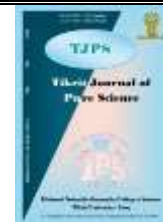




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ON Supra α - Compactness In Supra topological Spaces

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ABSTRACT

The purpose of this paper is to introduce the concept of strongly supra α - continuous function, perfectly supra α - continuous function and totally supra α - continuous function, The relationships among these functions are studied., and investigated some properties of them. Also we introduced the concepts of supra α - compact space, supra α - Lindelof spaces and countably supra α - compact spaces. Some basic properties are proved. At last the relationships among supra α - open, supra α - continuous maps and supra α - irresolute maps in supra topological spaces .

1. Introduction

$\gamma \subseteq \mu$, it implies that $\cup \gamma \in \mu$. The pair (X, μ) is called a supra topological space. Each element $A \in \mu$ is called a supra open set in (X, μ) and the complement of A denoted by $A^c = X - A$ is called a supra closed set in (X, μ) .

Definition 2.2. [1] Let (X, μ) be a supra topological space. The supra closure of a set A is denoted by supra $-cl(A) = \{B \subseteq X : B \text{ is a supra closed set in } X \text{ such that } A \subseteq B\}$.

The supra interior of a set A is denoted by supra $-Int(A)$ and is defined by supra $-Int(A) = \{U \subseteq X : U \text{ is a supra open set in } X \text{ such that } U \subseteq A\}$.

Definition 2.3. [1] Let $(X, \mathcal{T})$ be a topological space and μ be a supra topology on X . We call μ a supra topology associated with $\mathcal{T}$ if $\mathcal{T} \subseteq \mu$.

Definition 2.4. [2] Let (X, μ) be a supra topological space. A subset A of X is called a supra α - open set in X if $A \subseteq \text{supra} - Int [\text{supra} - cl [\text{supra} - Int (A)]]$. The complement of supra α - open set is called a supra α - closed set .

Definition 2.5. [2] Let (X, μ) be a supra topological space . The supra α - closure of a set A is denoted by supra $- \alpha - cl (A)$, and is defined as given in the following :

Supra $- \alpha - cl (A) = \{B \subseteq X : B \text{ is supra } \alpha - \text{closed set in } X \text{ such that } A \subseteq B\}$.

The supra α - interior of a set A is denoted by supra $- \alpha - Int (A)$, and is defined by supra

In 1983, Mashhour, A.S. et al [1] introduced the supra topological spaces and studied S - continuous maps and S^* - continuous maps. In 2008, Devi, R. et al [2] introduced and studied a class of sets and maps between topological spaces called supra α - open sets and $S\alpha$ - continuous maps. In 2008, Jassim, T.H. [3] came out with the concept of supra compactness in supra topological spaces. respectively. In 2012, Sekar, S. et al [4] introduced and investigated a new class of sets and function between topological spaces called supra I - open sets and supra I - continuous functions respectively. In 2013, Mustafa, J.M. [5] came out with the concept of supra b - compact and supra b - Lindelof spaces. In 2017, Krishnaveni and Vigneshwaran [6] came out with supra $b\mathcal{T}$ - closed sets and gave their properties. In 2018, Latif, R.M. [7] came out with the concept of supra I - compactness and supra I - connectdness. Now the study brings up with the new concepts of supra α - compact, countably supra α - compact, supra α - Lindelof spaces and present several properties and characteristics of these concepts.

2. Preliminaries

We recall some definitions which are needed in this work .

Definition 2.1. [1] Let X be a non-empty set and Let $\mu \subseteq P(X) = \{A : A \subseteq X\}$. Then μ is called a supra topology on X if $\varphi \in \mu, X \in \mu$ and for all

Theorem 3.9 Let $f: (X, \mu) \rightarrow (Y, \mu^*)$ be strongly supra α -continuous and $g: (Y, \mu^*) \rightarrow (Z, \mu^{**})$ be strongly supra α -continuous then their composition

Definition 3.1 A function $f: (X, \mu) \rightarrow (Y, \mu^*)$ is called i -supra α -continuous if $f^{-1}(V)$ of each

4. Supra α - Compactness

In this section, we present the concept of supra α -compactness and its properties .

Definition 4.1 A collection $\{A_i : i \in I\}$ of supra α -open sets in a supra topological space (X, μ) is called a supra α -open cover of a subset B of X if $B \subseteq \bigcup \{A_i : i \in I\}$ holds .

