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A study on SS- π- regular fuzzy ideals of semi groups

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ABSTRACT

In this paper, the notion of SS -π- regular fuzzy ideals of semi groups as a generalization of regular fuzzy ideal has been introduced and some of their important related properties have been investigated. Characterizations of fuzzy interior ideal , anti fuzzy ideal , anti fuzzy bi-ideal and anti fuzzy generalized –bi -ideal in terms of SS - π- regular fuzzy ideal have also been obtained .

1. Introduction

The fundamental concept of a fuzzy for short (F) set was first presented by [1], The concept F sets in the structure of groups, was first studied [2]. The concept of fuzzy ideals for short (FI) in semi group was developed by [3] On the other hand, the concept of anti fuzzy subgroups of groups introduced by [4]. The concept of anti fuzzy ideals in semi groups and characterized different classes of semi groups by the properties of their anti fuzzy ideals" explained by [5] and [6], gave some properties of anti fuzzy ideals for short (AN-F-I) in terms of regular and left (right) quasi regular semi group), where A F sub-set p of a semi group Q is called anti F sub-semi group of Q if $p(\alpha\beta) \leq p(\alpha) \vee p(\beta)\ \forall\, \alpha , \beta \in$ Q" .

"A F sub-set p of a semi group Q is called anti fuzzy left (right) for short(AN-F-L(R)-I) of Q if $p(\alpha\beta) \leq p(\beta)$, $(p(\alpha\beta) \leq p(\alpha))\ \forall\ \alpha , \beta \in$ Q ".

"A F sub-set p of a semi group Q is called a anti F ideal of Q if it is both (AN-F- L-I) and (AN-F- R-I)".

"A F sub-set p of a semi group Q is called anti fuzzy interior ideal for short (AN-F-IN-I) of Q if $p(\alpha\gamma\beta) \leq p(\gamma)$; $\forall\, \alpha, \gamma , \beta \in$ Q ".

"A F sub-set p of a semi group Q is called anti fuzzy generalized bi-ideal of Q for short (AN-F-G-BI-I) if $p(\alpha\, \beta\, \gamma) \leq p(\alpha) \vee p(\gamma)$; $\forall\, \alpha , \beta , \gamma \in$ Q",

"A F sub - semi group p is called anti Fuzzy bi-ideal of Q if $p(\alpha\, \beta\, \gamma) \leq p(\alpha) \vee p(\gamma)$, $\forall\, \alpha , \beta , \gamma \in$ Q ". The concept of" Intra-regular left almost semi group characterized by their AN-F-I " was studies by(Khan, Asif and Faisal,2010) where "A F sub-set p of a LA-semi group Q is called a F LA- sub-semi group if $p(\alpha\, \beta) \geq p(\alpha) \vee p(\beta)$; $\forall\, \alpha , \beta \in$ Q".

"A F sub-set p of LA- semi group Q is called a F left (right) ideal of Q if $p(\alpha\, \beta) \geq p(\beta)$, $(p(\alpha\, \beta) \geq p(\alpha))\ \forall\, \alpha , \beta \in$ Q" .

"A F LA- sub semi group p of a LA- semi group Q is called a F bi-ideal if

$p((\alpha\, \beta)\gamma) \geq p(\alpha) \wedge p(\gamma)\ \forall\, \alpha , \beta , \gamma \in$ Q" .

"A F LA- sub semi group p of a LA- semi group Q is AN-F-IN-I if

$p((\alpha\, \beta)\gamma) \geq p(\beta)\ \forall\ , \beta , \gamma \in$ Q ".

Now, we shall give the concepts of fuzzy sub-set and basic definitions with some related properties which will be used in this paper .

Let Q be a semi group , By a sub-semi group of Q we mean a nonempty C of Q s.t $C^2 \sqsubseteq$C , and by a left (right) ideal of Q we mean a nonempty sub-set C of Q s.t QC⊑C (CQ⊑C) . by two sided ideal or simply ideal , we mean a non-empty sub-set of Q which is a both "a left and right ideal of" Q, a sub-semi group C of a semi group Q is called bi- ideal of Q if CQC⊑C , by a F set p in anon empty Q . “we mean a function p: Q $\longrightarrow$[0 , 1] and the complement of p denoted by p' , is the fuzzy set in Q given by $p'(\alpha)$ = 1− $p(\alpha)\ \forall\ \alpha \in$Q.” , The concept of "fuzzy interior ideals in semi groups" was studied by (Hong and Jun , 1995) where a fuzzy sub-set p in a semi group Q is called a fuzzy sub-semi group of Q- if $p(\alpha\, \beta) \geq \min\{p(\alpha) , p(\beta)\}$; $\forall\, \alpha , \beta \in$ Q.

