DESIGN AND SIMLATION OF ROBUST FUZZY LOGIC CONTROLLER FOR PLANTS WITH HIGH NONLINEARITY+

تصميم ومحاكاة مسيطر المنطق الضبابي القوي للمنظومات ذات اللاخطية العالية

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Abstract:

The Inverted pendulum (IP) is a very popular plant for testing dynamics and control of highly non – linear plants. In the inverted pendulum control problem, the aim is to move the cart to the desired position and to balance a pendulum without the pendulum falling.

The goal of this paper is to design a robust Fuzzy Logic Controller (FLC) which is capable of driving the cart of the Inverted Pendulum to the desired position and balancing the ball around zero angles with respect to the vertical plane.

To accomplish this task several steps are passed through. First, a state - space model of the Inverted Pendulum has been analyzed and then builds using Simulink Software Package. Next, design of the FLC is performed using Fuzzy Inference System (FIS) within Matlab software package. Robustness of the designed control system has been tested for the response to different types of command signals and disturbance rejection points of view.

Simulation results show that the suggested FLC is robust against different environments conditions.

المستخلص:

البندول المعكوس هو المنظومة الشائعة الاستخدام في اختبار الديناميكية والسيطرة للمنظومات ذات اللاخطية العالية. تهدف السيطرة على البندول المعكوس الى تحريك العربة الى الموقع المطلوب مع موازنة البندول وعدم سقوطه.

الهدف من هذا البحث هو تصميم مسيطر المنطق الضبابي والذي يكون قادر على قيادة العربة للبندول المعكوس الى الموقع المطلوب مع موازنة الكرة ضمن زاوية الصفر نسبة الى المستوى العمودي.

انجاز هذه المهمة يتطلب المرور عبر عدة خطوات. اولا تم تحليل نموذج فضاء الحالة للبندول المعكوس ومن ثم بناءه باستخدام برنامج السميلنك. بعد ذلك تم تصميم مسيطر المنطق الضبابي باستخدام برنامج نظام الاستدلال الضبابي الموجود ضمن حزمة الماتلاب. تم اختبار متانة منظومة السيطرة من ناحية استجابتها لاشارات اوامر مختلفة ورفض الاضطرابات.

نتائج المحاكاة أظهرت متانة مسيطر المنطق الضبابي المقترح في مواجهة الشروط المحيطة المختلفة.

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Introduction:

Being an under-actuated mechanical system and inherently open loop unstable with highly non-linear dynamics, the IP system is a perfect test-bed for the design of a wide range of classical and contemporary control techniques. Its applications range widely from robotics to space rocket guidance systems. [1].

Modern control theory has made a modest inroad into practice, FLC has been rapidly gaining popularity among practicing engineers. This increased popularity can be attributed to the fact that fuzzy logic provides a powerful vehicle that allows engineers to incorporate human reasoning in the control algorithm. One of the most important problems while designing FLC is to derive the desired fuzzy rule base. Trial and error has been a natural choice to design FLC [2].

The strategy of the swing-up control of the pendulum and the stabilizing control of the whole system that consists of angular control of the pendulum at upright position and position control of the cart at origin of rail. First, swing up control is to bring the pendulum from the downwards position to the upright position. Thereafter, stabilizing control is to balance the pendulum in the upright position [3].

Until now, a lot of intelligent approaches about the swing-up and stabilizing control of inverted pendulum system have been proposed.

M. Bugeja [1] presents the design and implementation of a complete control system for the swing-up and stabilizing control of an inverted pendulum, based on feedback linearization and energy considerations. The controller exhibits stability for both distance and angle but the transient response is considerably long.

A.A. Saifizul et. al. [3], analyze the mathematical models of cart and single IP system. Then, the Position-Velocity controller is designed to swing up the pendulum considering physical behavior. For stabilizing the inverted pendulum, Adaptive Neuro-Fuzzy Inference System (ANFIS) architecture is used to guarantee stability at unstable equilibrium position. Simulation and experimental results show that the hybrid controllers take about 3.5 sec and 4 swings to bring the pendulum close to upright position.

Taku Komura et. al. [4], proposed a new method to simulate human gait motion when muscles are weakened. The method is based on the enhanced version of three-dimensional linear inverted pendulum model that is used for generation of gait in robotics.

Kent H. Lundberg et. al. [5], analyze and compare a dual-IP system with the single-IP system using classical linear methods. A classical controller is designed that stabilizes the pendulum in the inverted position with the cart at the center of the track. Simulations of the transient response to initial conditions are presented. Intuitive reasoning and an insightful approach to the control design are major emphases of this effort.

Osamu Fujita et. al. [6], presents a neural network approach for controlling nonlinear system. Trial-and-error correlation learning, which is a generally useful method for optimizing parameters, is applied to training a neural controller to balance an inverted pendulum. The controller is simplified by automatically pruning the hidden neurons to only two. Computer simulation shows that the trained neural controller can control the pendulum to recover quickly from a large deviation and to stabilize in a desired position with high accuracy.

This paper aims to design a FLC for driving the cart of the IP and stabilizing the pole in the upright position based on Simulink that unifying the theoretical and simulation aspects of the problem, along with simulation results demonstrating the effectiveness of the complete control system for different environment conditions.

This paper is organized as follows: Section 2 presents a brief overview of the complete system. Section 3 deals with the mathematical dynamic model of the IP used for the computer simulations. Section 4 goes through the principles of the FLC. The main steps in the design and implementation of the control algorithms using Fuzzy Inference System (FIS) was presented in section 5. Section 6 presents simulation results, outlining the system response to different environment conditions. Section 7 presents a conclusion.

System Overview:

The principle scheme of IP control system considered in this paper is shown in Figure 1.

