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Failure Lateral Distributed Load for Slender Composite Beam-Column

Asst. Lect. Nibras N. Al-Chalabi
Civil Engineering Department, College of Engineering
Al-Mustansiriya University, Baghdad, Iraq

Abstract

In this paper a method for computing the lateral deflection and the failure load for the slender composite columns of the type of concrete encased steel sections which is subjected to axial load with equal end eccentricities in addition to uniform lateral load is described. A computer program is used to calculate the deflection at the center of the column, hence it represents the maximum value, that corresponds to a specified properties of cross section(dimensions and strength of materials), length, and loading condition (axial load, eccentricity and uniform distributed lateral load which is taken as a proportion of the axial load).

Relationships between the failure lateral load/axial load ratio and the ratio of steal area to the concrete area of the composite section for different end eccentricities are shown. The relation between the central deflection and uniform distributed load for different axial loads values and end eccentricities is shown too.

The obtained results from the computer program for the failure lateral uniform load with the other affecting factors are used as an input data in program of "Statistica" to predicate a formula that represents the failure lateral uniform distributed load.

الخلاصـــة

في هذا البحث تم وصف طريقة لحساب الهطول والحمل عند الفشل للأعمدة المركبة من نوع المقاطع الحديدية المحاطة بالخرسانة المعرضة إلى حمل محوري لامركزي متساوي في نهايتي العمود المفصلي بالإضافة إلى الحمل العرضي المنتشر. تم استخدام برنامج حاسوبي لحساب الهطول في منتصف العمود ،حيث انه يمثل القيمة العظمى، لقيم معينة من خواص مقطع العمود (الأبعاد ومقاومة الحديد والخرسانة)، الطول، والأحمال المسلطة (المحورية اللامركزية والجانبية الموزعة بانتظام والتي تم أخذها كنسبة من الحمل المحوري).

تم توضيح العلاقة بين أعلى قيمة لنسبة الحمل الجانبي /الحمل المحوري المسموح به و مساحة الحديد/مساحة الخرسانة للمقطع المركب لقيم مختلفة من اللامركزية بالإضافة إلى العلاقة بين الهطول في المنتصف والحمل الجانبي الموزع بانتظام لقيم مختلفة من الأحمال المحورية اللامركزية.

النتائج المستحصلة من تطبيق البرنامج الحاسوبي للتحليل النظري لإيجاد قيم الفشل للحمل الجانبي مع مختلف العوامل المؤثرة تم إدخالها في برنامج إحصائي لاقتراح صيغة معادلة تمثل قيمة الفشل للحمل الجانبي الموزع بانتظام.

1. Introduction

A composite column is a compression member with a cross section consisting of concrete steel section and concrete, which act together to resist axial compression and very little transverse shear, the use of this type of columns appeared in the previous century in some places to get greater stiffness, higher load capacity against material damage, and also higher collapse capacity. Consequently, a composite section is generally smaller than alternative design to sustain the same load, resulting in the saving of material, weight, and headroom or construction depth [1]. The researches in this field were carried out on columns that subjected to axial load with or without end eccentricities only, except the research of (Abo-Hamd 1988) [2] who present analytical expression to describe a moment-thrust-curvature curves, then these expressions are used to develop typical interaction curves for composite beam column under various end and loading condition. In this paper a simple method to analyze a composite beam-column is presented to find the lateral deflection resulted from eccentric axial load with various equal end eccentricities and uniform distributed load, in addition to describe the effect of the steal area/concrete area on the maximum uniform distributed load/axial load for different end eccentricities. The present analytical solution depends on some assumptions:

1. A complete interaction takes place between steel and concrete, i.e. the strains in steel and concrete at their interfaces are equal.
2. The stress/strain curves for concrete and steel are assumed to be reversible.
3. Plane section before bending remains plane after the applications of the load. i.e. the strain distribution across the section is assumed to be linear.
4. The residual stresses in steel are negligible and the effect of strain hardening in steel is ignored, and the tensile strength of the concrete is neglected.
5. Failure due to local buckling does not occur.

The general step to analyze any column is the computation of the moment-thrust-curvature relationship (M-P-ϕ) by dividing the cross section of the column into strips in the direction of the applied moment, assuming specified values of curvature and depth of neutral axis the strain at every strip is calculated, hence the stress at the strip is obtained from the stress- strain relationship which is used in this paper as follows:

1-1 Concrete

It was concluded from the study presented by 'Luc Lachance' [3] that the shape of the stress-strain curve of concrete have a very little effect on the ultimate bearing moment capacity and ultimate curvature with a tolerance of (2%), so in the present analytical solution the calculations depend on a third order polynomial curve based on tests conducted by 'Barnard and Johnson' [4], as shown in **Fig.(1-a)** for the sake of comparison with the available theories.

$$\frac{f_c}{f_{c\max}} = 2.41\left(\frac{\varepsilon}{\varepsilon_{c\max}}\right) - 1.865\left(\frac{\varepsilon}{\varepsilon_{c\max}}\right)^2 + 0.5\left(\frac{\varepsilon}{\varepsilon_{c\max}}\right)^3 - 0.045\left(\frac{\varepsilon}{\varepsilon_{c\max}}\right)^4 \quad \text{.........} \; (1)$$