Definition 4.2 A supra topological space (X, μ) is called supra α -compact if every supra α -open cover of X has a finite sub cover .

Definition 4.3 A subset B of a supra topological space (X, μ) is said to be supra α -compact relative to (X, μ) if, for every collection $\{A_i : i \in I\}$ of supra α -open subsets of X such that $B \subseteq \bigcup \{A_i : i \in I\}$ there exists a finite subset I_0 of I such that $B \subseteq \bigcup \{A_i : i \in I_0\}$.

Definition 4.4 A subset B of a supra topological space (X, μ) is said to be supra α -compact if B is supra α -compact as a subspace of X .

Theorem 4.5 Every supra α -compact space (X, μ) is supra compact .

Proof : Let $\{A_i : i \in I\}$ be a supra open cover of X . Since every supra open set in X is a supra α -open set in X . So $\{A_i : i \in I\}$ is supra α -open cover of (X, μ) . Since (X, μ) is a supra α -compact. Therefore the supra α -open cover $\{A_i : i \in I\}$ of (X, μ) has a finite sub cover say $\{A_i : i = 1, 2, \dots, n\}$ for X . Hence (X, μ) is a supra compact space .

Theorem 4.6 Every supra α -closed subset of a supra α -compact space is supra α -compact with respect to X .

Proof : Let A be a supra α -closed subset of supra topological space (X, μ) . Then $A^c = X - A$ is supra α -open in (X, μ) . Let $S = \{A_i : i \in I\}$ be a supra α -open cover of A by supra α -open subsets in (X, μ) . Let $S^* = \{A_i : i \in I\} \cup \{A^c\}$ be a supra α -open cover of (X, μ) . That is $X = \bigcup S^* = (\bigcup \{A_i : i \in I\}) \cup A^c$. By hypothesis (X, μ) is supra α -compact and hence S^* is reducible to a finite sub cover of (X, μ) say $X = A_1 \cup A_2 \cup \dots \cup A_n \cup A^c$; But A and A^c are disjoint. Hence $A \subseteq A_1 \cup A_2 \cup \dots \cup A_n$; $A_i \in S$. Thus a supra α -open cover S of A contains a finite sub cover. Hence A is supra α -compact relative to (X, μ) .

Theorem 4.7 A supra α -continuous image of a supra α -compact space is supra compact .

Proof : Let $f : (X, \mu) \rightarrow (Y, \mu^*)$ be a supra α -continuous map from a supra α -compact space (X, μ) onto a supra topological space (Y, μ^*) . Let $\{A_i : i \in I\}$ be a supra open cover of (Y, μ^*) . Then $\{f^{-1}(A_i) : i \in I\}$ is a supra α -open cover of (X, μ) , as f is supra α -continuous. Since (X, μ) is supra α -compact, the supra α -open cover of (X, μ) , $\{f^{-1}(A_i) : i \in I\}$ has a finite sub cover say $\{f^{-1}(A_i) : i = 1, 2, \dots, n\}$. Therefore $X = \bigcup \{f^{-1}(A_i) : i = 1, 2, \dots, n\}$, which implies $f(X) = \bigcup \{A_i : i = 1, 2, \dots, n\}$, then $Y = \bigcup \{A_i : i = 1, 2, \dots, n\}$. That is $\{A_i : i = 1, 2, \dots, n\}$ is a finite sub

$gof : (X, \mu) \rightarrow (Z, \mu^{**})$ is strongly supra α -continuous function .

Proof : Let V be supra α -open set in (Z, μ^{**}) . Since g is strongly α -continuous, $g^{-1}(V)$ is supra open in (Y, μ^*) . We know that every supra open set is supra α -open set, $g^{-1}(V)$ is supra α -open in (Y, μ^*) . Since f is strongly α -continuous, $f^{-1}(g^{-1}(V))$ is supra open in (X, μ) , implies $(gof)(V)$ is supra open in (X, μ) . Therefore gof is strongly α -continuous .

