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[0,1] Truncated Half Logistic- Truncated Logistic Distribution

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Abstract

In this paper, we discuss a new generated class of continuous distribution, named Truncated [0,1] Half logistic- Truncated logistic distribution ([0,1] THL-TLD). Some of properties of our distribution will be presented. Such as the r^{th} moment function, the function of reliability, the function of hazard rate, the function of stress strength and the function of Shannon entropy.

Keywords: Half logistic distribution, Truncated Logistic Distribution, the r^{-th} moment, the function of Shannon entropy, stress& strength models, the function of reliability and hazard function.

1- Introduction

Here, we proposed a distribution with the hope it will attract wider applicability in other fields. "The generalization which is motivated by the work of Boshi [3] M.A.(2019)". "Will be our guide defined two ways to generate probability distributions by composition some generalized distributions, which are generalized gamma distribution, exponentiated Weibull distribution, generalized inverse Weibull distribution, and generalized Gompertz distribution". "In (2018) Abdulrazak [1] introduced a new generated family named, [0,1] truncated Fréchet-G generator of distributions based on replacing the Beta distribution in beta-G family", " by(Eugene et al, (2002)) [4] defined the beta normal (BN) distribution by taking G(x) to be the cdf of the normal distribution and derived some of its first moments". "General expressions for the moments of the BN distribution were derived (Gupta and Nadarajah, 2004 [5])". "An extensive review of scientific literature on this subject is available in Abid and Hassan (2015) [6]".

"The Half-logistic distribution (H-L) is introduced and studied by de Kank et al,[6] ". A random variable X has H-L distribution if the pdf and cdf. With two positive parameter $(\mu,\ \sigma)$ given respectively by,

$$f(x;\mu,\sigma)_{H-L} = \frac{2e^{\frac{-(x-\mu)}{\sigma}}}{\sigma\left(1+e^{\frac{-(x-\mu)}{\sigma}}\right)^2} \qquad , \quad x \geq \mu, \quad \sigma > 0 \qquad (1)$$

and CDF of the distribution can be written as [7],

$$F(x;\mu,\sigma)_{H-L} = \frac{1-e^{\frac{-(x-\mu_1)}{\sigma_1}}}{1+e^{\frac{-(x-\mu_1)}{\sigma_1}}} \qquad , \quad x \geq \mu, \quad \sigma > 0 \qquad (2)$$

2- Truncated$[\mathbf{0},\mathbf{1}]$ L-GD

This search, Generated new category of continuous distributions based on time period$[0,1]$ truncated cumulative distribution function L-G, named truncated [0,1] L-G (symbolized by [0,1] TL-G distributions, have been discussed..

Suppose that $G(x) and\ g(x)$ are any continuous and pdf of random variable x, assume that $L(.)\ and\ l(.)$ represents, the cdf and pdf of any continuous distribution on the interval $[0,\infty)$. The suggested general formula of cdf for this class depends composing L with G is[1],

$$F(x)_{TL-Q} = \frac{L[G(x)]-L[0]}{L[1]-L[0]} \qquad (3)$$

Now, let $L[0] = 0$ Then cdf in (3) can be rewritten as,

$$F(x)_{TL-G} = \frac{L[G(x)]}{L[1]} \qquad (4)$$

And its associated pdf, $f(x)\,\frac{d}{dx}[F(x)]$ will be,

$$f(x)_{TL-G} = \frac{l\,[G(x)]g(x)}{L[1]} \qquad (5)$$

3. **Truncated [0,1] Half Logistic- GD**

We introduce here a new generated family of truncated [0,1] based on Half logistic (HLD) .

Let $L(.)$ *and* $l(.)$ that mentioned in (4) and (5), be the cdf and pdf of (HL) distribution [7] recall (1) and (2) with tow positive parameters $(\mu = 0, \sigma > 0)$.We have L(0)=0 So,

$$L[G(x)] = \frac{1-e^{\frac{-G(x)}{\sigma}}}{1+e^{\frac{-G(x)}{\sigma}}} \quad , \quad L[1] = \frac{1-e^{\frac{-1}{\sigma}}}{1+e^{\frac{-1}{\sigma}}}$$

And $l[G(x)] = \dfrac{2e^{\frac{-G(x)}{\sigma}}}{\sigma\left(1+e^{\frac{-G(x)}{\sigma}}\right)^2}$

Then according to (4) and (5), the cdf and associated pdf for new family of distribution called [0,1] truncated HL- G (named [0,1] THL- GD) will be,

$$F(x)_{THL-G} = \frac{\left(1-e^{\frac{-G(x)}{\sigma}}\right)\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1+e^{\frac{-G(x)}{\sigma}}\right)\left(1-e^{\frac{-1}{\sigma}}\right)} \ , \ \sigma > 0,\ x \geq 0 \qquad (6)$$

and,

$$f(x)_{THL-G} = \frac{2e^{\frac{-G(x)}{\sigma}}\left(1+e^{\frac{-1}{\sigma}}\right)q(x)}{\sigma\left(1+e^{\frac{-G(x)}{\sigma}}\right)^2\left(1-e^{\frac{-1}{\sigma}}\right)} \ , \ \sigma > 0,\ x \geq 0 \qquad (7)$$

Before addressing some cases belonging to this family, it should be noted that we need to find the general expanded formula to $F(x)_{THL-G}$ and $f(x)_{THL-G}$ which is important for obtaining the basic statistical properties when dealing with some cases.