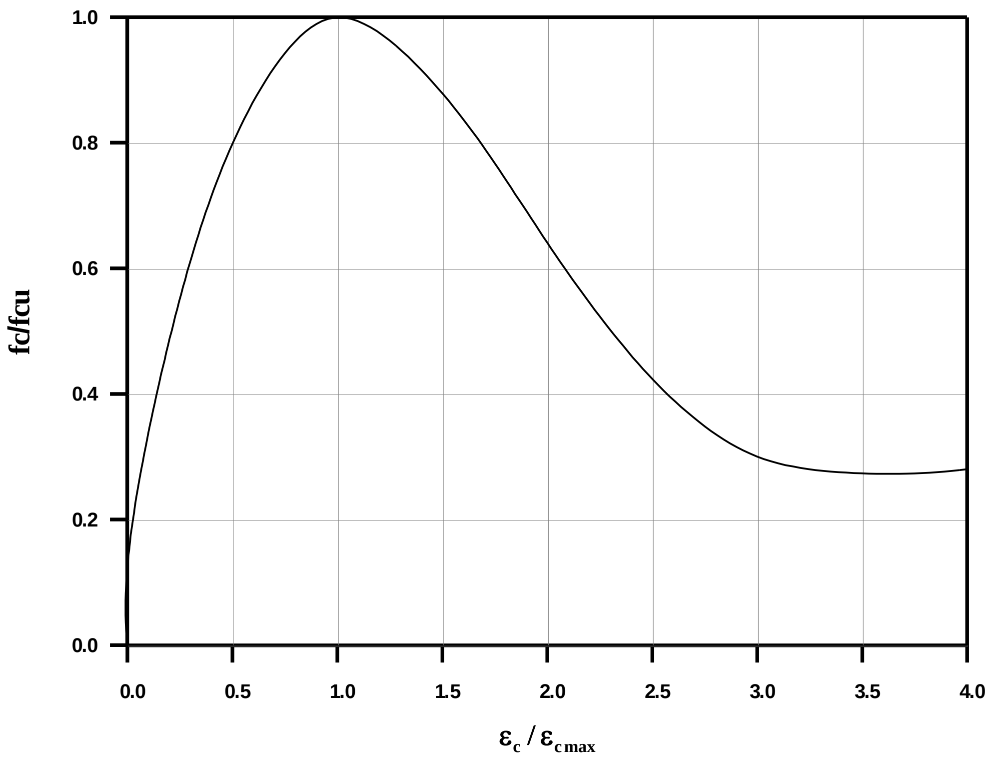
1-2 Steel

A typical stress/strain curve for structural steel is shown in **Fig.(1-b)**. For computations, the stress/strain relationship is idealized and an elastic-perfectly plastic (bilinear) curve is adopted for steel, ignoring the effect of strain hardening. The following equations were used to represent the required stress/strain relationship:

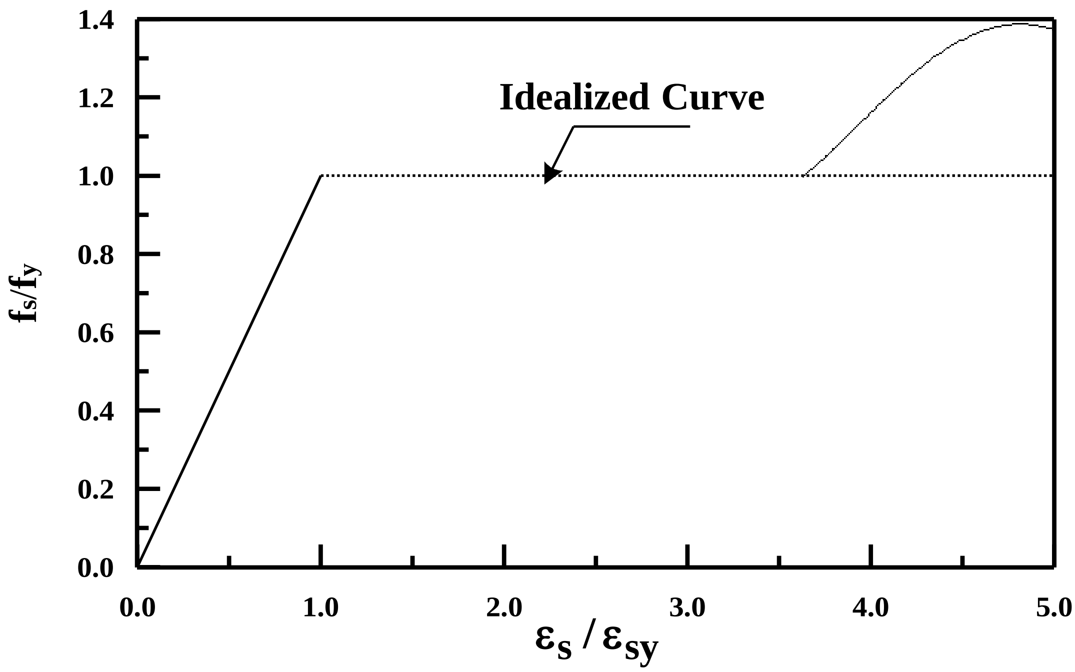
$$\mathbf{f_s = E_s \varepsilon_s} \quad \text{for } -\varepsilon_{sy} < \varepsilon_s < \varepsilon_{sy} \quad \text{.................................... \textbf{(2-a)}}$$

$$\mathbf{f_s = f_{sy}} \quad \text{for } \varepsilon_s \geq \varepsilon_{sy} \quad \text{... \textbf{(2-b)}}$$

$$\mathbf{f_s = -f_{sy}} \quad \text{for } -\varepsilon_{sy} \geq \varepsilon_s \quad \text{.. \textbf{(2-c)}}$$



(a) Concrete



(b) Steel

Figure (1) Stress/Strain Relationship for a: Concrete
b: Steel

The load and the moment is calculated for every stress. The computation for the given curvature ends either when the maximum compressive strain at concrete reaches a certain specified value or when (P) starts to decrease, whichever is earlier. From the values of these curvatures and the corresponding computed values of moments the curves of the (M-P-ϕ) for chosen set of values of (P) are generated.

