Using Modifying Fast Fourier Transform To Generating Fractal Landscapes.

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Abstract

This paper aims to provide an introduction to create fractal landscapes. In order to help the reader better understand the method presented we discuss the properties of real landscapes that make them fractals. Also we will present the modifying fast Fourier transform method for generating landscapes.

The purpose of this paper is to describe an algorithm for generating fractal landscapes. This paper also discusses some useful modifications and extensions to the algorithm.

المستخلص

هذا البحث يَهدَفُ لتزويد مقدمةً إلى خَلْق مناظر الطبيعية للكسوريات. لكي نساعد القارئ في تحسين فهمه حول النظرية المقدمة تم مناقشة خصائص المناظر الطبيعية الحقيقيةِ التي تَجْعلُهم كسوريات. كذلك تم توضيح طريقةِ تطوير تحويل فوريه السريع لتوليد المناظر الطبيعية.

الغرضُ من هذا البحث هو وصف خوارزمية لتوليد المناظر الطبيعية للكسوريات. كما يناقشُ هذا البحث كذلك بَعْض التعديلاتِ المفيدةِ وإمتداداتِ إلى الخوارزميةِ.

1- Introduction

Fractals discovered by Benoit Mandelbrot [5] in 1970s have changed the way we see everything. Natural objects such as clouds, plants, landscapes and many other objects, complex in shape, can be efficiently modeled using this new tool. The main property of fractals – self-similarity is the crucial feature used in finding fractal parameters needed in modeling process.

Fractals possess many different aspects through which they can be characterized. Among these are:

- **Self-similarity.**
- **Non-integer dimensionality.**
- **Being attractors of some peculiar dynamical system.**

Landscapes are self-similar. They are not self-similar in the sense that looking at a small portion of the landscape, you will see the same shape that you see looking at the entire landscape. Looking at a landscape at different scales, though, you will notice that it exhibits the same basic characteristics at every scale. Consider a mountainous terrain (*see fig. 1*), each mountain likely has several peaks. Think of these peaks as being the result of a smaller mountainous terrain imposed on top of one mountain in the larger terrain [1] [5].

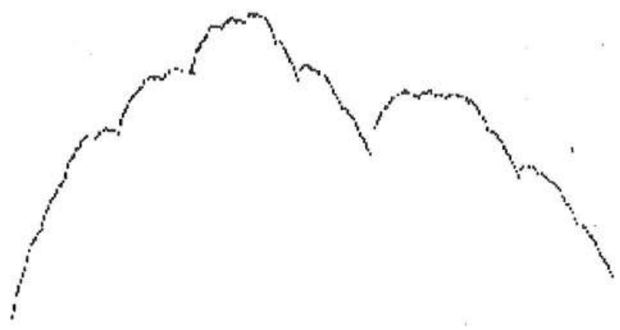

Figure (1) The Mountains

Further, each of these peaks contains more peaks but on a smaller scale. It is in this sense that landscapes are self-similar.

2- Fractal Landscapes

In 1967, Benoit Mandelbrot coined the popular question: "How long is the coast of Britain?" The answer is not as simple as it may appear. A measurement of the length of the coastline would depend entirely on the scale of the measurement device used [3]. If you were to measure the coastline one kilometer at a time, may think it would yield a reasonably accurate measurement. But we would have been ignoring all the rough detail within each kilometer. We would find that taking a meter stick and measuring the coastline one meter at a time would yield a far greater result. In fact, every measurement using a smaller scale would yield a greater result, without limit. The answer to the question is that the coastline of Britain does not have a length because it is not one-dimensional. It is a fractal [4], [5].

The goal in generating fractal landscapes is to make them look as realistic as possible. A generated landscape can be made quite realistic by adding colour, proper lighting, water, plants, atmospheric effects and other such things. This paper focuses on the process of creating the topological form of the landscape [1].

The output of the algorithm discussed is a set of altitudes assigned to a two-dimensional grid. Inputs into the algorithm are parameters that define certain desired characteristics of the generated landscape, mainly its roughness [1], [6].

3- Fourier Transform

A Fourier transform is based on the idea that a function can be expressed as a sum of sine or cosine waves at different frequencies. The Fourier transform expresses a function in this form; it converts the function to its frequency domain. A Discrete Fourier Transform (DFT) is a Fourier Transform applied to a set of discrete values. The result is a set of discrete complex numbers of the same size as the original set. The resulting values can be thought of as amplitudes of various frequencies in the original set [7]. The process can be reversed without any change to the original set. A DFT can be performed in O(N·log(N)) using a Fast Fourier Transform (FFT) algorithm making it a reasonably efficient process. A DFT can also be applied to multiple dimensions.

