Theoretical Design of a Ball Balancing on Plate Controller

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Abstract

The ball-on-plate balancing systems present a challenging design problem. In this study, the ball on plate balancing system has been analyzed. The main objective is to find a controller capable of controlling the ball position, and to tracking a certain preset complicated paths with precision and accuracy.

Two techniques are implemented in the design of this controller, full state feedback and output feedback. One of the modest methods is the Direct Eigenstructure Assignment, which is used for the first time in this study using MATLAB.

It has been shown that both regulation and tracking can be implemented in the Direct Eigenstructure Assignment algorithm with ease. The results show that the overall method is very quick and simple to use.

الخـلاصـــــة

تمثل منظومة الاتزان للكرة على السطح المستوي مشكلة و تحدي تصميمي. في هذه الدراسة، تم تحليل هذه المنظومة، إن الهدف الرئيسي لهذه الدراسة هو إيجاد مسيطر قادر على السيطرة على موقع الكرة، وإن يكون قادراً على جعل الكرة تتبع مسارات معقدة يتم تحديدها بدقة عالية.

تم استخدام تقنيتين لتصميم هذا المسيطر ، التقنية الأولى هي (Full State Feedback)، والتقنية الثانية هي (Output Feedback)، ولكل من هذه التقنيات عدة طرق للحل تصف تطور الستراتيجية المستخدمة لإتمام تصميم المسيطر لموازنة الكرة على السطح المستوي. إحدى طرق الحل هذه هي طريقة (Eigenstructure Assignment) التي لم يتطرق لها الباحثون الآخرون في هذه الدراسة تفصل الطريق لتصميم المسيطرات للأنظمة المختلفة بأستخدام طريقة (Eigenstructure Assignment) والتي تمت برمجتها بإستخدام لغة البرمجة MATLAB .

لقد تم التوضيح بأن كلاً من (Tracking and Regulating) يمكن أن ينفذ بطريقة (Direct Eigenstructure Assignment) بكل سهولة . وعليه فإن هذه الطريقة تمتاز بأنها سريعة و سهلة الاستعمال.

1. Introduction

The ball-on-plate balancing system shown in **Fig.(1)** is a complex multivariable control to the assumptions when the mathematical model is derived, and thus the model is approximated, some of the states of the system are decoupled from each other, so the system becomes as two SISO systems [1,2]. A new technique known as Eigenstructure Assignment is presented here. This technique helps the designer to keep the states or outputs of the system uncoupled, even when these states are coupled (if some of the neglected quantities are considered). So, this approach generates decoupled motion, which can be used to improve path tracking and accuracy. The gain obtained is not a function to motion condition only, but also to the mode selected [3]. The Direct Eigenstructure Assignment (DEA) method allows designers to shape the closed-loop response by choice of desired eigenvalues and eigenvectors. During this design effort DEA has been demonstrated to be a useful technique for aircraft control design. The control laws developed using DEA have demonstrated good performance, and robustness during simulations [4].
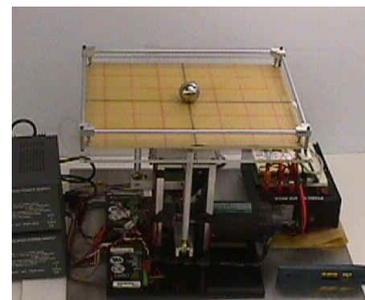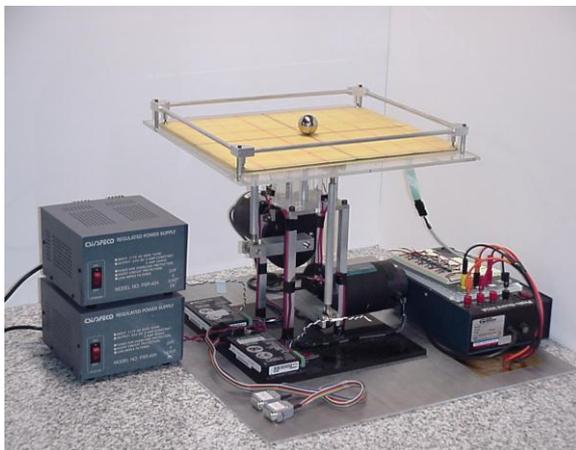

Figure (1) Ball on plate balancing system [1]

S. A. and Craig [1] discussed the conceptions and development of a ball on plate balancing system. Realization of the design is achieved with the simultaneous considerations toward constraints like cost, performance, functionality, extendibility, and educational merit. G. Andrew et. al. [2] described the proposed design and development strategy for implementing a control system to balance a ball on a plate. A pan-tilt device is placed on its side so as to create a tilt-tilt mechanism capable of moving a ball within an X-Y plane as shown in **Fig.(2)**. Dynamic modeling of this system allows the creation of a digital controller capable of placing the ball at certain locations or following a preset path. The project goal is to create a system capable of moving the ball at a rapid rate of speed in any of several predefined complex paths with precision and accuracy. L. F. Faleiro and R. W. Pratt [3] examined the use of eigenstructure assignment in the design of an aircraft stability augmentation system. It is seen that a static gain eigenstructure assignment controller itself may be insufficient if only a small number of outputs is available for measurement. J. W.

Choi [5] presents that the problem of eigenstructure assignment (simultaneous assignment of eigenvalues and eigenvectors) is of great importance in control theory and applications because the stability and dynamic behavior of a linear multivariable system are governed by the eigenstructure of the system. In general, the speed of response is determined by the assigned eigenvalues whereas the shape of the response is furnished by the assigned eigenvectors.
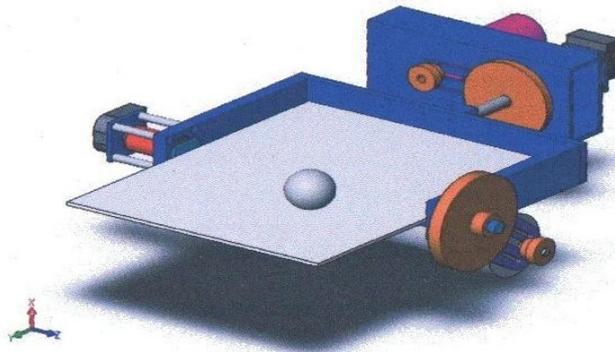

Figure (2) CAD model of a tilt-tilt mechanism [2]

2. Mathematical Model of the System

2-1 The State Differential Equation and System Modiling

The state of a system is described by the set of first-order differential equations written in terms of the state variables (x_1, x_2,…, x_n). These first-order differential equations can be written in matrix form as:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

The outputs of a linear system can be related to the state variables and the input signals by the output equation, see **Fig.(3)**.

$$\mathbf{y} = \mathbf{C}\mathbf{x} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$
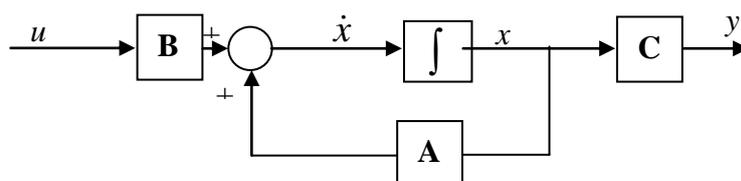