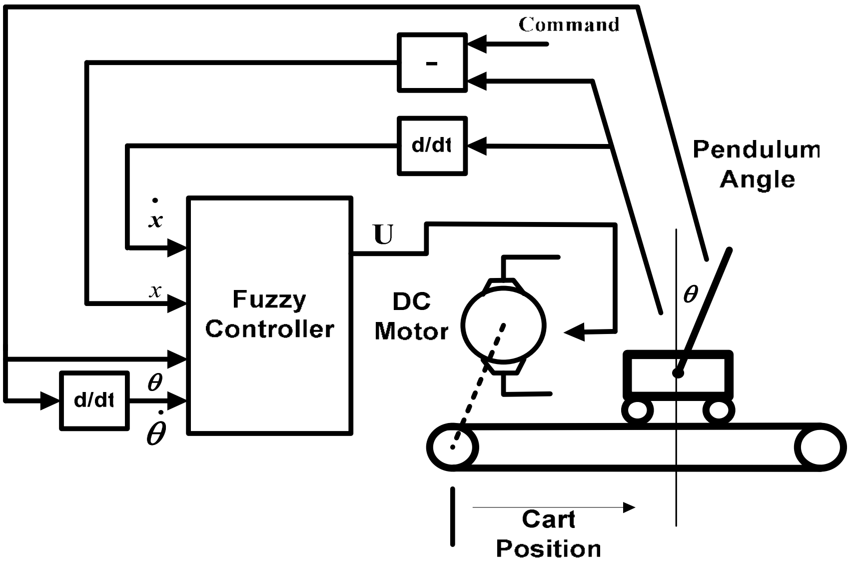

Figure 1. Inverted Pendulum Control System

The control system for the process consists of the following signals:

Process variables (inputs to the controller):

- Actual cart position [m] (signal name: 'x')
- Cart speed [m/s] (signal name: '$\dot{x}$')
- Cart position command signal [m] (signal name: 'x_ref')
- Actual angle of the pendulum pole measured from the upright position [rad] (signal name: θ).
- Angular speed of the pendulum pole [rad/s] (signal name: '$\dot{\theta}$')

Control variables (outputs from the controller):

- Control Signal (signal name: 'U').

Inverted Pendulum Model:

1 Mathematical Model

Given an inverted pendulum mounted on a motor driven cart as shown in Figure 1, the defining nonlinear equations can be derived as follows. It is assumed that the rod is mass less and that the cart mass and the point mass at the upper end of the inverted pendulum are denoted as M and m, respectively. There is an externally x-directed force on the cart, U, and a gravity force acts on the point mass at all times.

A force balance in the x-direction gives that the mass times acceleration of the cart plus the mass times the x-directed acceleration of the point mass must equal the external force on the system.

This can be written as [7,8]:

$$M\frac{d^2}{dt^2}x + m\frac{d^2}{dt^2}x_g = U \quad (1)$$

where the time-dependent center of gravity of the point mass is given by the coordinates, $(x_g,\ y_g)$. For the point mass assumed here, the location of the center of gravity of the pendulum mass is simply:

$$x_g = x + l\ sin\theta \quad and \quad y_g = l\ cos\theta \quad (2)$$

where l is the pendulum rod length, g is the gravity acceleration. Substitution of equation (2) into (1) gives:

$$M\frac{d^2}{dt^2}x + m\frac{d^2}{dt^2}(x + l\ sin\theta) = U \quad (3)$$

Therefore:

$$(M+m)\ \ddot{x} - \ ml\sin\theta\ \dot{\theta}^2 + ml\ cos\theta\ddot{\theta} = U \quad (4)$$

In a similar manner, performing a torque balance on the system is needed, where torque is the product of the perpendicular component of the force and the distance to the pivot point (lever arm length, l). In this case, the torque on the mass due to the acceleration force is balanced by the torque on the mass due to the gravity force. The force components are identified in Figure 2 and the resultant balance can be written as:

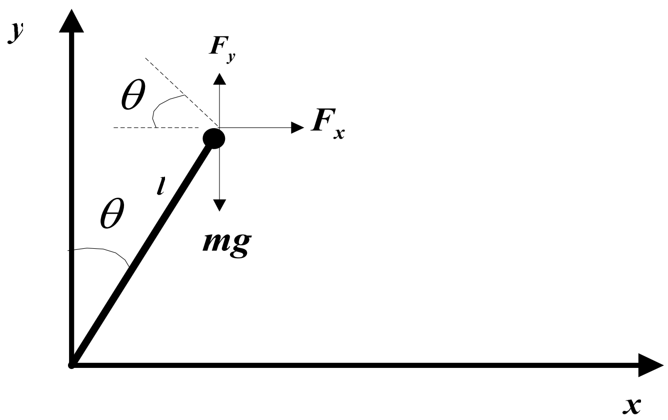

Figure 2 Vector diagram for force components in torque balance.

$$(F_x cos\theta)l - (F_y sin\theta)l = (mg\ sin\theta)l \quad (5)$$

Where:

$$F_x = m[\ddot{x} - lsin\theta\dot{\theta}^2 + lcos\theta\ddot{\theta}]$$

$$F_y = -m[lcos\theta\dot{\theta}^2 + lsin\theta\ddot{\theta}]$$

Substituting these expressions into equation (5) gives:

$$m\ddot{x}cos\theta + ml\ddot{\theta} = mgsin\theta \quad (6)$$

To numerically simulate the nonlinear model for the inverted pendulum it is needed to put it into standard state form,

$$\frac{d}{dt}\underline{z} = \underline{f}(\underline{z},u,t) \qquad (7)$$

To put equations (4) and (6) into this form, first manipulate the equations algebraically to only have a single second derivative term in each equation. From equation (6):

$$ml\ddot{\theta} = mgsin\theta - m\ddot{x}cos\theta$$

and putting this into equation (4) gives

$$(M+m)\ddot{x} - mlsin\theta\,\dot{\theta}^2 + mgcos\theta sin\theta - m\ddot{x}cos^2\theta = U$$

Or

$$(M+m-mcos^2\theta)\ddot{x} = U + mlsin\theta\dot{\theta}^2 - mgcos\theta sin\theta \qquad (8)$$

Similarly, from equation (6)

$$\ddot{x} = \frac{gsin\theta - l\ddot{\theta}}{cos\theta}$$

and putting this into equation (4) gives:

$$(M+m)\left(gsin\theta - l\ddot{\theta}\right) - mlcos\theta sin\theta\dot{\theta}^2 + mlcos^2\theta\ddot{\theta} = Ucos\theta$$

and

$$(mlcos^2\theta - (M+m)l)\ddot{\theta} = Ucos\theta - (M+m)gsin\theta + mlcos\theta sin\theta\,\dot{\theta}^2 \qquad (9)$$

Finally, dividing by the lead coefficients of equations (8) and (9) gives

$$\ddot{x} = \frac{U + mlsin\theta\,\dot{\theta}^2 - mgcos\theta sin\theta}{M+m-mcos^2\theta} \qquad (10)$$

$$\ddot{\theta} = \frac{Ucos\theta - (M+m)gsin\theta + ml(cos\theta sin\theta)\dot{\theta}}{mlcos^2\theta - (M+m)l} \qquad (11)$$

The standard state space form for nonlinear differential equations is

$$\dot{x} = f(x, U, t)$$

Therefore, the final state space model of the inverted pendulum is:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{U + ml(\sin(x_3)\,x_4^2 - mg\cos(x_3)\sin(x_3)}{M+m-mcos^2x_3} \\ x_4 \\ \dfrac{Ucosx_3 - (M+m)gsinx_3 + ml(cosx_3sinx_3)x_4}{mlcos^2x_3 - (M+m)l}\end{bmatrix} \qquad (12)$$

Where x_1 is the position of the cart (x), x_2 is the velocity of the cart $(\dot{x})$, x_3 is the angle of the pole (θ), x_4 is the angular velocity of the pole $(\dot{\theta})$. The control force is applied to the cart to prevent the pole from falling while keeping the cart within the specified bounds on the track.

