IMPLEMENTATION OF DOA-ESTIMATOR BASED ON PHASE MODE EXCITED UNIFORM CIRCULAR ARRAY +

بناء مخمن تحديد الاتجاه يعمل بمبدأ الإثارة الطورية للمصفوفة الدائرية للهوائيات

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Abstract:

In this paper the problems are presented and discussed in a comparative work and computer simulation test to study the performance of the two Eigenstracture algorithms, also this work presents the implementation of these algorithms to estimate the direction-of-arrival of the received signal relative to the uniform circular array.

The received signals are processed digitally using modern super resolution techniques like "Adaptive signal parameter estimation and classification technique" and "Multiple signal classification", and show the better work of the one of direction of arrival-Estimators in the worst conditions.

The problem of high frequency direction finding to resolve the multicomponent wavefeilds are generally compounded by signal coherence, bearing separation and the effects of noise and interference of other transmitted signals

This work introduces a theoretical research that is related to estimate the direction-of-arrival of HF surface wave impinging on uniform circular array with number of identical antennas uniformly distributed around a circle is considered, a phase mode excitation function with the uniform circular array is employed. The phase mode excitation in conjunction with subspace technique gives super resolution DOA estimation; also present the major advantages of uniform circular array arrangements over the uniform linear array arrangements.

المستخلص:

يمكن اعتبار منظومة إيجاد الاتجاه الراديوية ، ترتيب يتكون من (مصفوفة هوائيات + جهاز استلام متعدد القنوات+ معالج دقيق أو حاسوب) لتحديد اتجاه وموقع أجهزة الإرسال بالاستفادة من معاملات الإشارة المستلمة، مثل (الطور، الزمن، التردد).

يقدم هذا البحث بحثاً يتعلق بتخمين اتجاه وصول الإشارة للإشارات العاملة ضمن مدى الترددات العالية وباستخدام خوارزميات حديثة في معالجة الإشارة المستلمة رقمياً، مثل (MUSIC & ASPECT) ، إذ تمّ تصميم وبناء الخوارزميات والبرامج المتعلقة بهذه النظريات استناداً إلى استخدام مصفوفة الهوائيات الدائرية المتكونة من عدة هوائيات متماثلة ، وموزعة بشكل منتظم حول المصفوفة الدائرية ، وسميت هذه الطرائق الجديدة بـ (UCA-ASPECT & UCA-MUSIC) .

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تمّ تحليل أداء تلك الطرق، وإجراء عدة مقارنات فيما بينها، فيما يخص تخمـــين اتجـــاه الإشـــارة لمختلف العوامل المؤثرة في دقة وقرار التخمين للإشارات العاملة ضمن مدى الموجات القصيرة، ومن هذه العوامل تأثير نسبة الإشارة إلى الضوضاء، عدد ألقطات المستلمة، تشاكه الإشـــارات المـــستلمة، تقـــارب الموقع الزاوي لأجهزة الإرسال، وعدد العناصر في المصفوفة. إن برامج الحاسوب المعـــدة لهـــذا العمـــل تتضمن نموذج لتحديد الاتجاه باستخدام جهازين، ودراسة عمل هذه الطرائق، ومدى تأثير العوامل عليهـــا، عند تقارب هذين الجهازين من بعضهما البعض تقارباً شديداً.

Introduction:

The ability to estimate the direction of arrival of narrowband emitter signals is greater importance in many applications. One approach to this problem involves sampling the emitter signals with an array of sensors (the sensor can be any type of device that measures a spatially propagating wavefront, such as a microphone or antenna, then to get the important signal parameters) [1].

An important array-processing problem is to resolve fully correlated (coherent) sources, resolving incoherent sources and resolving closely spaced sources is an active area of research [2].

To solve direction of arrival problem, the underlying plane wave directions by modeling the delay pattern appears a cross the sensors of the array. Various modeling techniques that address the so-called narrow band array angle-of-arrival problem exist in which the signals present are modeled as being essentially sinusoidal [3].

Narrow band estimation techniques are generally associated with spatially small arrays while broadband or time difference of arrival TDOA method, are often associated with spatially large arrays.

The covariance matrix collected form the output of each antenna element is used to estimate angle-of-arrival of the sources applying the uniform circular array (UCA) based on sinusoidal waveform to the subspace spectral estimation method, MUSIC and ASPECT.

The phase mode excitation function for circular array will be explained and derived to provide the necessary phasing for pattern rotation with UCA, a phase mode excitation function is employed. The phase mode excitation in conjunction with subspace technique gives super resolution DOA estimation. Any excitation function is periodic with 2π and can hence be represented in terms of a Fourier series.

