measured values are observed and attributed to the given assumptions and approximations.

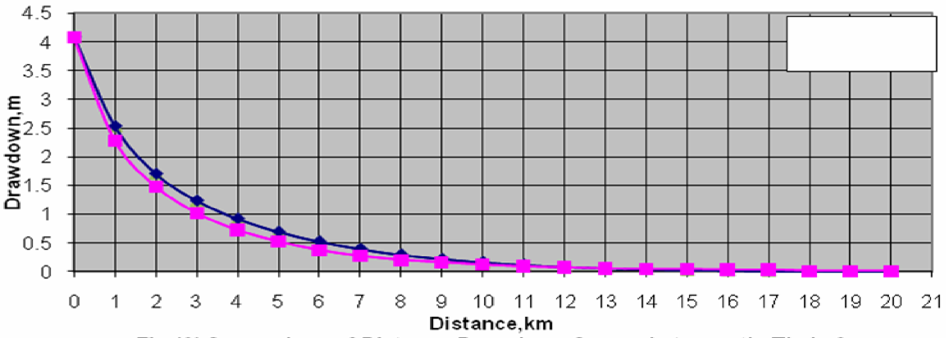

Fig.(6) Comparison of Distance-Drawdown Curves between theTheis & Numerical Solutions for an Abstraction well of 50L/s Continuous...

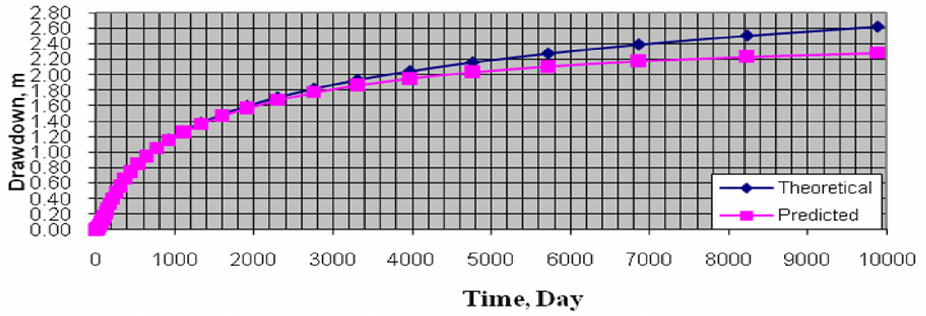

Fig (7) Comparison between the Theoretical & Numerical Time-...

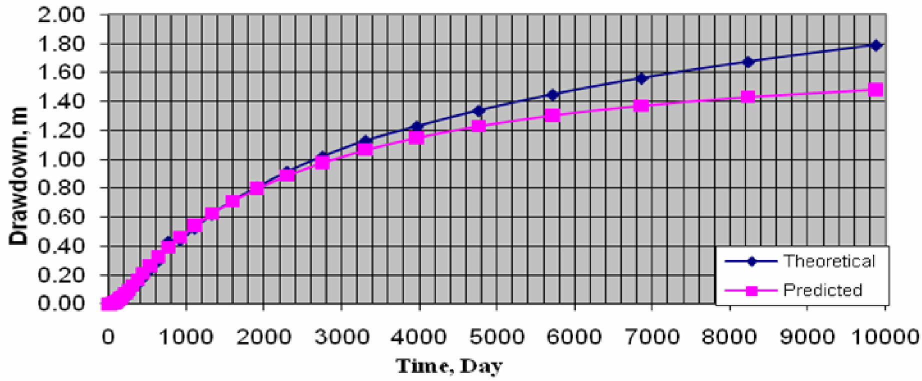

Fig (8) Comparison between the Theoretical & Numerical Time-Drawdown Curves at a distance of 2km from the Abstraction Well