2 Simulink Model:

According to equation 12, the Simulink model of the IP can be implemented from the basic building blocks provided by Simulink software package as shown in Figure 3.

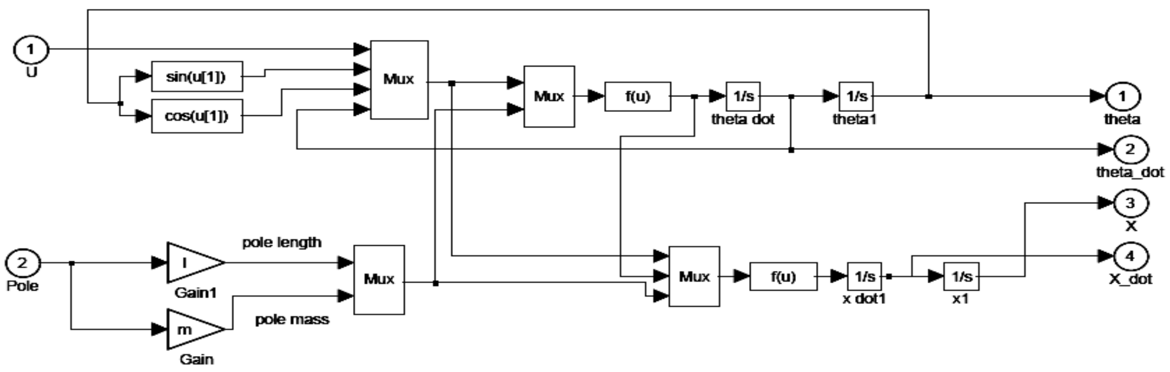

Figure 3. Simulink model of the Inverted Pendulum Based on Equation 12.

. The cart $l\&m$The inputs to the IP model are, the control signal U and the IP parameters mass M is included within the function block f(u). of equation 12 and the parameters of $\dot{x}_4$ and $\dot{x}_2$The function blocks contains the states these states are received from the multiplexers (MUX). The following numerical parameters has been used:

$l = 0.2m, \quad M = 1Kg, \quad m = 0.1Kg, \quad g = 9.8\ N/m^2$

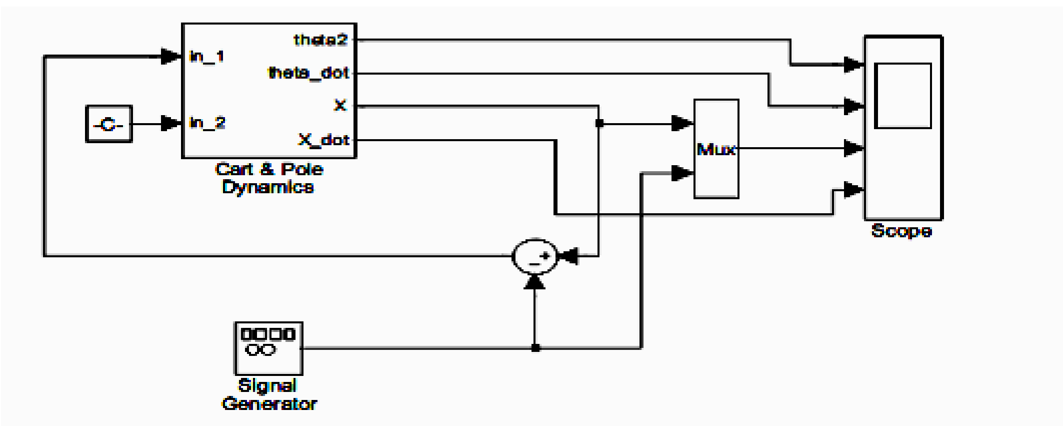

Figure 4. Simulink model of the Inverted Pendulum in closed loop without controller

Figure 4 shows the Inverted pendulum in closed loop system without controller. When a unit step command signal is applied to the closed loop system from signal generator, the time response shown in figure 5 has been produced.