By using the following Formula [1],

$$e^{-u} = \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} u^i \qquad (8)$$

the equation (6) can be rewritten as,

$$F(x)_{THL-G} = \frac{\left(1+e^{\frac{-1}{\sigma}}\right)\left(1-\sum_{i=0}^{\infty}\frac{(-1)^i}{\sigma^i i!}(G(x))^i\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\left(1+\sum_{i=0}^{\infty}\frac{(-1)^i}{\sigma^i i!}(G(x))^i\right)} \quad (9)$$

And,

$$f(x)_{THL-Q} = \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)e^{\frac{-G(x)}{\sigma}}g(x)}{\left(1-e^{\frac{-1}{\sigma}}\right)\sigma\left(1+e^{\frac{-G(x)}{\sigma}}\right)^2} \quad (10)$$

$$= \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\sigma}\left(1+e^{\frac{-G(x)}{\sigma}}\right)^{-2} e^{\frac{-G(x)}{\sigma}}g(x)$$

And by using the Formula [1],

$$(a+u)^{-n} = \sum_{i=0}^{\infty} C_i^{-n}\, a^{-n-i}u^i \quad (11)$$

$f(x)_{THL-G}$ can be rewritten as ,

$$f(x)_{THL-G} = \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\sigma}\sum_{k=0}^{\infty} C_k^{-2}\, e^{\frac{-kG(x)}{\sigma}}e^{\frac{-G(x)}{\sigma}}g(x)$$

$$= \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\sigma}\sum_{k=0}^{\infty} C_k^{-2}\, e^{\frac{-(1+k)G(x)}{\sigma}}g(x)$$

Accorded (8) $f(x)_{THL-G}$ will be,

$$f(x)_{THL-G} = \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\sigma}\sum_{k=0}^{\infty}\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}C_k^{-2}\left(\frac{(1+k)G(x)}{\sigma}\right)^i g(x)$$

$$= \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)}\sum_{k=0}^{\infty}\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}C_k^{-2}\sigma^{-(1+i)}(1+k)^i[G(x)]^i\, g(x) \quad (12)$$

4. Truncated [0,1] Half Logistic- Truncated Logistic Distribution

Assume that (x) *and* $g(x)$, the Truncated logistic distribution with tow positive parameter λ *and* β, cdf and pdf as[8],

$$G(x;\ \lambda,\beta) = \frac{\left(1+e^{\frac{\lambda}{\beta}}\right)}{e^{\frac{\lambda}{\beta}}}\left[\frac{1}{1+e^{\frac{-(x-\lambda)}{\beta}}} - \frac{1}{1+e^{\frac{\lambda}{\beta}}}\right] \qquad (13)$$

$$g(x;\ \lambda,\beta) = \frac{\left(1+e^{\frac{\lambda}{\beta}}\right)}{e^{\frac{\lambda}{\beta}}}\left(\frac{e^{\frac{-(x-\lambda)}{\beta}}}{\beta\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)^2}\right) \qquad (14)$$

According to (9), the cdf of new distribution named truncated [0,1] Half logistic-Truncated logistic (named [0,1] THL-TLD) will be,

$$F(x)_{THL-TL} = \frac{\left(1+e^{\frac{-1}{\sigma}}\right)\left(1-\sum_{i=0}^{\infty}\frac{(-1)^i}{\sigma^i i!}\left(\frac{1+e^{\frac{\lambda}{\beta}}}{e^{\frac{\lambda}{\beta}}\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)}-\frac{1}{e^{\frac{\lambda}{\beta}}}\right)^i\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\left(1+\sum_{i=0}^{\infty}\frac{(-1)^i}{\sigma^i i!}\left(\frac{1+e^{\frac{\lambda}{\beta}}}{e^{\frac{\lambda}{\beta}}\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)}-\frac{1}{e^{\frac{\lambda}{\beta}}}\right)^i\right)} \qquad (15)$$

The pdf [0,1] THL-TLD can be obtained, according to (12) as,

$$f(x)_{THL-TL} = \left\{\begin{array}{c} \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)}\sum_{k=0}^{\infty}\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}C_k^{-2}\sigma^{-(1+i)}(1+k)^i \\ \left[\frac{1-e^{-\frac{x}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right]^i\frac{\left(1+e^{\frac{\lambda}{\beta}}\right)}{\beta e^{\frac{\lambda}{\beta}}}\left(\frac{e^{\frac{-(x-\lambda)}{\beta}}}{\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)^2}\right)\end{array}\right\} \qquad (16)$$

Let, $I = \left[\frac{1-e^{-\frac{x}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right]^i$ *and* $II = \frac{e^{\frac{-(x-\lambda)}{\beta}}}{\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)^2}$

$$I = \left(\frac{1-e^{-\frac{x}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right)^i$$

$$= \frac{\left(1+e^{-\frac{x}{\beta}}\right)^i}{\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)^i}$$

$$= \left(1 - e^{-\frac{x}{\beta}}\right)^{i} \left(1 + e^{\frac{-(x-\lambda)}{\beta}}\right)^{-i}$$

By using the following Formula[1],

$$(1-u)^{b} = \sum_{i=0}^{\infty} (-1)^{i}\, C_{i}^{b} u^{i}; \qquad (17)$$

and (11) will be,

$$I = \sum_{n=0}^{\infty} (-1)^{n} C_{n}^{i}\, e^{\frac{-nx}{\beta}} \sum_{m=0}^{\infty} C_{m}^{-i}\, e^{\frac{-m(x-\lambda)}{\beta}}$$

Now,

$$II = \frac{e^{\frac{-(x-\lambda)}{\beta}}}{\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)^{2}}$$

$$= e^{\frac{-(x-\lambda)}{\beta}} \left(1 + e^{\frac{-(x-\lambda)}{\beta}}\right)^{-2}$$

According (11) will be,

$$II = e^{\frac{-(x-\lambda)}{\beta}} \sum_{j=0}^{\infty} C_{j}^{-2} e^{\frac{-j(x-\lambda)}{\beta}}$$

$$II = \sum_{j=0}^{\infty} C_{j}^{-2} e^{\frac{-(j+1)(x-\lambda)}{\beta}}$$

Substitutes I and II in (16) we get,

$$f(x)_{THL-TL} = \left\{ \begin{array}{c} \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)} \sum_{k=0}^{\infty} \sum_{i=0}^{\infty} \frac{(-1)^{i}}{i!} C_{k}^{-2} \sigma^{-(1+i)} (1+k)^{i} \\ \sum_{n=0}^{\infty} (-1)^{n} C_{n}^{i}\, e^{\frac{-nx}{\beta}} \sum_{m=0}^{\infty} C_{m}^{-i}\, e^{\frac{-m(x-\lambda)}{\beta}} \frac{\left(1+e^{-\frac{\lambda}{\beta}}\right)}{\beta} \\ \sum_{j=0}^{\infty} C_{j}^{-2} e^{\frac{-(j+1)(x-\lambda)}{\beta}} \end{array} \right\} \qquad (18)$$