A fuzzy sub-set p of a semi group Q is called a fuzzy interior ideal for short (F- IN - I) of Q if : $p(\alpha\beta\gamma) \geq p(\beta)$; $\forall\ \alpha, \beta, \gamma \in$ Q ."

The goal of this article is to characterize an SS-π-regular Semi group by the properties of their (AN-F-IN-I) , (AN-F-L(R)-I), anti fuzzy two sided ideal for short (AN-F-T-S-I), (AN-F-G-BI-I) . We also give some properties a SS -π- regular LA-semi group and their fuzzy (left (right), two sided) ideals.

2. The main results

In this section, we study the concept of SS -π-regular semi groups, and we give basic properties of this concept.

Now, we give the following definitions

Definition 2.1

A semi group Q is called SS –π- regular if for every a ∈ Q, ∃ b, c ∈ Q, s.t $a^n = a^n$ba^{2n}c, for some n ∈ Z^+, equivalently $a^n \in a^n$Qa^{2n} Q , for every a ∈ Q , for some $n \in Z^+$.

Example 2 . 2 :

Let Q={1, 2, 3, 4, 5, 6} be a semi group with then

.	1	2	3	4	5	6
1	1	2	3	4	5	6
2	4	1	5	2	6	3
3	5	3	1	6	4	2
4	2	4	6	1	3	5
5	3	6	2	5	1	4
6	6	5	4	3	2	1

Then it is clear that , Q is a SS- π - regular because if n = 2

1^2= $1^2$3 1^4 5 , 2^2= $2^2$4 2^4 2 , 3^2= 3^2 6 3^4 6 , 4^2= $4^2$6 4^4 6,

5^2= $5^2$1 $5^4$1 , 6^2= $6^2$3 $6^4$5

Definition 2.3:

Let p and φ be any F sub-set of a semi group Q, then the product $p \circ \varphi$ is $defined\ by$

$$(p \circ \varphi)_{(a^n)} = \begin{cases} \bigvee_{a^n = b^n c^n} \{p(b^n) \wedge \varphi(c^n)\}; \text{ if } \exists\ b, c \in Q \text{ s.t } a^n = b^n c^n \text{ for some } n \in Z^+ \} \\ 0 \ ; otherewise. \end{cases}$$

Definition 2.4:

Let p and φ be any F sub-set of a semi group Q. Then the anti product $p * \varphi$ is defined by

$$(p * \varphi)_{(a^n)} = \begin{cases} \bigwedge_{a^n = b^n c^n} \{p(b^n) \vee \varphi(c^n)\}; \text{ if } \exists\ b, c \in Q \text{ s.t } a^n = b^n c^n \} \\ 1 \ ; otherwise \ . \end{cases}$$

for some $n \in Z^+$.

Now we characterize SS- π - regular semi group by the properties of their FI

Theorem 2.5 :

In SS -π- regular semi group Q, every F-IN-I is idempotent .

Proof :

Suppose that p is a F-IN- I of a semi group Q , then it is clear that $p \circ p \subseteq p$

Let a ∈ Q , since Q is a SS -π- regular semi group, then ∃ x , y ∈ Q s.t $a^n = a^n$xa^{2n}y for some n∈ Z^+, we have

a^n= a^nxa^{2n}y= a^nx$a^n a^n$y =$a^n x(a^n x a^{2n} y)a^n y$ = $(a^n x)\, a^n$(xa^{2n}) ya^ny

$(p \circ p)_{(a^n)}$ =

$\bigvee_{a^n = (a^n x) a^n (x a^{2n}) y a^n y} \{p((a^n x) a^n (x a^{2n})) \wedge p(y a^n y)\}$

$\geq \{p((a^n x)\, a^n (x a^{2n})) \wedge p(y a^n y)\}$

$\geq p(a^n) \wedge p(a^n) = p(a^n)$ This implies that $p \subseteq p \circ p$,

Hence $p \circ p = p$.

