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Detection Performance of Time-Frequency Coded Signal

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Abstract

A comparison between nonparametric Wilcoxon tests with time-frequency coded signal in detecting a train of narrow-band signal pulses on a background of Gaussian distribution clutter is made. The detection performance is derived, and shows a gain in Signal-to-Noise.

Ratio about (5-10) dB for the same probability of detection P_d***, and a given probability of false alarm*** P_{fa}***, using the number of return pulses M as a parameter.***

الخلاصـــة

يتعرض البحث إلى مقارنة بين اختبار ولكوكسن الذي لايعتمد على معالم الضوضاء وبين الإشارة الزمنية الترددية الكودية عند كشف سلسلة من النبضات ذات المجال الترددي الضيق على خلفية ضوضاء ذات توزيع كاوس.

أداء الكشف تم استنتاجه، ويتضح أن هنالك ربحاً بحدود (٥-١٠) ديسبل لنفس احتمالية الكشف واحتمالية إنذار كاذب معطاة وباستخدام عدد النبضات كمعلم.

1. Introduction

A common assumption in the design of many Constant False Alarm Rate (CFAR) systems is the Probability Density Function (PDF) of the background noise amplitude is known (usually taken to be Gaussian) except for a scale factor. Clutter however is often nonhomogeneous and thus nonstationary, as well as being of unknown PDF in some cases. With such uncertainty in the background, a nonparametric method of detection must be used. A nonparametric detector also called a Distribution-Free Detector (DFD), in its most general form does not require prior knowledge of the PDF of the noise or the signal.

A nonparametric detector permits a CFAR to be achieved for a background noise that might be described by very broad classes of PDFs. It has a greater loss than when the character of the noise is known and an optimum detector can be designed, but it does keep the false alarm rate fixed [1,2,3,4].

2. Wilcoxon Test

One of the nonparametric detectors is the Wilcoxon test, which is a rank detector. The analysis of Wilcoxon test in comparison of optimum detector shows that for a train of 30 pulses and P_{fa} =1e-6 , the loss in Signal-to-Noise Ratio (SNR) about 2dB, while for 50 signal pulses the loss reduced to 1dB, **Figures (1) and (2)** [5].

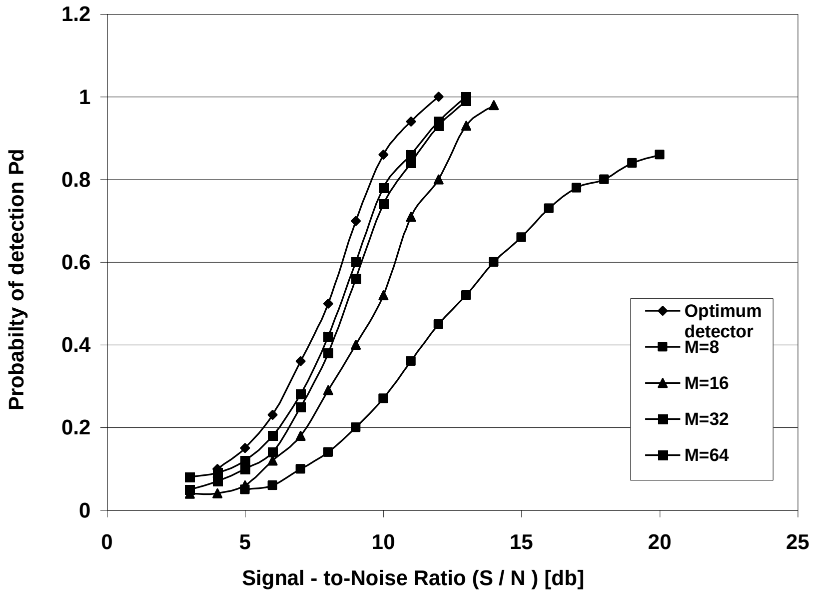

Figure (1) Detection Performance of Narrow-Band Wilcoxon Detector for Pfa=1e-3 and M as Parameter