The function of the reliability [0,1]THL-TLD can be obtained as,

$R(x)_{THL-TL} = 1 - F(x)_{THL-TL}$

$$R(x)_{THL-TL} = 1 - \frac{\left(1+e^{\frac{-1}{\sigma}}\right)\left(1-\sum_{i=0}^{\infty}\frac{(-1)^i}{\sigma^i i!}\left(\frac{1-e^{-\frac{x}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right)^i\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\left(1+\sum_{i=0}^{\infty}\frac{(-1)^i}{\sigma^i i!}\left(\frac{1-e^{-\frac{x}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right)^i\right)} \quad (19)$$

The hazard function of [0,1]THL-TLD as,

$$H(x)_{THL-TL} = \frac{f(x)_{THL-TL}}{R(x)_{THL-TL}}$$

$$H(x)_{THL-TL} = \frac{\left\{\begin{array}{c}\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)}\sum_{k=0}^{\infty}\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}C_k^{-2}\sigma^{-(1+i)}(1+k)^i \\ \sum_{n=0}^{i}C_n^i e^{\frac{-nx}{\beta}}\sum_{m=0}^{\infty}C_m^{-i}e^{\frac{-m(x-\lambda)}{\beta}}\frac{\left(1+e^{-\frac{\lambda}{\beta}}\right)}{\beta} \\ \sum_{j=0}^{\infty}C_j^{-2}e^{\frac{-(i+1)(x-\lambda)}{\beta}}\end{array}\right\}}{1-\frac{\left(1+e^{\frac{-1}{\sigma}}\right)\left(1-\sum_{i=0}^{\infty}\frac{(-1)^i}{\sigma^i i!}\left(\frac{1-e^{-\frac{x}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right)^i\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\left(1+\sum_{i=0}^{\infty}\frac{(-1)^i}{\sigma^i i!}\left(\frac{1-e^{-\frac{x}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right)^i\right)}} \quad (20)$$

5. properties of the [0,1] THL-TL distribution are given respectively as,

i- r-th Moment:

The r-th moment of [0,1]THL-TLD, can be obtained from $\int_0^{\infty} x^r f(x)_{THL-TL} dx$. According to (17). The r-th moment of [0,1] THL-TLD can be obtained as follows,

$$E(X^r)_{THL-TL} = \int_0^{\infty} x^r \left\{\begin{array}{c}\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)}\sum_{k=0}^{\infty}\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}C_k^{-2}\sigma^{-(1+i)}(1+k)^i \\ \sum_{n=0}^{\infty}(-1)^n C_n^i e^{\frac{-nx}{\beta}}\sum_{m=0}^{\infty}C_m^{-i}e^{\frac{-m(x-\lambda)}{\beta}}\frac{\left(1+e^{-\frac{\lambda}{\beta}}\right)}{\beta} \\ \sum_{j=0}^{\infty}C_j^{-2}e^{\frac{-(j+1)(x-\lambda)}{\beta}}\end{array}\right\} dx$$

$$E(X^r)_{THL-TL} = \int_0^\infty x^r \left\{ \begin{array}{c} \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)} \sum_{k=0}^\infty \sum_{i=0}^\infty \frac{(-1)^i}{i!} C_k^{-2} \sigma^{-(1+i)} (1+k)^i \\ \sum_{n=0}^\infty (-1)^n C_n^i e^{\frac{-nx}{\beta}} \sum_{m=0}^\infty C_m^{-i} e^{\frac{m\lambda}{\beta}} e^{\frac{-mx}{\beta}} \frac{\left(1+e^{-\frac{\lambda}{\beta}}\right)}{\beta} \\ \sum_{j=0}^\infty C_j^{-2} e^{\frac{(j+1)\lambda}{\beta}} e^{\frac{-(j+1)x}{\beta}} \end{array} \right\} dx$$

By using the formula $\int_0^\infty x^{\alpha-1} e^{-\beta x} dx = \frac{\Gamma(\alpha)}{\beta^\alpha}$, the r-th moment of the THL-TLD, is given by,

$$E(X^r)_{THL-TL} = \left\{ \begin{array}{c} \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)} \sum_{k=0}^\infty \sum_{i=0}^\infty \frac{(-1)^i}{i!} C_k^{-2} \sigma^{-(1+i)} (1+k)^i \\ \sum_{n=0}^\infty (-1)^n C_n^i \sum_{m=0}^\infty C_m^{-i} e^{\frac{m\lambda}{\beta}} \frac{\left(1+e^{-\frac{\lambda}{\beta}}\right)}{\beta} \\ \sum_{j=0}^\infty C_j^{-2} e^{\frac{(j+1)\lambda}{\beta}} \frac{\Gamma(r+1)\beta^{r+1}}{(j+n+m+1)^{r+1}} \end{array} \right\} \quad (21)$$

Depending on the particular $E(x^r)_{THL-HL}$; ($r = 1,2,3,4$), another properties of this distribution such as, $\left(mean\ \mu = E(x)\right), variance\ \left(var(x) = \sigma^2 = E(x^2) - \left(E(x)\right)^2\right)$ coefficient skewness $\left(SK = \frac{E(X-\mu)^3}{\sigma^3} = \frac{E(x^2) - 3\mu E(x^2) + 2\mu^2}{[\sigma^2]^{\frac{3}{2}}}\right)$ And coefficient of kurtosis $\left(Kr = \frac{E(X-\mu)^4}{\sigma^4} = \frac{E(x^4) - 4\mu E(x^3) + 6\mu^2 E(x^2) - 3\mu^3}{\sigma^4}\right)$