2. Computations of the Load/Deflection Relations

The fundamental problem involved in any method of computing the failure loads of columns is to determine their load/deflection relationships. The theoretical analysis used for obtaining the load/deflection relationships for composite beam-column subjected to a uniaxial bending with eccentric loading of equal end eccentricities and uniform lateral load is made by using computer program in “Fortran Language” and will be displayed here.

For the pin-ended column shown in the **Fig.(2)** of a length (ℓ) and subjected to eccentric load (P) with equal end eccentricities (e) at the ends of the column at the same direction, that the column bends in single curvature and subjected to uniform distributed load (W).

To find the maximum lateral deflection in the beam column, the energy method is used for the analysis.

The strain energy that is sorted in the member is:

$$\mathbf{U = \frac{EI}{2}\int_0^{\ell}\left(\frac{d^2y}{dx^2}\right)^2 dx} \quad \text{...} \quad \mathbf{(3)}$$

and the potential energy of the external load is:

$$\mathbf{V = -w\int_0^{\ell} y dx - \frac{P}{2}\int_0^{\ell}\left(\frac{dy}{dx}\right)^2 dx + \frac{Pe}{2}\int \frac{d^2y}{dx^2}} \quad \text{..} \quad \mathbf{(4)}$$

thus the total energy in the system is:

$$\mathbf{U + V = \frac{EI}{2}\int_0^{\ell}\left(\frac{d^2y}{dx^2}\right)^2 dx - w\int_0^{\ell} y dx - \frac{P}{2}\int_0^{\ell}\left(\frac{dy}{dx}\right)^2 dx + \frac{Pe}{2}\int \frac{d^2y}{dx^2}} \quad \text{..................} \quad \mathbf{(5)}$$

the deflection y is assumed to be of the form:

$$\mathbf{y = \delta \sin\frac{\pi x}{\ell}} \quad \text{...} \quad \mathbf{(6)}$$

Eq.(6) is substituted in Eq.(5) and after the integration Eq.(5) becomes:

$$\mathbf{U + V = \frac{EI\,\delta^2\pi^4}{4\ell^3} - \frac{2w\delta\ell}{\pi} - \frac{P\delta^2\pi^2}{4\ell} + \frac{Pe\delta\pi}{\ell}} \quad \text{...} \quad \mathbf{(7)}$$

To find the maximum deflection at the member Eq.(7) is differentiate with respect to δ and equals to zero, so the lateral deflection is obtained as:

$$\mathbf{\delta_{max} = \frac{4w\ell^4 + 2Pe\pi^2\ell^2}{EI\pi^5}\left(\frac{1}{1-(P/P_{cr})}\right)} \quad \text{...} \quad \mathbf{(8)}$$

Multiply the numerator and denominator of Eq.(8) by 5/384EI and arrange it, becomes:

$$\mathbf{\delta_{max} = \left(\frac{5w\ell^4}{384EI} + \frac{5Pe\ell^2}{78EI}\right)\left(\frac{1}{1-(P/P_{cr})}\right)} \quad \text{...} \quad \mathbf{(9)}$$

where $P_{cr} = \frac{\pi^2 EI}{\ell^2}$

and $$\mathbf{EI = \left[\left(0.3 - 0.2\frac{e}{D}\right)E_c\left(I_g - I_s\right) + 0.8E_sI_s\right]}$$ [5] …………………………..…. **(10)**

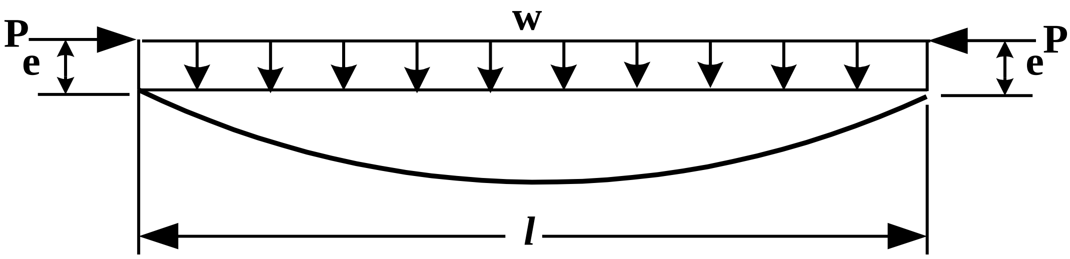


Figure (2) Deformed Shape of the Beam-Column under Study

The maximum moment resulted from this lateral deflection and an applied load is:

$$\mathbf{M_{max} = P(\delta_{max} + e) + \frac{w\ell^2}{8}}$$ ……………………………………………….... **(11)**

3. Procedure of Computing Load/Deflection Equations

1. Select specified values of the properties of the cross-section and the length of the composite column.

2. Assume value of the end eccentricities (smallest value e=0.2D).

3. Assume values of the axial load, and the uniform distributed load, which is taken as a proportion of the axial load.

4. Calculate the value of the lateral displacement at the center of the composite column from Eq.(9).

5. Calculate the resulted moment from Eq.(11).

6. From the (M-P-ϕ) relationship this moment is compared with the internal moment which corresponds to the applied axial load. If (M_{max}) is less than the internal moment increase the value of (W) until (M_{max}) reaches to the ultimate moment capacity, then this value of (W) will be the failure uniform load.