Figure (3) Block diagram description of dynamic relations

The linearized differential equations of the ball-on-plate balancing system shown in **Fig.(1)** [1] are:

$$\left.\begin{array}{l} \dfrac{7}{5}\ddot{x}_b + \left(\dfrac{7}{5}r_b + h\right)\ddot{q}_2 = gq_2 \\[3mm] \dfrac{7}{5}\ddot{y}_b - \left(\dfrac{7}{5}r_b + h\right)\ddot{q}_1 = -gq_1 \end{array}\right\} \quad \text{.. (3)}$$

The transfer function of the state-space system (output to input relation) G(s) is equal to [6]:

$$G(s) = C\,(sI\text{-}A)^{-1}\,B + D \text{ .... (4)}$$

Note that if the degree of the numerator of the transfer functions G(s) is equal to the degree of the denominator then a constant term can be split off first, which becomes the matrix D. Then Eq.(2) becomes:

$$y = Cx + Du \text{ .... (5)}$$

D: transmission matrix, $p \times m$.

Hence, for the system under consideration, the variables that can be measured are the position of the ball in the x,y directions, angular displacement and velocity of the motor and the plate.

The state variables are assumed to be:

$$u_2 = q_2, \quad u_1 = q_1, \quad x_1 = x_b + \left(r_b + \dfrac{5}{7}h\right)u_2, \quad x_2 = \dot{x}_1, \quad x_3 = y_b - \left(r_b + \dfrac{5}{7}h\right)u_1, \quad x_4 = \dot{x}_3$$

Sub these states into Eqs.(3), then,

$$\therefore \ddot{x}_1 = 7u_2 \quad \text{Or} \quad \dot{x}_2 = 7u_2, \qquad \ddot{x}_3 = -7u_1 \quad \text{Or} \quad \dot{x}_4 = -7u_1$$

2-2 Controllability and Observability

The system (A, B, C, D) is controllable if and only if the rank of the matrix:

$$C = (B,\ AB,\ A^2B...\ A^{n-1}B) \quad \text{is equal to n .... (7)}$$

and the system is observable if and only if the rank of

$$\mathbf{Q} = \begin{Bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA^2} \\ \vdots \\ \mathbf{CA^{n-1}} \end{Bmatrix} \quad \text{is equal to n} \quad \text{..} \quad \textbf{(8)}$$

The matrices C and Q are called controllability and observability matrices, respectively [7].

2-3 Feedback Control Theory

2-3-1 Controller Design with Output Feedback

Let a multivariable system be described by the state equations (Eqs.(1,5)), where y and u are $m \times 1$ vectors. The plant matrix relating the output Y(s) to the control U(s) is given by:

$$\mathbf{G_p(s) = C(sI - A)^{-1}B + D} \text{...} \textbf{(9)}$$

a compensator matrix G(s) is designed, so that

$$\mathbf{Y(s) = G_p(s)G(s)M(s)} \text{...} \textbf{(10)}$$

The matrix G(s) is chosen such that $G_p(s)G(s)$ is a diagonal matrix.

2-3-2 Controller Design with State Feedback

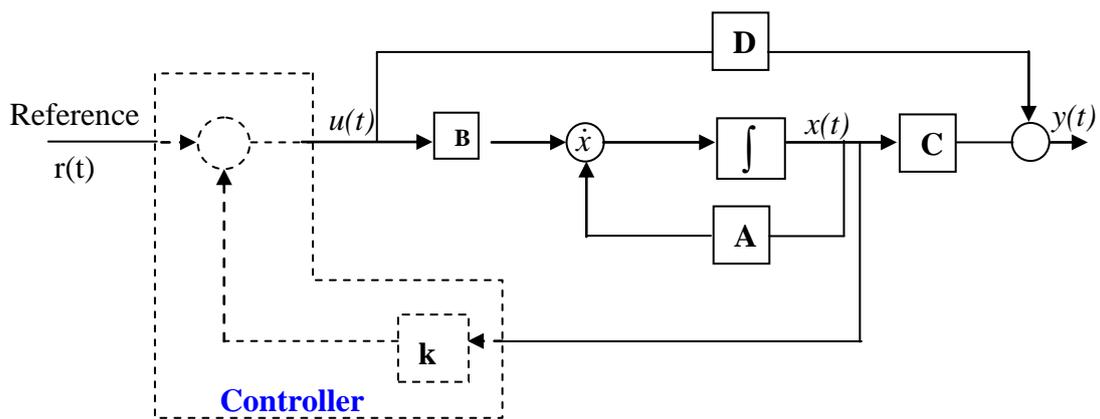**Figure (4)** shows the block diagram of state feedback controller

Figure (4) Block diagram description of state feedback

The following equation is only needed:

$$\mathbf{u = r - Kx} \quad\text{...} \quad (11)$$

and K is selected accordingly.

Combining eqs.(1) and (5) with eq.(10) the following is obtained:

$$\dot{\mathbf{x}} = (\mathbf{A - BK})\mathbf{x + Br} \quad\text{...} \quad (12)$$

it is clear that the state feedback matrix is:

$$\mathbf{A - BK} \quad\text{...} \quad (13)$$

therefore, the characteristic equation of the system is:

$$\left|\mathbf{sI - (A - BK)}\right| = \mathbf{0} \quad\text{...} \quad (14)$$

By equating the coefficient of the determinant with the desired roots, the controller will be obtained.

2-4 Eigenspace (Eigenstructure) Method

The system outputs are [1]:

$$\mathbf{y(t) = Ce^{At}x(0) + \int_0^t Ce^{A(t-\tau)}Bu(\tau)d\tau} \quad\text{...} \quad (15)$$

And the system dynamic matrix A can be represented by:

$$\mathbf{A = V\Lambda V^{-1} = V\Lambda L} \quad\text{..} \quad (16)$$

$$\mathbf{e^{At}Ve^{\Lambda t}L = \sum_1^n v_j e^{\lambda_j t}l_j} \quad\text{...} \quad (17)$$

Equation (13) can then be expressed as:

$$\mathbf{y(t) = \sum_{j=1}^n Cv_j e^{\lambda_j(t)}l_j x(0) + \sum_{j=1}^n Cv_j \int_0^t e^{\lambda_j(t-\tau)}l_j Bu(\tau)d\tau} \quad\text{...........................} \quad (18)$$

2-5 Direct Eigenspace Assignment Formulation For State Feedback

2-5-1 The Control Problem

Simple EA can be used to produce a ($m \times n$) static state feedback controller K, where

$$\mathbf{u} = -\mathbf{kx} \dotfill (19)$$
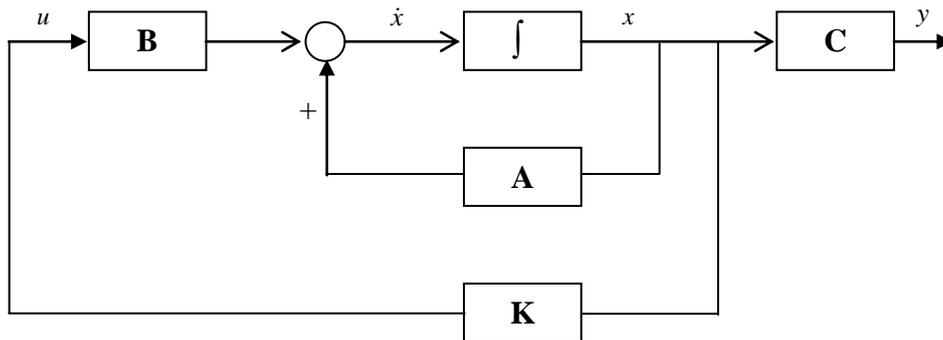
The regulator design that results is shown in **Fig.(5)**.