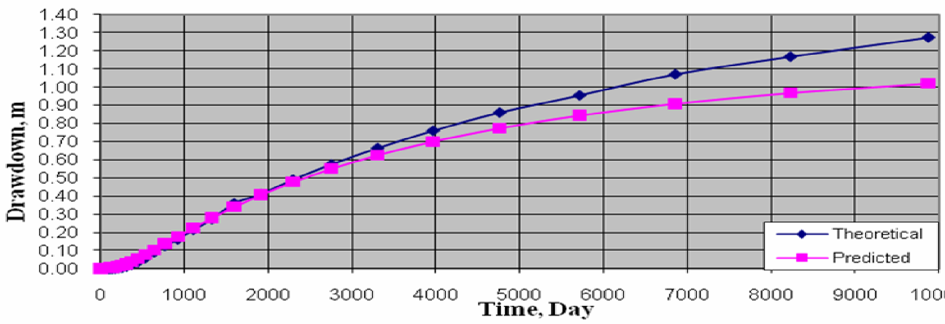

Fig (9) Comparison between Theoretical & Numerical Time-Drawdown Curves at a distance of 3km from the Abstraction Well

Simulation of the Confined Aquifer:

The same basic conceptual methodology of superposition remembered previously is also applied to the media of the confined aquifer. Fig.(10) presents the modified basic confined aquifer simulation flowchart.

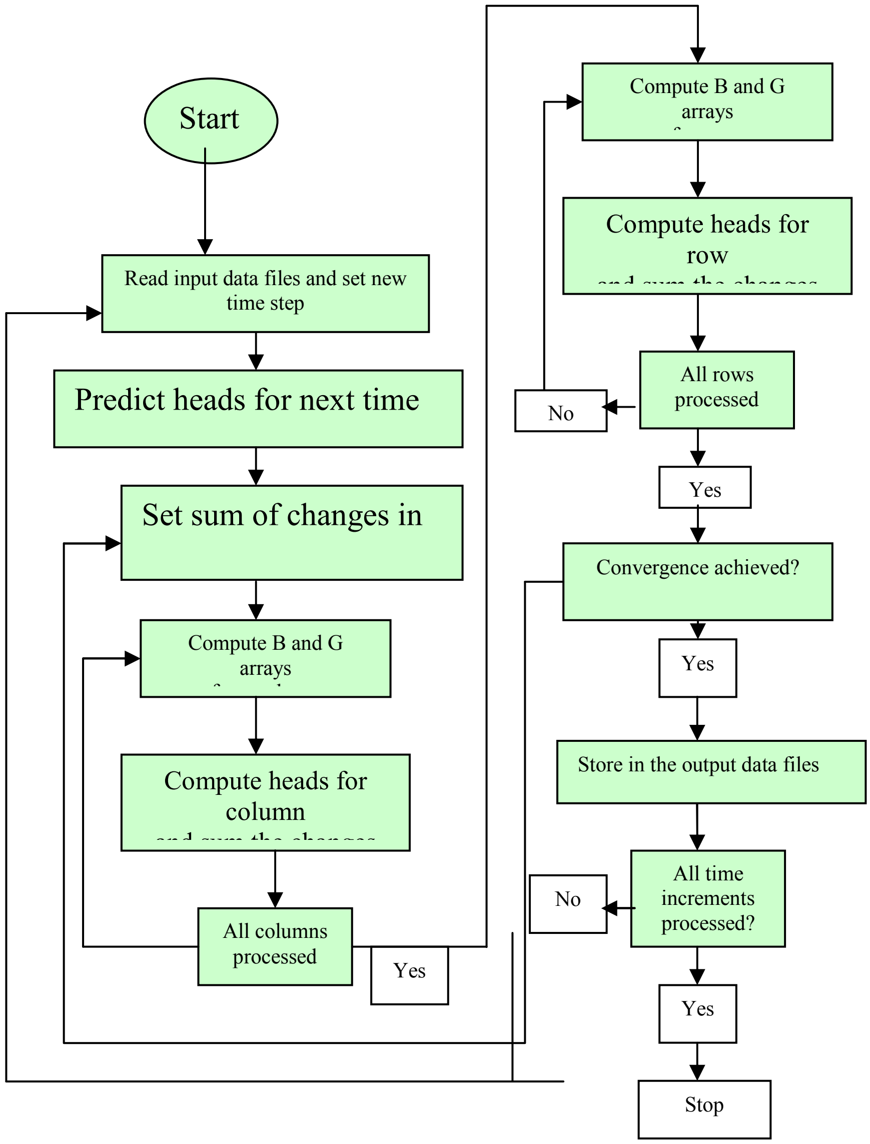

Fig.(10) Modified Basic Confined Aquifer Simulation Flowchart, after [5]