7. Repeat the steps (3-6) until failure uniform load converges to zero go to step (2) and make greater value of eccentricity.

4. Comparison with the Theoretical Results of Abo-Hamd [2]

To ensure the validity of the present analytical expression a comparison was made between the results obtained from the application of the present analytical solution with that obtained by (Abo-Hamd) as shown in **Fig.(3)**.

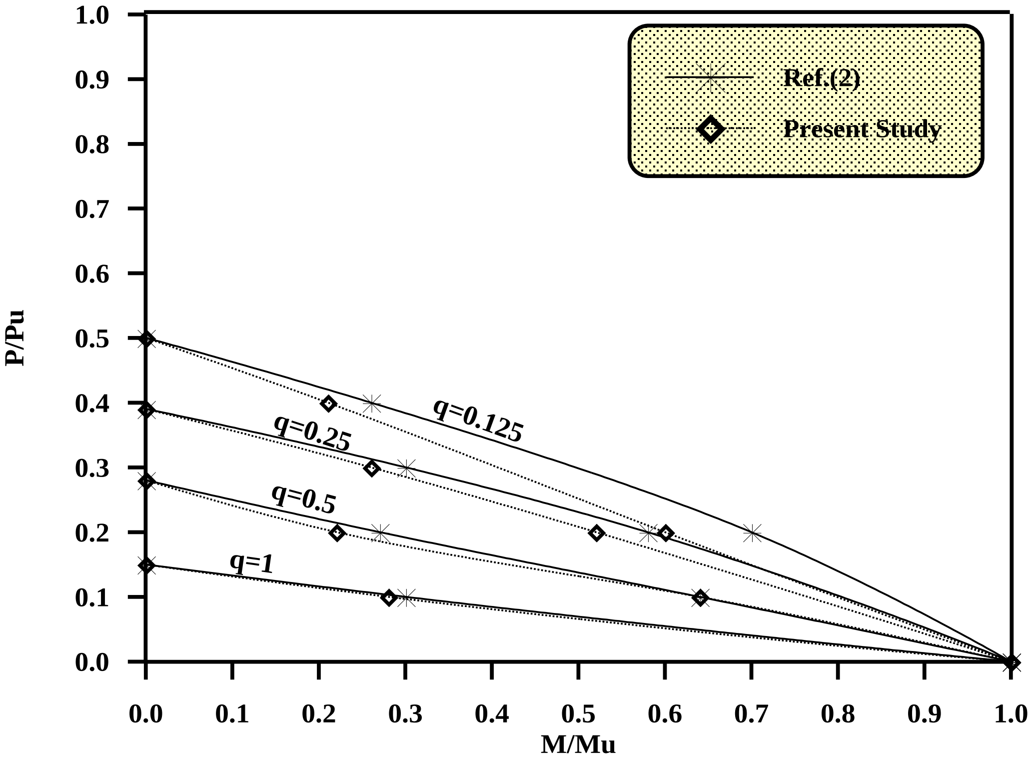


Figure (3) Comparison between Results of Present Study and Abo-Hamd Method

The figure shows the interaction curves for composite beam-column with equal end moments and lateral uniform load. The cross section used to develop these curves is a (W203x47.2kg/m) encased in a (303x303mm) concrete section. Other design parameters are f'_c=27.5 MPa, f_{sy}=248 MPa, ε_{cmax} =0.002 ε_{cu} =0.0035. with slenderness ratio ℓ / r of 75.The interaction curves are plotted in terms of a nondimensional lateral load, q, which is equal to $W/P\phi_u$. where ϕ_u is the ultimate curvature corresponding to the ultimate moment capacity at P equal zero. The comparison shows a good agreement as it is seen in the figure by a range of (88-100%).

5. Prediction of the Failure Uniform Distributed Load Equation

By applying the computer program to a number of composite columns of various steel area/concrete area and for each column the lateral deflection is determined for different values of column lengths, end eccentricity, axial load and uniform distributed load, these results was taken as input data in the program of "Statistica" to predicate an equation makes the failure uniform distributed load directly without using an analytical solution by the computer. This equation was obtained and makes very good correlation (R=0.91) given by:

$$\mathbf{W_f = 0.083\left(\frac{122.51+e}{D}\right)^{-1.687} + 473\left(\frac{A_S}{A_C}\right)^{3.804} - 224\left(\frac{P-136344}{\ell+659664.4}\right) - 45.78} \quad \textbf{... (8)}$$

the units are in kN and mm.

6. Applications

In order to study the effect of some factors on the value of the failure lateral uniform distributed load and the load/displacement relationship these applications were made to particular composite column section as follows.