Figure (5) Basic state feedback regulator

From Eqs.(1,5,19), $\dot{x} = (A - Bk)x$,

$$\therefore \mathbf{Ac} = \mathbf{A} - \mathbf{Bk} \dotfill (20)$$

Let a set of p desired self-conjugate eigenvalues and eigenvectors that define the behavior of the desired dynamic modes of the closed-loop system be defined by:

$$\Lambda_{\mathbf{d}} = \left[\lambda_{\mathbf{d1}} \dots \lambda_{\mathbf{di}} \dots \lambda_{\mathbf{dp}} \right] \dotfill (21)$$

and,

$$\mathbf{V_d} = \left[\mathbf{v_{d1}} \dots \mathbf{v_{di}} \dots \mathbf{v_{dp}} \right] \dotfill (22)$$

Where the closed-loop system eigenstructure can be given by

$$(\mathbf{A} - \mathbf{BK})\mathbf{V} = \mathbf{V}\Lambda_{\mathbf{d}} \dotfill (23)$$

Hence, if a set of desired eigenvalues Λ_d and a set of final eigenvectors V are defined, then Eq.(23) can be used to find the feedback gain K that will give this eigenstructure in the closed-loop system [4].

2-5-2 Determination of a Feedback Gain Matrix

As shown in Eqs.(20) to (23), the matrix equations that describe the system can be rearranged to give an expression for the feedback gain matrix K, given an achievable set of right eigenvectors, V:

$$\mathbf{K = B^{-1}(A - V\Lambda_d V^{-1})} \dots\dots\dots (24)$$

2-6 Direct Eigenspace Assignment Formulation for Output Feedback

Given the observable controllable system of Eq.(1) with system measurements given by Eq.(5), the total control input is u. The measurement (output) feedback control law is:

$$\mathbf{u=Ky} \dots\dots\dots (25)$$

Solving for u as a function of the system states:

$$\mathbf{u=KCx+KDu} \dots\dots\dots (26)$$

$$\mathbf{u= [I-KD]^{-1}KCx} \dots\dots\dots (27)$$

The system augmented with the control law is given by:

$$\begin{aligned}\mathbf{\dot{x} = Ax + B([I - KD]^{-1}KCx)} \\ \mathbf{\therefore \dot{x} = (A + B[I - KD]^{-1}KC)x}\end{aligned} \dots\dots\dots (28)$$

The spectral decomposition of the closed-loop system is given by:

$$\mathbf{(A + B[I - KD]^{-1}KC)\nu_i = \lambda_i \nu_i} \dots\dots\dots (29)$$

For i = 1,...,n where λ_i is the ith system eigenvalue and v_i is the associated i^{th} system eigenvector. Let w_i be defined by:

$$\mathbf{w_i \equiv [I - KD]^{-1}KC\nu_i} \dots\dots\dots (30)$$

Substituting this result into Eq.(29) and solving for v_i one obtains:

$$\mathbf{\nu_i = [\lambda_i I - A]^{-1}Bw_i} \dots\dots\dots (31)$$

Values of w_i that yield an achievable eigenspace that is as close as possible in a least square sense to a desired eigenspace can be determined by defining a cost function associated with the ith mode of the system:

$$\mathbf{j_i} = \frac{1}{2}(\mathbf{v_{ai}} - \mathbf{v_{di}})^{\mathbf{H}}\mathbf{Q_{di}}(\mathbf{v_{ai}} - \mathbf{v_{di}}) \quad\text{..} \quad (32)$$

for i = 1,...,l where v_{ai} is the ith achievable eigenvector associated with eigenvalue λ_i, v_{di} is the ith desired eigenvector, and Q_{di} is an $(n \times n)$ symmetric positive semi-definite weighting matrix on eigenvector elements. This cost function represents the error between the achievable eigenvector and the desired eigenvector weighted by the matrix Q_{di}. Values of w_i that minimize J_i are determined by substituting Eq.(31) into the cost function for v_{ai}. Taking the gradient of J_i with respect to w_i, setting this result equal to zero and solving for w_i, this yields:

$$\mathbf{w_i} = [\mathbf{L_i^H Q_{di} L_i}]^{-1}\mathbf{L_i^H Q_{di} v_{di}} \quad\text{..} \quad (33)$$

where,

$$\mathbf{L_i} = [\lambda_{di}\mathbf{I_n} - \mathbf{A}]^{-1}\mathbf{B} \quad\text{..} \quad (34)$$

and λ_{di} is the ith desired eigenvalue of the closed-loop system. Note in this development that λ_{di} cannot belong to the spectrum of A.

By concatenating the individual wi's column-wise to form W and v_{ai}'s column-wise to form V_a, Eq.(28) can be expressed in matrix form by:

$$\mathbf{W} = [\mathbf{I_m} - \mathbf{KD}]^{-1}\mathbf{KCV_a} \quad\text{..} \quad (35)$$

The feedback gain matrix that yields the desired closed-loop eigenvalues and achievable eigenvectors is given by:

$$\mathbf{K} = \mathbf{W}[\mathbf{CV_a} + \mathbf{DW}]^{-1} \quad\text{..} \quad (36)$$

3. Results

3-1 Controllability and Observability

The system must be checked if it is controllable and observable or not. This is done by checking the rank of Eqs.(7, 8). By using MATLAB, it is found that n is equal to four.

The rank of Q and C is equal to the number of states. i.e. the system is controllable and observable.

