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A Study on New Roulette and Special Forms of Cycloid and Laithoidal Curves

Laith H. M. Al-Ossmi*

Department of Civil Engineering, College of Engineering, University of Thi-Qar, Thi-Qar, IRAQ

*Corresponding author: laithhady@utq.edu.iq

*ORCID ID: https://orcid.org/0000-0002-6145-9478

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Keywords: Cycloid, Geometrical Method, Laithoid, Roulette.

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Abstract:

This article deals with a new roulette of special curves formed by a circle rolling along a line which are given the name of Laithoid curves. The new curve is a new special form of cycloid produced by rolling a circle along a horizontal line of 4 times the rolling circle's radius. It is the locus traced out by a point fixed to a circle (where the point may be on, inside, or outside the circle), as it rolls along a straight line. In this paper, a set of 6 forms of new curvatures within two groups are produced depending on a rolling circle on the Laithoid's curve, and their geometrical and algebra proportions are graphically formed. Furthermore, the article provides the coordinate equations that govern the points along these curves. With the potential to pave the way for exploring additional geometric aspects relevant to this class of curves, and to enable comparative analyses across diverse mathematical and geometric domains, particularly in three-dimensional contexts in the future.

Keywords: Cycloid, Geometrical Method, Laithoid, Roulette.

دراسة حول المنحنيات الناتجة من حركة نقطة، توليد حالات خاصة من المنحنيات السايكلويدية، حالة المنحنيات الليثويدية

ليث هادي منشد العصامي*

كلية الهندسة / جامعة ذي قار / محافظة ذي قار / العراق

laithhady@utq.edu.iq

الخلاصة:

يدرس هذا البحث حالة جديدة من المنحنيات الخاصة التي تتشكل عندما يتحرك دائرة على طول خط، وقد أُطلق عليها اسم منحنيات ليثويد. منحنى الليثويد هو حالة خاصة من منحنى السايكلويدي والذي ينتج عن طريق دوران دائرة بدون انزلاق على خط أفقي بطول 4 مرات نصف قطر الدائرة المتحركة. وهذا المنحني هو المسار الذي يرسمه نقطة ثابتة على دائرة (حيث يمكن أن تكون النقطة على الدائرة أو داخلها أو خارجها)، أثناء تحركها على خط مستقيم.

في هذا البحث، تم إنتاج مجموعة من 6 حالات جديدة من هذا المنحني ضمن مجموعتين تعتمد على دوران الدائرة على منحنى ليثويد، وتشكيلها بشكل هندسي وجبري بياني. علاوة على ذلك، يقدم المقال معادلات الإحداثيات التي تحكم النقاط على هذه المنحنيات. ويتيح ذلك الإمكانية لاستكشاف جوانب هندسية إضافية ذات صلة بهذا النوع من المنحنيات، وإجراء تحليلات مقارنة عبر مجموعات رياضية وهندسية متنوعة، لا سيما في السياقات ثلاثية الأبعاد في المستقبل.

الكلمات المفتاحية: منحنى الدويري، طريقة إنشاء هندسية، منحني الليث.

1. Introduction:

In algebraic geometry, a cycloid is the curve traced by a point on a circle as it rolls along a straight line without slipping. Also, a cycloid is a specific form of trochoid and is an example of a roulette, a curve generated by a curve rolling on another curve. The cycloid generated by a rolling circle, which can be produced by different methods using a fixed point traced on a circle's circumference with the cusps pointing upward, is the curve of fastest descent under uniform gravity, known as the brachistochrone curve **[1]**. It is also the form of a curve for which the period of an object in simple harmonic motion (rolling up and down repetitively along the curve) does not depend on the object's starting position, known as the tautochrone curve **[2]**. Before describing this new special case of the cycloid, some historical background is in order. Historians of mathematics have proposed several candidates for the discoverer of the cycloid: English mathematician John Wallis, writing in 1679 **[3]**, Nicholas of Cusa **[4]**, Père Marin Mersenne **[5]**, Galileo Galilei **[6]**, Marin Mersenne **[7]**, Moritz Cantor [8], Siegmund Günther **[9]**, and the works of Charles de Bovelles **[10]**. During the seventeenth century, Pascal proposed three questions relating to the center of gravity, area, and volume of the cycloid **[11]**. Fifteen years later, Christiaan Huygens deployed the cycloidal pendulum and discovered that a particle would traverse a segment of an inverted cycloidal arch in the same amount of time, regardless of its starting point **[12]**.The cycloid curve in algebraic geometry, through the origin, generated by a circle of radius r rolling over the x-axis on the positive side ($y \geq 0$), consists of the points (x, y), where it is a real parameter corresponding to the angle through which the rolling circle has rotated. For given t, the circle's center lies at (x, y) = (rt, r), with equations:

$$x = r\,(t - \sin t),$$

$$y = r\ (1-\cos t),$$

The *Laithoid*'s curve is a special case of cycloid; it is the curve traced by a point on a circle as it rolls along a straight line with the length of (2*a*) where (*a*) is the radius of the rolling circle **[13]**. Also, *Laithoid* is not an example of a roulette because it is not a curve generated by a curve rolling on another curve, hence it is generated by a rolling circle (Drawing Circle), on a straight line with a specific length related to a circle's radius **[14,15]**. However, all these derivate forms of curvatures produced from *Laithoid* are examples of a roulette as they all are generated by a circle rolling on the *Laithoid* curve.

2. Drawing Method:

The method for drawing the new curve, *Laithoid*, involves several steps:

1. **Drawing the Initial Circle**: by drawing a circle with a specified radius a.
2. **Dividing the Vertical Diameter**: Divide the vertical diameter of the circle into equal parts, n_1, n_2, …
3. **Drawing Horizontal Lines**: Draw a horizontal line from the center of the circle with a length equal to 2a. Extend this horizontal line to a length of 4a and divide it similarly into equal parts, c_1, c_2, …, matching the divisions of the vertical diameter.
4. **Drawing Moving Circles**: Draw circles representing the non-slipping movement of the initial circle, where the center moves sequentially to the points n_1, n_2, …,
5. **Drawing Rays**: From the uppermost point of the vertical diameter of each circle, draw a ray. This ray starts from the point that is a distance of 2a from the edge of the terminal circle.

The construction of a *Laithoid* using geometric tools is illustrated in **Figures 1** and **2**.