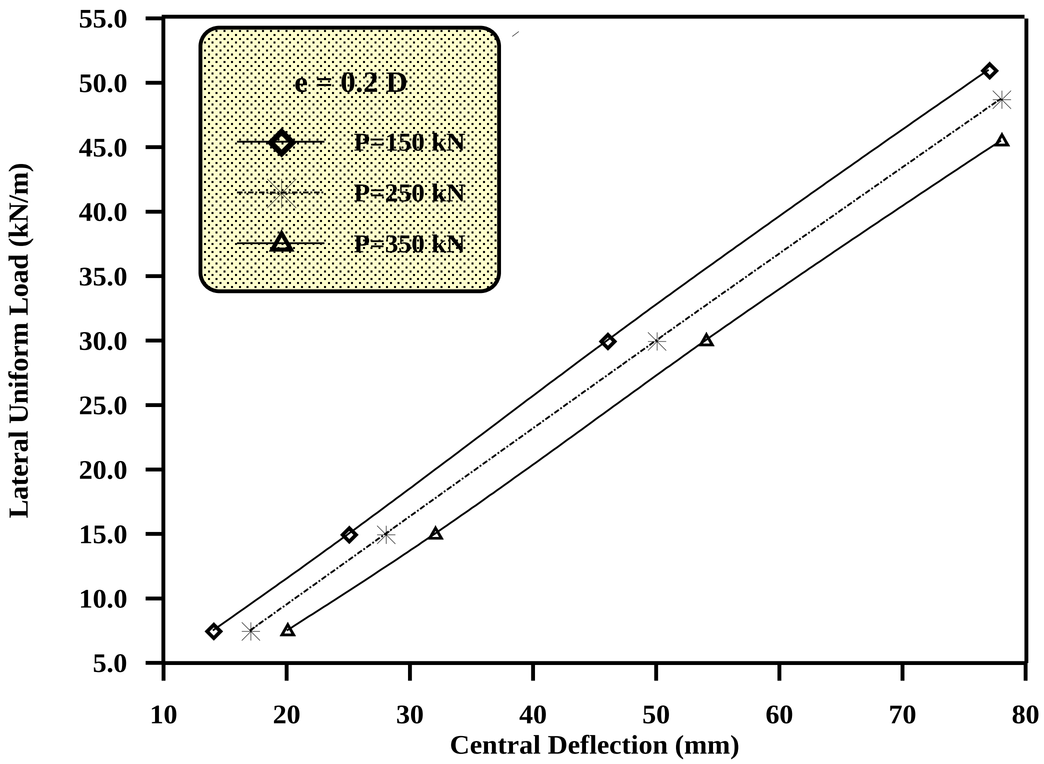
6-1 Effect of Eccentricity Ratios on the Load/Displacement Relationship

The column section used for this purpose was of concrete encased steel sections, the section is (203mm x 203mm x 47.2 kg/m) [6], steel encased to (303mm x 303mm) with effective length of (6 m), the material properties are as follows:

f_{sy} =386 MPa, E_s =200x10^3 MPa, ε_{sy} =0.0019

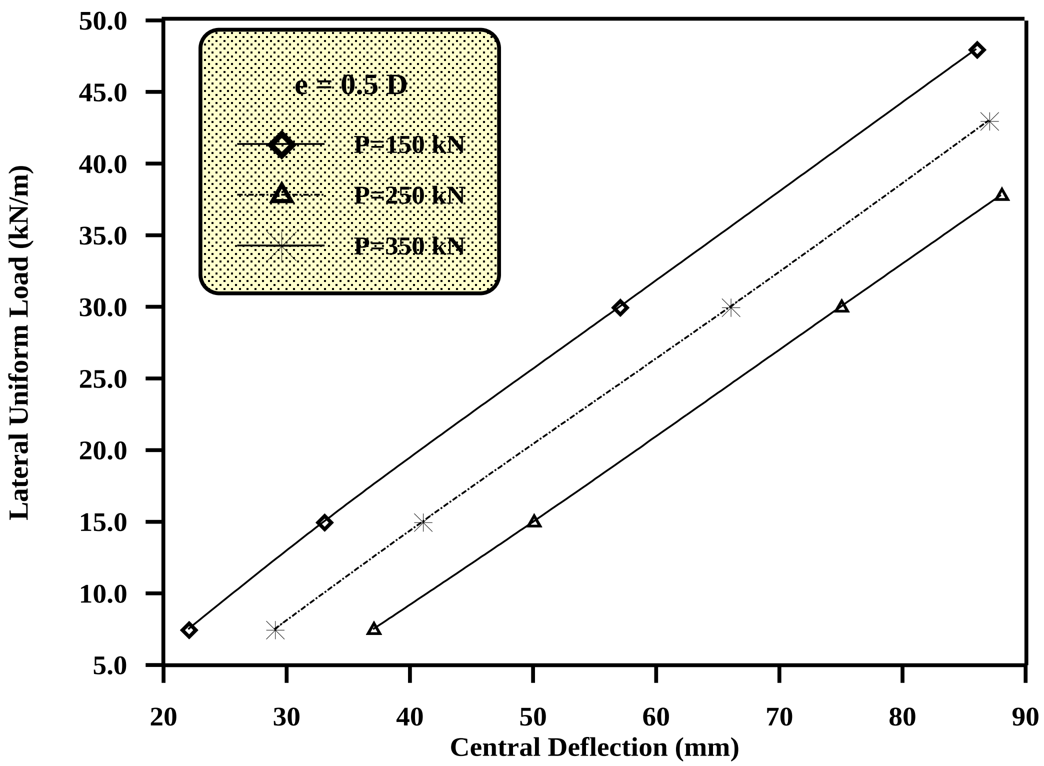
f_{cu} =44 MPa, E_c =670f_{cu}, ε_{cmax} =0.002

Figures (4-a), **(4-b)** and **(4-c)** show the lateral deflection corresponding to the combination of the axial load and uniform lateral load for different eccentricity values (e=0.2D, 0.5D and D) respectively (bending about major axis).