Simulation technique of the confined aquifer has few differences which needs data of the hydrogeologic structures, aquifer boundaries, natural ground flow regime, recharge and discharge boundaries, and hydrogeologic properties of the aquifer.

Simulation Technique:

1- Hydrogeologic_Aquifer Properties

Since the preceding pumping tests were carried out at the area without observation wells, then the available methods of analysis to find the aquifer coefficients cannot be used to estimate the storage coefficient (or specific yield for unconfined aquifer). A value of the storage coefficient for confined aquifer was selected to be .0001. For the nodes in the recharge areas where unconfined conditions exist, a value of 0.2 for the specific yield is assumed.

2- Initial Hydraulic Heads

Although any initial hydraulic head may be assumed, an initial hydraulic head of 360m is assigned (assumed according to natural hydraulic system of the area) to each nodal point within the modeled area in the initial head file. The nodes of the recharge areas were given measured head values which were fixed by assigning a high storage factor value. All the required data were prepared and stored in the appropriate input data files.

3- Model Verification

The predicted and natural hydraulic heads are found closely similar. They are falling in the range of (160-320) m.a.s.l. Fig. (11) shows the match between the natural and predicted hydraulic heads.

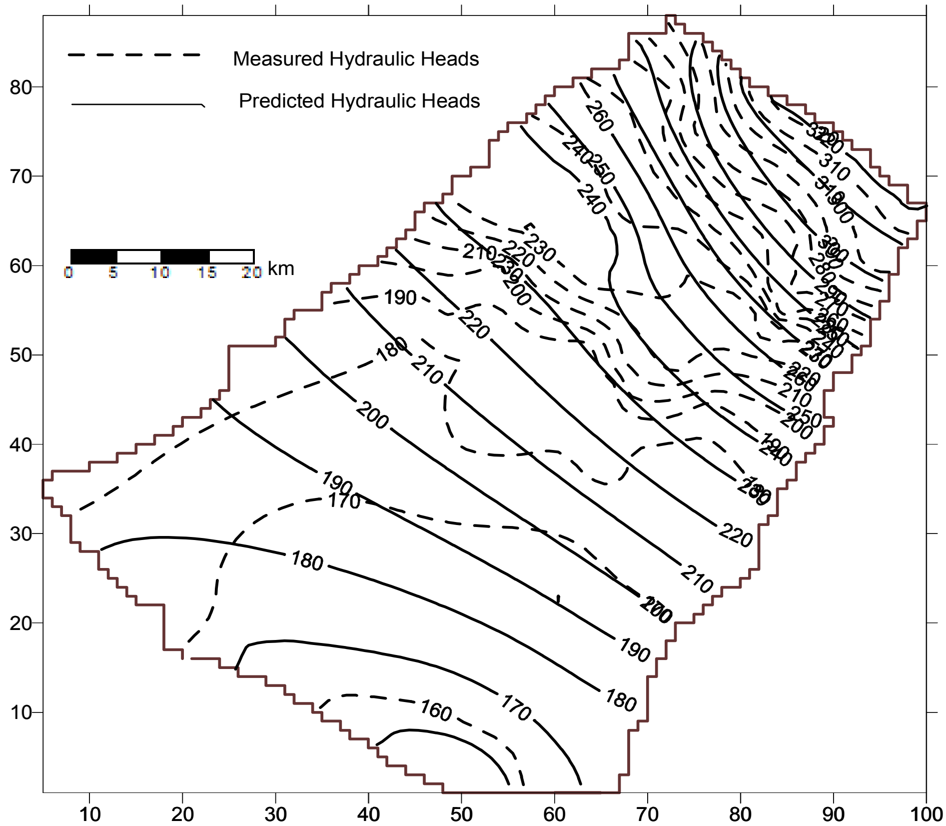

Fig.(11) Comparison between the Measured & Predicted Hydraulic HeadsUsing the Mathematical Model of the Confined Aquifer in (m) a.s.l