Spectral Estimation Methods:

Subspace methods or Eigen decomposition based methods have recently been extensively used in estimation the direction of arrival DOA of plane waves in noise. Most of the Eigen decomposition based method decomposes the observed covariance matrix into two orthogonal spaces, commonly referred to as the signal subspace, and noise subspace, as shown in Fig. (1) [4], and estimate the DOAs from one of these spaces.

These methods, often referred to as subspace based method, have been shown to perform very well and are capable of resolving closely spaced sources [5]. From these methods can be processing the received signal was treated by digital manner, giving so-called snapshot of the signal being received by the antenna array at a particular instant of time.

This work is concerned with the processing of such snapshots in digital form and in particular the use of the covariance matrix, which relates the signals from various combinations of antenna elements,
(a "snapshot" is defined as one simultaneous sampling of the aperture signals at each array elements) [3].

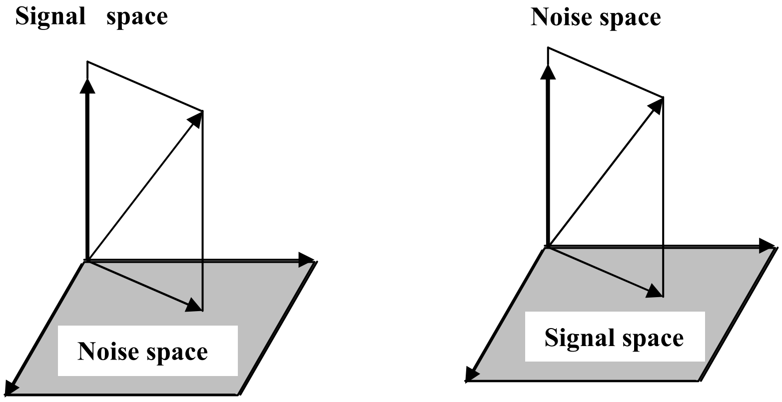


Figure (1) Signal and Noise spaces

Currently subspace techniques may be considered as the most powerful approach and they based on the Eigen decomposition of the array data covariance matrix R_{xx}.

<u>Uiform Circular Array and Digital Data matrix Preparation</u>:

The circular array, in which the elements are placed in a circular ring, is an array configuration of very practical interest. Its application is span radio direction finding (RDF), air and space navigation [6]. The antenna elements, assumed to be identical and omnidirectional, are uniformly distributed over the circumference of a circle of radius r in the x-y plane with it's center located at the origin of rectangular coordinates system.

The circular antenna array is composed of Q-elements from q=0 to Q-1, the first element is the zero element and it is on the x-axis; Thus Φ is zero. The locations of the antennas are at multiple of angle Φ_q.

Assume that a plane wave with amplitude (A_1) and frequency (f_1) is incident on the array with angles $\theta_1, \text{and}, \Phi_1$ [7]. The sources are assumed to be in far field of the array. Consequently, the radiation impinging of the array is in the form of a sum of waves.
The output induced at the q_{th} elements and M wavefronts from location of can be written as [7]:

$$m(q,t) = A_1 \exp(j2\pi f_1 (t - \frac{\tau_{q1}}{c}))$$ ……. (1)

Where:

τ_{q1} The delay time which is associated with received wave at antenna q with respect to the center of the array is [7]:

$$\tau_{q1} = -r\sin(\theta_1)\cos(\Phi_{q1})$$
………. (2)

Where:

r : the radius of the array and θ_1 is the angle between incident signal and the normal of the array.

In this case, because the antenna locations are discrete the angle Φ_{q1} can be written as:

$$\Phi_{q1} = q\Phi_q - \Phi_1$$
………. (3)

Where: q = 0, 1, ………….., Q-1

Φ_q : The angle between two adjacent antenna

$$\Phi_q = \frac{2\pi}{Q}$$
………. (4)

Substituting (2) through (4) into (1), the result is:

$$m(q,t) = A_1 \exp(j2\pi(f_1 t + \frac{r\sin(\theta_1)\cos(\frac{2\pi q}{Q} - \Phi_1)}{\lambda_1}))$$
………. (5)

$$m(q,t) = \sum_{m=1}^{M} |A| \exp(-j2\pi f_m t)\exp(j\beta_m)\exp(-j\eta_m)$$
……… (6)

Where:

$$\beta_m = \frac{2\pi r\sin(\theta_0)\cos(\Phi_0 - \gamma_q)}{\lambda_m}$$
……… (7)