Example 2 . 6 :

Let Q ={r^n, s^n, t^n, v^n} be a set with operation as follows :

.	r^n	s^n	t^n	v^n
r^n	r^n	r^n	r^n	r^n
s^n	r^n	r^n	r^n	r^n
t^n	r^n	r^n	s^n	r^n
v^n	r^n	r^n	s^n	s^n

Then we can easily see that (Q, .) is not SS -π-regular semi group

Define the F sub-set p of Q as :

$p(r^n) = 0.3$, $p(s^n) = 0.9$, $p(t^n) = 0.5$, $p(v^n) = 0.7$

Then it is clear that, p is AN-F-IN-I of Q but not an AN-F-T-S-I of Q, because {r^n, t^n} is not a 'two sided ideal of Q .

Theorem 2 . 7:

A F sub-set p of SS -π- regular semi group Q is an AN-F-T-S-I of Q if and only if is any AN-F-IN-I of Q .

Proof :

suppose that p be an AN-F-T-S-I of Q , obviously , p is AN-F-IN-I of Q.

Conversely :

suppose that p is any AN-F-IN-I of Q, let a , b ∈ Q, since Q is SS -π- regular semi group so ∃ x , y , w , z ∈ Q S.t $a^n = a^n$ x a^{2n} y; $b^n = b^n$ w b^{2n} z we have $p(a^n b^n) = p((a^n$ x a^{2n} y) $b^n) = p((a^n$x) $a^n(a^n$ y b^n))= $p((s^n\ a^n t^n) \leq p(a^n)$

where $s^n = a^n$ x and $t^n = a^n$ y b^n

Also $p(a^n b^n) = p((a^n b^n$ w $b^{2n})$ z) = $p((a^n b^n$ w) $b^n(b^n$ z)) =$p((d^n\ a^n u^n) \leq p(a^n)$

where $d^n = a^n\ b^n$w and $u^n = b^n$z . Hence, p is an AN-F-T-S-I of Q .

Proposition 2 . 8 :

Suppose that Q is SS -π- regular semi group, then :

1- Every AN-F-R-I is Idempotent.

2- Every AN-F-IN-I is Idempotent .

Proof :

1- Let p is any AN-F-R-I of semi group Q , then it is clear that $p \subseteq p * p$. since Q is a SS –π- regular so ; ∀ a ∈ Q , ∃ x , y ∈ Q , s.t $a^n = a^n$x a^{2n}y , for some n∈ Z^+, so we have

$(p * p)_{(a^n)} = \bigwedge_{a^n = a^n x\, a^{2n} y.} \{p(a^n x) \vee p(a^n a^n y)\}$

$= \bigwedge_{a^n = a^n x\, a^{2n} y.} \{p(a^n x) \vee p(a^n z)\}$ where z = a^ny

$\leq p(a^n$x) $\vee\ p(a^n z) \leq p(a^n) \vee p(a^n) = p(a^n)$ this implies that $p * p \subseteq p$ and

hence $p * p = p$.

2- Suppose that p is any AN-F-IN-I of semi group Q, then it is obvious that

$p \subseteq p * p$. since Q is a SS -π- regular so, ∀ a ∈ Q, ∃ x , y ∈ Q, s.t a^n = a^nx a^{2n}y, for some n ∈ Z^+, so we have:

a^n = a^nx a^{2n}y = a^nx$a^n a^n$y

= $(a^n$x$)a^n(xa^{2n})(ya^n y)$

= $\wedge_{a^n = a^n x\, a^{2n} y.}\{p((a^n x)a^n(xa^{2n})) \vee p(ya^n y)\}$

$\leq p((a^n x)a^n(xa^{2n})) \vee p(ya^n y)$

$\leq p(a^n) \vee p(a^n) = p(a^n)$,

this implies that $p * p \subseteq p$. hence $p * p = p$.