4- Prediction of Aquifer Responses to Pumping Effects

After the mathematical model has been verified, a pumping rate of 50 ℓ/sec was assumed for the sake of comparison between model prediction and Theis Formula, the program was run for a long period (10000 days). The resulting drawdown values of the model are shown in Fig. (12). The results obtained by the two techniques (numerical & analytical) show a close similarity in the drawdown values. Figs. (13 to 17) show the comparison of the distance-drawdown curves between the Theis and numerical solutions at different periods. Time drawdown curves of the process are represented in Figs.(18 to 21)

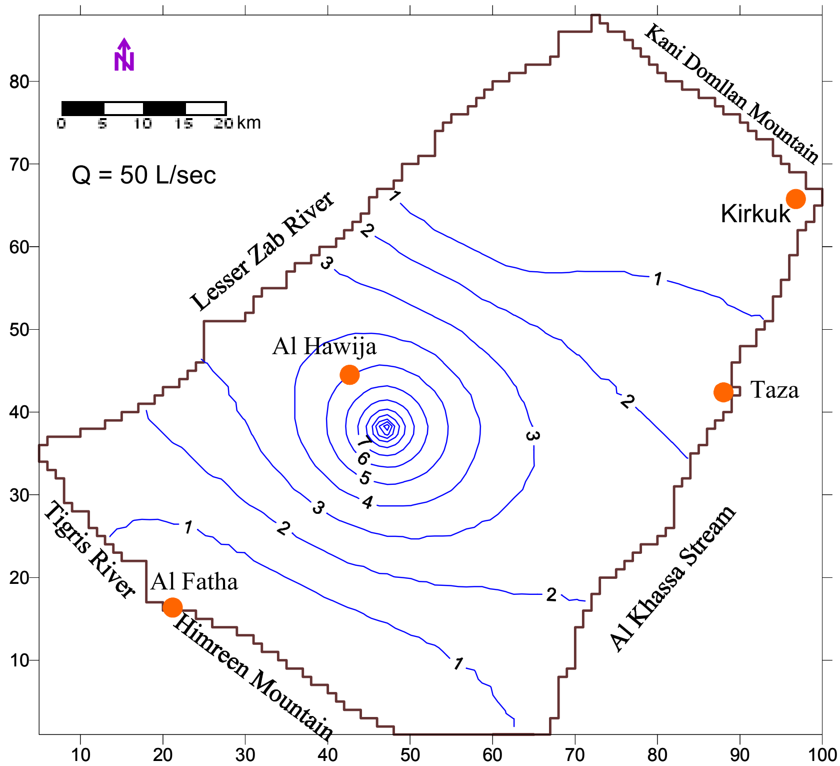

Fig (12) Drawdown Values of the Confined Aquifer as a Result of 50 l/sec Withdrawal Rate at a Production Well after 10000 days