Then, |A| =absolute value of signal amplitude

$\eta_m = \frac{2\pi r}{\lambda_m}\sin(\theta_m)\cos(\Phi_m - \gamma_q)$, Where: $\gamma_q = \frac{2\pi q}{Q}$ Angular position

β_m = Phase shift must apply to have maximum reception from

a desired direction defined by angles (θ_o, Φ_o) The voltage signal of each antenna of M sources adding to it an additive white Gaussian noise AWGN is expressed below:

$$X_q(t) = \sum_{m=1}^{M} |A| \exp(-j2\pi f_m t)\exp(j\beta_m)\exp(-j\eta_m) + n_q(t)$$

……… (8)

Thus, equation (8) is represented the received signal from emitter located on far-field point

$$X_q(t) = X_q(\ell t_s) \qquad (\ell = 1,2,\ldots\ldots,L)$$ ……… (9)

Equation (8) may be rewritten in the sampling form as:

$$X_q(\ell t_S) = \sum_{m=1}^{M} |A| \exp(-j2\pi f_m \ell t_S)\exp(-j\eta_m) + n_q(\ell t_S)$$

……… (10)

Where: $n_q \ell t_s = L - \text{Sample WGN}$, Equation (10) represents the received data at output of ADC.

$$X(t) = S.m(t) + n(t)$$ ………

(11)

Where: S: denote the source-position-vector (SPV) or manifold vector.

m(t): the base band message signal vector, and n(t): represent AWGN.

Equation (11) can be expressed in matrix form: $X = S.M + N$

The matrix S is (Q×M) matrix called direction matrix.

The observed signal vector X_ℓ is (Q×1) vector, the noise vector N_ℓ is (Q×1) vector, and the vector M_ℓ is (M×1) vector called signal in space vector.

However, in many cases in practice, the estimated covariance matrix is defined as [8]:

$$R_{xx} = E[X(t).X(t)^H] = S.R_{mm}.S^H + \sigma^2 I_Q$$ ………

(12)

$$R_{mm} = E[m(t).m(t)^H]$$ ………

(13)

Where: R_{mm} **the source covariance matrix,**

:σ the Standard deviation is computed and related to the signal power to noise ratio (SNR in dB) [9].

The estimated covariance matrix, instead of the ensemble average, given by equation (12), is used. In the case for which L snapshots, or observations, (X (tℓ): ℓ=1, 2,…, L).The data covariance matrix:

$$R_{xx} = \frac{1}{L}\sum_{\ell=1}^{L} E[X(t\ell).X(t\ell)^H] = S.R_{mm}.S^H + R_{nn} = \frac{1}{L} X.X^H$$

……… (14)

.1 Phase Mode Excitation of UCA:

The phase mode excitation function for circular array will be explained and derived to provide the necessary phasing for pattern rotation with UCA, a phase mode excitation function is employed [7].The arbitrary excitation function Z(δ) (δ∈[0, 2π]) can be represented:

$$Z(\delta) = \sum_{b=-\infty}^{\infty} c_b \exp(jb\delta)$$ ……….
(15)

Where: δ: b_{th} phase mode. , c_b: Fourier series coefficient.

The normalized far-field pattern resulting from exciting the aperture with b_{th} phase mode is:

$$g_b^c = \frac{1}{2\pi}\int_0^{2\pi} Z(\delta)\exp(\ j\mu\cos(\ \Phi-\delta))\, d\delta$$ **…….. (16)**

Where: $\mu = \dfrac{2\pi r\sin(\theta)}{\lambda}$, c_a: continuous aperture.

By substituting Z (δ), the far-field pattern can be expressed as:

$$g_b^c(\theta) = j^b J_b(\mu)\exp(jb\Phi)$$ …
…….. (17)

Where: $J_b(\mu)$: Bessel function of the first kind of order b.

This probably allows attractive direction pattern to be synthesized using phase mode excitation. The amplitude and elevation dependent on the far field pattern and sensitive to variations of the phase, The source position function can be extending with phase mode excitation to obtain:

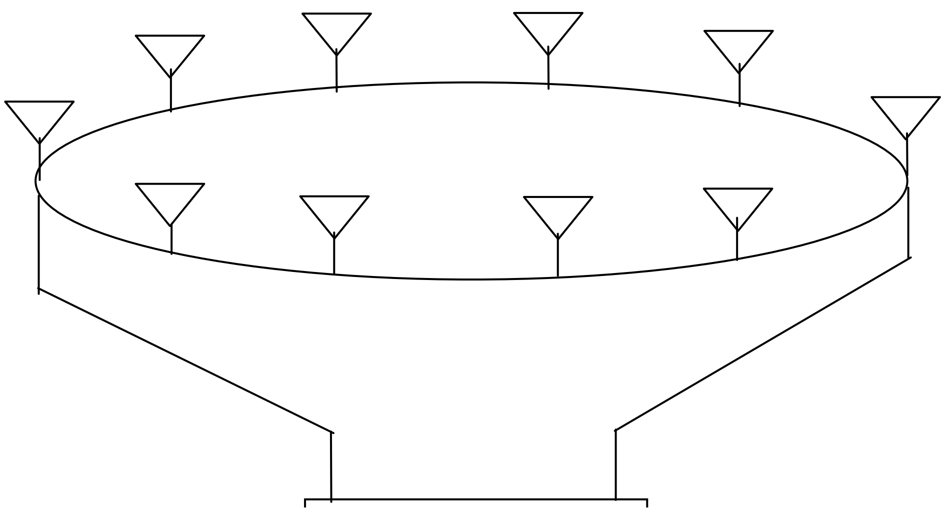
$$\bar{s}(\theta) = g_b^c(\theta)s(\theta)$$ ……… (18)

Also data covariance matrix in eq. (14) must be:

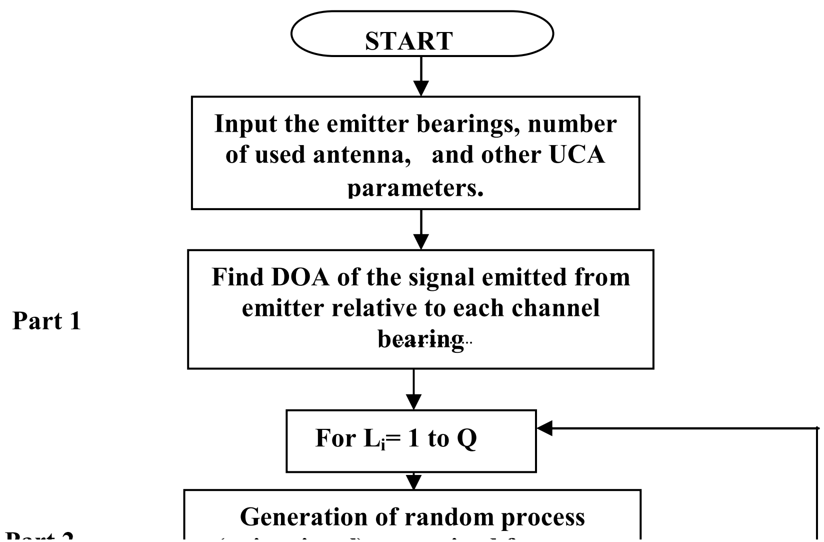
$$\bar{R}_{xx} = \bar{S}.R_{mm}.\bar{S}^H + R_{nn}$$ **……… (19)**

System Modeling:

In this section the data covariance matrix will be considered as the input data to the UCA-subspace algorithms. The data covariance matrix is will processed to obtain the emitters angles, these algorithms are explained in next subsections, such as UCA-MUSIC, and UCA-ASPECT. The DOA system model can be shown in Figure (2).



The general flowchart of subspace methods and flowchart of generation received data can be shown in figure (3).



MUSIC Algorithm

The term MUSIC has been became for, multiple signal classification. The estimated covariance matrix from the collected data is solved for the Eigen system to satisfy:

$$\overline{R}_{xx}E = \sigma^2 I\overline{E}V \quad \text{……….(20)}$$

Where: $V = \text{diag}\lfloor v_1 \quad v_2 \quad \dots \quad v_Q \rfloor$ and $v_1 \geq v_2 \geq \dots \geq v_Q$ the Eigen values of $\overline{R}_{xx}$

$\overline{E} = \lfloor e_1 \;\vdots\; e_2 \;\vdots\; \dots \;\vdots\; e_Q \rfloor$ are the Eigenvectors of $\overline{R}_{xx}$

$$O_{NEV} = \sum_{i=M+1}^{Q} e_i e_i^{H} = E_N E_N^{H} \quad \text{……….(21)}$$

Where: O_{NEV}: Outer products of noise Eigen values (noise covariance matrix) $E_N = \lfloor e_{M+1} | \dots | \quad e_Q \rfloor$. Finally the power angular spectra can be computed as:

$$F(\Phi,\theta)_{MUSIC} = \frac{1}{\overline{S}(\Phi,\theta) O_{NEV} \overline{S}(\Phi,\theta)^{H}} \quad \text{……….(22)}$$

Then one can be find the largest peak of $F(\Phi,\theta)_{MUSIC}$ and then determine the estimated DOA.