3-2 System Transfer Function

The transfer function of the open loop state-space system G(s) is found by applying Eq.(4):

$$\therefore G(s) = \begin{bmatrix} 0 & \dfrac{7 - (r_b + \dfrac{5}{7}h)s^2}{s^2} \\ \dfrac{-7 + (r_b + \dfrac{5}{7}h)s^2}{s^2} & 0 \end{bmatrix} \dots\dots\dots\dots\dots\dots\dots (37)$$

$$\therefore g_{11} = g_{22} = 0 \;,\; g_{12} = \dfrac{7 - (r_b + \dfrac{5}{7}h)s^2}{s^2} \;,\; g_{21} = \dfrac{-7 + (r_b + \dfrac{5}{7}h)s^2}{s^2} \dots\dots\dots\dots (38)$$

3-3 Stability of the System

Let, $h = 3$ cm, $r_b = 1.36$ cm , By substituting these values in Eq.(37), then:

$$\therefore G(s) = \begin{bmatrix} 0 & \dfrac{7 - 0.035s^2}{s^2} \\ \dfrac{-7 + 0.035s^2}{s^2} & 0 \end{bmatrix} \dots\dots\dots\dots\dots\dots\dots\dots (39)$$

the roots are: $s_1 = s_2 = s_3 = s_4 = 0$

The roots lie at origin, i.e. at a critical unstable point, see **Fig.(6)**, then the system is unstable.
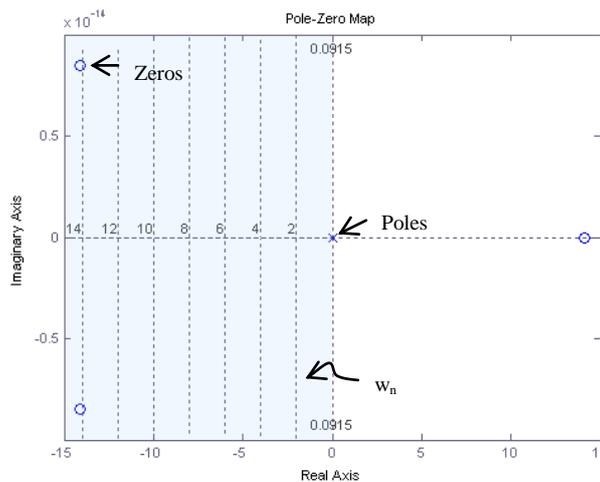
Figure (6) Open loop poles and zeros of the system

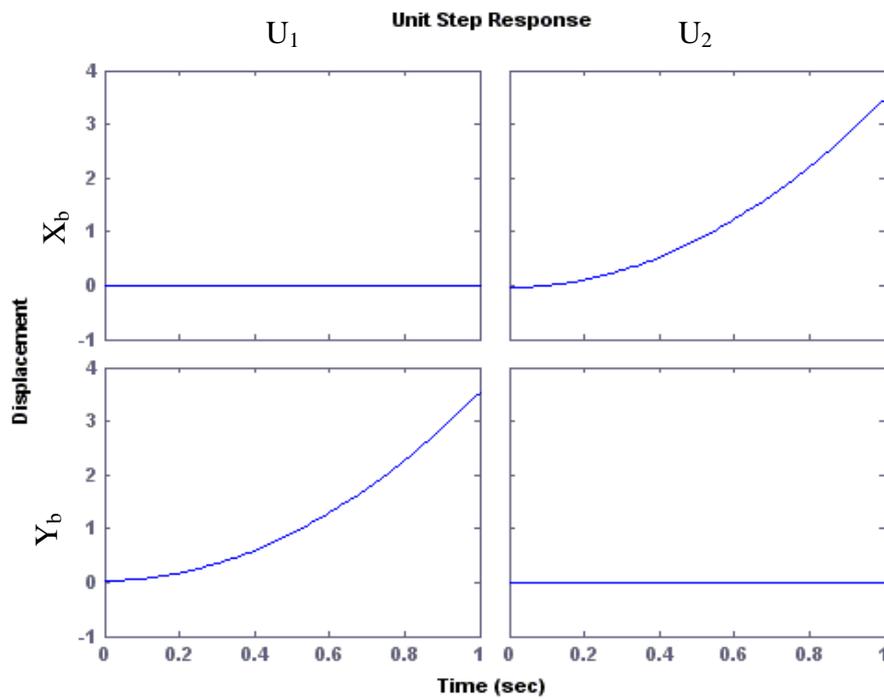Using MATLAB [7], a time response plot for a unit step input can be obtained. See **Fig.(7)**.

Figure (7) Time response to a unit step input of the open loop system

3-4 Control Design Techniques

3-4-1 State Feedback

This method is applicable to SISO systems. The system is decoupled into two systems, i.e. each input affects one of the outputs as shown in **Fig.(8)**.
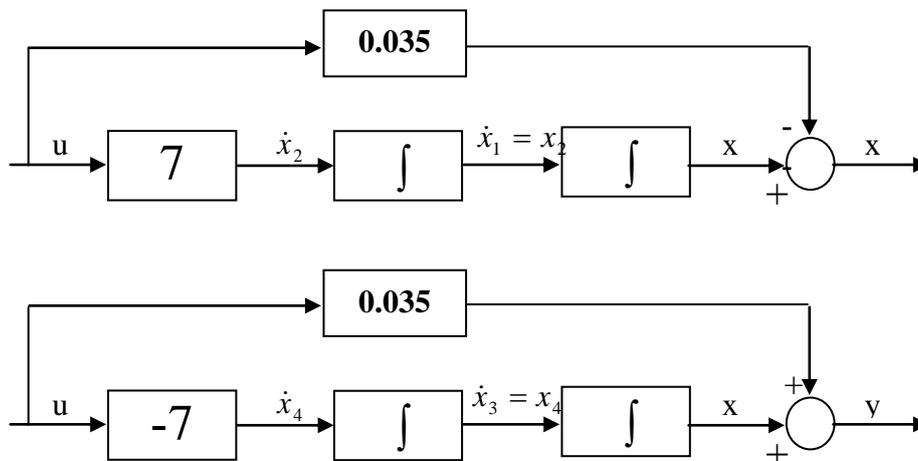

Figure (8) Open loop MIMO system decoupled into two SISO systems

3-4-2 Controller Design

Figure (9) shows the system with the controller then:
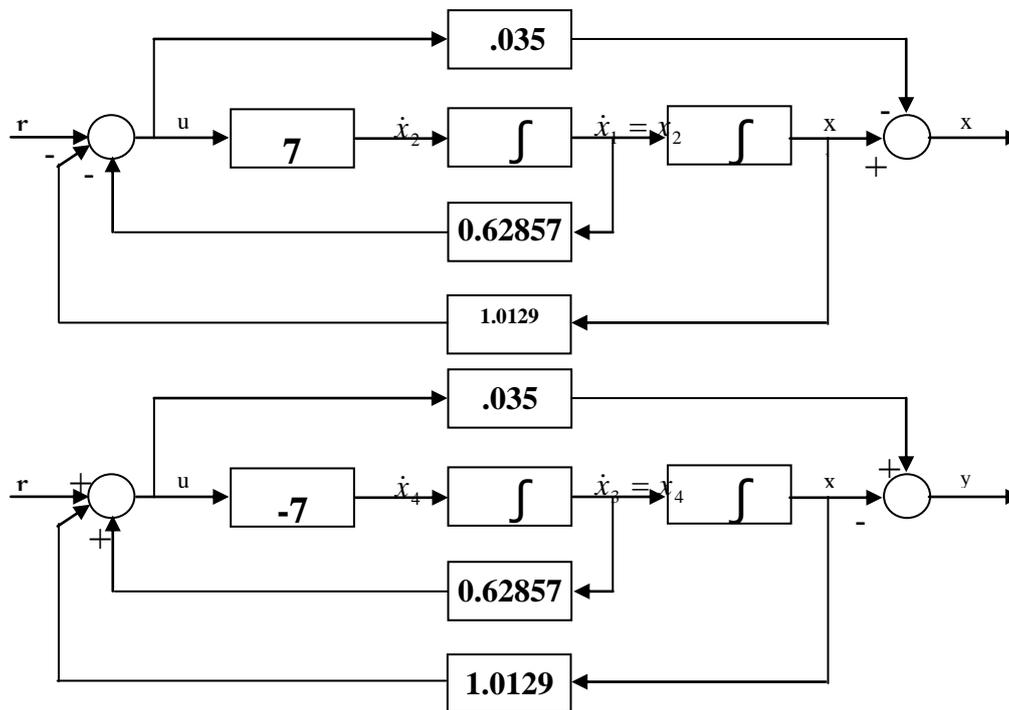
Figure (9) Block diagram of the two SISO systems with the controllers