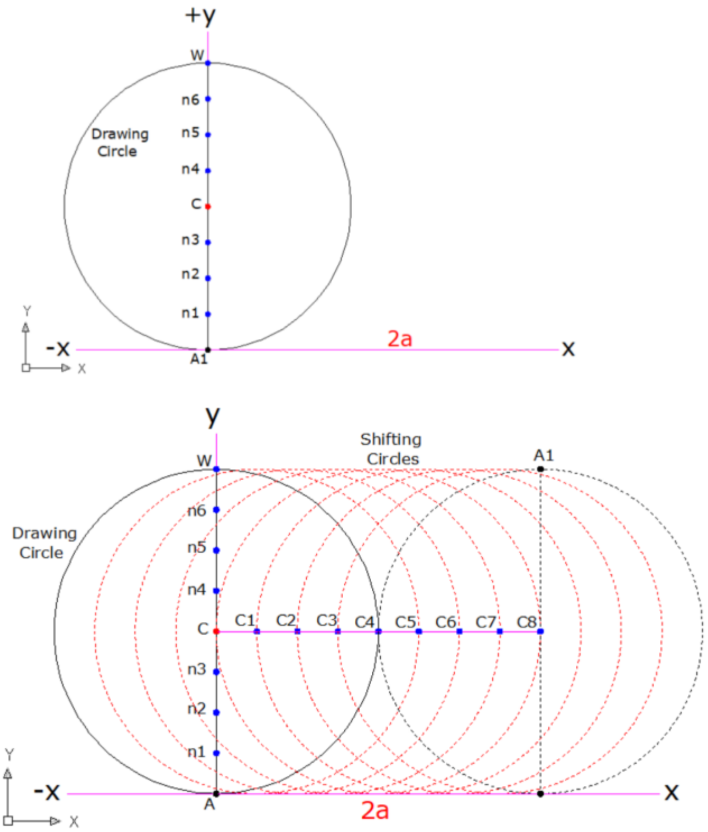
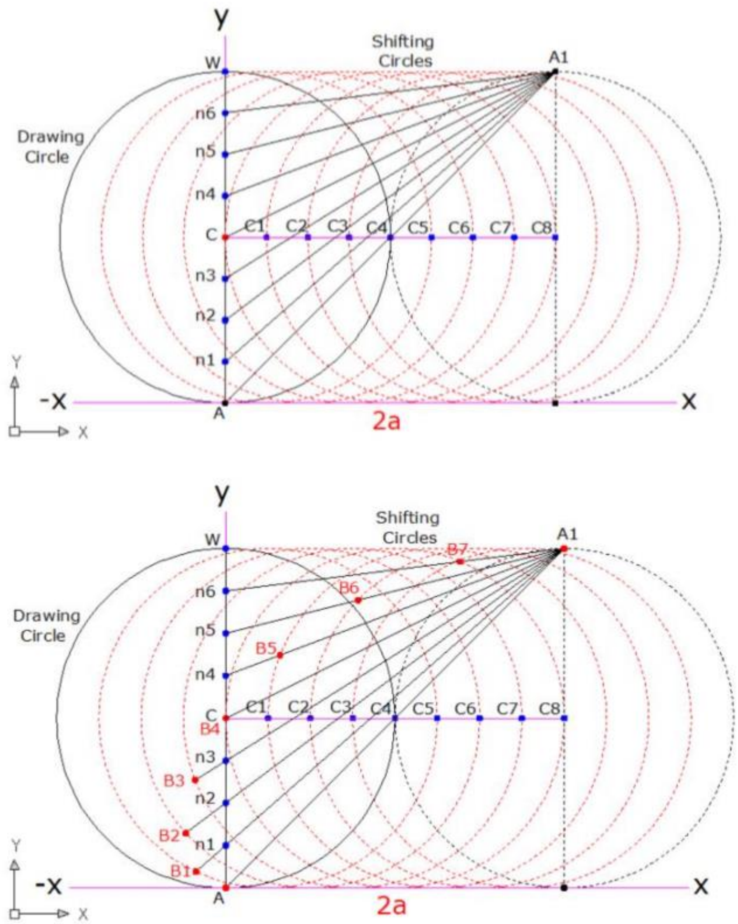


Figure 1: Construction of a *Laithoid* using a rolling disk's radius (*a*) along a path of (4*a*)



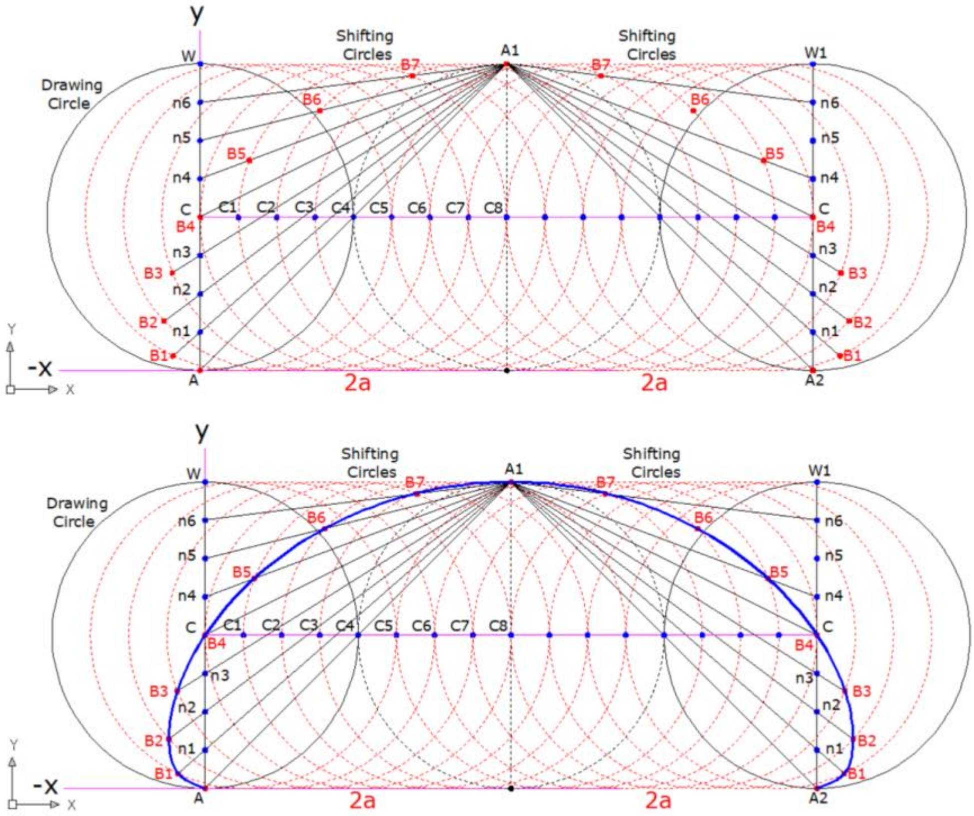


Figure 2: The plot of the construction of a *Laithoid* using a geometric method

2.1 Laithoid's Equations: Let a drawing circle with radius (a), and point (A) be the origin, (0,0). By drawing the Laithoid's arch passing from point (A) and (A_1) till (A_2), the horizontal distance is ($4a$). Each point on the curve is a result of rolling the drawing disk with angles; ($BNC_1= t$) and ($BC1D_1= u$), where point (B) lies at *Laithoid*, (see **Figure 3**).