ASPECT Algorithm

ASPECT short term means the (Adaptive Signal Parameter Estimation and Classification Technique) [10].

ASPECT is based on an adaptive rotation of a given initial subspace up to the point at which rotated subspace coincides with true signal subspace. Then, the number of incident signals together with information relating to their DOAs.

This algorithm can be expressed as a number of steps as follows:

Step1: Forms the data covariance matrix $\overline{R}_{xx}$.

Step2: Estimate eigenvectors E that correspond to largest Eigen values, if there is confidence that these Eigenvectors belong to the signal subspace (E is the [Q × M] matrix).

Step3: Estimate the projection (P_E) operators of the subspace spanned by the columns of

E.

Step4: Choose K initial direction with k to be certainly greater than, or equal the number of incident signals (M), but less than or equal to array manifold dimensionality.

Step5: Compute the cost function.

$$\xi = \sum_{i=1}^{M} \left(\frac{1}{\sqrt{E_i^H . P_s . E_i}}\right) \text{ or } \xi = \text{trace}(P_s . P_E) \quad \dots\dots\dots (23)$$

Where: $P_s = I_Q - \overline{S}(\overline{S}^H \overline{S})^{-1} \overline{S}^H$ ……..…. (24)

Step6: Minimize the cost function.

$$(\hat{\Phi}, \hat{\theta})_{ASPECT} = \arg\min(\xi) \quad \dots\dots\dots (25)$$

Step7: Find the nonzero elements of the vector α.

$$\alpha = ((\hat{\Phi}, \hat{\theta})^H . (\hat{\Phi}, \hat{\theta}))^{-1} . (\hat{\Phi}, \hat{\theta})^H . E_1 \quad \dots \quad \dots\dots\dots (26)$$

And accept the direction-of-arrival, to be the directions which correspond to non-zero elements of α.

<u>Initial Environment and RMS-Error</u>:

In theory, it is assumed that the second order statistics of the received signal vector x(t) are available the two algorithms under investigation will operate on a simulated data covariance matrix of (L)snapshots on a particular signal environment.

The RMS-error for each direction and algorithm will be estimated, then the root mean square error (RMS) in degree, was obtained for bearing as follows:

$$\text{RMS-error (DOA)} = \sqrt{\frac{(\Phi_1 - \hat{\Phi}_1)^2 + (\Phi_2 - \hat{\Phi}_2)^2 + \cdots + (\Phi_M - \hat{\Phi}_M)^2}{M}}$$

$$= \sqrt{\frac{\sum_{m=1}^{M} (\Phi_m - \hat{\Phi}_m)^2}{M}}$$

Where: Φ : the actual DOA, $\hat{\Phi}$: the estimated DOA.

$(\Phi_m - \hat{\Phi}_m)$ = Estimating bias [it can be defined as the amount of deviation between the estimated DOA and actual one].

The estimator will be unbiased if it's estimation angle equal to the true angle. The initial signal environment involves two equipower coherent sources originally located at $(350^\circ, 0^\circ)$ and $(300^\circ, 0^\circ)$ measured clockwise with respect to the NORTH.

Computer Simulation Tests:-

The performance of two methods are compared, i.e. how accurate can these methods estimate the DOA, by using uniform circular array. These algorithms were carried out under various condition including coherent and incoherent signals of

1. DOA Resolution:

Resolution is defined as the ability of the estimator to resolve two closely separated sources; however, resolution becomes very difficult to be achieved as sources become more and more closely separated.

The directions of the simulation studies so far are the directions of two initial sources $(350^\circ, 0^\circ)$ and $(300^\circ, 0^\circ)$, measured clockwise with respect to NORTH. This implies an angle separation of 50°, i.e. $\Delta\Phi = 50^\circ$. To assess the performance of the two algorithms in resolving closely spaced sources, $\Delta\Phi$ is reduced from 50° to $25^\circ, 15^\circ, 10^\circ$ and 5° angle separation. This done by moving the second sources from $(300^\circ, 0^\circ)$ to $(325^\circ, 0^\circ), (335^\circ, 0^\circ), (340^\circ, 0^\circ)$, $(345^\circ, 0^\circ)$, also take the angle separation in case of less than 2°. Each situation is examined for different values of snapshots and SNR=25dB. See the table (1) for results.

2. Observation Intervals Effects:-

In many practical situations the observation intervals is small. To examine how good are the estimation provided by the, UCA-MUSIC and UCA-ASPECT as a function of observation interval (number of snapshots), the initial number of snapshots L=10 is increased successively to 50, 250, and 500. Again, in each RMS-error for each direction is estimated. The results are presented in Figures. (4 to 7).