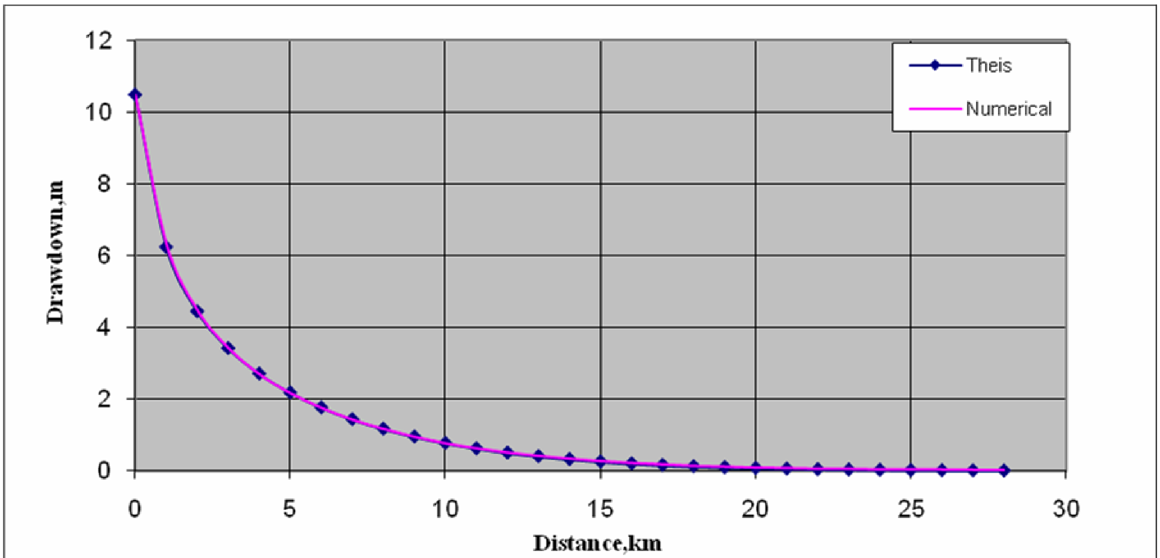

Fig.(13) Comparison of the Distance-Drawdown Curves between Theis & the Numerical Solutions for an Abstraction well of 50L/sec Continuous Withdrawal Rate after 20days

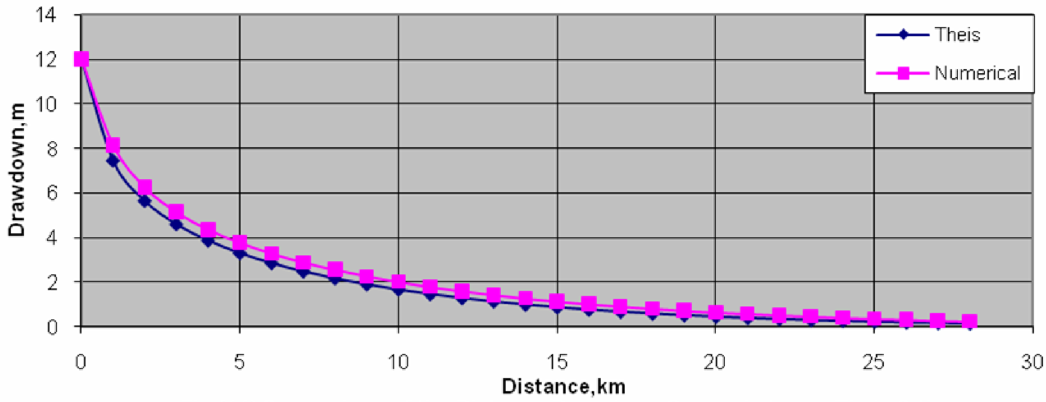

Fig.(14) Comparison of the Distance-Drawdown Curves between Theis & the Numerical Solutions for an Abstraction well of 50L/sec Continuous Withdrawal Rate after50days