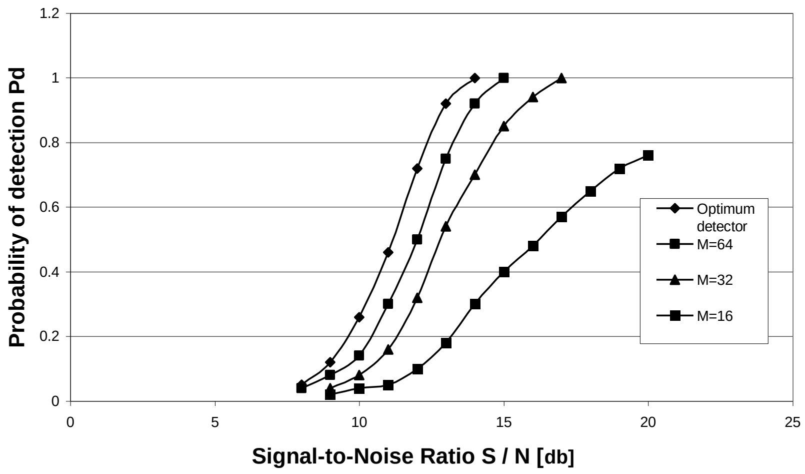

Figure (2) Detection Performance of Narrow-Band Wilcoxon Detector for Pfa=1e-6 and M as Parameter

3. Transmitter Waveform Selection

Another approach to achieve CFAR conditions is the transmitter waveform selection, and the frequency agility [6]. To represent this approach a time-frequency coded waveform is selected.

4. Signal Detection

The quality of signal detection is characterized by the conditional probabilities of detection P_{dN} , and false alarm P_{fMN} , for M resolution cells (or M-successive return pulses from the target) and N scan cycles.

For one scan (N=1), and at one resolution cell:

$$p_d = p_{dN} \qquad ; p_f = \sqrt[N]{\frac{p_{fMN}}{M}} \qquad \text{.......... (1)}$$

and they determined by:

$$p_d = \frac{\Gamma\left(n_1, \frac{n_1\lambda}{1+\rho}\right)}{\Gamma(n_1)} \qquad ; \; p_f = \frac{\Gamma(n_o, n_o\lambda)}{\Gamma(n_o)} \qquad \text{.......... (2)}$$

where:

$\Gamma(n, X)$ - not complete Gamma function;

$\Gamma(n)$ -complete Gamma function;

$$n_o = 1 + T_{ni}\Delta F_{ci} \qquad ; n_1 = 1 + T_{ni}\frac{\Delta F_{ci} + \rho(\Delta F_{LS})}{1+\rho} \quad \text{.....................} \textbf{(3)}$$

where:

$n_\circ, n_1$ - half number of freedom for χ^2-signal distribution at the output of noncoherent integrator for the signal presence n_1 , and its absence $n_\circ$;

λ - normalized detection threshold;

T_{ni} - time interval for noncoherent integrator;

ΔF_{ci} - the width of frequency response characteristic for coherent integrator;

ΔF_{ls} - the lobe width of reflected signal energy spectrum;

ρ - Signal-to-Clutter ratio (S / C).

From (2) it can be seen that the choice of the transmitted signal depends not only on S/C, but also from $n_\circ, n_1, \lambda$, M and N.

The target observation time T_{obs} (which is equal to coherent signal integration interval T_{ci}) equals to:

$$T_{obs} = T_{ci} = \frac{V_{scan}}{V} \quad \text{...} \textbf{(4)}$$

where:

V_{scan} - the volume of detection zone;

V- angular dimensions, or the transmitted signal volume, i.e. the transmitting antenna beam width $\Delta\theta_x, \Delta\theta_y$.

In this case the detection performance expressed by:

$$p_d = p_f^{\frac{1}{1+\rho}} \quad \text{..} \textbf{(5)}$$

The discussion will be limited for the case , when the detection decision is made for one scan (N =1), to find a rule to determine the transmitted signal dimensions X, Y ,Z .and its spectrum width (base band) $\Delta f_\circ$,based on the Nyman-Pearson criteria.

Taken into consideration the above discussion:

$$p_d(V, \Delta f_o) = \left[\frac{P_{fM1}}{M(V, \Delta f_o)}\right]^{\frac{1}{1+\rho(V,\Delta f_o)}} \quad \text{...} \textbf{(6)}$$

From (6) it can be seen that the choice of V, Δf_o must be oriented toward the decrement of M and the increment of ρ.