3. SNR Effects:-

To consider the effects of the additive noise on the performance of the algorithms the signal to noise power ratio is brought up 25dB to 5dB below the power of each of the sources. Also shows how different SNR (high, middle, low) affects on the performance of two estimators. In this subsection the effects of SNR variation on UCA-MUSIC and UCA-ASPECT was studied and the results obtained are illustrated in Figures (8 and 9).

4. Effect Number of Sensors

In this subsection the number of sensors in array, it is also affected on the accuracy principle of the subspace algorithms.

Theoretically, only three elements can determine the sources DOA (minimum number of sensors to detect two sources [5]), but the DOA accuracy is rather poor. Thus, the effect of the number of sensors is restricted with the aperture of the array. See table.2 for results.

Performance Comparisons and Results Discussion:-

The performance comparisons are made in terms of the number of snapshots, SNR, angle separation, and number of sensors for each estimator.

In this investigation, computer-simulated signals have been used to identify the strengths and weaknesses of the UCA-MUSIC and UCA-ASPECT estimators, operating in a coherent signal environment.

Initially, the SNR is taken to 25dB below the power of each of the two coherent sources.

The two coherent sources are widely separated (angle separation of 50°), then the two estimators successfully resolve to estimate the DOAs, and for high value of snapshots (L=1000) the two estimators have same RMS-error value (see Table 1).

The performance of the two estimators for different angle separation, however, when the angle separation between two sources is reduced from 50° to 25° (and smaller to 5°) the estimators are also able to resolve to estimate the direction-of-arrival, but the RMS-error increased with the value of angle separation become small, but the UCA-MUSIC estimator have been give worst results than UCA-ASPECT.

UCA-MUSIC estimator give poor results when $\Delta\phi=5^{o}$. When the two sources are very close together (to an angle separation of less than 2°), then the UCA-MUSIC fails to resolve and estimate the two sources. Thus, the performance of the two methods will be influence not only by the angular separation, but also by the number of snapshots.

Figures from (4 to 7) shows the effects of the snapshots, snapshots take for same SNR value for different values (L=10, 50, 250, and 500), the most interesting observation here is that UCA-ASPECT was capable to estimating the DOA and resolving two sources when the number of snapshots was small, and it is give results better than UCA-MUSIC, also in this figures can be shows the best and narrow beam without or less effect of aliasing which result in the two coherent sources for UCA-ASPECT estimator than other.

The effect of the SNR variations on two estimators was studied also, and the results are obtained, by different values of SNR (high = 25dB, middle = 15dB, low = 5dB).the calculated RMS-error for different values of (Q) can be seen in Table (2).

Figure (8) Represent the RMS-error versus the number of snapshots for UCA-ASPECT estimator for different SNR (25 to 5) dB, whereas Fig. (9) is represented the SNR effects for the UCA-.MUSIC estimator.

For two figures (8) and (9) can be seen the best work and small value of RMS-error for case of UCA-ASPECT, also show the two estimators influence with the variation of SNR values, and give good results at high value (25) dB, whereas UCA-MUSIC give worst results than UCA-ASPECT.

It is clearly shown that the larger number of sensors provide finer DOA resolution (narrow beam), also the effect of number of sensors (Q) at the performance of two estimators the UCA-ASPECT gives the best performance with increasing the value of (Q) (see the figures 8&9).

The relative merits of two estimator's technique under investigation for 25dB can be seen in Table (3). This table shows that the UCA-ASPECT has first (1^{st}) resolution for different separation angles, whereas UCA-MUSIC has second (2^{nd}) resolution and fails in case of ($\Delta\phi < 2^{o}$).

Conclusions:-

The performance of these algorithms was investigated by means of Several simulation example getting:

1. Intensive study is made to test the algorithms. It is found that UCA-ASPECT is better than UCA-MUSIC to solve the problem related to AOA of the emitters. The performance of the signal-subspace-based ASPECT method is superior in resolving two sources even when the sources are close together.

2. In every experiment tests are performed the RMS-error for each direction is estimated. It is found that the RMS-error increases with decreasing the value of the observation interval, and the number of the sensors.

3. The ASPECT method is a signal-subspace-type multidimensional search algorithm involving different cost functions. In this investigation, the cost functions will be considered both of which are capable of handling coherent and incoherent sources.

4. The investigation of the subspace estimators for different numbers of sensors shows that the large values of Q, the number of elements in the circular array produces a narrow beam and super DOA-Resolution, whereas a small number of elements produce a wide beam and poor DOA-Resolution.