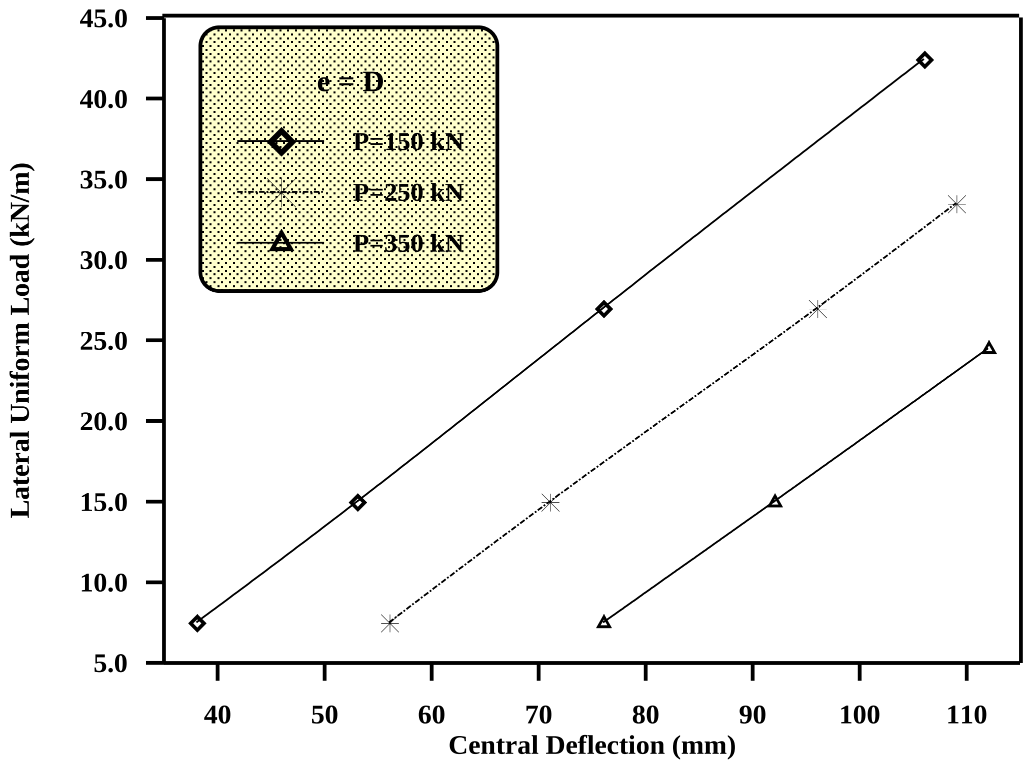
e = 0.2 D
P=150 kN
P=250 kN
P=350 kN
Lateral Uniform Load (kN/m)
Central Deflection (mm)
55.0
50.0
45.0
40.0
35.0
30.0
25.0
20.0
15.0
10.0
5.0
10
20
30
40
50
60
70
80

(a)



e = 0.5 D
P=150 kN
P=250 kN
P=350 kN
Lateral Uniform Load (kN/m)
Central Deflection (mm)
50.0
45.0
40.0
35.0
30.0
25.0
20.0
15.0
10.0
5.0
20
30
40
50
60
70
80
90

(b)



(c)

Figure (4) Load/Displacement Relationships

It is seen from these figures that the maximum lateral deflection (δ_{max}) increases with the increase of the applied uniform load for specified values of axial load and eccentricity.

Also it is seen from these figures that for a specified value (w) and (e) the value of (δ_{max}) increases by the increase of the axial load, but this increment differs with the difference of the end eccentricity. (i.e. for e=0.2D, δ_{max} increases by an average of (10.8%) with the increase of the axial load from (150 to 250), while with the increase of P from (150 to 350), δ_{max} increases by an average of (35%), for e=0.5D, δ_{max} increases by an average of (24%) with the increase of the axial load from (150 to 250), while with the increase of P from (150 to 350), δ_{max} increases by an average of (50%) and for e=D, δ_{max} increases by an average of (35%) with the increase of the axial load from (150 to 250), while with the increase of P from (150 to 350), δ_{max} increases by an average of 87%).

It is concluded from the above observation that the value of (δ_{max}) increases by a larger rate while the end eccentricity increases.