Proposition 2 . 9 : [8]

Let p is any AN-F-R-I and φ AN-F- L-I of a semi group Q,then $p * \varphi \supseteq p \cup \varphi$

It is clear that from Proposition 2.9 $p * \varphi \supseteq p \cup \varphi$, but the converse needs not at all be true . Consider the following example

Example 2.10 :

Consider the semi group Q=$\{r^n, s^n, t^n, v^n\}$ with the operation as follows :

.	r^n	s^n	t^n	v^n
r^n	r^n	r^n	r^n	r^n
s^n	r^n	r^n	r^n	r^n
t^n	r^n	r^n	s^n	r^n
v^n	r^n	r^n	s^n	s^n

The ideals of Q are $\{r^n\}$, $\{r^n, s^n\}$, $\{r^n, s^n, t^n\}$ and $\{r^n, s^n, t^n, v^n\}$

Let us define two F sub set p and φ of Q as follows

$p(r^n) = 0.5$, $p(s^n) = 0.6$, $p(t^n) = 0.7$, $p(v^n) = 0.8$,

$\varphi(r^n) = 0.6$, $\varphi(s^n) = 0.7$, $\varphi(t^n) = 0.8$, $\varphi(v^n) = 0.9$.

$\Longrightarrow$ p and φ are an anti F ideal of Q , and we note that:

$(p * \varphi)_{(S^n)} = \wedge_{S^n = x^n y^n}\{p((r^n x)r^n(xr^{2n})) \vee \varphi(yr^n y)\}$

$= \wedge\{0.8, 0.8, 0.9\} = 0.8 \geq (p \cup \varphi)_{(S^n)} = 0.7$

To consider the converse of proposition 2.9 , we need to streng then the condition of semi group Q.

Theorem 2.11 :

If p , φ are an AN-F-T-S-I of SS -π- regular semi group Q, Then $p * \varphi = p \cup \varphi$.

proof :suppose that p and φ be an AN-F-T-S-I of Q, $\Longrightarrow$ obviously $p * \varphi \supseteq p \cup \varphi$. since Q is a SS –π- regular so for each element a ∈ Q, ∃ x , y ∈ Q , s.t a^n =a^nxa^{2n}y, for some n ∈ Z^+,

so we have $(p * \varphi)_{(a^n)} = \wedge_{a^n = a^n x\, a^n a^n y.}\{p(a^n x) \vee \varphi(a^n a^n y)\}$

$\leq p(a^n x) \vee \varphi(a^n a^n y) \leq p(a^n) \vee \varphi(a^n) = (p \cup \varphi)(a^n) \Longrightarrow (p * p) \subseteq p \cup \varphi$.

Hence, $p * p = p \cup \varphi$.

Example 2.12 :

Let Q= $\{r^n, s^n, t^n\}$be a semi group with the following table :

.	r^n	s^n	t^n
r^n	r^n	s^n	t^n
s^n	s^n	s^n	t^n
t^n	t^n	t^n	t^n

Define a F p sub - set of Q by $p(r^n) = 0.6$, $p(s^n) = 0.5$, $p(t^n) = 0.4$.

By routine calculation , we can check that p is an AN-F-I , AN- F-IN-I and anti F bi- ideal , because Q is SS -π- S regular semi group .

Now , we give other F characterizations of SS -π - regular semi group .

Proposition (2 . 13) :

For a F sub-set p of an SS -π- regular semi group Q , the following conditions are equivalent :

1- p is anti F bi- ideal of Q .

2- p is an AN-F-G-BI-I of Q.

Proof :

1⟶2 suppose that p is any anti F bi-ideal of Q the obviously p is an AN-F-G-BI-I of Q.

2⟶1 suppose p be any AN-F-G-BI-I of Q, and x, y ∈ Q, Then since Q is an SS -π- regular of a semi group, So; ∀ x ∈ S, ∃ a, b ∈ Q s.t x^n = x^n a x^{2n}b . thus we have :

$p(x^n y^n) = p((x^n$ a x^{2n}b$)y^n) = p(x^n($ a x^{2n}b$)y^n) = p((x^n w^n y^n)$

$\leq p(x^n) \vee p(y^n)$ where w^n = a x^{2n}b .

Therefore, p is an anti F sub-semi group of Q . Hence , p is an anti F bi- ideals of Q.

Theorem 2. 14 :

In SS -π- regular semi group Q, $p * \varphi \leq p \vee \varphi$, for anti fuzzy bi-ideal p and AN-F-R-I φ .