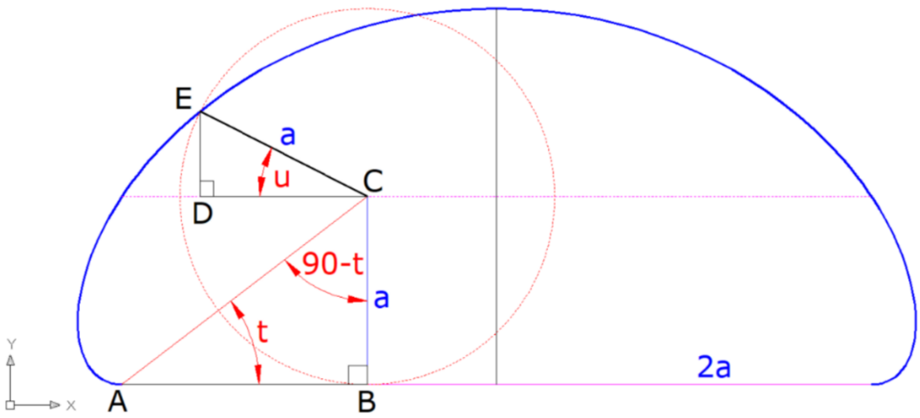


Figure 3: The plot of the Laithoid curve using a geometric method

Therefore, from **Figure 3** the *Laithoid* is the path traced out by a fixed point (B) on the boundary of a circular disk whose radius (a) rolls along a horizontal line ($4a$), to produce an arch with the length of (7.000700558440277713 a), as it is shown in **Table 1**.

Table 1: *Laithoid's* key properties

Properties Values	Properties Values
Rolling disk's radius, a	Rolling disk's radius, a
Start point (0,0)	Start point (0,0)
Top point (2a,2a)	Top point (2a,2a)
Cusp's intersection	(a,0)
Two arches cusp	$\pm(a, 0)$ *and* $\pm(2a, a)$
End point (4a,0)	End point (4a,0)

From **Figure 3**, let (a) be the radius of the rolling circle and (t) be an angle between the segment line from the circle center (C) and the horizon line (AB) drawn from the origin point (A), this gives the Cartesian equations of the Laithoid's curve:

$$x = (AB - 2a) - DC, \tag{1}$$

$$y = ED + a, \tag{2}$$

$$\sin u = \frac{DE}{a},$$

$$\cos u = \frac{DC}{a},$$

$$DC = a\cos u,$$

Then by angle (t) and a simple trigonometry, we find:

$$\tan t = \frac{a}{AB},$$

$$AB = \frac{a}{\tan t},$$

Then for any point (x,y), we can find:

$$x = \left(\frac{a}{\tan t} - 2a\right) - a\cos u, \tag{3}$$

$$x = \left(\frac{a - 2a\tan t}{\tan t}\right) - a\cos u, \tag{4}$$

$$y = a\sin u + a, \tag{5}$$

$$y = a(\sin u + 1), \tag{6}$$

According to *Laithoid's* proportions, it shades region of area which can also be done by calculus, which is not exactly inconsequential. Then it must integrate this to get the required area covered by *Laithoid*:

$$A = \int_{x=0}^{2\pi a} y dx = \int_{u=0}^{2\pi} a^2 (1 + \sin u)^2 du, \tag{7}$$

$$A = \int_{x=0}^{2\pi} a^2 (1 + 2\sin u + sin^2 u)\, du, \tag{8}$$

$$Laithoid's\ Area = 2.2797365122\ \pi a^2 \ , \quad (9)$$

This region under one *Laithoidal* arch and the horizontal line of (4*a*) is more than double times the area of the rolling circle. Obtained data from Table 1 indicates that the ratio between Laithoid's length and its rolling disk's circumference is precisely (1.1143102520478548*a*), nevertheless, this ratio related to cycloid is found to be (1.2978725139014964*a*), as shown in **Figure 3**. Along the coordinators of the y-axis and x-axis, (**Figure 3**), there is only one point (A) in which both curves, Cycloid and *Laithoid*, are intersected. Using the same rolling disk that rolls along a horizontal line, both curves make up an intersection point (V_1) which lies at (±2.56483 a, 1.94521 a), precisely it is:

$$AV_2 = 2.564834658174025\ a, \quad (10)$$

$$V1V_2 = 1.945206239700822\ a, \quad (11)$$

Similarly, to Trochoid **[12]**, the *Laithoidal* arches also have a cusp at point (A) whose intersection point (0, *a*) lies at the y-axis, whereas cycloid arches have not, **Figure 4**. Also, for each rolling of the drawing disk along the horizontal line (A_1A_2), the cycloid's arch requires a horizontal distance of (6.5188394907830363470*a*), which is precisely (1.631668317375) time of *Laithoid's*, hence *Laithoid's* arch requires a horizontal distance of (4*a*), (as it is listed in **Table 2**).

Table 2: Comparison of the key proportions of Cycloid and Laithoid.

Key proportions	Cycloid	Laithoid	ratio
Curve's length.	$8a$	7.000699277711867699 a	1.142742986471309640
Covered area.	$3\pi a^2$	2.27973675122 πa^2	1.315941412268127658
Horizontal distance.	$2a\pi$	$4a$	1.570796326721501765
The hump height.	(3.2590a, 2a)	(2a,2a)	-
Cusp's length.	-	1.17507941583 a	-
Cusp's area.	-	2.4809340098320 πa^2	-
Cusp's intersection P.	(0, 0)	± (0, a)	1.000000000000000000
Where (a) is the rolling disk's radius.			