Can be obtained, where,

$$E(X)_{THL-TL} = \left\{ \begin{array}{c} \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)} \sum_{k=0}^\infty \sum_{i=0}^\infty \frac{(-1)^i}{i!} C_k^{-2} \sigma^{-(1+i)} (1+k)^i \\ \sum_{n=0}^\infty (-1)^n C_n^i \sum_{m=0}^\infty C_m^{-i} e^{\frac{m\lambda}{\beta}} \frac{\left(1+e^{-\frac{\lambda}{\beta}}\right)}{\beta} \\ \sum_{j=0}^\infty C_j^{-2} e^{\frac{(j+1)\lambda}{\beta}} \frac{\Gamma(2)\beta^2}{(j+n+m+1)^2} \end{array} \right\}$$

$$E(X^2)_{THL-TL} = \left\{ \begin{array}{c} \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)} \sum_{k=0}^{\infty} \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} C_k^{-2} \sigma^{-(1+i)} (1+k)^i \\ \sum_{n=0}^{\infty} (-1)^n C_n^i \sum_{m=0}^{\infty} C_m^{-i} e^{\frac{m\lambda}{\beta}} \frac{\left(1+e^{-\frac{\lambda}{\beta}}\right)}{\beta} \\ \sum_{j=0}^{\infty} C_j^{-2} e^{\frac{(j+1)\lambda}{\beta}} \frac{\Gamma(3)\beta^3}{(j+n+m+1)^3} \end{array} \right\}$$

$$E(X^3)_{THL-TL} = \left\{ \begin{array}{c} \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)} \sum_{k=0}^{\infty} \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} C_k^{-2} \sigma^{-(1+i)} (1+k)^i \\ \sum_{n=0}^{\infty} (-1)^n C_n^i \sum_{m=0}^{\infty} C_m^{-i} e^{\frac{m\lambda}{\beta}} \frac{\left(1+e^{-\frac{\lambda}{\beta}}\right)}{\beta} \\ \sum_{j=0}^{\infty} C_j^{-2} e^{\frac{(j+1)\lambda}{\beta}} \frac{\Gamma(4)\beta^4}{(j+n+m+1)^4} \end{array} \right\}$$

$$E(X^4)_{THL-TL} = \left\{ \begin{array}{c} \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)} \sum_{k=0}^{\infty} \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} C_k^{-2} \sigma^{-(1+i)} (1+k)^i \\ \sum_{n=0}^{\infty} (-1)^n C_n^i \sum_{m=0}^{\infty} C_m^{-i} e^{\frac{m\lambda}{\beta}} \frac{\left(1+e^{-\frac{\lambda}{\beta}}\right)}{\beta} \\ \sum_{j=0}^{\infty} C_j^{-2} e^{\frac{(j+1)\lambda}{\beta}} \frac{\Gamma(5)\beta^5}{(j+n+m+1)^5} \end{array} \right\}$$

ii- Characteristic Function:

The function of the characteristic of the THL-TLD can be obtained from,

$$E\left(e^{itx}\right) = \sum_{r=0}^{\infty} \frac{(it)^r}{r!} E(X^r)_{THL-TL}.$$

Therefore, the characteristic function of the THL-TLD is given by,

$$\emptyset_X\,(t)_{THL-TL}=\sum_{r=0}^{\infty}\frac{(it)^r}{r!}\left\{\begin{array}{c}\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)}\sum_{k=0}^{\infty}\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}C_k^{-2}\sigma^{-(1+i)}(1+k)^i\\ \sum_{n=0}^{\infty}(-1)^n C_n^i\sum_{m=0}^{\infty}C_m^{-i}\,e^{\frac{m\lambda}{\beta}}\frac{\left(1+e^{-\frac{\lambda}{\beta}}\right)}{\beta}\\ \sum_{j=0}^{\infty}C_j^{-2}e^{\frac{(j+1)\lambda}{\beta}}\frac{\Gamma(r+1)\beta^{r+1}}{(j+n+m+1)^{r+1}}\end{array}\right\} \qquad (22)$$

iii- Shannon Entropy:

The Shannon entropy of the THL-TLD can be obtained from,

$$-\int_0^{\infty}\ln(f(x)_{THL-TL})f(x)_{THL-TL}\,dx$$

By using the natural logarithm of the pdf in (10), and according (13) and (14), we get.

$$ln(f(x)_{THL-TL})=ln\left\{\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)e^{\frac{-Q(x)}{\sigma}}q(x)}{\left(1-e^{\frac{-1}{\sigma}}\right)\sigma\left(1+e^{\frac{-Q(x)}{\sigma}}\right)^2}\right\}$$

$$ln(f(x)_{THL-TL})=\left\{ln\left(\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\sigma\left(1-e^{\frac{-1}{\sigma}}\right)}\right)-\frac{Q(x)}{\sigma}+ln\big(q(x)\big)-ln\left(1+e^{\frac{-Q(x)}{\sigma}}\right)^2\right\}$$

$$=\left\{\begin{array}{c}ln\left(\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\sigma\left(1-e^{\frac{-1}{\sigma}}\right)}\right)-\frac{\left(1-e^{\frac{-x}{\beta}}\right)}{\sigma\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)}\\ +ln\left[\frac{1+e^{\frac{\lambda}{\beta}}}{e^{\frac{\lambda}{\beta}}}\left(\frac{e^{\frac{-(x-\lambda)}{\beta}}}{\beta\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)^2}\right)\right]-\quad ln\left(1+e^{\frac{-Q(x)}{\sigma}}\right)^2\end{array}\right\}$$

$$ln(f(x)_{THL-TL})=\left\{\begin{array}{c}ln\left(\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\sigma\left(1-e^{\frac{-1}{\sigma}}\right)}\right)-\frac{\left(1-e^{\frac{-x}{\beta}}\right)}{\sigma\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)}+ln\left(\frac{1+e^{\frac{\lambda}{\beta}}}{e^{\frac{\lambda}{\beta}}}\right)\\ -\frac{(x-\lambda)}{\beta}-ln(\beta)-2ln\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)-2ln\left(1+e^{\frac{-Q(x)}{\sigma}}\right)\end{array}\right\}$$