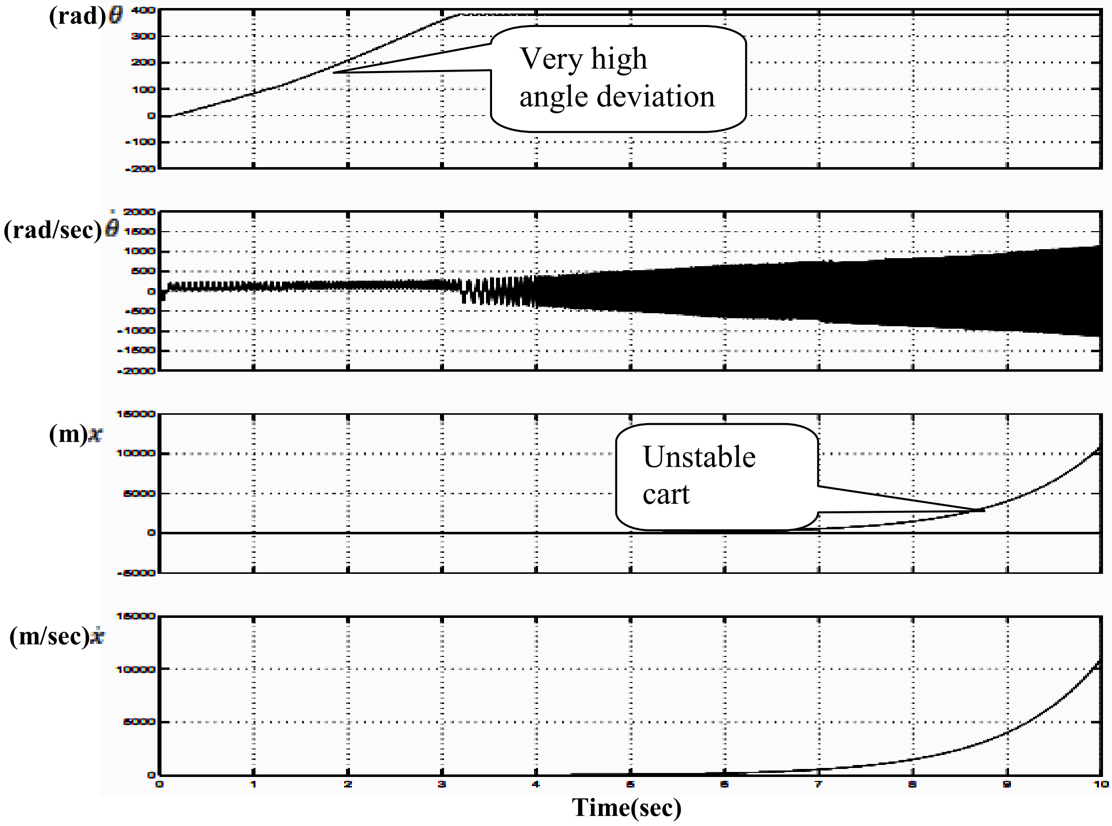

Figure 5. Closed loop System response to the unit step input.

From figure 5 it can be notice that the cart position is unstable and high fluctuation of the pendulum angle. Practically, the cart will be stop when it reaches the end of the track by disabling the limit switch which turning the DC motor OFF and the pole will downright position.

Fuzzy Logic Controller:

FLC is a simple and effective technique for the control of nonlinear systems [8]. In particular, the FLC is well suited to model-based nonlinear control. In the model-based fuzzy control, the dynamics of a nonlinear system is represented as a fuzzy blending of multiple local models. Then, a fuzzy controller is designed such that not only the stabilities of the corresponding local closed-loop systems but also the stability of the overall closed-loop system is guaranteed.

A block diagram of a fuzzy control system is shown in Figure 6. The fuzzy controller is composed of the following four elements [2]:

- A rule-base (a set of if-then rules), which contains a fuzzy logic quantification of the expert's linguistic description of how to achieve good control.
- An inference mechanism (also called a fuzzy inference engine), which emulates the expert's decision making in interpreting and applying knowledge about how best to control the plant.
- A fuzzification interface, which converts controller inputs into information that the inference mechanism can easily use to activate and apply rules.
- A defuzzification interface, which converts the conclusions of the inference mechanism in to actual inputs for the process.

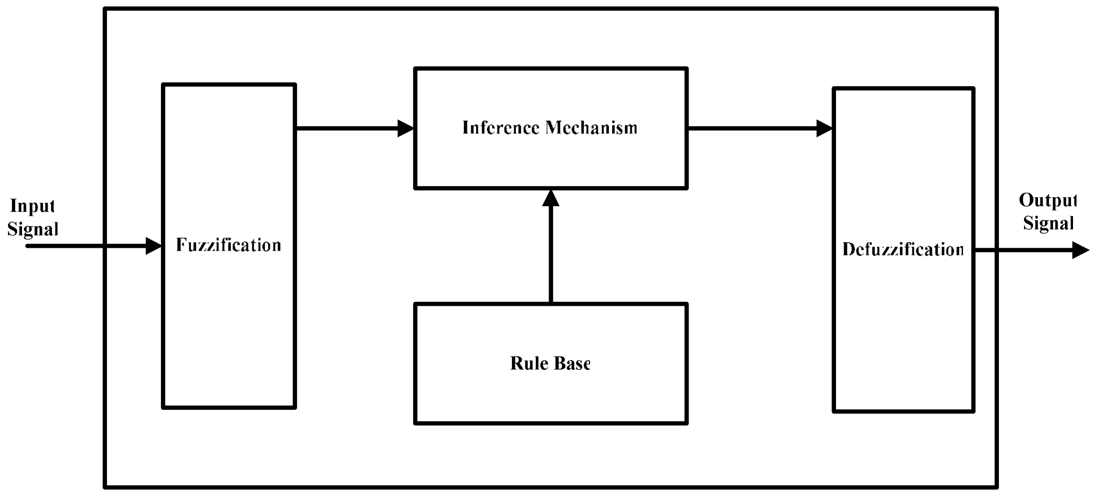

Figure 6. Block diagram of (SISO) Fuzzy controller.

For choosing the inputs and outputs of fuzzy controller, the controller is to be designed to automate how a human expert who is successful for controlling the system.

Design of Control System:

Figure 7 shows the Simulink model of the control system, the IP represented by the Cart&Pole Dynamics block which includes the Simulink model of the IP showed in Figure 3.

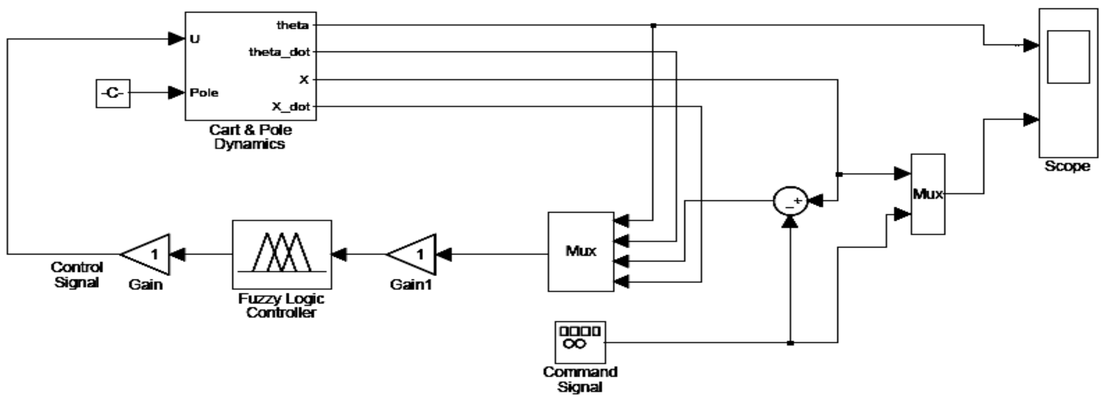

Figure 7. Simulink model of the Inverted Pendulum controlled by FLC

Therefore, FLC contains 4 – inputs and one output. Sugeno FIS has been used with linear functions for defuzzification process, the membership functions for the fuzzification of the inputs and their limits are shown in Figure 8.