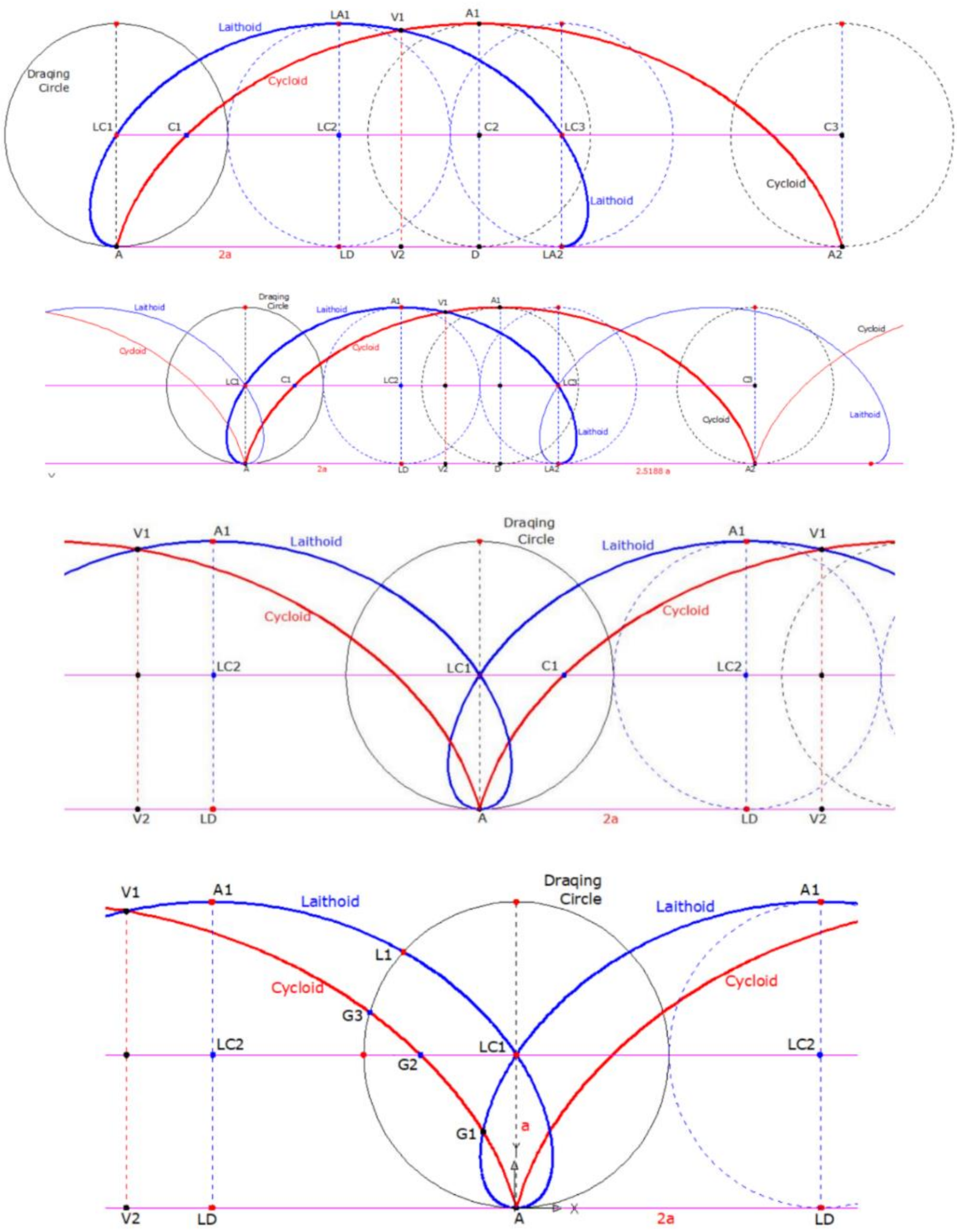


Figure 4: The plot of the comparison of the key points of Cycloid and *Laithoid*, where (*a*) is the rolling disk's radius

It is perceptible from **Figure 4** that from the origin point (A), the cycloid intersects with the drawing disk at two points,(G_1) and (G_2) in addition to point (G_3) through the horizontal distance from the center of the disk. While 3 points of Laithoid are; (A), (LC_1), and (L_1) with which the curve intersects with the disk's circumference. However, regardless of point (A), both curves intersect at two points; (G_2) and (V_1), more proportions are shown in **Table 3**.

Table 3: Comparison between the intersection points of Cycloid and Laithoid

Points	Cycloid	Laithoid
A	(0,0)	

G_1	± (0.2184*a*, 0.4993*a*)	
G_2	± (0.6297*a*, *a*)	-
G_3	± (0.9606a, 1.2776*a*)	-
LC_1	-	(0,*a*)
L_1	-	± (0.7398*a* ,1.6727*a*)
V_1	±(1.9399*a*, 2.5562*a*)	
Where (a) is the rolling disk's radius.		

In this paper, Laithoid's curve has been used to produce a set of new forms of curves, which are graphically studied and figured using AutoCad utility, (Version 2020).

3. Laithoid's family

There is a part of a new family of curves invented from the Laithoid curve that will be described here presently. In this paper, a new set of 6 curves are produced. In general, these 6 curves can be tabulated into two groups, the closed and opened curves. The first group is those curves that complete a full rolling at one point, (2*a*,0) without a cusp, while the second group is those that complete a full rolling at two points, (0,0) and (4*a*,0).

A- Group One: in this paper 3 forms of new curves were produced. Let a Laithoid and a given drawing circle with radius (*a*), let the drawing circle lie from its center at point (2*a*, *a*), where angle (*u*) is between line segment (BD) and x-axis. Let point (B) be a point at the Laithoid, then construct segment (BD), and from (B) draw a horizontal ray to intersect the circle at (B_1), then from point (B_1) draw a perpendicular to intersect the circle's circumference at (n_1), shown in **Figure 4**. Then;

$$Dn_1 = 2a\ sin(90 - u), \tag{12}$$

$$x = 2a\ \sin(90 - u)\ \sin(90 - u), \tag{13}$$

$$\boldsymbol{x} = sin(90 - u)\ (2a - 1), \tag{14}$$

$$\tan(90 - u) = 2a\ \cos(90 - u)\ \sin(90 - u)\ y\ , \tag{15}$$

And then we can obtain:

$$y = \left(\frac{2\ a\cos(90-u)\sin(90-u)}{\tan(90-u)}\right), \tag{16}$$

$$y = \left(\frac{2\ a\cos^2(90-u)\sin(90-u)}{\sin(90-u)}\right), \tag{17}$$

$$y = 2\ a\ cos^2(90 - u), \tag{18}$$

$$\left(\frac{x}{y}\right) = \left(\frac{\sin(90-u)(2a-1)}{2\ a\ cos^2(90-u)}\right), \tag{19}$$

Then it is obtained that any point of the *laithoid's* curve can be described by ;

$$\boldsymbol{y} = \left(\frac{x\ 2\ a\ cos^2(90-u)}{sin(90-u)(2a-1)}\right), \tag{20}$$

$$x = \left(\frac{y\ sin\ (90-u)}{2\ a\ cos^2(90-u)}\right), \quad (21)$$

Figure 5 shows that the heart-like curve is a closed curve without a cusp, and it intersects with the x-axis at (2*a*,0), and tangents with Laithoid by a single point (D1) at the top which is (2*a*,*a*). The coordination of any point of this curve can be determined by the ray drawn from a point (B) at laithoid and point (D), the intersection point between (BD) and the drawing disk (n_1), the horizontal and vertical segments from (B) and (n_1) will determine a point of the curve, (B_1). This new curve has a shaded region with an area of ($0.8223858140642\pi a^2$), and a length of (6.127795789824198*a*), where (*a*) is the rolling disk's radius, (**Figure 5**).