Proof :

Let p and φ be any AN-F-BI-I and AN-F-R-I of Q , respectively and let a ∈ Q . Then since Q is an SS -π- regular of a semi group, ∃ x , y ∈ S s.t

a^n = a^n x a^{2n}y then we have :

$(p * \varphi)_{(a^n)} = \wedge_{a^n = b^n c^n.}\{p(b^n) \vee \varphi(c^n)\} \leq p(a^nx\, a^n) \vee \varphi(a^n y)$

$\leq p(a^n) \vee \varphi(a^n) = (p \vee \varphi)(a^n)$

And so we have $p * \varphi \leq p \cup \varphi$.

Theorem 2.15 :

Let p and φ be any AN-F-IN-I of SS- π- regular of a semi group Q, then $(p * \varphi) \vee (\varphi * p) \leq p \vee \varphi$

Proof :

suppose that p , φ be any AN-F-IN-I of Q, and a ∈ Q, $\Longrightarrow$ since Q is SS -π- regular,

∃ x , y ∈ Q s.t a^n = a^n x a^{2n}y = $((a^n x)a^n$ (xa^{2n}))(y$a^n y$) . Hence

$(p * \varphi)_{(a^n)} = \wedge_{a^n = b^n c^n.}\{p(b^n) \vee \varphi(c^n)\} \leq p((a^n x)a^n$ (xa^{2n})$) \vee \varphi(ya^n y)$

$\leq p(a^n) \vee \varphi(a^n) = (p \vee \varphi)(a^n)$

And so we have $p * \varphi \leq p \vee \varphi$ .similarly, we have $(\varphi * p) \leq p \vee \varphi$

Therefore $(p * \varphi) \vee (\varphi * p) \leq p \vee \varphi$.

Theorem 2. 16 :

Let Q be an SS -π- regular semi group. Then $\varphi * \alpha * p \subseteq \varphi \cup \alpha \cup p$. for every AN-F-L–I α , every AN-F-G-BI-I φ and every AN-F-IN-I p of Q .

Proof :

Let φ and p be any AN-F-L-I, AN-F-G-BI-I, and AN-F-IN-I p of Q, respectively and Let a ∈ Q, since Q is an SS -π- regular,

∃ x, y ∈ Q s.t a^n = a^n x a^{2n}y = $(a^n x a^n$ xa^{2n}y$a^n y)$, Then we have $:(\varphi * \alpha * p)_{(a^n)}$ $= \wedge_{a^n = a^n x a^n \, x a^{2n} y a^n y.}\{\varphi(a^n x a^n) \vee (\alpha * p)(x a^{2n} y a^n y)\}$

$\leq \varphi(a^n) \vee \{\wedge_{x a^{2n} y a^n y.}\{\alpha(x a^{2n}) \vee p(y a^n y)\}$

$\leq \varphi(a^n) \vee \alpha(a^n) \vee p(a^n) = (\varphi \cup \alpha \cup p)(a^n)$

And so we have $\varphi * \alpha * p \subseteq \varphi \cup \alpha \cup p$.

Now , we characterized SS –π- regular LA- semi groups by the properties of their F left (right , two sided) ideals ".

Let Q be a groupoid . Then

1- Q is called left almost semi group if (rs)t=(ts)r , ∀ r ,s ,t ∈ Q

2- Medial law of a left almost semi group means (r s) (t v) = (r t) (s v) ; ∀ r ,s ,t ,v ∈ Q

3- In additional if Q has a left identity (necessary unique) the paramedical law mean (r s) (t v) = (v s) (t r) ; ∀ r ,s ,t ,v ∈ Q

4- An left almost semi group with" right identity" becomes a commutative semi group with identity . if an LA- semi group contains" left identity" , the following law holds r (s t) = s (r t) ; ∀ r ,s ,t ∈ Q.

Definition 2.17 :

Any element a of LA-semi group Q is called SS –π- regular if ∃ x , y ∈ Q ,

s.t a^n =(a^nx a^{2n}) y, for some n ∈ Z^+,

and Q is called SS -π- regular if every elements of Q is SS –π- regular .