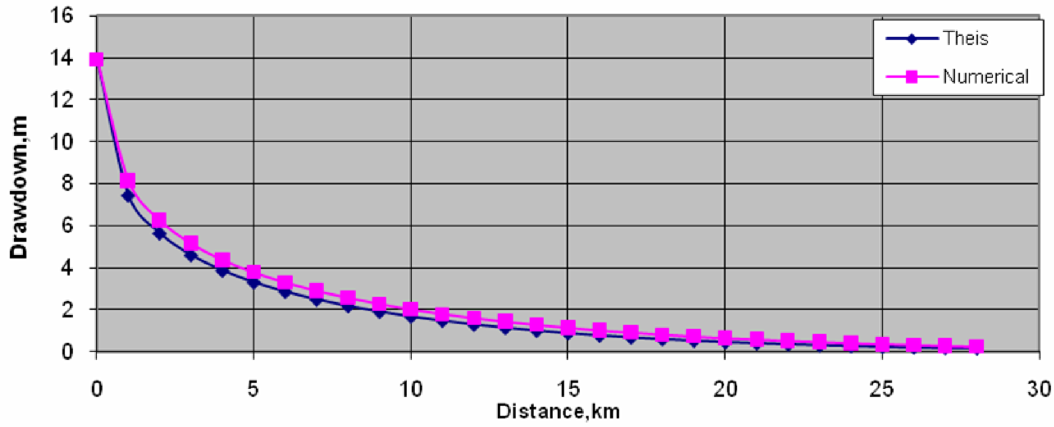

Fig.(15) Comparison of the Distance-Drawdown Curves between Theis & the Numerical Solutions for an Abstraction well of 50L/sec Continuous Withdrawal Rate after103days

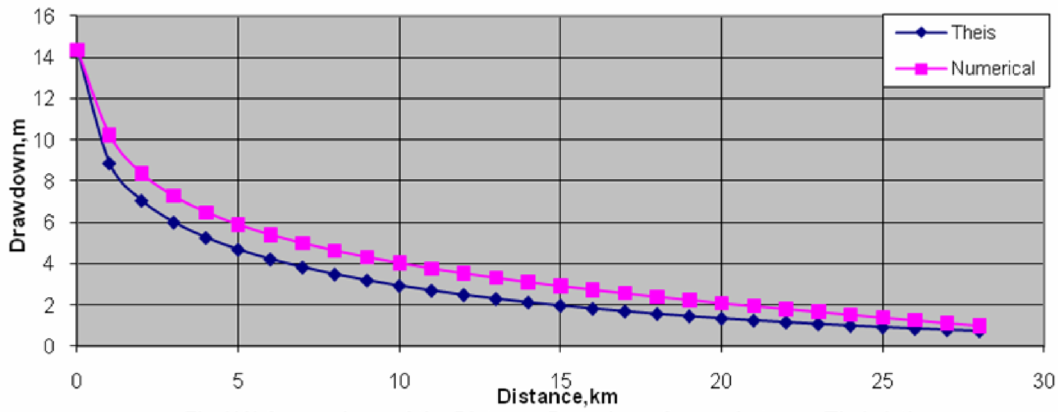

Fig.(16) Comparison of the Distance-Drawdown Curves between Theis& the Numerical Solutions for an Abstraction well of 50L/sec Continuous Withdrawal...

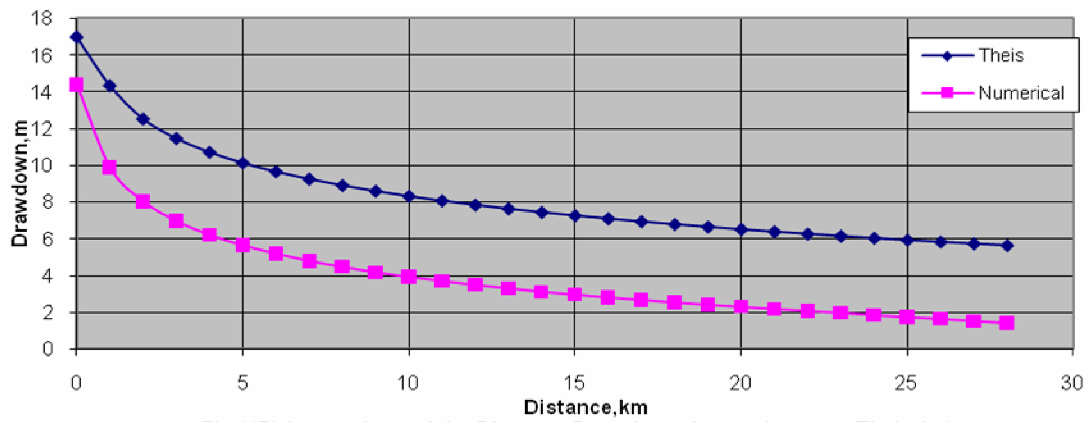

Fig.(17) Comparison of the Distance-Drawdown Curves between Theis & the Numerical Solutions for an Abstraction well of 50L/sec Continuous Withdrawal Rate after 10000days

