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# *Analysis and Formulation of Deception Capabilities against Monopulse Tracking Systems*

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## Abstract

*Monopulse technique is the most common used technique in the tracking systems because it has the ability to obtain complete angle-error information on a single pulse. For one target, the antenna boresight of the monopulse tracking system will track the target while for two targets (or more), the antenna boresight will tracks the direction of the center of energy that transmitted or reflected by these targets.*

*This paper introduces the analysis of this phenomena and how to utilize it to deceive angularly the monopulse tracking system. This analysis consists of the direction determination of the center of energy transmitted (or reflected) by the two targets before reaching the resolution angle, then the correction in the direction toward one of these targets after the resolution angle. The optimum separation between the two targets for deception has been calculated.*

*The results of this analysis and calculation show that, there is an acceptable miss distance from the target when the distance between the two targets was selected to be an optimum distance.*

## الخـلاصـــــــة

*إن تقنية القياس بالإعتماد على نبضة واحدة هي من أهم التقنيات المستخدمة في منظومات المتابعة كونها لها الإمكانية لإعطاء المعلومات الزاوية المطلوبة خلال نبضة واحدة. في حالة وجود هدف واحد فأن محور الهوائيات لمنظومة المتابعة أحادية النبضة يقوم بمتابعة الهدف بينما في حالة وجود هدفين متقربين فأن محور الهوائيات يقوم بالمتابعة بأتجاه مركز الطاقة المرسلة أو المنعكسة من هذين الهدفين.*

*يتناول البحث تحليل لهذه الظاهرة وكيفية استثمارها التمويه من متابعة منظومات المتابعة الأحادية النبضة، يتضمن التحليل حساب مركز الطاقة المرسلة أو المنعكسة من الهدفين قبل الوصول إلى نقطة التمييز ثم إمكانية التصحيح للاتجاه بعد نقطة التمييز. إن مقدار الإنحراف لمنظومة المتابعة عن الهدف وكذلك المسافة المثالية بين الهدفين لإعطاء أفضل انحراف ثم احتسابه في هذا البحث.*

*إن نتائج هذه الحسابات والتحليلات تشير إلى إمكانية الحصول على مسافة انحراف مقبولة عندما تكون المسافة بين الهدفين مساوية للمسافة المثالية.*

## 1. Introduction

The susceptibility of scanning and lobbing techniques to echo-amplitude fluctuations was the major reason for developing a tracking radar that provides simultaneously all the necessary lobes for angle-error sensing. The output from the lobes may be compared simultaneously on a single pulse, eliminating any effect of time change of the echo amplitude [1].

There are several methods by which angle-error information might be obtained with only a single pulse. The angle of arrival of the echo signal may be determined in a single-pulse system by measuring the relative phase (pulse comparison method) or the relative amplitude (amplitude comparison method) of the echo pulse received in each beam. The names simultaneous lobbing and monopulse are used to describe those tracking techniques which derive angle-error information on the basis of a single pulse [2, 3, and 4].

The presence of two or more unresolved targets causes the indicated in-phase angle of arrival to wander, sometimes far beyond the separation between targets. As a result, a tracking system which measures and tracks the direction of arrival of a radio wave might actually be tracking a point in-between these targets, or at certain conditions this point might be in-phase unrelated to the true target position [5].

In the following sections, analysis will be developed for the resolution angle of the monopulse tracking system, direction of the tracking-beam before and after the resolution process, and the calculation of the miss distance from the target.

## 2. Two Point Targets in the Monopulse Tracking-Beam

In the monopulse tracking system, two squinted beams are combined to form the sum channel (S) and difference channel (D) in the elevation or in the azimuth planes [5].

For a single target and for small error angles the sum signal (S) is constant, while the difference signal (D) is linear, it can be found that the amplitude of the difference signal is given by [1,2]:

$$\mathbf{D = k_s \varepsilon\, S} \quad \text{.......} \quad \mathbf{(1)}$$

where:

$k_s$ = constant depends on the sequent angle between the two beams.

$\varepsilon$ = the angle between the target and the reference.

The echoes from the two targets will be represented by two complex signals having amplitudes and phases generally different between them. The signals in the S and D channels can be obtained by adding the individual signals due to each target, and it can be written as:

$$\mathbf{S = S_1 + S_2} \quad \text{.......} \quad \mathbf{(2)}$$

$$\mathbf{D = D_1 + D_2 = k_s\, \varepsilon_1 S_1 + ks\, \varepsilon_2 S_2} \quad \text{.......} \quad \mathbf{(3)}$$

where:

$\varepsilon_1$ = the angle between target 1 and the reference.
$\varepsilon_2$ = the angle between target 2 and the reference.