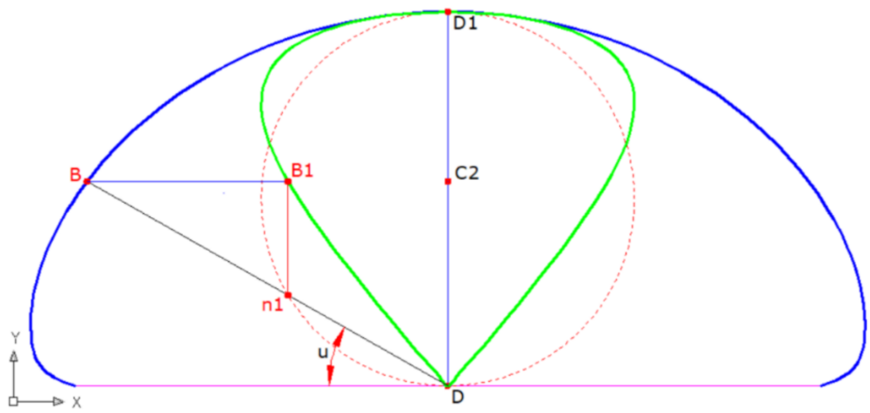
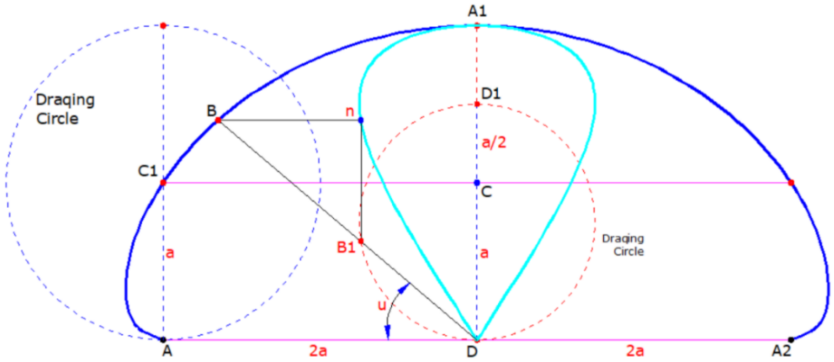


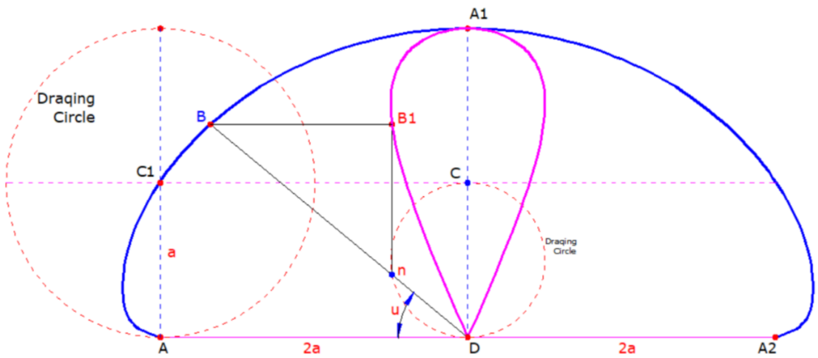
Figure 5: Construct a new form of curves from a point on Laithoid.
Where (*a*) is the rolling disk's radius, and (*u*) is the angle between the segment line from the origin (D) and the horizon line.

By changing the radius value of the rolling disk, another set of heart-like curves is formed. When the value of the radius of the rolling disk declined to (0.5 *a*), the curve tended to shrink from the top taking a shape of drop close to the y-axis, which decreased its shaded region area. In general, the length value of the curves depends on the value of the radius of the rolling disk regardless of that of drawing the circle of Laithoid. Correspondingly, the horizontal width of the curve is determined by the value of its rolling disk's radius.

When the radius of the rolling disk (0.5*a*) then the produced curve has a length of (4.7836971666054*a*) and a shaded region area of ($0.4110219882904975\ \pi a^2$), while in case radius of the rolling disk is (1.5*a*) the new curve is more inflated and has a length value of (5.412244557765046*a*), and more area in its shaded region of ($0.6156407068172649\pi a^2$), as it is shown in **Figure 6**.



Where the rolling disk's radius is (1.5a)



Where the rolling disk's radius is (0.5a)

Figure 6: Constructing a new form of curves from a point on Laithoid,
where the rolling disk's radius is (1.5a) and (0.5a)

B- Group Two: In this group, 3 forms of new curves were produced. Let a *laithoid* and a given drawing circle with radius (a), let the drawing circle lie from its center at point (2a, a), where angle (u) is between line segment (BD) and the x-axis. Let point (B) be a point at the Laithoid, then construct segment (BD), and from (B) draw a horizontal ray to intersect the circle at (B_1), then from point (B_1) draw a perpendicular ton intersect with the circle's circumference at (n_1), shown in **Figure 6**. Then;

$$B_1(x) = B_1 D \cos u, \quad (22)$$

$$DB_1 = 2a \cos(90 - u), \quad (23)$$

Then the value of y can be obtained by:

$$\boldsymbol{y} = 2a\, cos^2\,(90 - u), \quad (24)$$

Figure 7 shows that this form of the curve is an opened curve with a cusp between two arches along the y-axis, as it intersects with the x-axis at (0,0) and (2a,0), and tangents with *Laithoid* by a single point (D_1) at the top which is (2a, a). The coordination of any point of this curve can be determined by the ray drawn from a point (B) at *Laithoid* and point (D), the intersection point between (BD) and the drawing disk is (n_1), the horizontal and vertical segments from (B) and (n_1) will determine a point of the new curve, (B_1).

This new curve has a shaded region and a length, where (a) is the rolling disk's radius, a set of these curves is illustrated in **Figures 6** and **7**. It is noticeable that all points of this form of curve are the intersection of these rays drawn between (D) and (B). Also, the produced curve (green in **Figure 7**), has a cusp with *Laithoid's* curve which is formed by two intersection points:

$$\pm(0.12968796501a, 0.2086584604810a),$$

$$\text{and } (4.129687965a, 0.2086584604810a).$$

Correspondingly, the intersection point of the curve's cusp lies at point (0, 0.416009485062a), in addition, its cusp length is made up of (1.37384011705a), hence this form of curve correspondingly shaded region area of (0.0357779584131759 πa^2), (see **Figure 7**, and **Tables 4**,**5**,**6**, and **7**).