Example 2.18 :

Let Q = {1, 2, 3, 4, 5} be an LA - semi group with the "left identity (5)" with Then

.	1	2	3	4	5
1	5	1	2	3	4
2	4	5	1	2	3
3	3	4	5	1	2
4	2	3	4	5	1
5	1	2	3	4	5

Then it is clear that, Q is a SS –π- regular because if n = 2

$1^2 = 1^2 3\ 1^4\ 2$, $2^2 = 2^2 4\ 2^4\ 1$, $3^2 = 3^2\ 5\ 3^4\ 5$, $4^2 = 4^2 2\ 4^4\ 3$, $5^2 = 5^2 1\ 5^4 4$.

Proposition 2 . 19 :

A F sub - set p of an SS -π- regular semi group Q is a F right ideal iff it is a F left ideal .

Proof : suppose that p is a F right ideal of Q. since Q is SS – π- regular so, ∀ a ∈ Q, ∃ x , y ∈ Q , s.t a^n =(a^nx a^{2n}) y ,for some n ∈ Z^+ So by (1)

$p(a^n b^n) = p(((a^n x\, a^{2n})\ y) b^n\qquad)$

$=p((b^n y)(a^n x\, a^{2n})) \geq p(b^n y) \geq p(b^n)$

Conversely :

suppose that p is a F left ideal of Q, Then by (1)

$p(a^n b^n) = p(((a^n x\, a^{2n})\ y) b^n\qquad) = p((b^n y)(a^n x\, a^{2n}))$

$\geq p(d\, a^{2n}) \geq p(a^{2n}) \geq p(a^n)$.

Lemma 2.20 :

Every F two sided of SS -π- regular LA - semi group Q with " left identity " is idempotent .

Proof :

suppose that p is a F two sided ideal of Q Then it is clear that $p \circ p \subseteq p \circ Q \subseteq p$, since Q is SS – π - regular so ; ∀ a ∈ Q , ∃ x , y ∈ Q , s.t a^n =(a^nx a^{2n}) y,

for some n ∈ Z^+ So by (1) a^n =(a^nx a^{2n}) y = (y x $a^n a^n$)a^n, thus we have

$(p \circ p)_{(a^n)} = \vee_{a^n = (y\, x\, a^n a^n) a^n}\, p(y\, x\, a^n a^n) \wedge p(a^n)$

$\geq p(y\, x\, a^n a^n) \wedge p(a^n)$

$\geq p(a^n) \wedge p(a^n) = p(a^n)$.

And this implies that $p \circ p \supseteq p$, hence $p \circ p = p$.

Theorem 2 . 21 :

For a F sub-set p of SS -π- regular LA-semi group Q ,with "left identity" the following conditions are equivalent :

1- p is a F two sided ideal of Q .

2- p is a F-AN-I of Q .

Proof :

1⟶2 suppose p be a F two sided ideal of Q, ⟹ obviously p is a F-IN-I of Q.

2⟶1 Let p is a F-IN-I of Q , and a , b ∈ Q ⟹ , since Q is an SS -π- regular of LA - semi group , So ∃ x , y , u , v ∈ Q s.t $a^n = a^n$ x a^{2n}y, $b^n = b^n$ u b^{2n}v , we have :

$p(a^n b^n) = p((a^n\, x\, a^{2n}) y) b^n)$ by (1)

$= p((b^n y)(a^n x a^n a^n)\,)$ by (2)

$= p((b^n a^n)(y x a^n a^n)\,) = p((b^n a^n) z^n) \geq p(a^n)$, where $z^n = y x a^n a^n$

Also $p(a^n b^n) = p(a^n (b^n \text{u}\, b^n)\ \text{v}))$ by (4)

$= p((b^n \text{u} b^{2n})(a^n \text{v})\,)$

$= p((w^n b^n t^n) \geq p(b^n)$,

where $w^n = b^n u b^n$ and $t^n = a^n$v

Hence, p is a F two sided ideal

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دراسة حول المثاليات المنتظمة المضببة من النمط -π -SS على شبه الزمرة

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الملخص

في هذا البحث تم تقديم تعريف المثاليات المضببة المنتظمة من النمط -π -SS على اشباه الزمر كتعميم للمثاليات المضببة المنتظمة على اشباه الزمر. وتم دراسة بعض الخصائص الاساسية لها والحصول على بعض النتائج الجديدة .