For the composite target, this is formed by the set of the two targets, a mathematical center. This center will be chosen midway between the two targets. The choice of the center is arbitrary, and is not guaranteed that the tracker will point to the center. However, it results that in particular conditions the monopulse tracks the center, but this does not happen in general, as it will be seen in the following.

The angular error relative to the target center will be indicated by ε, it can be written as:

$$\boldsymbol{\varepsilon = (\varepsilon_1 + \varepsilon_2) / 2} \quad \textbf{(4)}$$

The angular width between the two targets $\varepsilon_o$ is:

$$\boldsymbol{\varepsilon_o = \varepsilon_1 - \varepsilon_2} \quad \textbf{(5)}$$

Equation (2) can be rewritten, by using equations (1), (3) and (4), as:

$$\mathbf{D = ks\varepsilon\, S + k_s\, \varepsilon_o\, (\varepsilon_1 - \varepsilon_2) / 2} \quad \textbf{(6)}$$

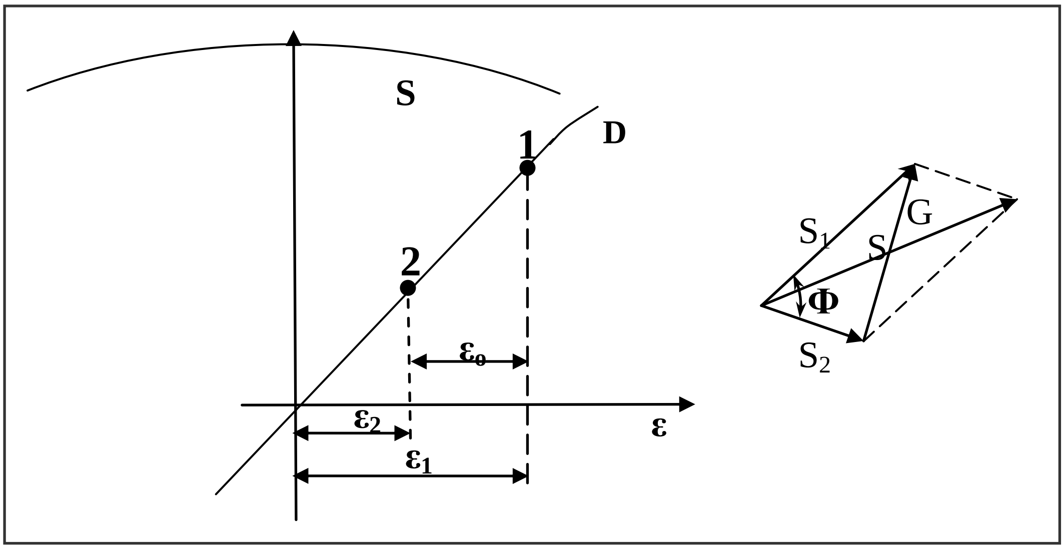


**Figure (1) S and D curves for two targets**

By comparing equations (1) and (6) it results that in the two target case a new term was added, which is proportional to the angular extension of the target, is present in the difference signal. The term (S1 – $S_2$) was called the glint term, G:

$$\mathbf{G = S_1 - S_2} \quad \textbf{(7)}$$

Equation (6) can be rewritten as:

$$\mathbf{D = k_s \varepsilon\, S + k_s\, \varepsilon_o\, G / 2} \quad \text{.......... (8)}$$

When two targets are present, the left side of equation (8) is still measurable in the difference channel, while the two individual terms at the right side of equation (8) are not measurable. Thus the tracker can not be aware that two targets are present, since the quantities $\varepsilon_{o}$, $\varepsilon$, G are individually measurable. In other words the tracker has S and D channels, but it has no G (glint) channel.