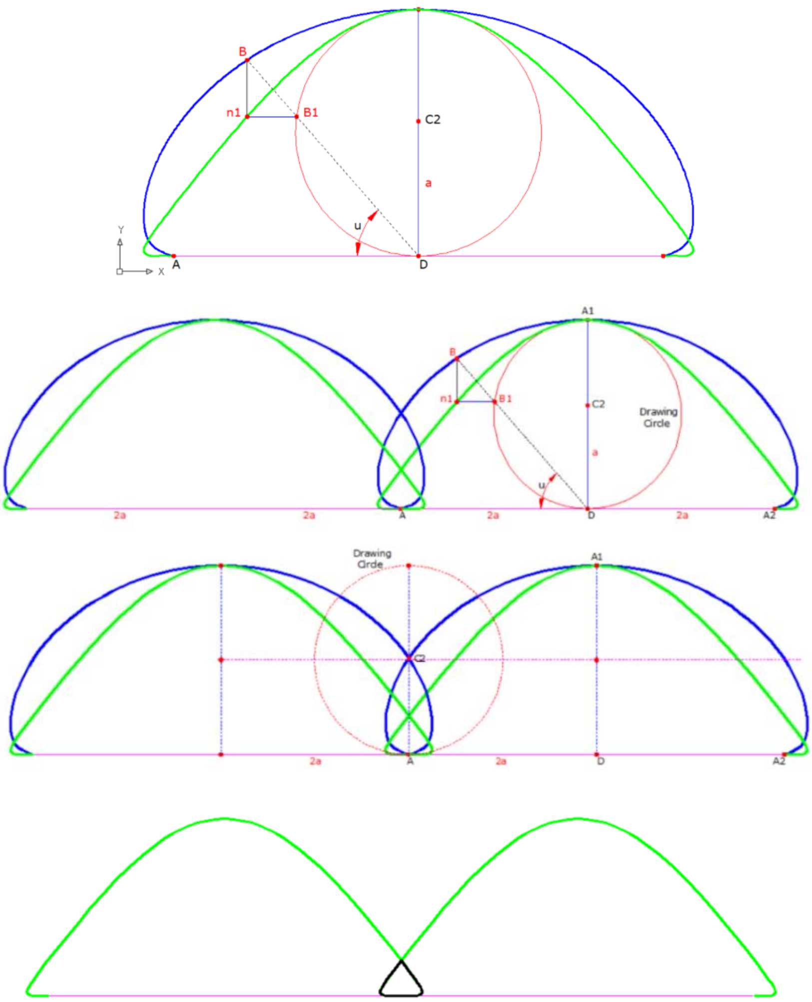
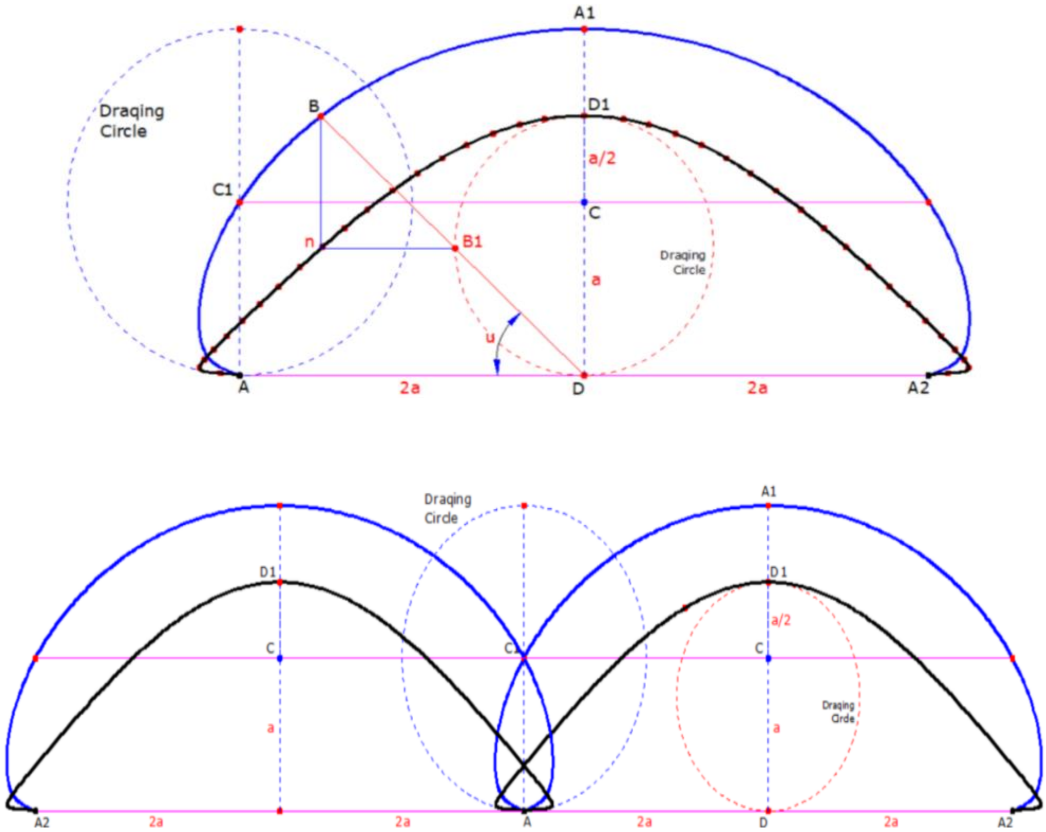
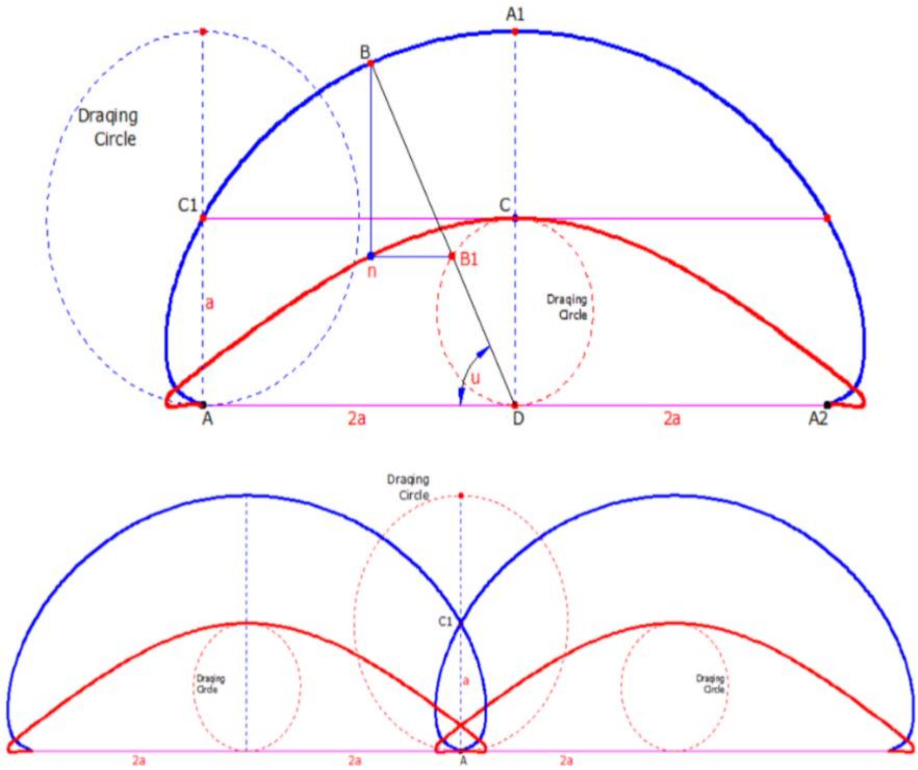


Figure 7: Constructing a new form of curves from a point on *Laithoid* where the rolling disk's radius is (*a*) and (n_1) a point on the curve



Where the rolling disk's radius is (1.5a).



where the rolling disk's radius is (0.5a).

Figure 8: Constructing a new form of curves from a point on Laithoid, where the rolling disk's radius is (*a*) and (*n*) a point on the curve.