By using MATLAB, The s-plane of the closed loop poles and zeros is obtained, and the time response plot to a unit step input is also obtained to each one of the two systems separately, see **Figs.(10,11)**.
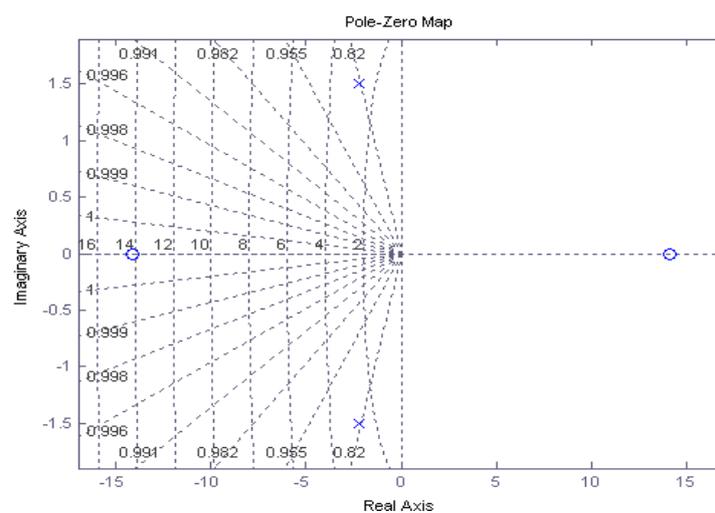

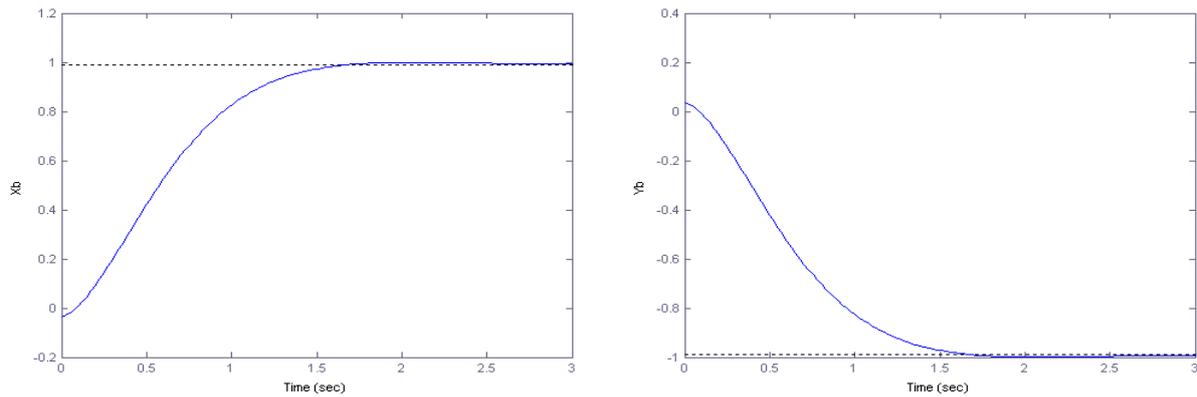**Figure (10) Closed loop poles and zeros of the system**

Figure (11) Time response to a unit step input (state feedback)

3-4-3 Output Feedback

The controller obtained by this method is based on feeding back all the outputs with constant gains, see **Figs.(12,13)**.
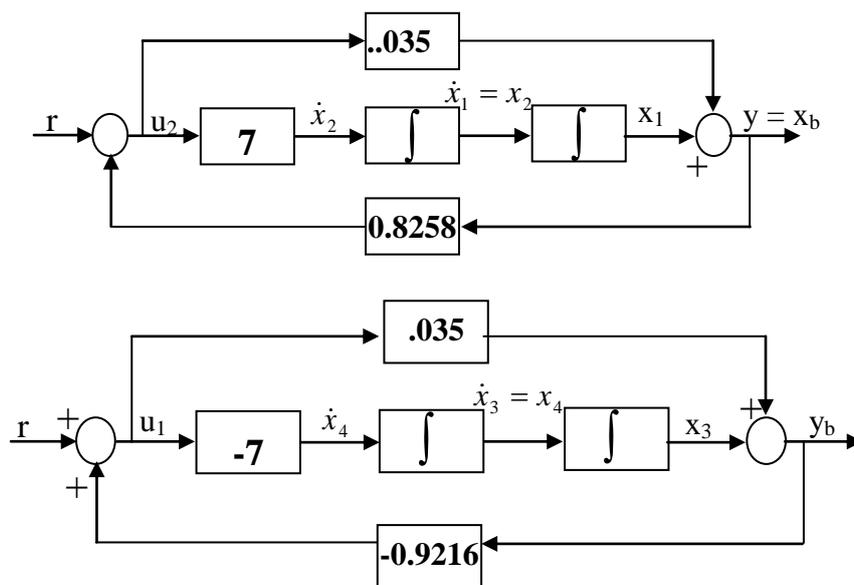