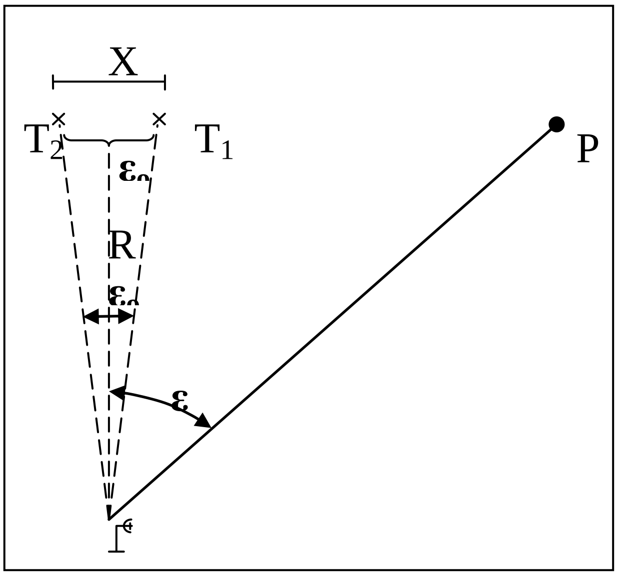


**Figure (2) Two targets geometry**

## 3. Resolution Angle

When a monopulse system tracking a group of targets, it tracks the center of energy reflected or continuously transmitted by these targets before reaching the resolution angle point. After this point, some of these targets being out of the tracking beam.

For two targets, after the resolution angle point, the tracking system will track one of these targets and leave another one. The resolution angle is approximately equal to the beam width at half-power points [6]:

$$\theta_r = \theta_{3dB} \quad \text{.......... (9)}$$

## 4. Direction of the Tracking-Beam before the Resolution Process

The Automatic Gain Control (AGC) signal which is derived by detecting and averaging (time average) the sum signal will be indicated by $<|S|>$. The AGC functional action is the one of dividing the signals of S and D channels by the AGC voltage as follows:

$S / <|S|>,\ D / <|S|>$

The phase sensitive detector (PSD) performs the scalar (or dot) product between the two complex terms ($S / <|S|>$) and ($D / <|S|>$) producing the output voltage:

$$\mathbf{e = (|S| / <|S|>)\ (|D| / <|S|>) \cos \Phi} \quad \text{.......... (10)}$$

where:

$\Phi$ = the phase angle between sum and difference signals.

The phase sensitive detector output is processed and sent to the servo circuit which controls the boresight position. The control law for the simple first order servo is the following [7]:

$$\mathbf{T_s.\ d\,\varepsilon_b(t) / dt = (|S|.\ |D|.\ \cos \Phi) / [<|S|>]^2} \quad \text{.......... (11)}$$

Or:

$$\mathbf{\varepsilon_b (t) = (1/T_s) \int_{-\infty}^{t} (|S|.\ |D|.\ \cos \Phi)\, dt / [<|S|>]^2} \quad \text{.......... (12)}$$

Therefore, the direction of the tracker is obtained simply by integrating the output of the PSD. For two targets in the tracking beam, the signal after AGC and PSD is given by:

$$\mathbf{e = (|S|.|D|.\ \cos \Phi) / [<|S|>]^2}$$
$$\mathbf{= k_s\ [\varepsilon\ |S|^2 + (\varepsilon o / 2)|\ S|.|G|.\ \cos (S, G)] / [<|S|>]^2} \quad \text{.......... (13)}$$

The static solutions are found when all the quantities which could vary with time are constant. In this case equation (11) it results $d\,\varepsilon_b(t) / dt = 0$, then the output of the PSD must be zero. From equation (13) it results:

$$\mathbf{\varepsilon = -\ (\varepsilon o / 2)\ |S|.|G|.\ \cos (S,G) / |S|^2} \quad \text{.......... (14)}$$

Hence a non-zero steady state error results, and the tracker does not point in general to the midpoint between the two targets. When $\varepsilon_o = 0$ , the glint term and the error are equal to zero, so the tracker points to the unique point target, as it must be.

Suppose that the phase delay between the echoes of targets 1 and 2 is constant:

$$\mathbf{\Psi = \Psi_1 + \Psi_2} \quad \text{.......... (15)}$$

The glint effect will be better clarified by the following example:

- amplitude of target 1 = 1
- amplitude of target 2 = k

It can be assumed a Cartesian reference frame (x,y) in which $S_2$ vector has length k and is parallel to $x_1$.