Example 3.10 Let $X = Y = Z = \{a, b, c\}$, $\mu = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$, $\mu^* = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$, $\mu^{**} = \{\emptyset, Z, \{a, b\}, \{b, c\}\}$, $f : (X, \mu) \rightarrow (Y, \mu^*)$ be the function defined by $f(a) = b$, $f(b) = a$, $f(c) = c$. $g : (Y, \mu^*) \rightarrow (Z, \mu^{**})$ be a function defined by $g(a) = c$, $g(b) = b$, $g(c) = a$. Here f and g are strongly supra α -continuous and gof is supra open in (X, μ) . Then gof is strongly supra α -continuous function .

Theorem 3.11 If a function $f : (X, \mu) \rightarrow (Y, \mu^*)$ a strongly supra α -continuous then it is i -supra α -continuous but not conversely .

Proof : Let $f : (X, \mu) \rightarrow (Y, \mu^*)$ be strongly supra α -continuous function. Let G be a supra α -open set in (Y, μ^*) . Since f is strongly supra α -continuous, $f^{-1}(G)$ is supra open in (X, μ) . Since every supra open set is supra α -open. So every supra closed set is supra α -closed set, $f^{-1}(G)$ is α -open in (X, μ) . Hence f is i -supra α -continuous .

The converse of the above theorem need not be true as seen from the following example .

Example 3.12 Let $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$, $\mu = \{\emptyset, X, \{a\}\}$ and $\mu^* = \{\emptyset, Y, \{1\}, \{1, 2\}\}$. Let $f : (X, \mu) \rightarrow (Y, \mu^*)$ be a function defined by $f(a) = 1$, $f(b) = 2$ and $f(c) = 3$. Then f is i -supra α -continuous but not strongly supra α -continuous, since for the supra α -open set $\{1, 2\}$ in Y , $f^{-1}(\{1, 2\}) = \{a, b\}$ is not supra open in X .

Theorem 3.13 If a function $f : (X, \mu) \rightarrow (Y, \mu^*)$ is perfectly supra α -continuous then it is i -supra α -continuous but not conversely .

Proof : Let $f : (X, \mu) \rightarrow (Y, \mu^*)$ be perfectly supra α -continuous function. Let G be a supra α -open set in (Y, μ^*) . Since f is perfectly supra α -continuous, $f^{-1}(G)$ is both supra open and supra closed in (X, μ) . "Since every supra open set is supra α -open set and so $f^{-1}(G)$ is supra α -open in (X, μ) . Hence f is i -supra α -continuous .

The converse of the above theorem need not be true as seen from the following example .

Example 3.14 Let $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$, $\mu = \{\emptyset, X, \{a\}\}$ and $\mu^* = \{\emptyset, Y, \{1\}, \{1, 2\}\}$. Let $f : (X, \mu) \rightarrow (Y, \mu^*)$ be a function defined by $f(a) = 1$, $f(b) = 2$, and $f(c) = 3$. Then f is i -supra α -continuous but not perfectly supra α -continuous, since for the supra α -open set $\{1, 2\}$ in Y , $f^{-1}(\{1, 2\}) = \{a, b\}$ is not both supra open and supra closed in X .

$\{(X - A_i) : i \in I\}$ of X has a finite sub cover say $\{(X - A_i) : i = 1, 2, \dots, n\}$. This implies that $X = \bigcup \{(X - A_i) : i = 1, 2, \dots, n\}$, which implies $X = X - I\{A_i : i = 1, 2, \dots, n\}$, which implies $X - X = I\{A_i : i = 1, 2, \dots, n\}$, and which implies $\varphi = I\{A_i : i = 1, 2, \dots, n\}$. This disproves the assumption. Hence $I\{A_i : i \in I\} \neq \varphi$.

Conversely, suppose (X, μ) is not supra α -compact. Then there exist a supra α -open cover of (X, μ) say $\{G_i : i \in I\}$ having no finite sub cover. This implies that for any finite subfamily $\{G_i : i = 1, 2, \dots, n\}$ of $\{G_i : i \in I\}$, we have $\bigcup \{G_i : i = 1, 2, \dots, n\} \neq X$, which implies $X - (\bigcup \{G_i : i = 1, 2, \dots, n\}) \neq X - X$, hence $I\{X - G_i : i = 1, 2, \dots, n\} \neq \varphi$. Therefore the family $\{X - G_i : i \in I\}$ of supra α -closed sets has a finite intersection property. Then by assumption $I\{X - G_i : i \in I\} \neq \varphi$, which implies $X - (\bigcup \{G_i : i \in I\}) \neq \varphi$, so that $\bigcup \{G_i : i \in I\} \neq X$. This implies that $\{G_i : i \in I\}$ is not a cover of (X, μ) . This disproves the fact that $\{G_i : i \in I\}$ is a cover for (X, μ) . Therefore any supra α -open cover $\{G_i : i \in I\}$ of (X, μ) has a finite sub cover $\{G_i : i = 1, 2, \dots, n\}$. Hence (X, μ) is supra α -compact.