Figure (12) Block diagrams of the output feedback of the two SISO systems
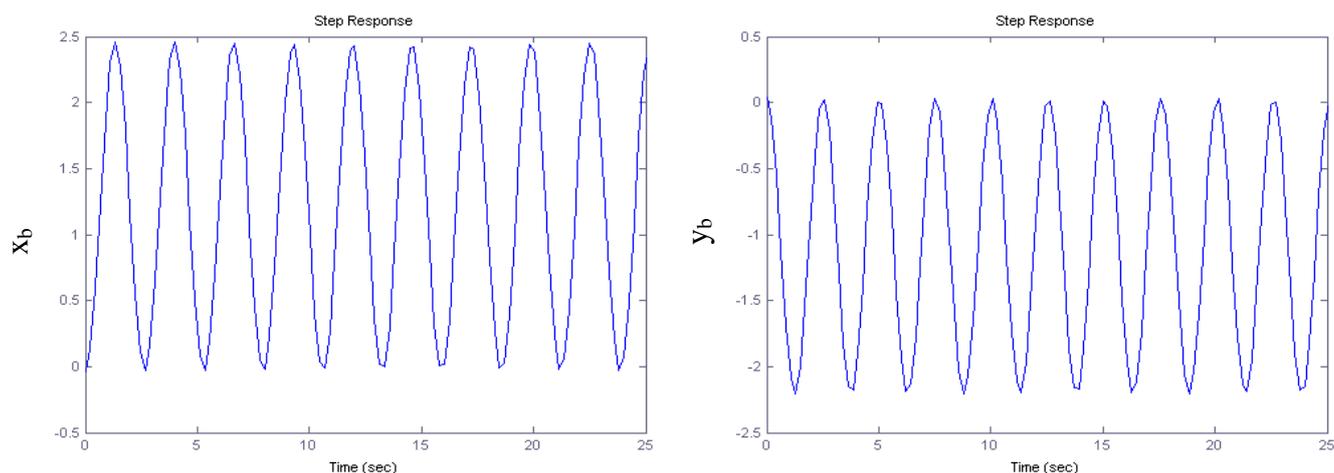
**Figure (13) Time response to a step input of the output feedback
of the two systems**

3-5 Eigenstructure Method for Full State Feedback

3-5-1 Eigenvalue Requirements

The desired eigenvalues are determined by their trigonometric relationship to damping ratio and frequency, as shown in **Table (1)**.

Table (1) Determination of the desired eigenvalues

Mode	Minimum Damping ratio	Minimum frequency	Open-loop eigenvaluse	Desired closed-loop eigenvalues
First mode	0.826	2.66	0 0	$-2.2 \pm 1.5\,j$
Second mode	0.826	2.66	0 0	$-2.2 \pm 1.5\,j$

The open-loop poles and zeros, and the desired closed-loop eigenvalues are shown in **Fig.(14)**.
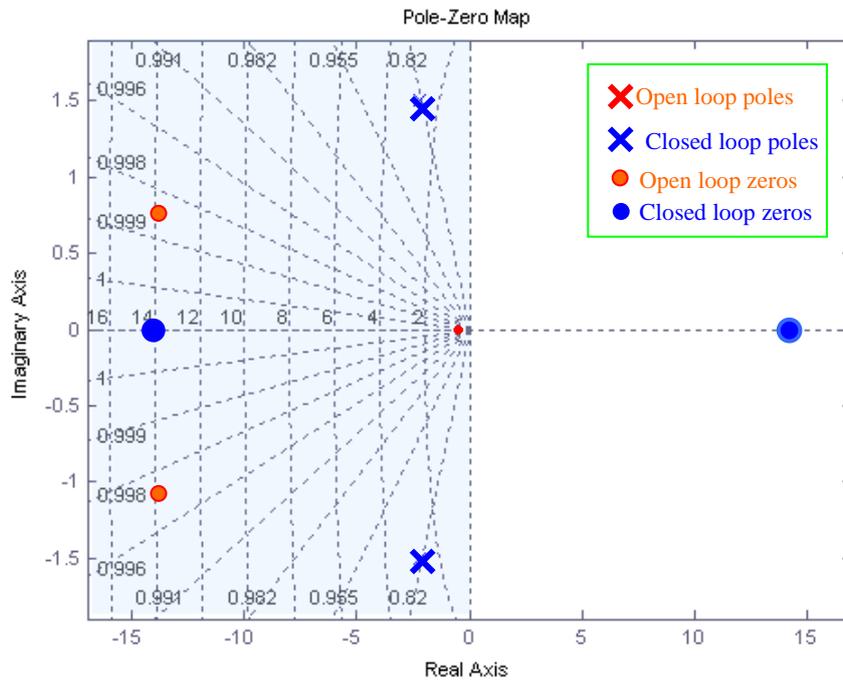
Figure (14) Open-loop and closed loop poles and zeros

3-5-2 Mode-state Coupling Vector Requirements

Table (2) shows the modes of the system. For a stability augmentation system, it is desirable to keep x_1 and x_3 uncoupled from each other.

Table (2) Mode state coupling vectors of the open loop system

States	First Mode		Second Mode	
x_1	1	1	0	0
x_2	x	x	x	x
x_3	0	0	1	1
x_4	x	x	x	x

3-5-3 Choice of Desired Eigenvectors

If the linearized equations of the system are considered, and the state vector is, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

There are 4 states and 2 inputs in this system. This implies freedom to assign 4 eigenvectors and decouple one element in each eigenvector, coupling any of the remaining elements. Therefore, with this assignment, all the freedom available to specify the closed-loop eigenvectors is used up, and so the desired eigenstructure will be achieved exactly.

Then, a good choice of closed loop eigenvectors might be:

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$$
\begin{bmatrix} 1 \\ \times \\ 0 \\ \times \end{bmatrix}
\begin{bmatrix} \times \\ 1 \\ 0 \\ \times \end{bmatrix}
\begin{bmatrix} 0 \\ \times \\ 1 \\ \times \end{bmatrix}
\begin{bmatrix} 0 \\ \times \\ \times \\ 1 \end{bmatrix} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{ (40)}
$$

$$
\quad \mathbf{v}_1 \qquad \mathbf{v}_2 \qquad \mathbf{v}_3 \qquad \mathbf{v}_4
$$

3-5-4 Direct Eigenstructure Assignment (DEA) Design (State Feedback)

The time response of the open loop system to a step input is shown in **Fig.(7)**, the time response to a unit step input of the closed loop system is shown in **Fig.(15)**, and to impulse input in **Fig.(16)**.
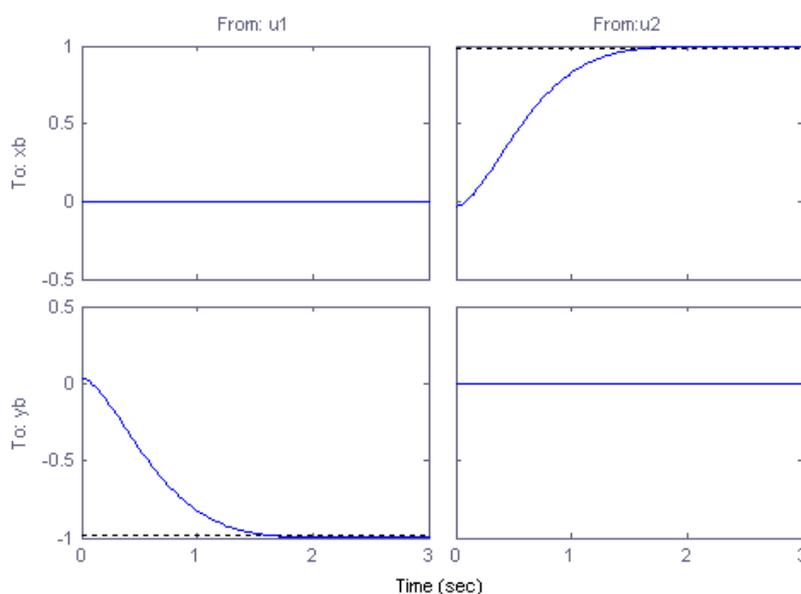