Table 4: Comparison of group one's curves related to Laithoid

Points	Group one's curves	Laithoid
Start point	$(2a,0)$	$(0,0)$
Top point	$(2a,a)$	$(2a,a)$
Endpoint	$(2a,0)$	$(4a,0)$
Length	(varied)	$(7.000690251107a)$
Shaded area	(varied)	$(2.278887062739374\ \pi\ a^2)$
Cusp's area	-	$(0.105913393319132167\ \pi a^2)$
Where (a) is the rolling disk's radius of Laithoid.		

Table 5: Comparison of group two's curves related to Laithoid

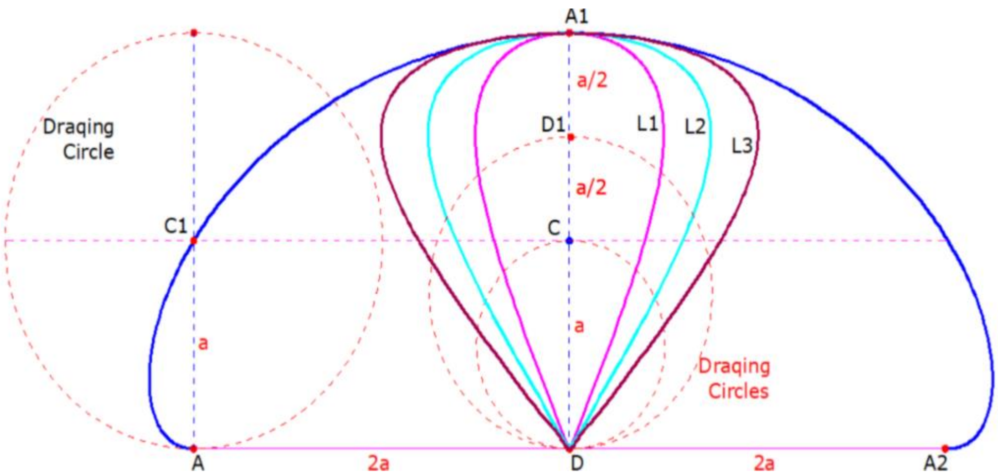
Points	Group Two's curves	Laithoid
Start point	$(0,0)$	
Top point	$(2a,a)$	
Endpoint	$(4a,0)$	
Length	(varied)	(7.000690251107a)
Shaded area	(varied)	$(2.278887062739374\ \pi\ a^2)$
Cusp's area	(varied)	$(0.105913393319132167\ \pi a^2)$
Where (a) is the rolling disk's radius of Laithoid.		

Table 6: Key properties of Group One's curves

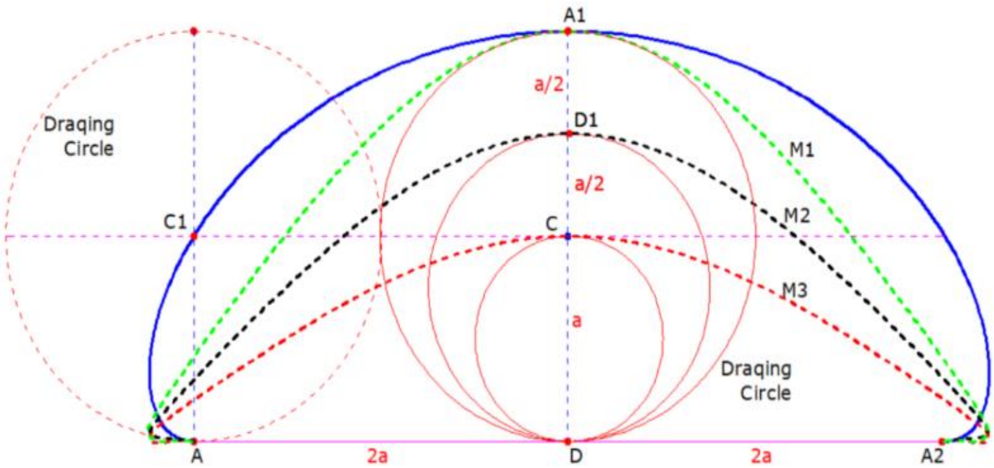
Properties	L 1	L 2	L 3
Length	$4.7836971666054\ a$	$5.41224455776504\ a$	$6.127795789824198\ a$
Shaded area	$0.411021988290\ \pi a^2$	$0.61564070279121\ \pi a^2$	$0.82238581422668\pi a^2$
Cusp's length	-	-	-
Cusp's area	-	-	-
Tangent point*	$(2a,\ a)$		
Intersection point*	-	-	-
**Shaded points with a Laithoid.* *Where (a) is the rolling disk's radius of Laithoid.*			

Table 7: Key properties of Group Two's curves

Properties	L 1	L 2	L 3
Length	6.654806579093416a	$5.9604358927946a$	$5.47020227329001a$
Shaded area	$1.84053446580540\pi a^2$	$1.3774307432443\pi a^2$	$0.92072136146895\pi a^2$
Cusp's length	$1.3740375859635581a$	$1.10172429250216\ a$	$0.551372160842206\ a$
Cusp's area	$0.0188500927561\ \pi a^2$	$0.0125071773814\pi a^2$	$0.00917801064989\pi a^2$
Tangent point*	$(2a,\ a)$		
Intersection point*	$\pm\ (0.2008444685a,$ $0.141708219704a)$	$\pm\ (0.1857509648141a,$ $0.1114055079789a)$	$\pm\ (0.169865928825a,$ $0.08678455016606a)$
**Shaded points with a Laithoid.* *Where (a) is the rolling disk's radius of Laithoid.*			



Group one; 3 curves of heart-like; (L_1), (L_2), and (L_3).



Group two; 3 curves of cusp; (M_1), (M_2), and (M_3).

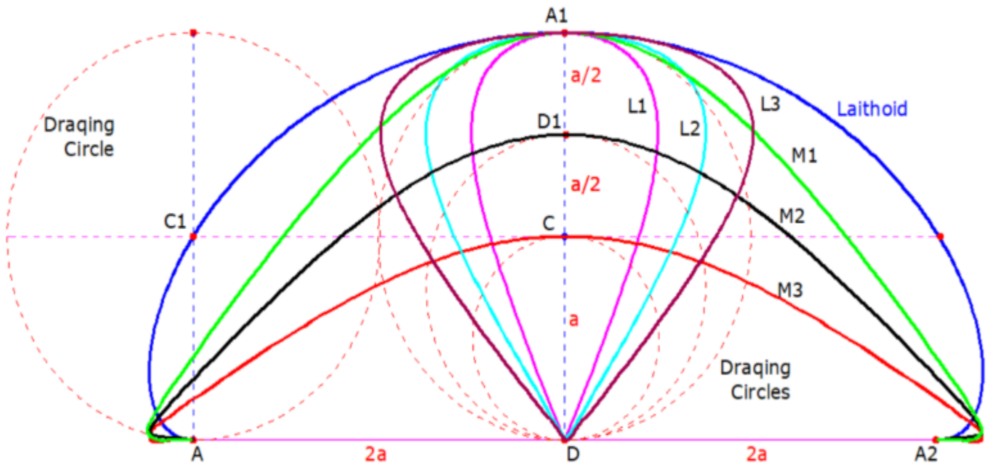


Figure 9: Constructing a set of two groups of new curves from a point on *Laithoid*, Where the rolling disk's radius is (*a*) and (*n*) a point on the curve