Theorem 4.12 Let $f: (X, \mu) \rightarrow (Y, \mu^*)$ is i -supra α -continuous and a subset B of X is supra α -compact relative to X . Then $f(B)$ is supra α -compact relative to Y .

Proof: Let $\{A_i : i \in I\}$ be a cover of $f(B)$ by supra α -open subsets of Y . Since f is i -supra α -continuous. Then $\{f^{-1}(A_i) : i \in I\}$ is a cover of B by supra α -open subsets of X . Since B is supra α -compact relative to X , so $\{f^{-1}(A_i) : i \in I\}$ has a finite subcover say $\{f^{-1}(A_1), f^{-1}(A_2), \dots, f^{-1}(A_n)\}$ for B . Then it implies that $\{A_i : i = 1, 2, \dots, n\}$ is a finite subcover of $\{A_i : i \in I\}$ for $f(B)$. So $f(B)$ is supra α -compact relative to Y .

5. Countably Supra α -Compactness

In this section, we present the concept of countably supra α -compactness and its properties.

Definition 5.1.[8] A supra topological space (X, μ) is said to be countably supra compact if every countable supra open cover of (X, μ) has a finite subcover.

Definition 5.2 A supra topological space (X, μ) is said to be countably supra α -compact if every countable supra α -open cover of X has a finite subcover.

Theorem 5.3 If (X, μ) is a countably supra α -compact space, then (X, μ) is countably supra compact.

Proof: Let (X, μ) be a countably supra α -compact. Let $\{A_i : i \in I\}$ be a countable supra open cover of (X, μ) . Since every supra open set in X is always supra α -open set in X . So $\{A_i : i \in I\}$ is a countable supra α -open cover of (X, μ) . Since (X, μ) is countable supra α -compact, so the countable supra α -open cover $\{A_i : i \in I\}$

cover of $\{A_i : i \in I\}$ for (Y, μ^*) . Hence (Y, μ^*) is supra compact.

Theorem 4.8 Let $f: (X, \mu) \rightarrow (Y, \mu^*)$ is strongly supra α -continuous map from a supra compact space (X, μ) onto a supra topological space (Y, μ^*) , then (Y, μ^*) is supra α -compact.

Proof: Let $\{A_i : i \in I\}$ be a supra α -open cover of (Y, μ^*) . Then $\{f^{-1}(A_i) : i \in I\}$ is a supra open cover of (X, μ) , since f is strongly supra α -continuous. Since (X, μ) is supra compact, the supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, μ) has a finite sub cover say $\{f^{-1}(A_i) : i = 1, 2, \dots, n\}$. Therefore $X = \bigcup \{f^{-1}(A_i) : i = 1, 2, \dots, n\}$, which implies $f(X) = \bigcup \{A_i : i = 1, 2, \dots, n\}$, so that $Y = \bigcup \{A_i : i = 1, 2, \dots, n\}$. That is $\{A_1, A_2, \dots, A_n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y, μ^*) . Hence (Y, μ^*) is supra α -compact.

Theorem 4.9 Let $f: (X, \mu) \rightarrow (Y, \mu^*)$ is perfectly supra α -continuous map from a supra compact space (X, μ) onto a supra topological space (Y, μ^*) , then (Y, μ^*) is supra α -compact.

Proof: Let $\{A_i : i \in I\}$ be a supra α -open cover of (Y, μ^*) . Then $\{f^{-1}(A_i) : i \in I\}$ is a supra open cover of (X, μ) , since f is perfectly supra α -continuous. Since (X, μ) is supra compact, the supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, μ) has a finite subcover say $\{f^{-1}(A_i) : i = 1, 2, \dots, n\}$. Therefore $X = \bigcup \{f^{-1}(A_i) : i = 1, 2, \dots, n\}$, which implies $f(X) = \bigcup \{A_i : i = 1, 2, \dots, n\}$, so that $Y = \bigcup \{A_i : i = 1, 2, \dots, n\}$. That is $\{A_1, A_2, \dots, A_n\}$ is a finite subcover of $\{A_i : i \in I\}$ for (Y, μ^*) . Hence (Y, μ^*) is supra α -compact.