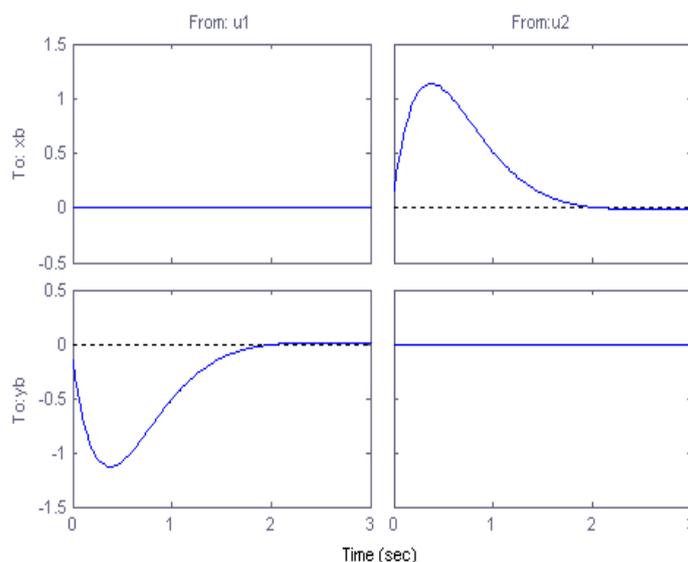**Figure (15) Step response to unit step input for the MIMO system**

Figure (16) Response to unit Impulse input for the decoupled MIMO system

3-6 Eigenstructure Method for Output Feedback

3-6-1 Eigenvalue Requirements

The performance requirements can be translated into the required position of the closed-loop poles of the system, and hence the desired eigenvalues are as shown in **Table (3)**.

Table (3) Determination of the desired eigenvalues

Mode	Minimum Damping ratio	Minimum frequency	Open-loop eigenvaluse	Desired closed-loop eigenvalues
First mode	0.8	2.66	0 0	$-2.2 \pm 1.5\,j$
Second mode	0.8	2.66	0 0	$-2.2 \pm 1.5\,j$

3-6-2 Mode-Output Coupling Vector Requirements

There are 4 states and 2 inputs in this system. This implies freedom to assign 2 eigenvectors and decouple one element in each eigenvector, coupling any of the remaining elements. Therefore, the desired eigenstructure may not be achieved exactly. From **Table (4)**, it can be seen that x_1 is decoupled from x_3 and is equal to one when x_3 is zero at the first mode, and x_3 is equal to one when x_1 is zero for the second mode.

For a stability augmentation system, it is desirable to keep x_b and y_b uncoupled from each other. x_1 represents x_b with the aid of the input, and x_3 represents y_b with the aid of the input. Thus, the state coupling vectors are chosen such that x_3 is decoupled from the first mode, and x_1 is decoupled from the second mode so that any correction in displacement in one direction will not affect the other.

Table (4) Mode state coupling vectors of the open loop system

States	First Mode	Second Mode
x_1	1	0
x_2	x	x
x_3	0	1
x_4	x	x

The time response to a unit step input of the closed loop system is shown in **Fig.(17)**, and to impulse input in **Fig.(18)**.
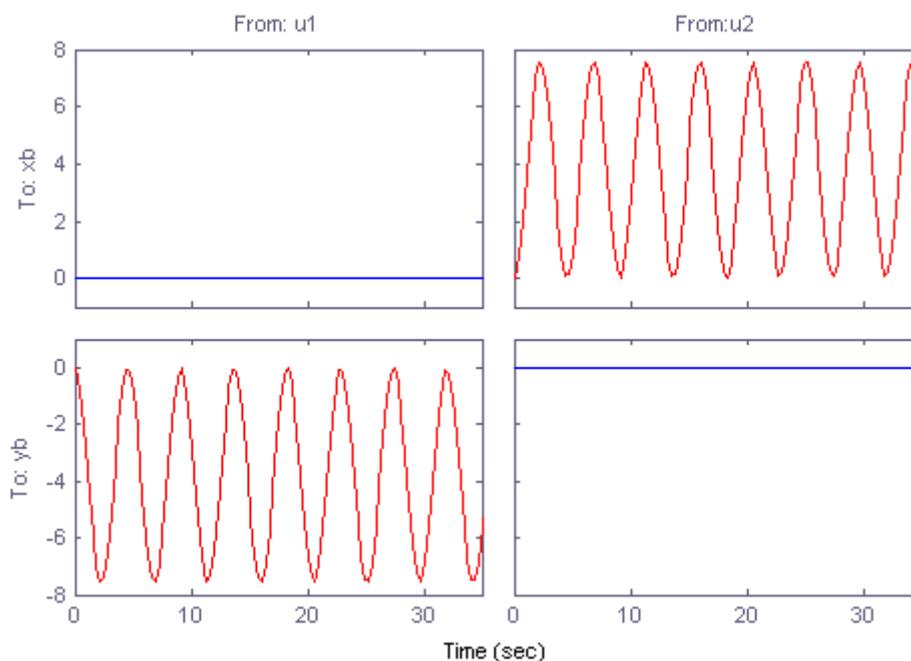
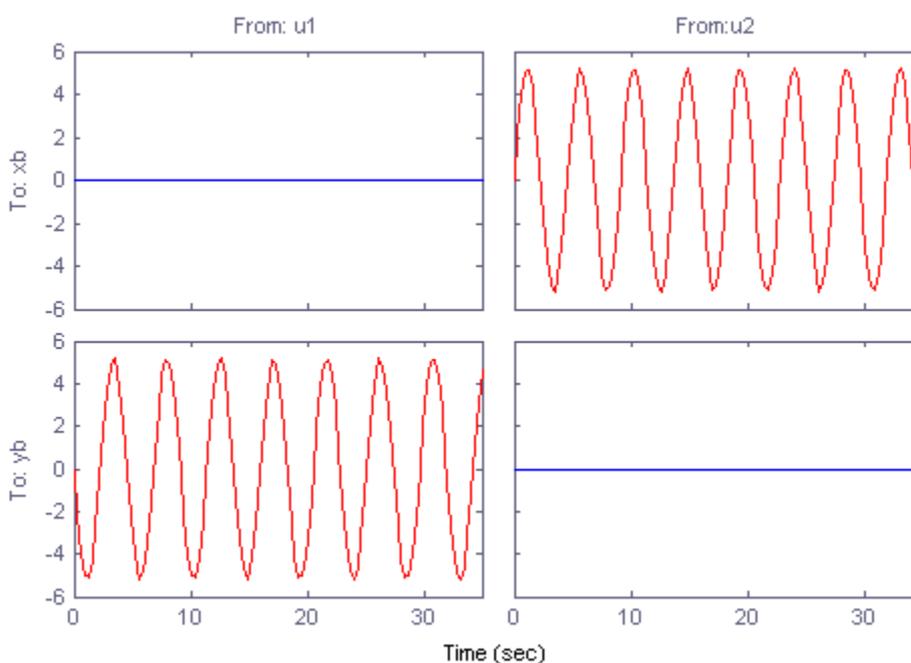**Figure (17) Response to unit step input for the output feedback system**

Figure (18) Response to unit impulse input for the output feedback system

The original idea for the ball on plate system for Greg Andrews [2] is from the Labyrinth game **Fig.(19)**. Therefore, the system will be checked to follow a similar path to the path of the game. **Figure (20)** is a schematic diagram to the game top surface, which shows that the line represents the path.