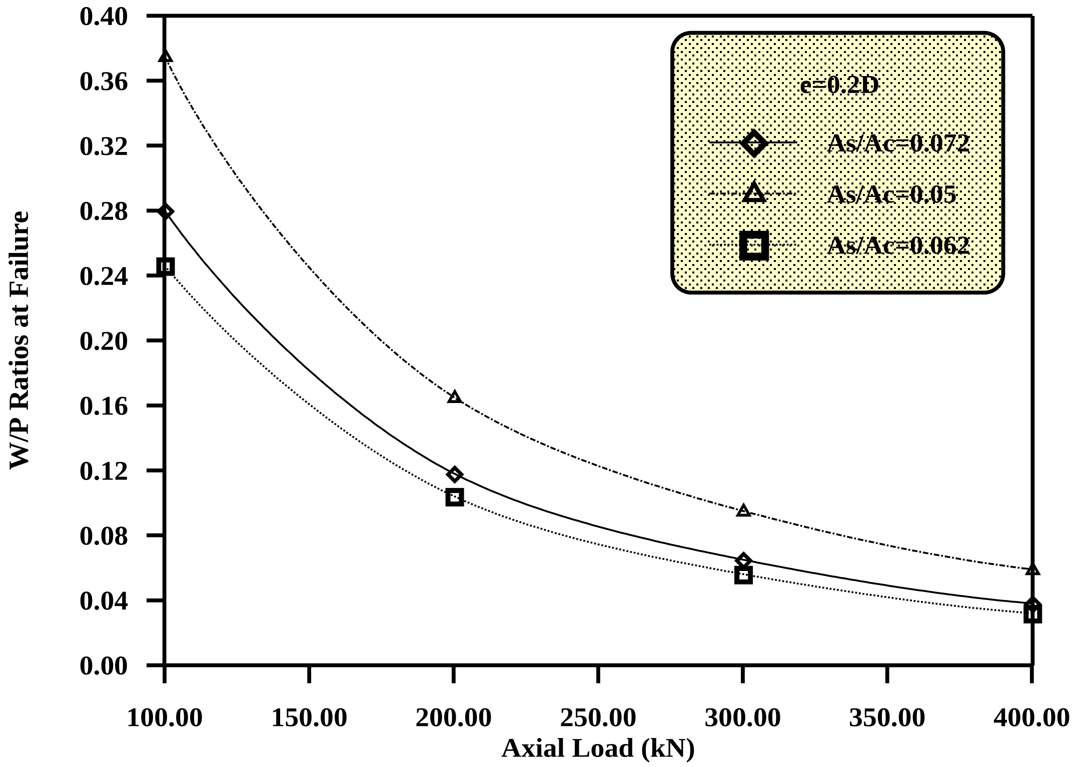
6-2 Effect of the Area Ratios on Failure Uniform Distributed Load

For the purpose of the identification of the effect of steel area with respect to the concrete area, three sections of the concrete encased steel sections were used. The steal sections are (152mm x26.27kg/m), (178mm x 21.54kg/m) and (127mm x24.37kg/m) encased in concrete with a cover of (50mm) at all sides. (A_s/A_c =0.072, 0.062, 0.05), respectively of length (5m), the material properties are as follows:

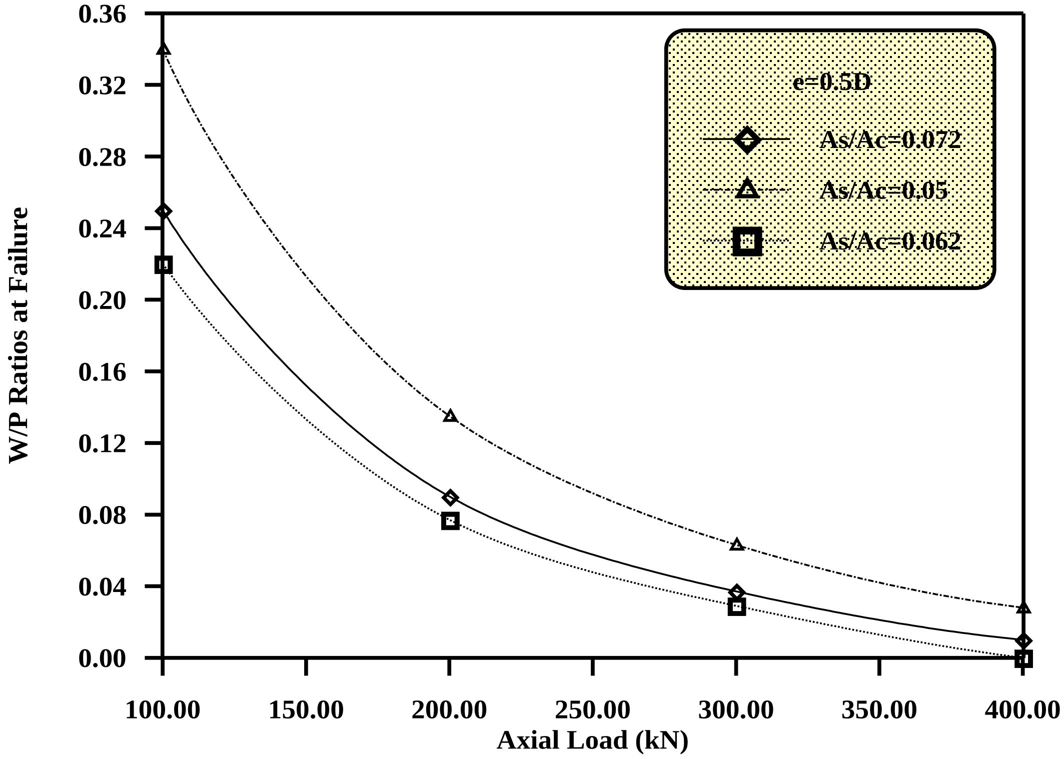
f_{sy} =368 MPa, E_s =200x10^3 MPa, ε_{sy} =0.0016