By using the following Formula[1],

$$ln(u) = \sum_{i=0}^{\infty} \frac{(-1)^i (u-1)^{i-1}}{i+1} \; ; \qquad 0 < u \leq 2, \quad \textit{will be,} \tag{23}$$

$$ln(f(x)_{THL-TL}) = \left\{ \begin{array}{c} ln\left(\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\sigma\left(1-e^{\frac{-1}{\sigma}}\right)} \right) - \frac{\left(1-e^{\frac{-x}{\beta}}\right)}{\sigma\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)} \\ +ln\left(\frac{1+e^{\frac{\lambda}{\beta}}}{e^{\frac{\lambda}{\beta}}} \right) - \frac{(x-\lambda)}{\beta} - ln(\beta) \\ -2\sum_{i=0}^{\infty} \frac{(-1)^i \left(e^{\frac{-(x-\lambda)}{\beta}}\right)^{i-1}}{i+1} - 2\sum_{k=0}^{\infty} \frac{(-1)^k \left(e^{\frac{-Q(x)}{\sigma}}\right)^{k-1}}{k+1} \end{array} \right\}$$

(24)

Now let, $I = \frac{\left(1-e^{\frac{-x}{\beta}}\right)}{\sigma\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)}$, $II = 2\sum_{k=0}^{\infty} \frac{(-1)^k \left(e^{\frac{-Q(x)}{\sigma}}\right)^{k-1}}{k+1}$ *and* $III =$

$2\sum_{i=0}^{\infty} \frac{(-1)^i \left(e^{\frac{-(x-\lambda)}{\beta}}\right)^{i-1}}{i+1}$

$$I = \frac{\left(1-e^{\frac{-x}{\beta}}\right)}{\sigma\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)} = \frac{1}{\sigma}\left(1-e^{\frac{-x}{\beta}}\right)\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)^{-1}$$

According (17) and (11) we get (III),

$$I = \frac{1}{\sigma}\sum_{c=0}^{\infty}(-1)^c C_c^1 \, e^{\frac{-cx}{\beta}} \sum_{b=0}^{\infty} C_b^{-1} \, e^{\frac{-b(x-\lambda)}{\beta}}$$

$$= \frac{1}{\sigma}\sum_{c=0}^{\infty}(-1)^c C_c^1 \sum_{b=0}^{\infty} e^{\frac{b\lambda}{\beta}} C_b^{-1} \, e^{\frac{-(c+b)x}{\beta}}$$

According (8) will be,

$$I = \frac{1}{\sigma}\sum_{c=0}^{\infty}(-1)^c C_c^1 \sum_{b=0}^{\infty} C_b^{-1} \, e^{\frac{b\lambda}{\beta}} \sum_{f=0}^{\infty} \frac{(-1)^f}{f!}\left(\frac{c+b}{\beta}\right)^f x^f$$

$$II = 2\sum_{k=0}^{\infty} \frac{(-1)^k \left(e^{\frac{-Q(x)}{\sigma}}\right)^{k-1}}{k+1} = 2\sum_{k=0}^{\infty} \frac{(-1)^k e^{\frac{-(K-1)Q(x)}{\sigma}}}{k+1}$$

According (8) will be,

$$II = 2\sum_{k=0}^{\infty}\frac{(-1)^k}{k+1}\sum_{n=0}^{\infty}\frac{(-1)^n(k-1)^n}{n!\sigma^n}(G(x))^n \qquad (25)$$

Now, according (13) we get,

$$(G(x))^n = \left(\frac{1-e^{\frac{-x}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right)^n = \frac{\left(1-e^{\frac{-x}{\beta}}\right)^n}{\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)^n}$$

According (17) and (11) $(Q(x))^n$ will be,

$$(G(x))^n = \sum_{z=0}^{\infty}(-1)^z C_z^n\, e^{\frac{-zx}{\beta}} \sum_{s=0}^{\infty} C_s^{-n}\, e^{\frac{-s(x-\lambda)}{\beta}}$$

According (1.12) will be,

$$(G(x))^n = \left\{\begin{matrix}\sum_{z=0}^{\infty}(-1)^z C_z^n \sum_{m=0}^{\infty}\frac{(-1)^m z^m}{m!\beta^m}x^m \\ \sum_{s=0}^{\infty} C_s^{-n}\, e^{\frac{s\lambda}{\beta}} \sum_{t=0}^{\infty}\frac{(-1)^t s^t}{t!\beta^t}x^t\end{matrix}\right\} \qquad (26)$$

Substitute (26) in (25) we get,

$$II = \left\{\begin{matrix}2\sum_{k=0}^{\infty}\frac{(-1)^k}{k+1}\sum_{n=0}^{\infty}\frac{(-1)^n(k-1)^n}{n!\sigma^n}\sum_{z=0}^{\infty}(-1)^z C_z^n \\ \sum_{m=0}^{\infty}\frac{(-1)^m z^m}{m!\beta^m}x^m \sum_{s=0}^{\infty} C_s^{-n}\, e^{\frac{s\lambda}{\beta}} \sum_{t=0}^{\infty}\frac{(-1)^t s^t}{t!\beta^t}x^t\end{matrix}\right\}$$

Now,

$$III = 2\sum_{i=0}^{\infty}\frac{(-1)^i\left(e^{\frac{-(x-\lambda)}{\beta}}\right)^{i-1}}{i+1} = 2\sum_{i=0}^{\infty}\frac{(-1)^i e^{\frac{-(i-1)(x-\lambda)}{\beta}}}{i+1}$$

According (8) will be,

$$I = 2\sum_{i=0}^{\infty}\frac{(-1)^i}{i+1}e^{\frac{(i-1)\lambda}{\beta}}\sum_{j=0}^{\infty}\frac{(-1)^j}{j!}\frac{(i-1)^j}{\beta^j}x^j$$