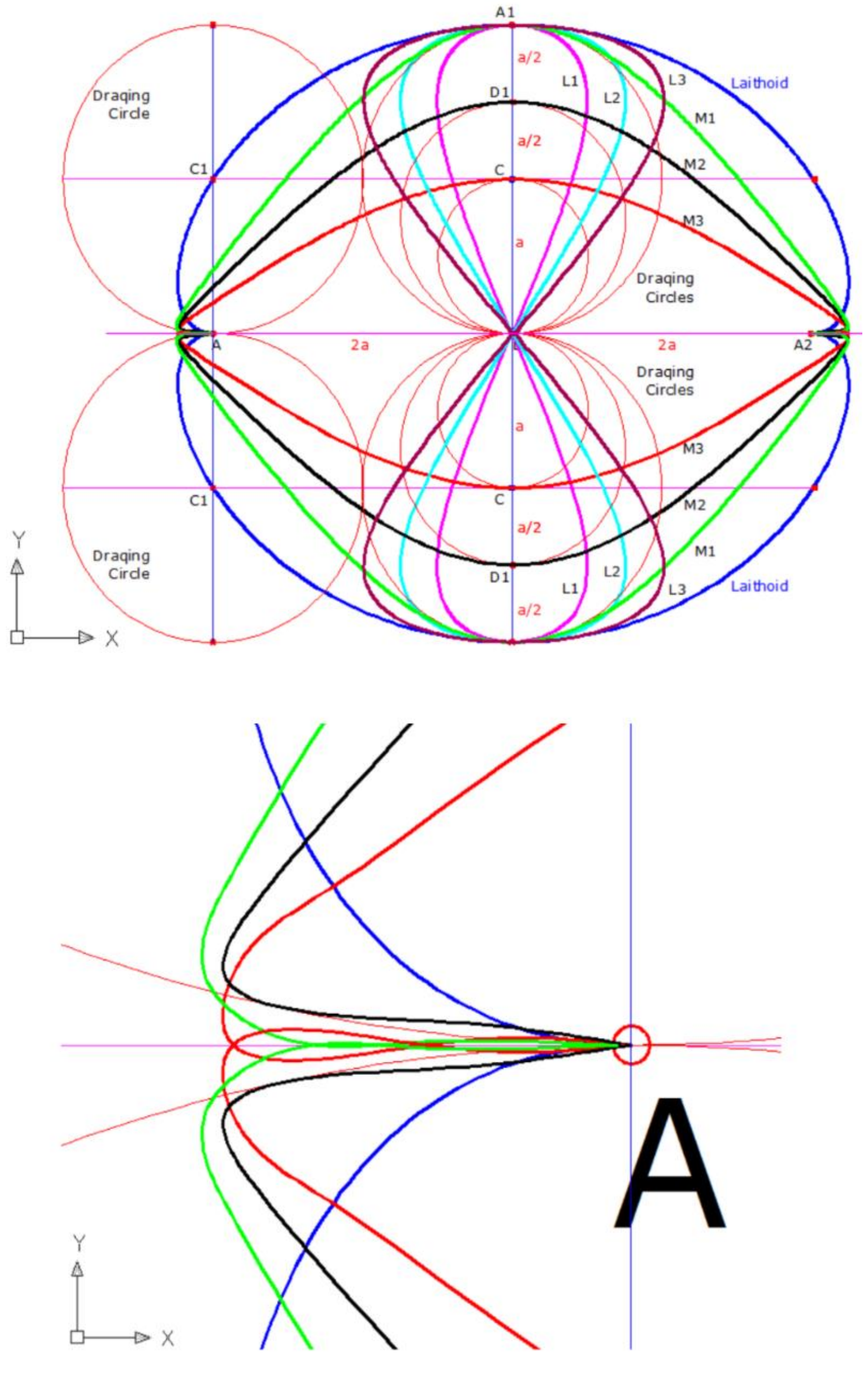


Figure 10: Plot all two groups of *Laithoid's* curves over $\pm y$ and x axis zooming in point of (A). Where the rolling disk's radius is (*a*)

4. Conclusion:

This paper delves into a specific aspect of 2D plane geometry, focusing on a special case of the cycloid, termed the "*Laithoid*". The paper utilizes geometric methods and mathematical analyses to investigate various aspects of the *Laithoid* curves. It explores equalities between different metrics in specific instances of the cycloid, indicating a rigorous analytical approach. The introduction of new curves and their properties demonstrates a comprehensive investigation of various geometrical aspects. The study explores various equalities between the Cassinian metric and other related metrics in specific instances of the cycloid. Additionally, the paper introduces six new curves and thoroughly investigates their properties. A rolling disk with a constant radius of 'a' is utilized to develop a geometric method for identifying points of curvature. These curves are categorized into two groups based on their rolling characteristics, with some completing a full rolling at one point ($2a$, 0) without a cusp, while others complete a full rolling at two points (0, 0) and ($4a$, 0).

The paper includes studying equalities between metrics related to the cycloid, introducing six new curves, investigating their properties, and examining the invariance and distortion properties of these curves using geometric methods. Additionally, the paper explores the creation of 3D surfaces from these curves through consistent revolution. The research also employs geometric methods to examine the invariance and distortion properties of these curves. Notably, certain curves are referred to as "heart-like *Laithoids*" due to their semi-cyclic nature, featuring arches with cusps along the y-axis. The study considers a more general form of relative distortion properties, incorporating the rolling disk's radius (a), distance ratio ($4a$), and ray segment angle (u). Furthermore, it is found that the curves generated in this study possess a cusp along with the *Laithoid*, where the cusp's intersection point lies on the y-axis. This observation defines a specific shaded region of area associated with the curve's cusp. The creation of 3D surfaces derived from these curves offers new insights into their geometric properties and applications. Furthermore, this study explores the generation of novel 3D surfaces (M_1, M_2, and M_3) from these curves by consistently revolving their shapes. The surface resulting from the revolution of the Laithoid's curves around the y-axis exhibits a simple and symmetric form, albeit requiring supplementary steps to adapt them to complex notation and operational calculus.

Regarding the limitations and future directions, further exploration of 3D and 2D practical applications and the need for adaptation to complex notation and operational calculus are taken into account. In conclusion, the findings presented in this paper hold potential for significant practical applications. The Laithoid's curves, especially in architectural engineering, show promise for future practical use. The author's selection of conclusions is based on their extensive experience in this field.

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