A Fast Fourier Transform (FFT) is an efficient algorithm to compute the discrete Fourier transform (DFT) and its inverse [7]. FFTs are of great importance to a wide variety of applications, from digital signal processing and solving partial differential equations to algorithms for quick multiplication of large integers. This article describes the algorithms, of which there are many; see discrete Fourier transform for properties and applications of the transform[2].

Let $x_0, \dots, x_{N-1}$ be complex numbers, where N is the number of iteration . The DFT is defined by the formula

$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N}nk} \qquad k = 0, \dots, N-1.$$

Evaluating these sums directly would take O(N2) arithmetical operation. An FFT is an algorithm to compute the same result in only O(Nlog N) operation [7]. In general, such algorithms depend upon the factorization of N, but (contrary to popular misconception) there are FFTs with O(N log N) complexity for all N, even for prime N.

Many FFT algorithms only depend on the fact that $e^{-\frac{2\pi i}{N}}$ is an Nth primitive root of unity, and thus can be applied to analogous transform over any finite field, such as number – theoretic transforms.

Since the inverse DFT is the same as the DFT, but with the opposite sign in the exponent and a 1/N factor, any FFT algorithm can easily be adapted for it as well [6].

4- Modifying Fast Fourier Transform Algorithm

The idea of a Fourier Transform ties into fractal landscapes by thinking of a landscape as a sum of waves at different frequencies. Observe that the amplitude of these waves has a decreasing trend as the frequency increases. This is essentially the same as the observation used in the previous algorithms that the size of perturbations should decrease as the scale at which they are being applied decreases [4].

The underlying cogitation behind the Fourier Transform algorithm is that by taking a set of random numbers, we can scale down the higher frequencies to create a landscape. The curious mathematician would wonder what other sorts of rules can be applied while in the frequency domain to alter the characteristics of the landscape. An interesting and useful result is achieved by scaling down the frequencies by a larger amount in one direction than the other. The resulting features are skewed in one direction, similar to the way mountain ranges typically form in a line. Figure 2 was generated by scaling the frequencies down in one direction by twice the scaling factor. Figure 3 was generated by combining this method with the multiplication technique.

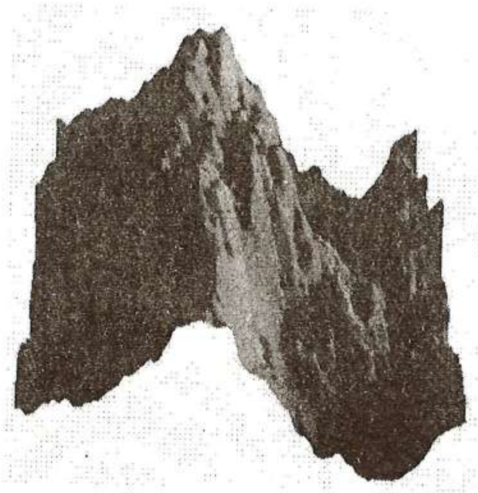

Figure 2

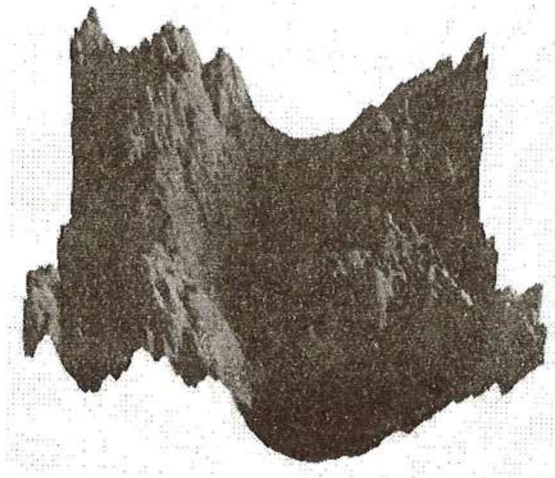

Figure 3

5- Conclusion

The resulting fractal landscape does not contain any artificial ridges or peaks as the modifying fast Fourier transform algorithms did. In addition, the landscape has the interesting (and sometimes desirable) property that it can be tiled. One side of the landscape flows perfectly into the opposite side.

Also the landscapes generated by the modifying fast Fourier transform algorithms have the best result from the Fourier transform algorithms[4]; where the landscapes generated by the Fourier transform algorithms described fail to mimic certain characteristics of real landscapes[4]. An easy way to create a more realistic fractal landscape is to multiply two generated landscapes together, creating smoother valleys and rougher peaks. More advanced methods exist to convert a generated landscape into a realistic landscape such as simulating the erosion process.

6- Future Work

1- Attempting find the collecting between any two algorithms having properties generating fractal landscapes. Such as, collecting between modify fast Fourier transform algorithms and Triangle Division Algorithm, Multi-dimensional Triangle Division Algorithm algorithms to generating best value of fractal landscapes.
2- Solutions the inverse problem of modify fast Fourier transform algorithms.

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