The relationship input and output of the FLC is described by 16 Fuzzy – if – then rules which are represented in the following form:

Rule i: if (theta is A_j) and (theta_dot is B_j) and (x is C_j) and (x_dot is D_j) then (U is out_k)

Where:

$$j = 1, 2 \text{ and } i, k = 1, 2, \ldots\ldots, 16$$

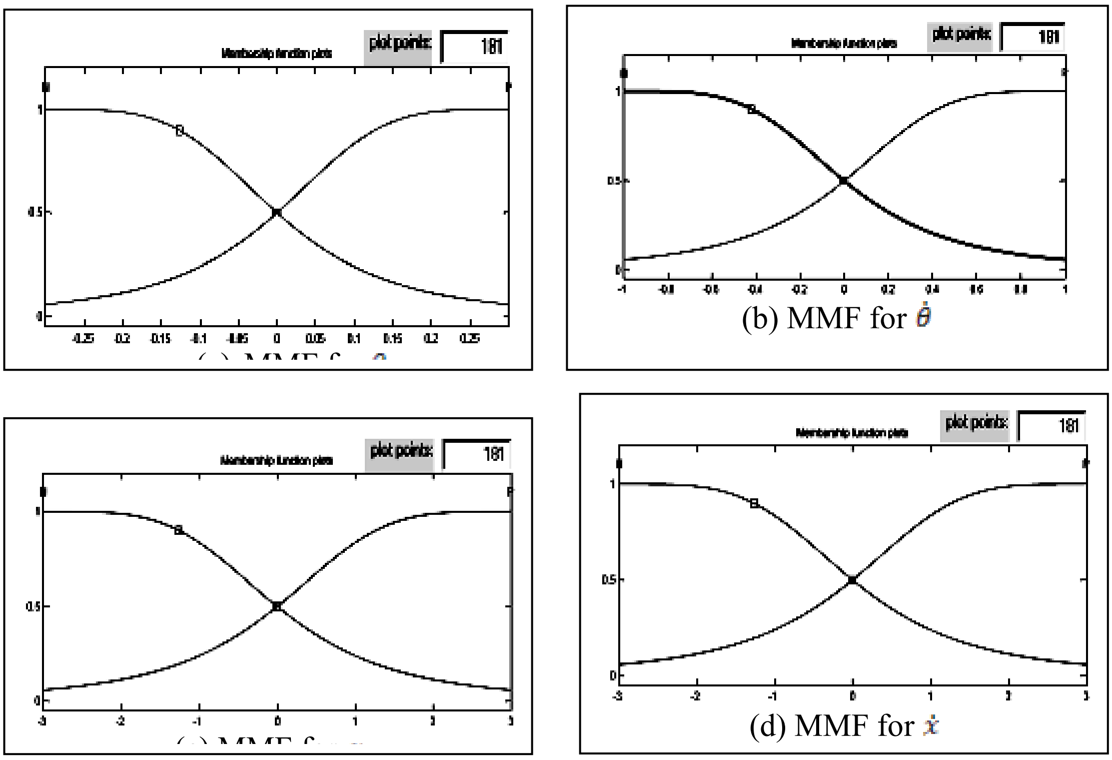

Figure 8. Membership functions for Fuzzy input variables

The rule – base set shown in table1 has been obtained by trial and error iterations:

Table 1: Rule - Base

Rule no.	θ **If**	$\dot{\theta}$ **and**	x **and**	$\dot{x}$ **and**	**Then U**
1	NG	NG	NG	NG	U1
2	NG	NG	NG	PS	U2
3	NG	NG	PS	NG	U3
4	NG	NG	PS	PS	U4
5	NG	PS	NG	NG	U5
6	NG	PS	NG	PS	U6
7	NG	PS	PS	NG	U7
8	NG	PS	PS	PS	U8
9	PS	NG	NG	NG	U9
10	PS	NG	NG	PS	U10
11	PS	NG	PS	NG	U11
12	PS	NG	PS	PS	U12
13	PS	PS	NG	NG	U13
14	PS	PS	NG	PS	U14
15	PS	PS	PS	NG	U15
16	PS	PS	PS	PS	U16

Simulation Results:

1 System Response to unit step input:

The time response of the closed loop control system to the unit step input is shown in figure 9.

Comparing figure 9 with figure 5, it can be shown that employing of FLC in controlling inverted pendulum improved the performance of the system from cart position response and stabilizing pendulum angle at upright position points of view.

Table 2 shows the comparison of the time response parameters and integral of the square error (ISE) of the closed loop system with and without controller.

Table 2: Response Parameters

Controller	ISE	Time response prameters			
		t_r(sec)	t_p(sec)	t_s(sec)	M_p%
Without controller	∞	Un observable	∞	Not reach	∞
With FLC	1.1373	1.68	3.36	4.64	9.07%

The ISE is obtained by integration of the square error using backward trapezoidal method. Table 2 is obtained by collecting the data of simulation in .mat file format.