5. The potential advantages of a circular array over several separate linear arrays may only be realized if the number of elements is kept to minimum.

6. Intensive study was made of the ASPECT algorithm which provides superior performance in resolving coherent or incoherent sources using the proposed techniques of uniform circular array of sensor. It is less sensitive **to noise at a low number of snapshots** than MUSIC algorithm

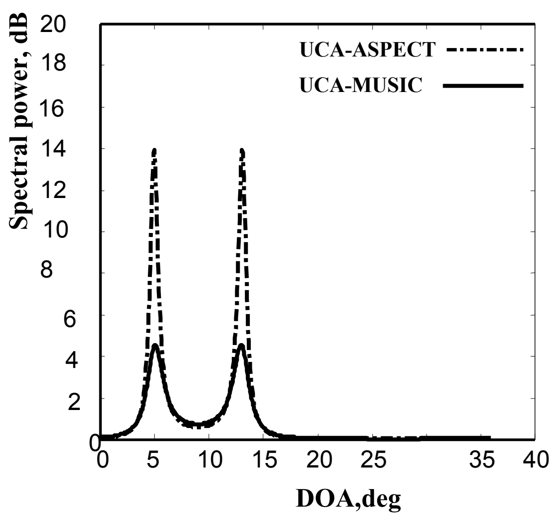


Fig. (4) Shows the effect of number of snapshots at L = 50

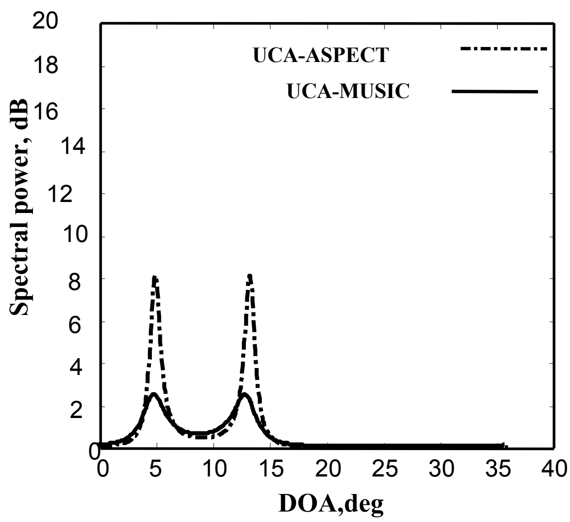


Fig. (5) Shows the effect of number of snapshots at L = 10

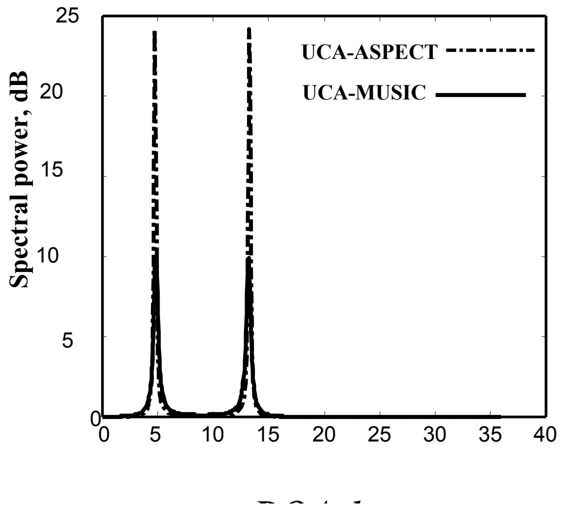


Fig. (6) shows the effect of the number of snapshots at L = 500.

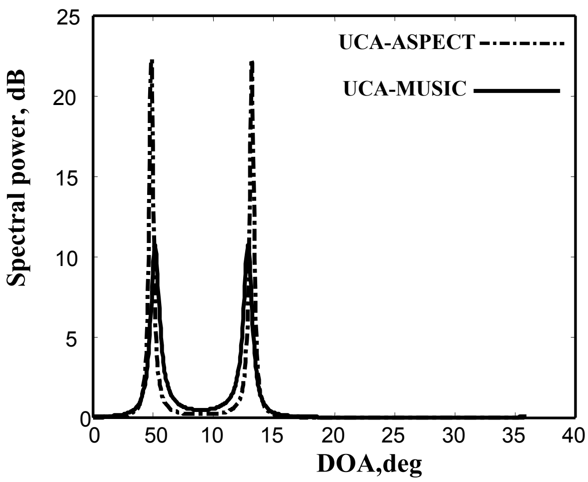


Fig. (7) shows the effect of the number of snapshots at L = 250.