Theorem 4.10 Let $f: (X, \mu) \rightarrow (Y, \mu^*)$ is i -supra α -continuous from a supra α -compact space (X, μ) onto a supra topological space (Y, μ^*) , then (Y, μ^*) is supra α -compact.

Proof: Let $\{A_i : i \in I\}$ be a supra α -open cover of (Y, μ^*) . Then $\{f^{-1}(A_i) : i \in I\}$ is a supra α -open cover of (X, μ) , since f is i -supra α -continuous. Since (X, μ) is supra α -compact, the supra α -open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, μ) has a finite sub cover say $\{f^{-1}(A_i) : i = 1, 2, \dots, n\}$. Therefore $X = \bigcup \{f^{-1}(A_i) : i = 1, 2, \dots, n\}$, which implies $f(X) = \bigcup \{A_i : i = 1, 2, \dots, n\}$, so that $Y = \bigcup \{A_i : i = 1, 2, \dots, n\}$. That is $\{A_1, A_2, \dots, A_n\}$ is a finite subcover of $\{A_i : i \in I\}$ for (Y, μ^*) . Hence (Y, μ^*) is supra α -compact.

Theorem 4.11 A supra topological space (X, μ) is supra α -compact if and only if every family of supra α -closed sets of (X, μ) having finite intersection property has a non-empty intersection.

Proof: Suppose (X, μ) is supra α -compact, Let $\{A_i : i \in I\}$ be a family of supra α -closed sets with finite intersection property. Suppose $\bigcap_{i \in I} A_i = \varphi$, then $X - I\{A_i : i \in I\} = X$. This implies $\bigcup \{(X - A_i) : i \in I\} = X$. Thus $\{(X - A_i) : i \in I\}$ is a supra α -open cover of (X, μ) . Then as (X, μ) is supra α -compact, the supra α -open cover

compact space (X, μ) onto a supra topological space (Y, μ^*) , then (Y, μ^*) is countably supra α -compact .
 Proof : Let $\{A_i : i \in I\}$ be a countable supra α -open cover of (Y, μ^*) . Since f is perfectly supra α -continuous map, $\{f^{-1}(A_i) : i \in I\}$ is countable supra open cover and countable supra closed cover of (X, μ) . Again since (X, μ) is countably supra compact space the countable supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, μ) has a finite subcover $\{f^{-1}(A_i) : i = 1, 2, \dots, n\}$. Therefore $X = \cup_{i=1}^n f^{-1}(A_i)$, which implies $f(X) = \cup_{i=1}^n (A_i)$. Then $Y = \cup_{i=1}^n (A_i)$ is finite sub cover of $\{A_i : i \in I\}$ for (Y, μ^*) . Hence (Y, μ^*) is countably supra α -compact .

Theorem 5.9 If $f: (X, \mu) \rightarrow (Y, \mu^*)$ is i -supra α -continuous from a countably supra α -compact space (X, μ) onto a supra topological space (Y, μ^*) , then (Y, μ^*) is countably supra α -compact .

Proof : Let $\{A_i : i \in I\}$ be a countable supra α -open cover of (Y, μ^*) . Since f is i -supra α -continuous, $\{f^{-1}(A_i) : i \in I\}$ is countable supra α -open cover of (X, μ) . Again since (X, μ) is countably supra α -compact space, the countable supra α -open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, μ) has a finite subcover $\{f^{-1}(A_i) : i = 1, 2, \dots, n\}$. Therefore $X = \cup_{i=1}^n f^{-1}(A_i)$, which implies $f(X) = \cup_{i=1}^n (A_i)$. Then $Y = \cup_{i=1}^n (A_i)$ is finite sub cover of $\{A_i : i \in I\}$ for (Y, μ^*) . Hence (Y, μ^*) is countably supra α -compact .

6. Supra α -Lindelof Space

In this section, we concentrate on the concept of supra α -Lindelof space and its properties .

Definition 6.1. [8] A supra topological space (X, μ) is said to be supra Lindelof space if every supra open covre of (X, μ) has a countable sub cover .