f_{cu} =44 MPa, E_c =29480 MPa, ε_{cmax} =0.002

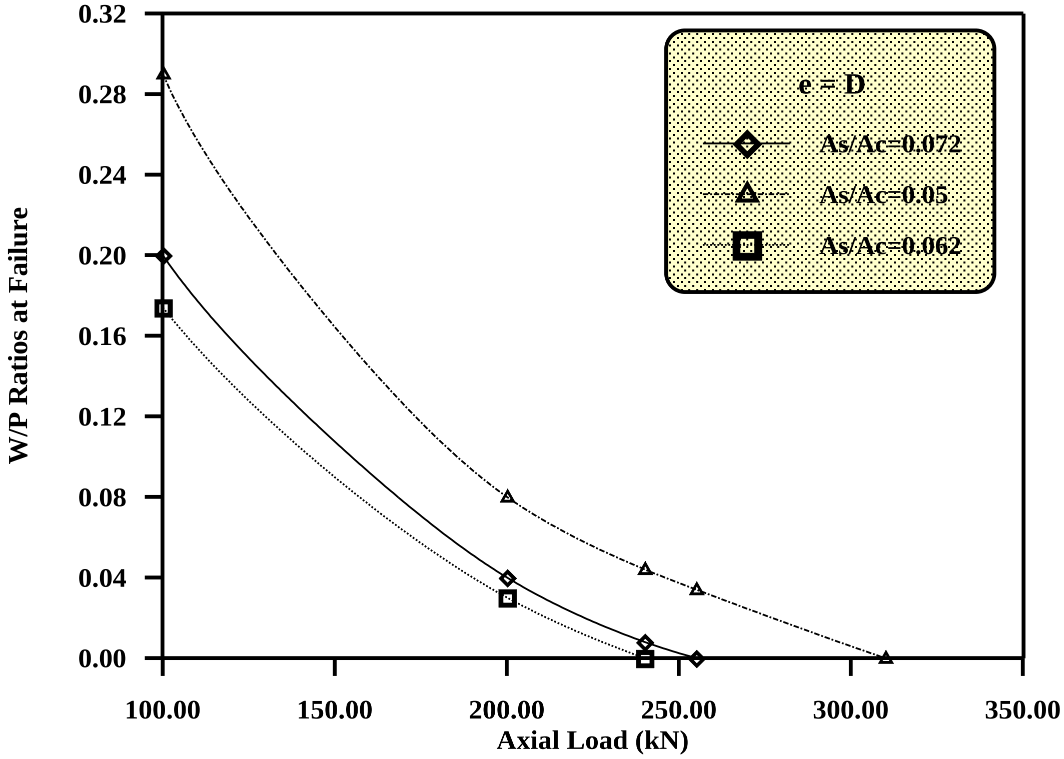
Figures (5-a), **(5-b)** and **(5-c)** show the relationship between the axial load (P) and the ratios of (W/P) values at the failure of the three sections for values of eccentricity (e=0.2D, 0.5D and D) respectively.



(a)



(b)



(c)

Figure (5) Relationship between (W/P) at Failure Values with (A_S/A_C)

It is seen from the figures that there is no fixed relationship between the different values of (A_s/A_c) and (W/P) at failure. (e.g. for e=D, P=100 kN and A_s/A_c =0.072 (W/P) =0.2, by decreasing A_s/A_c to 0.062 (W/P) decreases to 0.174 while by more decrease of A_s/A_c to 0.05 (W/P) increases to 0.29). But this relationship between the values of the (A_s/A_c) ratios is similar for the three different figures.

Also it is noticed from the figures that while the axial force increases the decrease of (W/P) ratio increases for the change of (A_s/A_c) from (0.05 to 0.072) and from (0.05 to 0.062), but this increment decreases while the value of the eccentricity decreases. (e.g. for e=0.2D, P=100 kN (W/P) decreases by a rate of 25.33% for the change of (A_s/A_c) from 0.05 to 0.072, and for P=200 kN (W/P) decreases by a rate of 28.48% for the change of (A_s/A_c) from 0.05 to 0.072 while for e=D, P=100 kN (W/P) decreases by a rate of 31% for the change of (A_s/A_c) from 0.05 to 0.072, and for P=200 kN (W/P) decreases by a rate of 50% for the change of (A_s/A_c) from 0.05 to 0.072.

7. Conclusions

From this study the following points are concluded:

The comparison between the simple analytical expression that used at this study and that of **Ref.(2)** shows a good agreement that ensuring the validity of this method.

1. The predicated formula for the failure uniform lateral load that obtained from the nonlinear regression converges to (0.91) that make the use of this formula is possible for any pin-ended composite beam column.
2. The maximum lateral deflection (δ_{max}) increases with the increase of the applied uniform load for specified values of axial load and eccentricity.
3. The value of (δ_{max}) increases by a larger rate while the end eccentricity increases for a specified W and P.
4. The relation between the failure (W/P) value for any two values of (As/Ac) is constant for the same axial load and eccentricity.
5. The difference between the values of (W/P) at failure and any two ratios of (As/Ac) increases by the increase of the end eccentricity and same axial load.

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Notations

A_c,A_s	= Area of concrete and steal section respectively
D	= Depth of the cross section
e	= End eccentricity
E_c, E_s	= Module of elasticity of concrete and steal
EI	= Effective flexural stiffness of column
f_{cu},f_{sy}	= Ultimate compressive strength of concrete and yield strength of steel respectively
I_g, I_s	= Moment of inertia of gross concrete section and of the structural steel section
ℓ	= Length of the composite column
M	= Applied bending moment
M_{max}	= Maximum bending moment at the mid span of the column
M_u	= Ultimate moment capacity of the cross section
P	= Applied axial load
r	= Radius of gyration
W,W_f	= Applied uniform lateral distributed load and its failure value, respectively
δ_{max}	= Maximum lateral deflection at the composite column
ε_c, ε_s	= Strain of concrete and steal, respectively