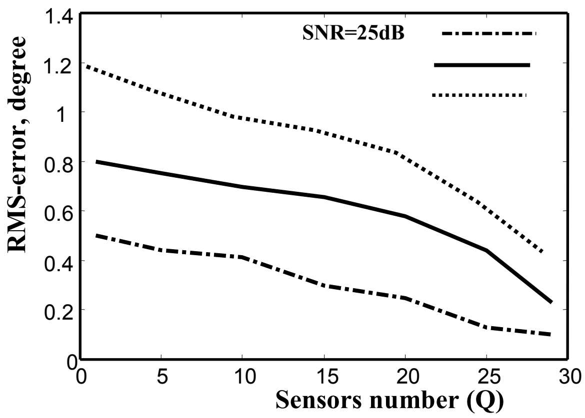


Fig. (8.) shows (RMS-error) versus (Q) for different value of SNR for UCA-

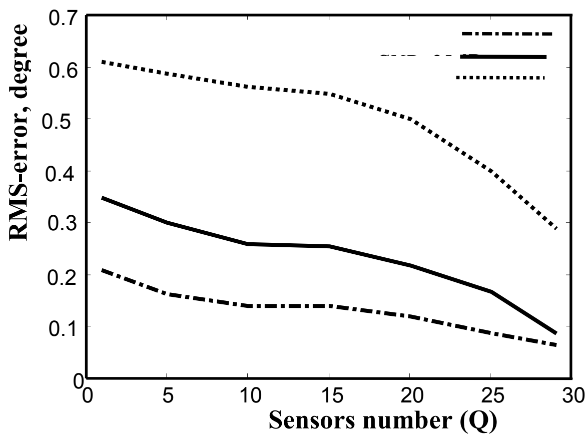


Fig. (9) shows (RMS-error) versus (Q) for different values of SNR for UCA-ASPECT

Table (1) The calculated RMS-error values for different values of snapshots (L) and separation angles for two estimators

$\Delta\phi$	50^o		25^o		15^o		10^o		5^o	
L-Snapshots	RMS-error (degree)		RMS-error (degree)		RMS-error (degree)		RMS-error (degree)		RMS-error (degree)	
	ASPECT	MUSIC	ASPECT	MUSIC	ASPECT	MUSIC	ASPECT	MUSIC	ASPECT	MUSIC
1	0.1502	0.1680	0.2033	0.3200	0.457	0.5910	0.7000	1.1010	1.2500	2.4000
10	0.1412	0.1640	0.1510	0.2513	0.3814	0.5000	0.4970	0.8310	0.8660	2.0110
50	0.0611	0.0781	0.0862	0.1250	0.1411	0.2980	0.3880	0.7340	0.7500	1.8400
100	0.0433	0.0535	0.0611	0.0834	0.0921	0.2130	0.3000	0.5500	0.6630	1.6210
300	0.0171	0.0221	0.0321	0.0551	0.0480	0.0901	0.1000	0.3000	0.3160	0.8470
500	0.0099	0.0140	0.0111	0.0340	0.0261	0.0501	0.0610	0.2550	02510	0.7743
700	0.0073	0.0750	0.0089	0.0101	0.0132	0.0261	0.0520	0.1130	0.2330	0.6671
1000	0.0054	0.0054	0.0065	0.0069	0.0099	0.0203	0.0350	0.1000	0.1250	0.5416

Table (2) The calculated RMS-error values for different values of SNR and Sensors (Q)

SNR	25dB		15dB		5dB	
Q-Sensors	RMS-error (degree)		RMS-error (degree)		RMS-error (degree)	
	ASPECT	MUSIC	ASPECT	MUSIC	ASPECT	MUSIC
2	0.2102	0.5000	0.3500	0.8000	0.6120	1.1999
5	0.1509	0.4440	0.2981	0.7703	0.5800	1.0332
10	0.1492	0.4170	0.2603	0.7522	0.5510	0.9962
15	0.1900	0.3691	0.2599	0.7150	0.5327	0.9046
20	0.1241	0.3811	0.2139	0.6623	0.4980	0.8416
25	0.0966	0.1788	0.2001	0.4111	0.4010	0.7030
28	0.0791	0.1129	0.1500	0.2557	0.3402	0.5110

Table (3) DOA- resolution performance of the two estimators for different ($\Delta\phi$)

Overall ordering for SNR=25dB						
$\Delta\phi$	50^o	25^o	15^o	10^o	5^o	$< 2^o$
UCA-ASPECT Estimator	Super resolution	1st	1st	1st	1st	Super resolution at few No. of snapshots
UCA-MUSIC Estimator	High resolution	2nd	2nd	2nd	Poor resolution	Fails

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