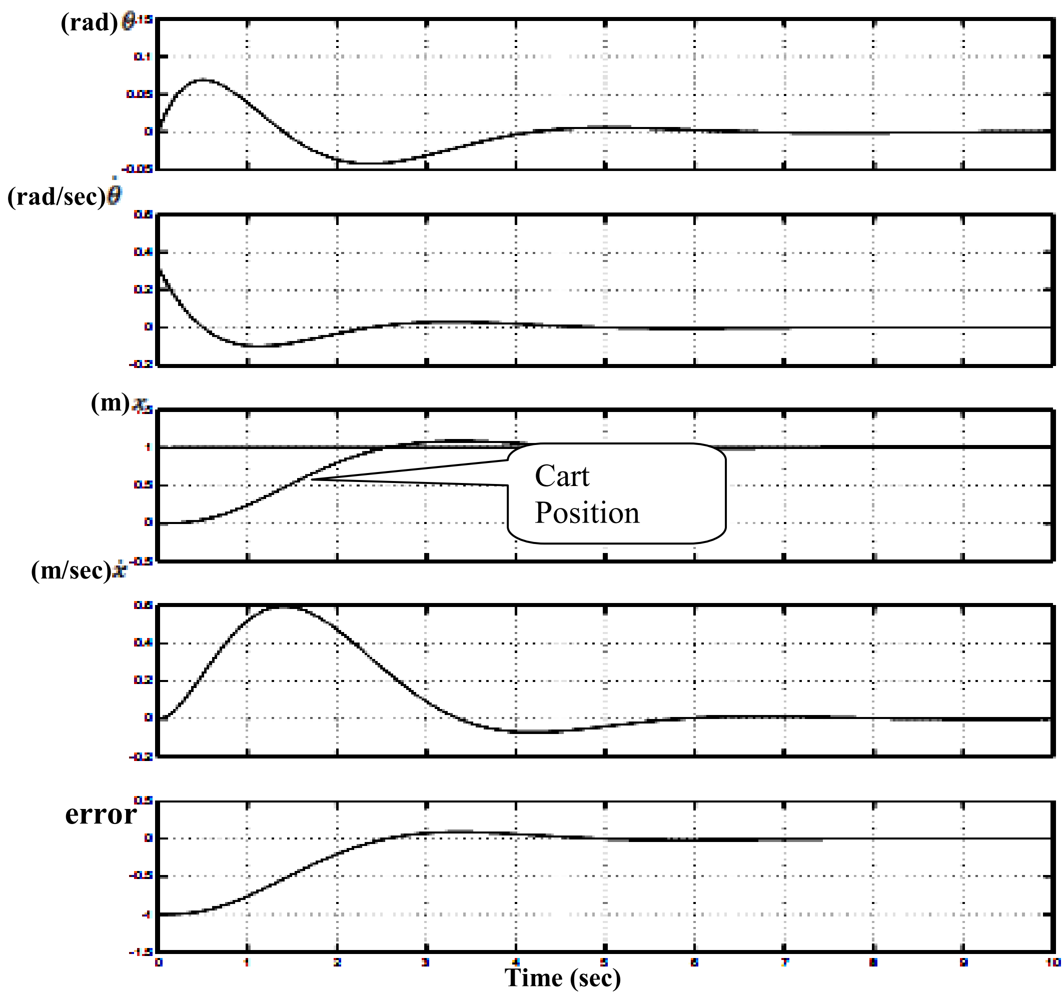

Figure 9. Closed loop control System response to the unit step input.

2 Different Command signals:

In order to test the robustness of the designed control system, simulation task has been performed for different command signals

Figure 10 shows the simulation results of the control system responding to the square command signal. The upper part of the Figure represents the stabilization process of the IP angle, while the lower part of the Figure represents the cart position response to the command signal. It can be seen from Figure 10 that the cart perform dual tasks, the

first task is to keep track to the variations of command signal. This task was achieved with delay time of 1 sec, rise time of 0.5 sec, peak overshoot of 5% and steady state time of 5 sec with zero steady state error. The second task is to balance the angle of the IP, and this task is achieved with low angle variations at the transient.

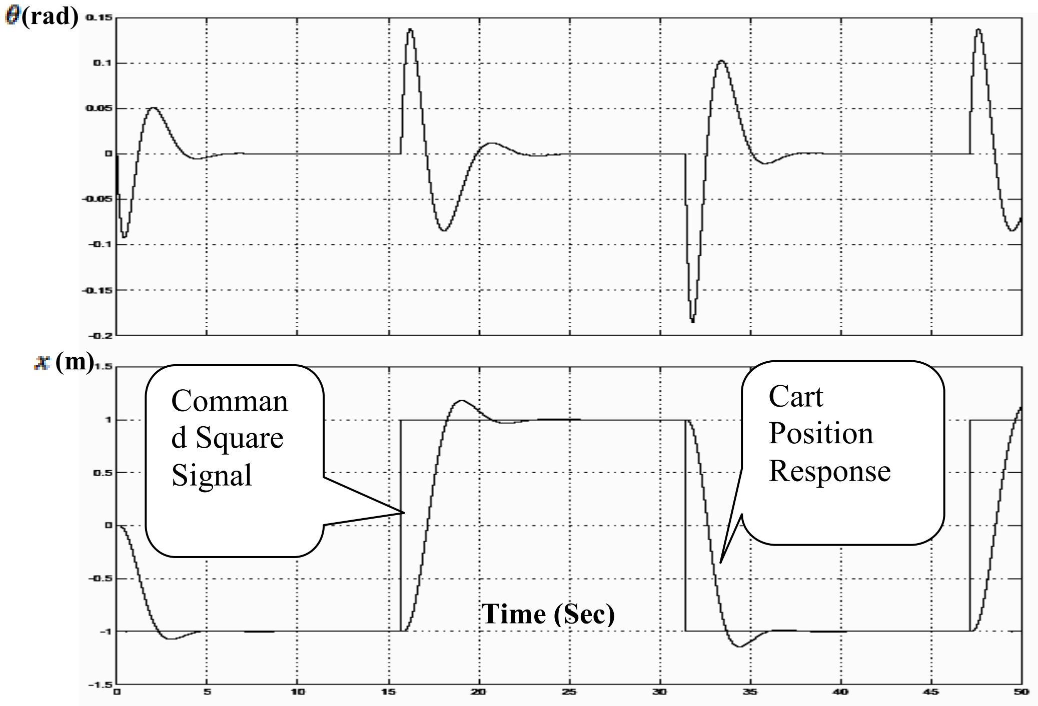

Figure 10. Control System response to the square wave.

Figure 11 represents simulation results of the control system to the saw tooth command signal. The cart position response to this signal with time delay of 1 sec, it can be seen that the variations of angle is acceptable in transient as shown in the Figure.