For non-modulated RF pulses, the number of resolution cells equals to:

$$M = \frac{V_{scan}}{V} \cdot \frac{F_{scan}}{\Delta F_{ci}} = \frac{V_{scan}}{V} \cdot \frac{F_{scan} \cdot V \cdot T_{scan}}{V_{scan}} = F_{scan} \cdot T_{scan} \quad \text{(7)}$$

thus, M is not depend on V. And the dimensions of the transmitted signal in X, Y and its radial expansion must be selected to obtain maximum S/C.

For signals with large base band, the number of range resolution cells increases by the signal base, in addition S/C increases by the same value, when the clutter exist only.

$$M = F_{scan} \cdot T_{scan} \cdot \Delta f_o \cdot T_o = m_o \cdot \Delta f_o \cdot T_o \quad \text{(8)}$$

$$\rho = \rho_o \cdot \Delta f_o \cdot T_o \quad \text{(9)}$$

where m_o, ρ_o, T_o are the number of resolution cells, S/C ratio, and the transmitted pulse duration for noncomplex signal.

Taken into consideration the above discussion, expression 6, could be written as [7]:

$$p_d(V, \Delta f_o) = \left[\frac{P_{fM1}}{m_o \cdot \Delta f_o \cdot T_o} \right]^{\frac{1}{1+\rho_o \cdot \Delta f_o \cdot T_o}} \quad \text{(10)}$$

5. Probability of Detection Analysis

The analysis of expression (10) shows that the detection probability at clutter existence is a monotonically increasing function with the signal base band. Thus to obtain highest values of P_d it is recommended to implement a transmitted signal with large base band.

If expression (10) is differentiated by the signal base band, we obtain:

$$\frac{\partial p_d}{\partial(\Delta f_o, T_o)} = \left(\frac{p_{fM1}}{m_o, \Delta f_o, T_o} \right)^{\frac{1}{1+\rho_o \cdot \Delta f_o \cdot T_o}} \frac{\rho_o}{(1+\rho_o \cdot \Delta f_o \cdot T_o)^2} \left[Ln\left(\frac{m_o \cdot \Delta f_o \cdot T_o}{p_{fM1}} \right) - \left(\frac{1+\rho_o \cdot \Delta f_o \cdot T_o}{\rho_o \cdot \Delta f_o \cdot T_o} \right) \right] \quad \text{(11)}$$

Analyzing the terms of (11) yields:

1. For any finite base band values $m_\circ, (P_{fM1})^{-1} >> 1$, the first two terms accept nonzero positive value.

2. To evathe sign of the third term, the first sub-term under Ln sign $m_\circ, (P_{fM1})^{-1} >> 1$, and the signal base band $\Delta f_\circ \cdot T_\circ$ is not less 1, so the first subterm will accept a value more than one. While the second subterm $m_\circ \cdot \Delta f_\circ \cdot T_\circ >> 1$, thus this subterm approximately equals to one and the difference of these sub-terms is positive.

The analyzed function derivative for any finite base band values will be always a positive value. It means that the function $P_d(\Delta f_\circ, T_\circ)$ is an increasing function.

6. Evaluation of Detection Performance

To evaluate the implementation of complex signal compared with Wilcoxon nonparametric test the detection performance was derived using (10) for a time-frequency coded waveform, consisting of a train of M pulses, each one transmitted at a different frequency.

The pulse width is τ and the waveform duration $T = M \cdot \tau$. The time-bandwidth product $T \cdot B = M^2$[8].

The detection performance is considered for a single scan (N=1), and different number of return pulses from the target (or cells under test) M= 8, 16, 32 and 64 for a given value of false alarm $P_{fa1} = 10^{-3}, P_{fa2} = 10^{-6}$. The results are plotted in **Figures (3)** and **(4)**.