Substitute (I), (II) and (III) in (24) we get,

$$ln(f(x)_{THL-TL}) = \left\{ \begin{array}{c} ln\left(\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\sigma\left(1-e^{\frac{-1}{\sigma}}\right)}\right) - \frac{1}{\sigma}\sum_{c=0}^{\infty}(-1)^c C_c^1 \sum_{b=0}^{\infty} C_b^{-1}\, e^{\frac{b\lambda}{\beta}} \\ \sum_{f=0}^{\infty}\frac{(-1)^f}{f!}\left(\frac{c+b}{\beta}\right)^f x^f + ln\left(1+e^{-\frac{\lambda}{\beta}}\right) - \frac{(x-\lambda)}{\beta} - ln(\beta) \\ -2\sum_{i=0}^{\infty}\frac{(-1)^i}{i+1} e^{\frac{(i-1)\lambda}{\beta}} \sum_{j=0}^{\infty}\frac{(-1)^j}{j!}\frac{(i-1)^j}{\beta^j} x^j \; - \\ 2\sum_{k=0}^{\infty}\frac{(-1)^k}{k+1}\sum_{n=0}^{\infty}\frac{(-1)^n(k-1)^n}{n!\sigma^n}\sum_{z=0}^{\infty}(-1)^z C_z^n \\ \sum_{m=0}^{\infty}\frac{(-1)^m z^m}{m!\beta^m} x^m \sum_{s=0}^{\infty} C_s^{-n}\, e^{\frac{s\lambda}{\beta}} \sum_{t=0}^{\infty}\frac{(-1)^t s^t}{t!\beta^t} x^t \end{array} \right\}$$

The function of the Shannon entropy ,THL-TLD can be obtained as:

$$SH_{THL-TL} =$$

$$-\int_0^{\infty} \left\{ \begin{array}{c} ln\left(\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\sigma\left(1-e^{\frac{-1}{\sigma}}\right)}\right) - \frac{1}{\sigma}\sum_{c=0}^{1} C_c^1 \sum_{b=0}^{\infty} C_b^{-1}\, e^{\frac{b\lambda}{\beta}} \\ \sum_{f=0}^{\infty}\frac{(-1)^f}{f!}\left(\frac{c+b}{\beta}\right)^f x^f + ln\left(1+e^{-\frac{\lambda}{\beta}}\right) - \frac{x}{\beta} + \frac{\lambda}{\beta} - ln(\beta) \\ -2\sum_{i=0}^{\infty}\frac{(-1)^i}{i+1} e^{\frac{(i-1)\lambda}{\beta}} \sum_{j=0}^{\infty}\frac{(-1)^j}{j!}\frac{(i-1)^j}{\beta^j} x^j \; - \\ 2\sum_{k=0}^{\infty}\frac{(-1)^k}{k+1}\sum_{n=0}^{\infty}\frac{(-1)^n(k-1)^n}{n!\sigma^n}\sum_{z=0}^{\infty}(-1)^z C_z^n \\ \sum_{m=0}^{\infty}\frac{(-1)^m z^m}{m!\beta^m}\sum_{s=0}^{\infty} C_s^{-n}\, e^{\frac{s\lambda}{\beta}} \sum_{t=0}^{\infty}\frac{(-1)^t s^t}{t!\beta^t} x^{m+t} \end{array} \right\} f(x)_{THL-TL}\, dx$$

Therefore, the function of the Shannon entropy , THL-HLD in (10) will be,

$$SH_{THL-TL} = \left\{ \begin{array}{c} -ln\left(\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\sigma\left(1-e^{\frac{-1}{\sigma}}\right)}\right) + \frac{1}{\sigma}\sum_{c=0}^{1} C_c^1 \sum_{b=0}^{\infty} C_b^{-1}\, e^{\frac{b\lambda}{\beta}}\; \sum_{f=0}^{\infty}\frac{(-1)^f}{f!}\left(\frac{c+b}{\beta}\right)^f \\ \int_0^{\infty} x^f f(x)_{THL-TL}dx - ln\left(1+e^{-\frac{\lambda}{\beta}}\right) + \frac{1}{\beta}\int_0^{\infty} x f(x)_{THL-TL}dx - \frac{\lambda}{\beta} + ln(\beta) \\ +2\sum_{i=0}^{\infty}\frac{(-1)^i}{i+1} e^{\frac{(i-1)\lambda}{\beta}} \sum_{j=0}^{\infty}\frac{(-1)^j}{j!}\frac{(i-1)^j}{\beta^j}\int_0^{\infty} x^j f(x)_{THL-TL}dx \; + \\ 2\sum_{k=0}^{\infty}\frac{(-1)^k}{k+1}\sum_{n=0}^{\infty}\frac{(-1)^n(k-1)^n}{n!\sigma^n}\sum_{z=0}^{\infty}(-1)^z C_z^n \sum_{m=0}^{\infty}\frac{(-1)^m z^m}{m!\beta^m} \\ \sum_{s=0}^{\infty} C_s^{-n}\, e^{\frac{s\lambda}{\beta}} \sum_{t=0}^{\infty}\frac{(-1)^t s^t}{t!\beta^t}\int_0^{\infty} x^{m+t} f(x)_{THL-TL}dx \end{array} \right\}$$

$$SH_{THL-TL} = \left\{ \begin{array}{c} -ln\left(\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\sigma\left(1-e^{\frac{-1}{\sigma}}\right)}\right) + \frac{1}{\sigma}\sum_{c=0}^{\infty}(-1)^{c}C_{c}^{1}\sum_{b=0}^{\infty}C_{b}^{-1}e^{\frac{b\lambda}{\beta}} \\ \sum_{f=0}^{\infty}\frac{(-1)^{f}}{f!}\left(\frac{c+b}{\beta}\right)^{f}E(X^{f}) - ln\left(1+e^{-\frac{\lambda}{\beta}}\right) + \frac{1}{\beta}E(X) - \frac{\lambda}{\beta} + ln(\beta) \\ +2\sum_{i=0}^{\infty}\frac{(-1)^{i}}{i+1}e^{\frac{(i-1)\lambda}{\beta}}\sum_{j=0}^{\infty}\frac{(-1)^{j}}{j!}\frac{(i-1)^{j}}{\beta^{j}}E\left(X^{j}\right) + \\ 2\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k+1}\sum_{n=0}^{\infty}\frac{(-1)^{n}(k-1)^{n}}{n!\sigma^{n}}\sum_{z=0}^{\infty}(-1)^{z}C_{z}^{n} \\ \sum_{m=0}^{\infty}\frac{(-1)^{m}z^{m}}{m!\beta^{m}}\sum_{s=0}^{\infty}C_{s}^{-n}e^{\frac{s\lambda}{\beta}}\sum_{t=0}^{\infty}\frac{(-1)^{t}s^{t}}{t!\beta^{t}}E(X^{m+t}) \end{array} \right\}$$