Definition 6.2 A supra topological space (X, μ) is said to be supra α -Lindelof space if every supra α -open cover of (X, μ) has a countable sub cover .

Theorem 6.3. Every supra α -Lindelof space (X, μ) is supra Lindelof space .

Proof: Let (X, μ) be a supra α -Lindelof space . Let $\{A_i : i \in I\}$ be a supra open cover of (X, μ) . Since every supra open set in X is always supra α -open set in X . Therefore $\{A_i : i \in I\}$ is supra α -open cover of (X, μ) . Since (X, μ) is supra α -Lindelof, so the supra α -open cover $\{A_i : i \in I\}$ of (X, μ) has a countable subcover say $\{A_i : i = 1, 2, \dots, n\}$ for X . Hence (X, μ) is a supra Lindelof space .

Theorem 6.4. If (X, μ) is supra α -Lindelof space, then $(X, \mathcal{T})$ is Lindelof space .

Proof : Let $\{A_i : i \in I\}$ be an open cover of X . Since every open set in X being a supra open set in X is also supra α -open set in X . Therefore $\{A_i : i \in I\}$ is a supra α -open cover of (X, μ) . Since (X, μ) is supra α -Lindelof, so the supra α -open cover $\{A_i : i \in I\}$ of (X, μ) has a countable subcover say $\{A_i : i = 1, 2, \dots, n\}$ for X . Hence $(X, \mathcal{T})$ is a Lindelof space .

has a finite sub cover say $\{A_i : i = 1, 2, \dots, n\}$ for X . Hence (X, μ) is a countably supra compact space.

Theorem 5.4 If (X, μ) is countably supra compact and every supra α -closed subset of X is supra closed in X , then (X, μ) is countably supra α -compact .

Proof : Let (X, μ) be a countably supra compact space . Let $\{A_i : i \in I\}$ be a countable supra α -open cover of (X, μ) . since every supra α -closed subset of X is supra closed in X . Thus every supra α -open set in X is supra open in X . Therefore $\{A_i : i \in I\}$ is a countable supra open cover of (X, μ) . Since (X, μ) is countably supra compact . So the countable supra open cover $\{A_i : i \in I\}$ of (X, μ) has a finite sub cover say $\{A_i : i = 1, 2, \dots, n\}$ for X . Hence (X, μ) is a countably supra α -compact space .

Theorem 5.5. Every supra α -compact space is countably supra α -compact .

Proof : Let (X, μ) be supra α -compact space . Let $\{A_i : i \in I\}$ be a countable supra α -open cover of (X, μ) . Since (X, μ) is supra α -compact, so the supra α -open cover $\{A_i : i \in I\}$ of (X, μ) has a finite sub cover say $\{A_i : i = 1, 2, \dots, n\}$ for (X, μ) . Hence (X, μ) is countably supra α -compact space .

Theorem 5.6 If $f: (X, \mu) \rightarrow (Y, \mu^*)$ is supra α -continuous map from a countably supra α -compact space (X, μ) onto a supra topological space (Y, μ^*) , then (Y, μ^*) is countably supra compact .

Proof : Let $\{A_i : i \in I\}$ be a countable supra open cover of (Y, μ^*) . Since f is supra α -continuous map, $\{f^{-1}(A_i) : i \in I\}$ is countable supra α -open cover of (X, μ) . Again since (X, μ) is countably supra α -compact space, then $\{f^{-1}(A_i) : i \in I\}$ of (X, μ) has a finite subcover $\{f^{-1}(A_i) : i = 1, 2, \dots, n\}$. Therefore $X = \cup_{i=1}^n f^{-1}(A_i)$. which implies $f(X) = \cup_{i=1}^n (A_i)$. Then $Y = \cup_{i=1}^n (A_i)$ is finite subcover of $\{A_i : i \in I\}$ for (Y, μ^*) . (Y, μ^*) is countably supra compact .

Theorem 5.7 If $f: (X, \mu) \rightarrow (Y, \mu^*)$ is strongly supra supra α -continuous map from a countably supra compact space (X, μ) onto a supra topological space (Y, μ^*) , then (Y, μ^*) is countable supra α -compact .