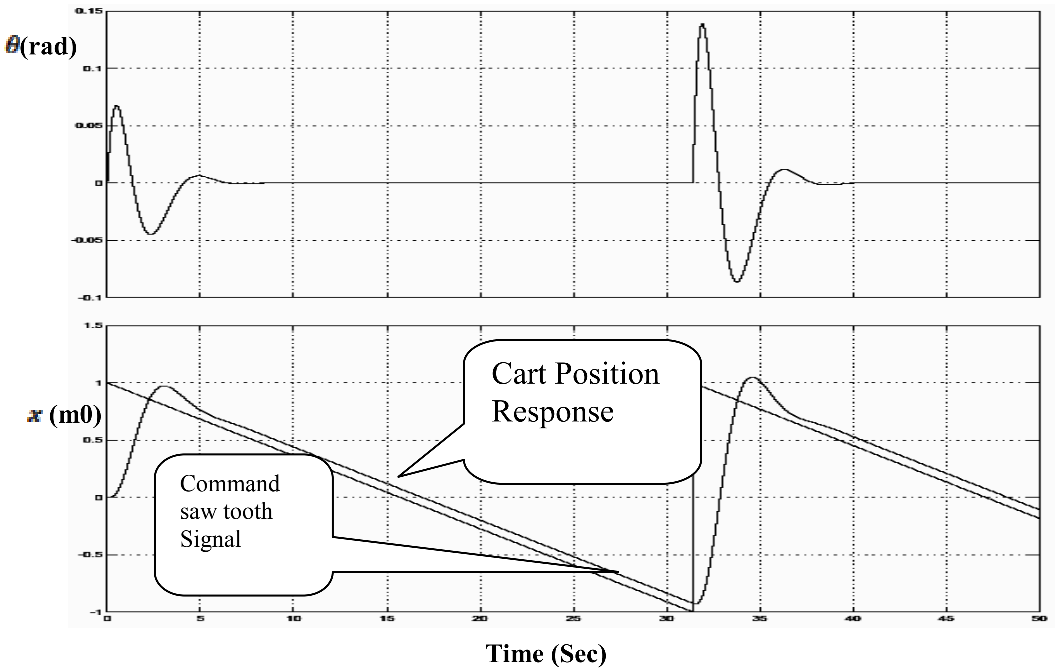

Figure 11. Control System response to the saw tooth wave.

Figure 12 shows the control system response to the sinusoidal command signal with time delay of 2 sec. It be seen from this Figure that the pendulum angle has very low variations and this is due to the variations of command signal with one frequency component.

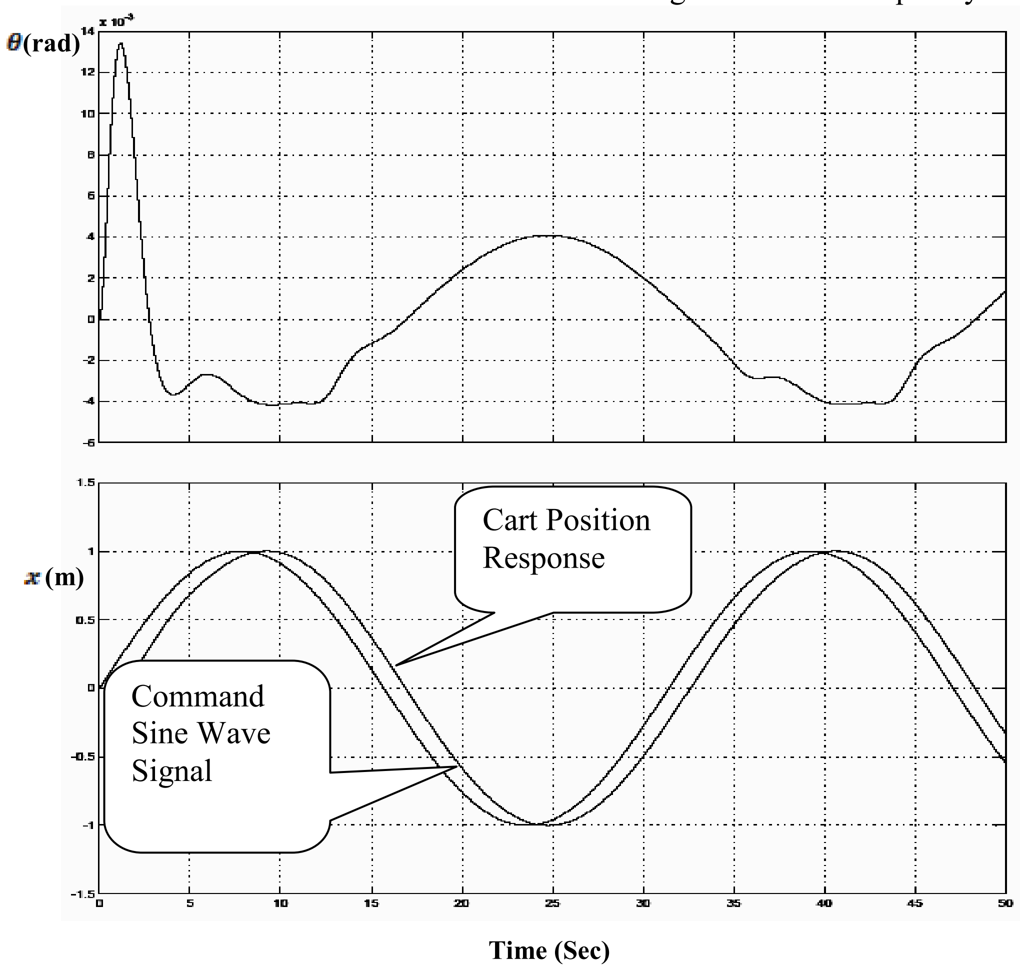

Figure 12. Control System response to the sinusoidal wave.

2 Disturbance Rejection:

The second task of the simulation process is to test the robustness of the control system from disturbance rejection point of view. Figure 13 shows applying of deterministic output disturbance signal. Figure 14 illustrates that the control system easily and quickly overcomes this disturbance effect.

Figure 15 shows the robustness of the control system in rejecting deterministic input disturbance signal.

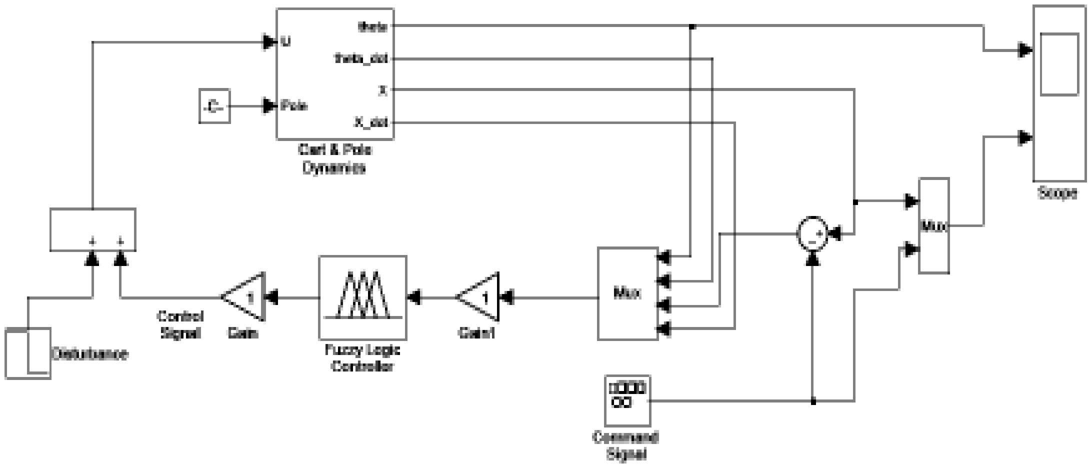

Figure 13. Simulink model of the Inverted Pendulum controlled by Fuzzy Logic Control with deterministic disturbance