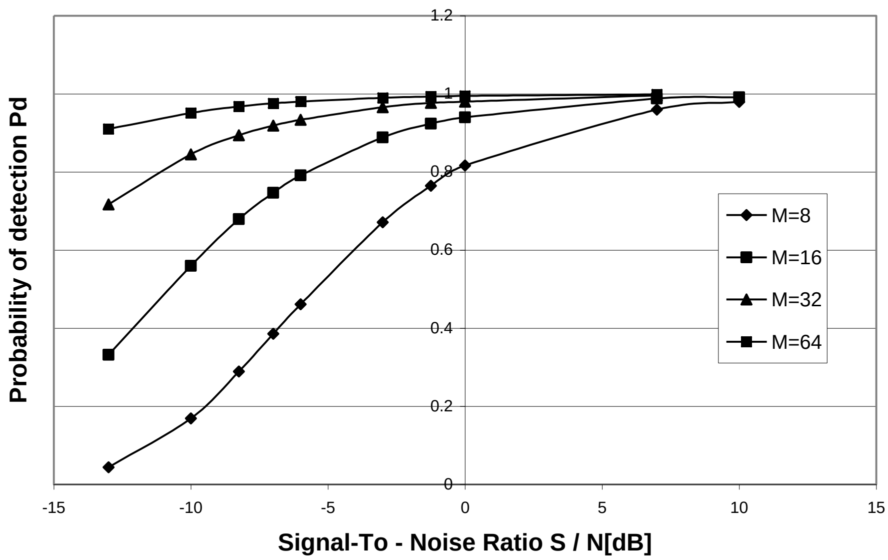

Figure (3) Detection Performance of Time-Frequency Coded Signal for Pfa=1e-3 and M as Parameter

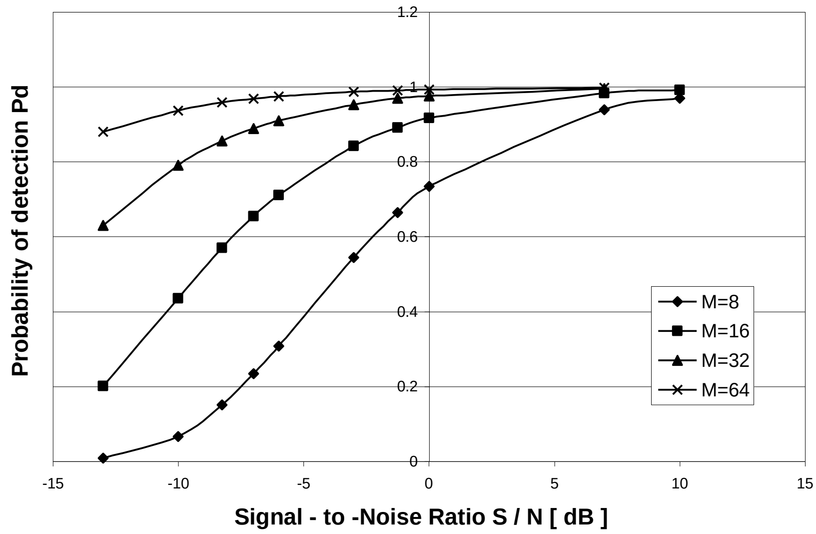

Figure (4) Detection Performance of Time-Frequency Coded Signal for Pfa=1e-6 and M as Parameter

From **Figures (1, 2, 3)** and **(4)** it can be seen that:

1. The results are shown, show the variation in detection performance as M is varied;

2. As M becomes large (greater 32), the detection performance of time-frequency coded signal is superior to the Wilcoxon nonparametric rank test, but increment of M leads to degradation of target detection information rate;

3. Show the way of the performance of the detector varies as the deviation from minimum probability of false alarm P_{fa} increases;

4. The necessary S/N to achieve required probability of detection P_d for both detectors, is better for time-frequency coded signal, the gain in S/N is about 10 dB;

5. The gain in S/N, could overcome the loss in S/N due to the variation of clutter probability density function;

6. When the product of the bandwidth B and the pulse width τ is greater than unity, as in pulse compression radar, the loss is determined by the product $M \cdot B \cdot \tau$ rather than M.

Thus a loss of 1dB corresponds to MBτ =100 .Both coded pulse waveforms and frequency-modulated waveforms have been considered for pulse compression radar with CFAR [1].

7. Conclusion

1. The detection performance of time-frequency coded signal is superior to that of the nonparametric rank Wilcoxon test;

2. Radar with a complex signal is not without disadvantages. It requires a transmitter can be readily modulated, and a matched filter more sophisticated than that of conventional pulse radar;

3. In spite of its limitation, radar with complex signal has been an important part of radar system technology.

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