Discussion of the Results:

Figures (13 to 17) show a comparison of drawdown variation with time as obtained by *Theis solution* and those obtained numerically. A relatively good coincidence between the time-drawdown curves of the analytical and that of the model at the early period of pumping is obtained. However, it starts departing after 2500 days since pumping started as shown in Fig (18). This deviation in the drawdown values may be attributed to the effect of the recharge areas of the confined aquifer at *Bteawa Mountain Recharge Area* which is located at a distance of 28km to the north of the abstraction well. This effect has been taken into account through the calculations of the numerical model but it is not considered through Theis solution. After the model have been run with a withdrawal rate of 50 l/sec for a period of 150days, the resulting drawdown of (.84m) is produced at the recharge boundary of Bteawa Mountain. The induced drawdown value at the recharge boundary produces this departure in the drawdown values between the theoretical and numerical solutions.

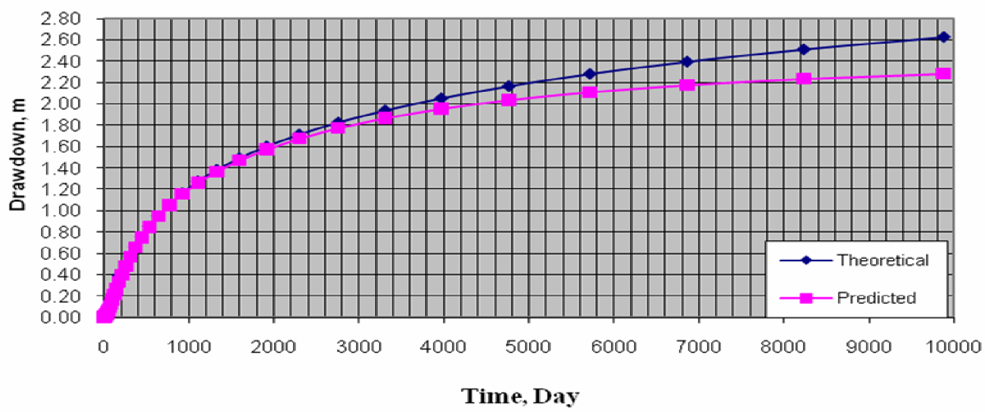

Fig (18) Comparison between the Theoretical & Numerical Time-Drawdown Curves at Distance of 1km from the Abstraction Well

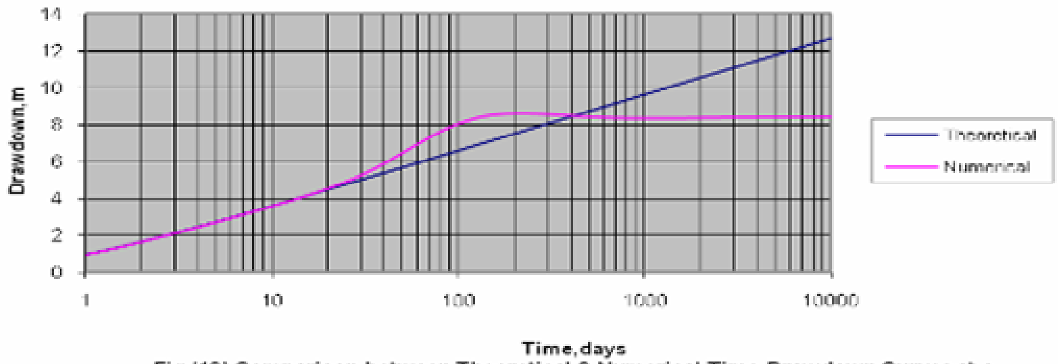

Fig (19) Comparison between Theoretical & Numerical Time-Drawdown Curves at a Distance of 2km from the Abstraction