Proof : Let $\{A_i : i \in I\}$ be a countable supra α -open cover of (Y, μ^*) . Since f is strongly supra α -continuous map, $\{f^{-1}(A_i) : i \in I\}$ is countable supra open cover of (X, μ) . Again since (X, μ) is countable supra compact space, the countable supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, μ) has a finite subcover $\{f^{-1}(A_i) : i = 1, 2, \dots, n\}$. Therefore $X = \cup_{i=1}^n f^{-1}(A_i)$, which implies $f(X) = \cup_{i=1}^n (A_i)$. Then $Y = \cup_{i=1}^n (A_i)$ is finite subcover of $\{A_i : i \in I\}$ for (Y, μ^*) . Hence (Y, μ^*) is countably supra α -compact .

Theorem 5.8 If $f: (X, \mu) \rightarrow (Y, \mu^*)$ is perfectly supra α -continuous map from a countably supra

α – Lindelof space, the supra α – open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, μ) has a countable subcover $\{f^{-1}(A_i) : i = 1, 2, \dots, n\}$. Therefore $X = \bigcup_{i=1}^n f^{-1}(A_i)$, which implies $f(X) = \bigcup_{i=1}^n f(f^{-1}(A_i))$. Then $Y = \bigcup_{i=1}^n A_i$ is a countable sub cover of $\{A_i : i \in I\}$ for (Y, μ^*) . Hence (Y, μ^*) is supra α – Lindelof.

Theorem 6.9 If $f: (X, \mu) \rightarrow (Y, \mu^*)$ is perfectly supra α – continuous map from a supra Lindelof space (X, μ) onto a supra topological space (Y, μ^*) , then (Y, μ^*) is supra α – Lindelof.

Proof: Let $\{A_i : i \in I\}$ be a supra α – open cover of (Y, μ^*) . Since f is perfectly supra α – continuous map, $\{f^{-1}(A_i) : i \in I\}$ is supra open cover and supra closed cover of (X, μ) . Again since (X, μ) is supra Lindelof space, the supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, μ) has a countable sub cover $\{f^{-1}(A_i) : i = 1, 2, \dots, n\}$. Therefore $X = \bigcup_{i=1}^n f^{-1}(A_i)$, which implies $f(X) = \bigcup_{i=1}^n f(f^{-1}(A_i))$. Then $Y = \bigcup_{i=1}^n A_i$ is a countable sub cover of $\{A_i : i \in I\}$ for (Y, μ^*) . Hence (Y, μ^*) is supra α – Lindelof.

Theorem 6.10 If (X, μ) is supra α – Lindelof space and countably supra α – compact space, then (X, μ) is supra α – compact space.

Proof: Suppose (X, μ) is supra α – Lindelof space and countably supra α – compact space. Let $\{A_i : i \in I\}$ be a supra α – open cover of (X, μ) . Since (X, μ) is supra α – Lindelof space, $\{A_i : i \in I\}$ has a countable sub cover say $\{A_i : i \in I_0\}$ for some $I_0 \subseteq I$ and I_0 is countable. Therefore $\{A_i : i \in I_0\}$ is a countable supra α – open cover of (X, μ) . Again, since (X, μ) is countably supra α – compact space $\{A_i : i \in I_0\}$ has a finite sub cover and say $\{A_i : i = 1, 2, \dots, n\}$. Therefore $\{A_i : i = 1, 2, \dots, n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (X, μ) . Hence (X, μ) is a supra α – compact space.

Theorem 6.11. If $f: (X, \mu) \rightarrow (Y, \mu^*)$ is i – supra α – continuous and a subset B of X is supra α – Lindelof relative to X , then $f(B)$ is supra α – Lindelof relative to Y .

Proof: Let $\{A_i : i \in I\}$ be a cover of $f(B)$ by supra α – open subsets of Y . By hypothesis f is supra α – irresolute and so $\{f^{-1}(A_i) : i \in I\}$ is a cover of B by supra α – open subsets of X . Since B is supra α – Lindelof relative to X , $\{f^{-1}(A_i) : i \in I\}$ has a countable subcover say $\{f^{-1}(A_i) : i \in I_0\}$ for B , where I_0 is a countable subset of I . Now $\{A_i : i \in I_0\}$ is a countable subcover of $\{A_i : i \in I\}$ for $f(B)$. So $f(B)$ is supra α – Lindelof relative to Y .

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Theorem 6.5. Every supra α – compact space is supra α – Lindelof space.