Where,

$E(X)\ E(X^{f}), E\left(X^{j}\right), and E(X^{m+t})$ *as in (21) with* $(r = 1, f, j, m + t)$

v- Stress Strength reliability model

Let Y and X be the Stress- Strength model random variables that independent of each other follows respectively [0,1] THL-TL with different Parameters, the Stress- Strength model can be obtained by,

$SS_{THL-HL} = P(Y < X) = \int_{0}^{\infty} f_X(x)_{THL-TL}\, F_y(x)dx$,

where,

$$F_y(x) = \frac{\left(1+e^{\frac{-1}{\sigma_1}}\right)\left(1-\sum_{i=0}^{\infty}\frac{(-1)^{i}}{\sigma_1{}^{i}i!}\left(\frac{1-e^{-\frac{x}{\beta_1}}}{1+e^{\frac{-(x-\lambda_1)}{\beta_1}}}\right)^{i}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)\left(1+\sum_{i=0}^{\infty}\frac{(-1)^{i}}{\sigma_1{}^{i}i!}\left(\frac{1-e^{-\frac{x}{\beta_1}}}{1+e^{\frac{-(x-\lambda_1)}{\beta_1}}}\right)^{i}\right)} \qquad (27)$$

Let $J = \left(\frac{1-e^{-\frac{x}{\beta_1}}}{1+e^{\frac{-(x-\lambda_1)}{\beta_1}}}\right)^{i}$

$$= \left(1 - e^{-\frac{x}{\beta_1}}\right)^{i}\left(1 + e^{\frac{-(x-\lambda_1)}{\beta_1}}\right)^{-i}$$

According (17) and (11) we get (J),

$$J = \sum_{j=0}^{\infty}(-1)^{j}C_{j}^{1}\, e^{-\frac{jx}{\beta_1}}\sum_{k=0}^{\infty}C_{k}^{-i}\, e^{\frac{-k(x-\lambda_1)}{\beta_1}}$$

According (8) will be,

$$J = \sum_{j=0}^{\infty}(-1)^{j} C_{j}^{1}\, e^{\frac{k\lambda_1}{\beta_1}} \sum_{k=0}^{\infty} C_{k}^{-i} \sum_{n=0}^{\infty} \frac{(-1)^{n}(k+j)^{n}}{n!{\beta_1}^{n}}\, x^{n} \qquad (28)$$

Substitute (28) in (27) we get $F_y(x)$,

$$F_y(x) = \frac{\left(1+e^{\frac{-1}{\sigma_1}}\right)\left(1-\sum_{i=0}^{\infty}\frac{(-1)^{i}}{{\sigma_1}^{i} i!}\sum_{j=0}^{\infty}(-1)^{j}C_{j}^{1}e^{\frac{k\lambda_1}{\beta_1}}\sum_{k=0}^{\infty}C_{k}^{-i}\sum_{n=0}^{\infty}\frac{(-1)^{n}(k+j)^{n}}{n!{\beta_1}^{n}}x^{n}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)\left(1+\sum_{i=0}^{\infty}\frac{(-1)^{i}}{{\sigma_1}^{i} i!}\sum_{j=0}^{\infty}(-1)^{j}C_{j}^{1}e^{\frac{k\lambda_1}{\beta_1}}\sum_{k=0}^{\infty}C_{k}^{-i}\sum_{n=0}^{\infty}\frac{(-1)^{n}(k+j)^{n}}{n!{\beta_1}^{n}}x^{n}\right)}$$

$$= \frac{\left(1+e^{\frac{-1}{\sigma_1}}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)}\left[\frac{2}{\left(1+\sum_{i=0}^{\infty}\frac{(-1)^{i}}{{\sigma_1}^{i} i!}\sum_{j=0}^{\infty}(-1)^{j}C_{j}^{1}e^{\frac{k\lambda_1}{\beta_1}}\sum_{k=0}^{\infty}C_{k}^{-i}\sum_{n=0}^{\infty}\frac{(-1)^{n}(k+j)^{n}}{n!{\beta_1}^{n}}x^{n}\right)} - 1\right]$$

$$= \frac{\left(1+e^{\frac{-1}{\sigma_1}}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)}\left[2\left(1+\sum_{i=0}^{\infty}\frac{(-1)^{i}}{{\sigma_1}^{i} i!}\sum_{j=0}^{\infty}(-1)^{j}C_{j}^{1}\, e^{\frac{k\lambda_1}{\beta_1}}\sum_{k=0}^{\infty}C_{k}^{-i}\sum_{n=0}^{\infty}\frac{(-1)^{n}(k+j)^{n}}{n!{\beta_1}^{n}}x^{n}\right)^{-1} - 1\right]$$

According (11) $F_y(x)$ will be,

$$F_y(x) = \left\{\frac{2\left(1+e^{\frac{-1}{\sigma_1}}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)}\left(\sum_{m=0}^{\infty} C_{m}^{-1}\left(\sum_{i=0}^{\infty}\frac{(-1)^{i}}{{\sigma_1}^{i} i!}\sum_{j=0}^{\infty}(-1)^{j}C_{j}^{1}\, e^{\frac{k\lambda_1}{\beta_1}}\sum_{k=0}^{\infty}C_{k}^{-i}\sum_{n=0}^{\infty}\frac{(-1)^{n}(k+j)^{n}}{n!{\beta_1}^{n}}\right)^{m} x^{mn}\right) - \frac{\left(1+e^{\frac{-1}{\sigma_1}}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)}\right\} \qquad (29)$$