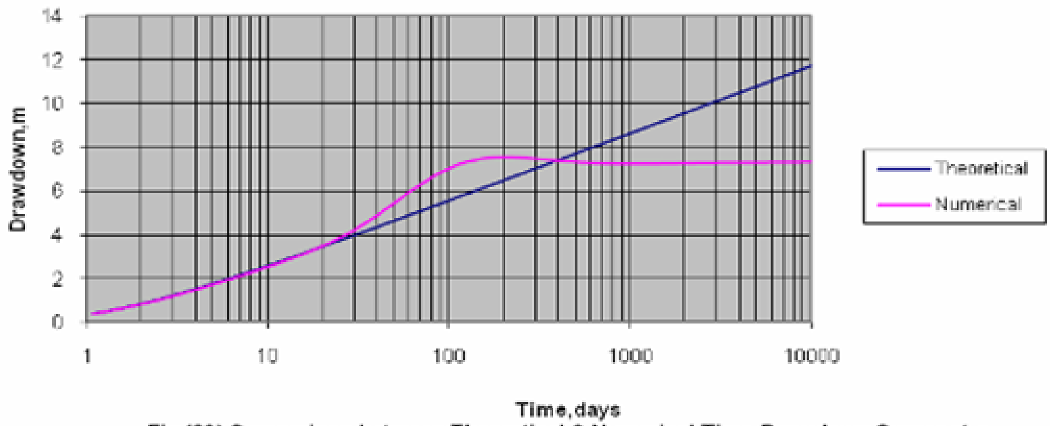

Fig (20) Comparison between Theoretical & Numerical Time-Drawdown Curves at a Distance of3km from the abstraction

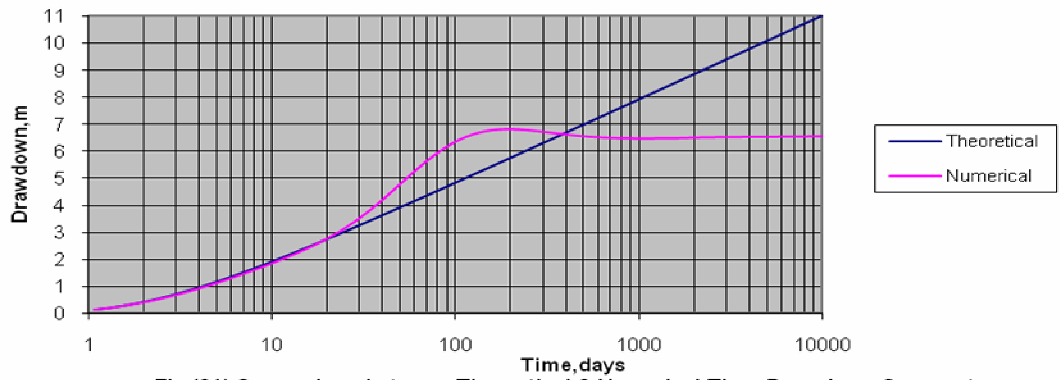

Fig (21) Comparison between Theoretical & Numerical Time-Drawdown Curves at a Distance of4km from the abstraction

Conclusion:

The study of superposition in heterogeneous hydrogeologic media shows a good coincidence between the time and distance drawdown curves of theoretical (Theis) and numerical solutions for both the confined and unconfined aquifers.

Recommendations:

1- It is recommended to use this new technology of superposition theory in hydrogeologic heterogeneous media wherever and whatever it is encountered.
2- It is recommended to extend this analysis to comprise the allowable limit of errors and acceptable fitting of the numerical and theoretical output data.

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List of Symbols

Symbol	Definition	Dimension
U	Theis Factor	-
r	**Distance from pumping well**	**L**
S	**Storage coefficient**	**-**
t	Time	T
s	Drawdown	L
T	**Transmissivity**	L^2/T
W(u)	**Well Function of Theis**	**L**
Q	**Discharge**	L^3/T