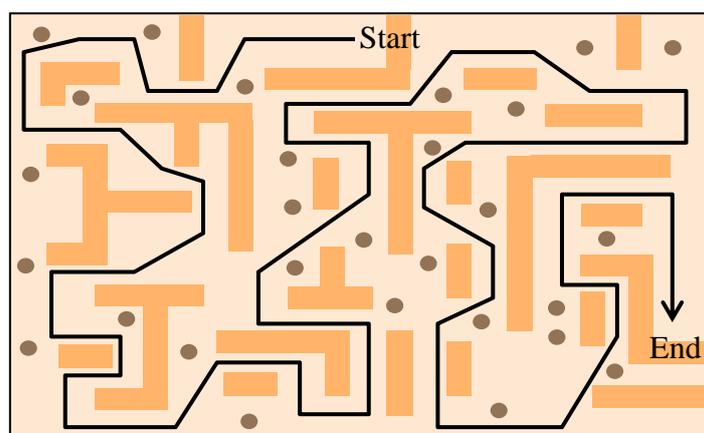**Figure (19) Photo for labyrinth game**

Figure (20) Top surface with the path line of the Labyrinth game

The time response for the system for this path is shown in **Fig.(21)**, the x_b,y_b plot for the desired and simulated paths are shown in **Figs.(22,23)**. The difference between them due to neglecting some parameters such as friction.
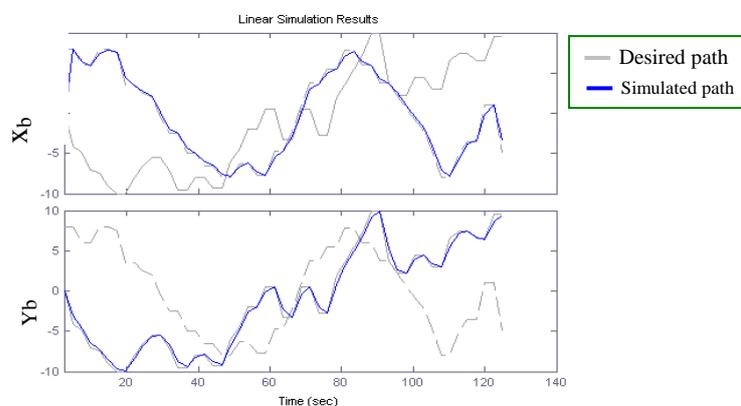

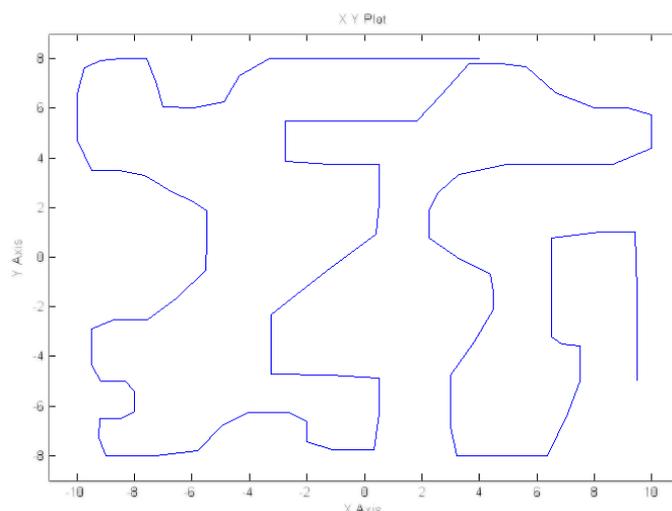**Figure (21) Time response to the desired and simulated paths**

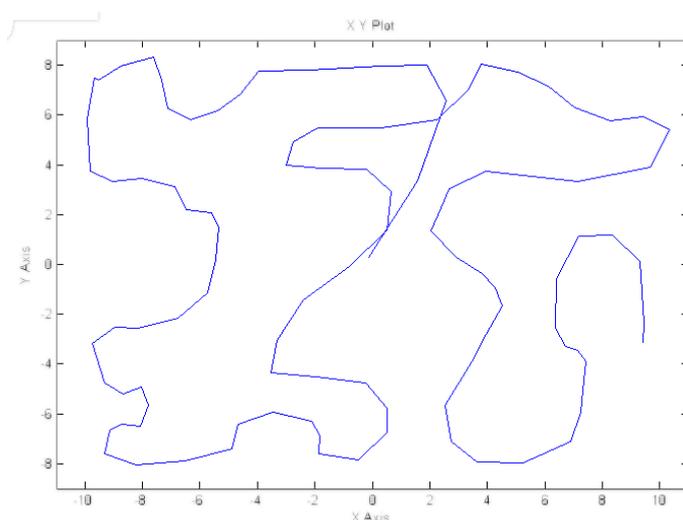**Figure (22) Desired xb, yb path of the Labyrinth game**

Figure (23) Simulated xb, yb path of the Labyrinth game

4. Conclusions

The main conclusions are:

1. EA can be used for regulation which can be implemented in the DEA algorithm easily. The resulting overall method is very quick and simple to use.

2. Eigenstructure analysis adds a measure of system dynamics that can enhance a designer's understanding of some of the interactions of the system, beyond classical techniques.

3. The strength of eigenstructure analysis lies in its stability to describe input-mode-output instruction easily.

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5. References

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Matrices, Vectors and Variables

h	Displacement between plate and U-joint center (m).
J	Performance index.
L	Left eigenvector "inverse of V".
l_i	the ith row of L.
p	Number of system outputs.
p	transformation matrix in DEA.
$\dot{q}_r$	Velocity in the r-direction (m/s).
u	System input vector.
V	Matrix of right eigenvectors.
v	Right eigenvector.
x	System state vector.
×	Unspecified component of desired vector.
y	System output vector.
Λ	Eigenvalue matrix.
λ_i	i^{th} eigenvalues.
η_r	Displacement in the r-direction (m).
υ	Velocity of the ball (m/s).

System Variables

I_b	Mass moment of inertia (Kg.m^2).
m_b	Mass of the ball (Kg).
q_1	Angle of rotation of the plate about x-axis (rad.).
q_2	Angle of rotation of the plate about y-axis (rad.).
r_b	Radius of the ball (m).

Abbreviations

MIMO   Multi-Input, Multi-Output
SISO   Single-Input, Single-Output