Therefore, based on (29), the stress- strength model of the [0,1] THL-TLD can be obtained as,

$$SS_{THL-TL} = \left\{ \begin{array}{c} \frac{2\left(1+e^{\frac{-1}{\sigma_1}}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)} \left(\sum_{m=0}^{\infty} C_m^{-1} \left(\sum_{i=0}^{\infty} \frac{(-1)^i}{\sigma_1{}^i i!} \sum_{j=0}^{\infty} (-1)^j C_j^1 \, e^{\frac{k\lambda_1}{\beta_1}} \sum_{k=0}^{\infty} C_k^{-i} \sum_{n=0}^{\infty} \frac{(-1)^n (k+j)^n}{n! {\beta_1}^n}\right)^m\right) \\ \int_0^{\infty} x^{nm} f_X(x)_{THL-HL} - \frac{\left(1+e^{\frac{-1}{\sigma_1}}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)} \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \frac{2\left(1+e^{\frac{-1}{\sigma_1}}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)} \left(\sum_{m=0}^{\infty} C_m^{-1} \left(\sum_{i=0}^{\infty} \frac{(-1)^i}{\sigma_1{}^i i!} \sum_{j=0}^{\infty} (-1)^j C_j^1 \, e^{\frac{k\lambda_1}{\beta_1}} \sum_{k=0}^{\infty} C_k^{-i} \sum_{n=0}^{\infty} \frac{(-1)^n (k+j)^n}{n! {\beta_1}^n}\right)^m\right) \\ E(X^{mn}) - \frac{\left(1+e^{\frac{-1}{\sigma_1}}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)} \end{array} \right\}$$

Where, $E(X^{nm})$ as in (21) with $(r = nm)$.

6- Simulated Data:

A random variable X has the THL-TLD can be simulated by solving numerically the following nonlinear equation,

$$U\left(1-e^{\frac{-1}{\sigma}}\right)\left(1+\sum_{i=0}^{\infty}\frac{(-1)^i}{\sigma^i i!}\left(\frac{1+e^{\frac{\lambda}{\beta}}}{e^{\frac{\lambda}{\beta}}\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)}-\frac{1}{e^{\frac{\lambda}{\beta}}}\right)^i\right) - \left(1+e^{\frac{-1}{\sigma}}\right)\left(1-\sum_{i=0}^{\infty}\frac{(-1)^i}{\sigma^i i!}\left(\frac{1+e^{\frac{\lambda}{\beta}}}{e^{\frac{\lambda}{\beta}}\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)}-\frac{1}{e^{\frac{\lambda}{\beta}}}\right)^i\right) = 0$$

Where U has the standard uniform distribution.

A simulation study was carried out to compare the performance of different estimates from [0,1] THL-TLD. We create random variable from [0,1] THL-TLD for different sample sizes and different parameter values. The simulation study is repeated (1000 N) times each with sample size $(n = 20, 50, 150)$ and the selected parameter values. The estimators of (σ, λ, β) . Mean square errors.. Through the R program, we obtained the results shown in the tables (1), we can conclude the following:

Table 1. Empirical MSE for the parameters estimation of $THL-TLD$.

Default values of parameters			Sample size	Empirical MSE		
σ	λ	β	n	$\hat{\sigma}$	$\hat{\lambda}$	$\hat{\beta}$
0.5	0.5	0.5	20	0.001335	0.000937	0.001357
			50	0.001289	0.000652	0.001572
			150	0.001122	0.000279	0.001404
		1.2	20	0.001446	0.001112	0.002321
			50	0.001393	0.000814	0.1768
			150	0.001242	0.000332	0.001514
	1.2	0.5	20	0.001256	0.001004	0.2308
			50	0.001328	0.000814	0.001879
			150	0.001252	0.000476	0.001734
		1.2	20	0.001653	0.001115	0.002354
			50	0.001386	0.000913	0.001968
			150	0.001259	0.000447	0.001813
1.2	0.5	0.5	20	0.001518	0.001207	0.002356
			50	0.001443	0.000942	0.001986
			150	0.001355	0.000613	0.001798
		1.2	20	0.001664	0.001308	0.002483
			50	0.001539	0.001168	0.002138
			150	0.001473	0.000823	0.001793
	1.2	0.5	20	0.001775	0.001353	0.002565

			50	0.001613	0.001063	0.002086
			150	0.001573	0.000874	0.001829
		1.2	20	0.001877	0.001511	0.002643
			50	0.001789	0.001284	0.002286
			150	0.001634	0.000845	0.001975

from this table, we observe that:

i- The MSE's will be decrease as a sample size increases for all cases. This result is compatible with statistical theory.

ii- the increase in scale parameters give us increase for MSE of estimated parameters , for all sample sizes .

iii- the increase in location parameters given us decrease for MSE of estimated parameters, for all sample sizes .

7 - Conclusion

We created a new family of distributions with random variable of continues type, named truncated [0,1] HL-TLD, is discussed as special case.

We provide form the function of the characteristic, the r^{-th} moment, the function of reliability , hazard function, the function of the Shannon entropy and stress- strength model.

References

[1] Abdulrazak R.K. (2018), New Class of probability densities, M.Sc .Thesis, Department of Mathematics, College of Education. Mustansiriyah University.

[2] Abid, S., Hassan, H. (2015) "The Beta Marshall-Olkin Extended Uniform Distribution"; Journal of Safety Engineering 2015, 4(1): 1-7.

[3] Boshi M.A.(2019)," New classes of probability distribution". Thesis submitted to the council of the college of science at the Mustansiriya University.

[4] Eugene, N., Lee, C., Famoye, F. (2002) "Beta-normal distribution and its applications" Commun. Statist. - Theory and Methods 31:497-512.

[5] Gupta, A.K., Nadarajah, S. (2004) "On the moments of the beta normal distribution" Commun. Statist. - Theory and Methods 33:1-13.

[6] Kang S.K; Park Y.K. (2005),"Estimation for the half- logistic distribution based on multiply Type-II Censored Samples. Journal of Korean, Vol.16,No.L;145-156.

[7] Kehinde O.A; Samuel A.F.(2019), "Astudy on Transmuted Half Logistic Distribution: properties and Application". International Journal of statistical distribution and Applecation;5(3):54-59.

[8] Mohammed. M. A; and Taqi. A.Y.(2015), Estimate parameter and Reliability function for Truncated logistic distribution: simulated study. No. 93,ISSN 0215-9511.