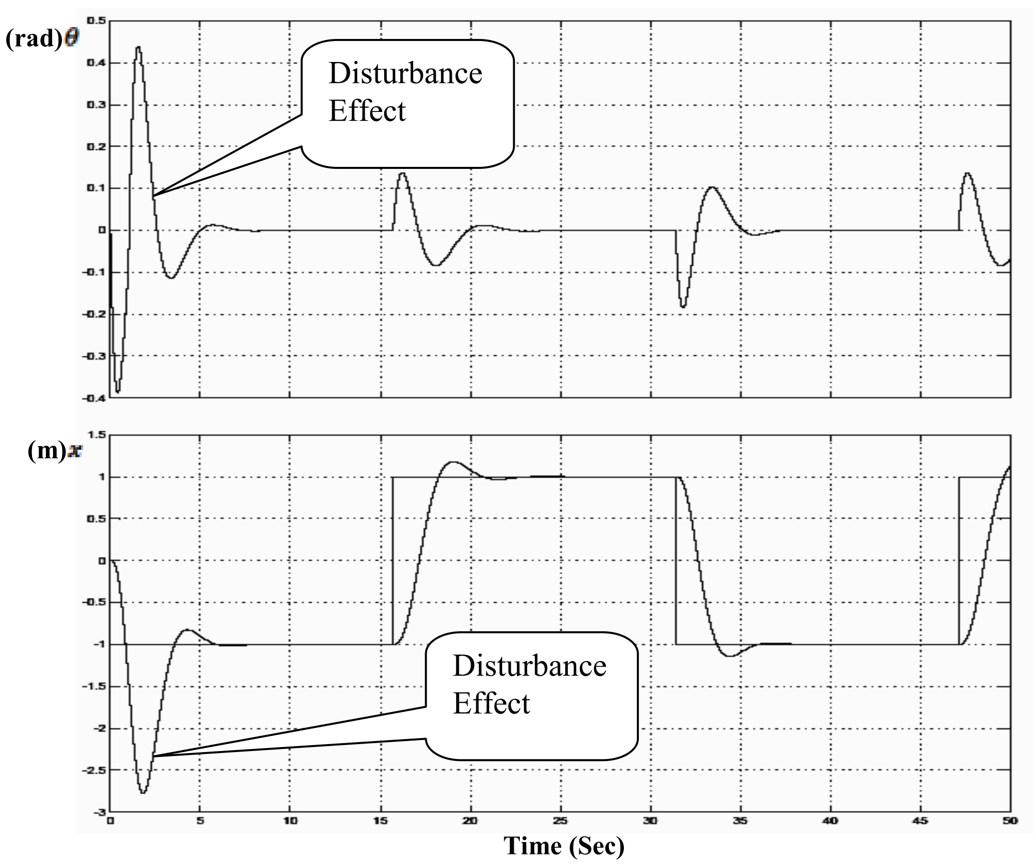

Figure 14. Output disturbance rejection

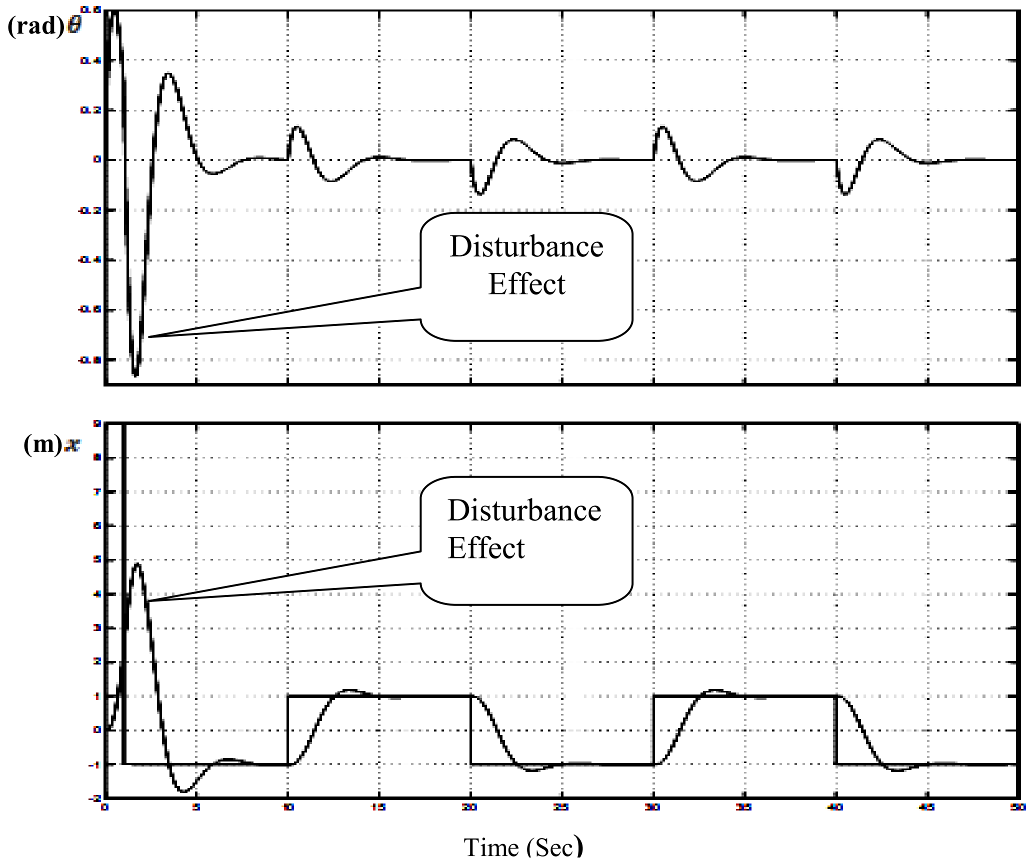

Figure 15. Input disturbance rejection

Conclusions:

It can be shown from simulation results in section 3 that the behavior of the inverted pendulum without including FLC controller posses unstable characteristics these are, the cart diverge very far from the desired position as well as fluctuation of the pole angle is very high. When the designed FLC included, the behavior of the system has been considerably improved as shown from the results presented in section 6 and table2. These results show that the designed FLC successfully drives the cart of the IP to the desired position as well as stabilizing the pole of the pendulum in the upright position with minimum ISE and acceptable time response parameters. The controlled IP was proven to be highly robust for driving the cart to the desired position according to different command signals with or without external disturbances.

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