Proof: Let (X, μ) be a supra α – compact space. Let $\{A_i : i \in I\}$ be a supra α – open cover of (X, μ) . Since (X, μ) is supra α – compact space. Then $\{A_i : i \in I\}$ has a finite sub cover say $\{A_i : i = 1, 2, \dots, n\}$. Since every finite sub cover is always countable sub cover and therefore $\{A_i : i = 1, 2, \dots, n\}$ is a countable sub cover of $\{A_i : i \in I\}$. Hence (X, μ) is a supra α – Lindelof space.

Theorem 6.6 If $f: (X, \mu) \rightarrow (Y, \mu^*)$ is supra α – continuous map from a supra α – Lindelof space (X, μ) onto a supra topological space (Y, μ^*) then (Y, μ^*) is supra Lindelof space.

Proof: Let $\{A_i : i \in I\}$ be a supra open cover of (Y, μ^*) . Then $\{f^{-1}(A_i) : i \in I\}$ is a supra α – open cover of (X, μ) , as f is supra α – continuous. Since (X, μ) is supra α – Lindelof space, so the supra α – open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, μ) has a countable sub cover say $\{f^{-1}(A_i) : i \in I_0\}$ for some $I_0 \subseteq I$ and I_0 is countable. Therefore $X = \bigcup \{f^{-1}(A_i) : i \in I_0\}$, which implies $f(X) = \bigcup \{A_i : i \in I_0\}$, then $Y = \bigcup \{A_i : i \in I_0\}$. That is $\{A_i : i \in I_0\}$ is a countable sub cover of $\{A_i : i \in I\}$ for (Y, μ^*) . Hence (Y, μ^*) is a supra Lindelof space.

Theorem 6.7 If $f: (X, \mu) \rightarrow (Y, \mu^*)$ is i – supra α – continuous from a supra α – Lindelof space (X, μ) onto a supra topological space (Y, μ^*) , then (Y, μ^*) is supra α – Lindelof space.

Proof: Let $\{A_i : i \in I\}$ be a supra α – open cover of (Y, μ^*) . Then $\{f^{-1}(A_i) : i \in I\}$ is a supra α – open cover of (X, μ) . Since f is i – supra α – continuous. As (X, μ) is supra α – Lindelof space, so the supra α – open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, μ) has a countable sub cover say $\{f^{-1}(A_i) : i \in I_0\}$ for some $I_0 \subseteq I$ and I_0 is countable. Therefore $X = \bigcup \{f^{-1}(A_i) : i \in I_0\}$, which implies $f(X) = \bigcup \{A_i : i \in I_0\}$, so that $Y = \bigcup \{A_i : i \in I_0\}$. That is $\{A_i : i \in I_0\}$ is a countable sub cover of $\{A_i : i \in I\}$ for (Y, μ^*) . Hence (Y, μ^*) is a supra α – Lindelof space.

Theorem 6.8 If $f: (X, \mu) \rightarrow (Y, \mu^*)$ is strongly supra α – continuous map from a supra α – Lindelof space (X, μ) onto a supra topological space (Y, μ^*) , then (Y, μ^*) is supra α – Lindelof.

Proof: Let $\{A_i : i \in I\}$ be a supra α – open cover of (Y, μ^*) . Since f is strongly supra α – continuous map, $\{f^{-1}(A_i) : i \in I\}$ is supra open cover of (X, μ) , which implies $\{f^{-1}(A_i) : i \in I\}$ is supra α – open cover of (X, μ) . Again since (X, μ) is supra

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حول التراص الفوقي من النمط α في الفضاءات التبولوجية الفوقية

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الملخص

الغرض من البحث هو تقديم مفهوم الدالة المستمرة الفوقية بقوة من النمط α والدالة المستمرة الفوقية التامة من النمط α والدالة المستمرة الفوقية الكلية من النمط α . قد درست العلاقات بين هذه الدوال وتحرينا بعض خواصها. وكذلك قدمنا مفهوماً الفضاء المرصوص الفوقي من النمط α والفضاء الندلوفي الفوقي من النمط α والفضاءات المرصوصة المعدودة الفوقية من النمط α وبعض الخصائص الأساسية قد برهنت. وأخيراً، العلاقات بين الدوال المفتوحة الفوقية من النمط α والدوال المستمرة الفوقية من النمط α والدوال المحيرة الفوقية من النمط α في الفضاءات التبولوجية الفوقية قد درست.