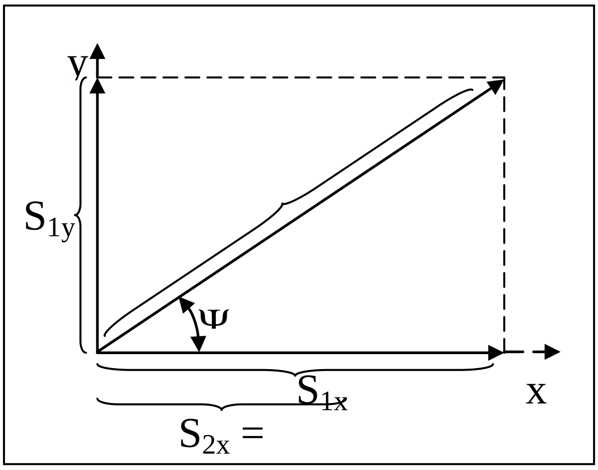


**Figure (3) Phasor representation for two targets**

$S_1 = (\cos\Psi, \sin\Psi)$

$S_2 = (k, 0)$

$S = S_1 + S_2 = (k+\cos\Psi, \sin\Psi)$

$G = S_1 - S_2 = (-k+\cos\Psi, \sin\Psi)$

$|S|^2 = (k+\cos\Psi)^2 + \sin^2\Psi = 1+k^2 + 2k\cos\Psi$

$S.G = |S|.|G|.\cos(S,G) = \cos^2\Psi - k^2 + \sin^2\Psi = 1 - k^2$

Using these equations in to equation (14) to get:

$$\boldsymbol{\varepsilon = -(\varepsilon o / 2)(1-k)(1+k) / (1+k^2 + 2k\cos\Psi)} \quad \text{.............................................} \quad \mathbf{(16)}$$

***For Ψ = 0:***

$\varepsilon = -(\varepsilon o / 2)(1-k) / (1+k)$

When k = 1, the error is zero, and the tracker points exactly to the center between the two targets.

When k = 0, the error is $(-\varepsilon_o / 2)$ and this means that the tracker points to target 1, as it should be.

Note that when $(-\varepsilon_o / 2) \le \varepsilon \le (\varepsilon o / 2)$ the monopulse tracks some point whose angular position is comprised between $\varepsilon_1$ and $\varepsilon_2$. This point will be closer to the target having higher amplitude.

***For Ψ = π:***

$\varepsilon = -\ (\varepsilon o / 2)\ (1+k) / (1-k)$

When k = 1, the error tends to +∞, and the tracker will tracks some point can be external to the interval defined by $\varepsilon_1$, $\varepsilon_2$.

## 5. Direction of the Tracking-Beam after the Resolution Process

For a distance greater than the resolution distance, R, (distance at which the resolution or discrimination between the two targets occurs), the two targets will be within the tracking beam, and the tracker will be directed to a point in the midway between the two targets. For approaching targets, when the distance becomes less than the resolution distance, R, the tracking system directed to one of the targets and leaves the other. The tracker is directed to which one of the targets, this depends on the amplitude of the echoes from these targets, if they have the same amplitude, they have equal probable to be tracked by the tracker. If one of them has higher amplitude than the other, this target will have more chance to be tracked by the tracker.

## 6. Calculation of the Miss Distance from the Target

After the resolution of the pair sources, the antenna will tracks the direction towards one of them, and the tracking system will home on that source.

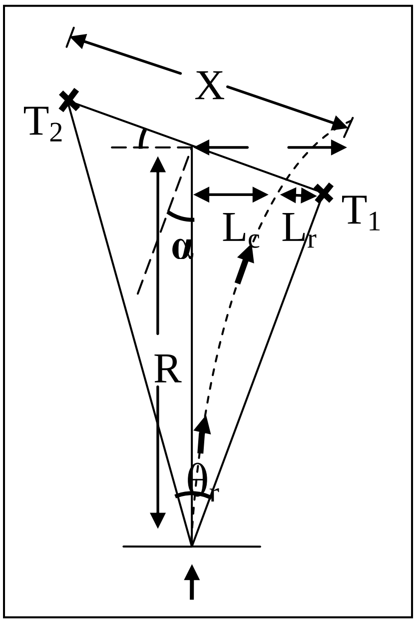


**Figure (4) Tracker trajectory after the resolution point**

In **Fig.(4)**, at the resolution point, the tracker will tracks a point at a distance L from the target 1, an angular error proportional to this distance now will be appear at the output of the phase sensitive detector, this error voltage is processed and sent to the servo system to correct the trajectory of the vehicle which carrying the tracking system. This correction in the trajectory can not be done immediately, but it can be implemented gradually depending on the mechanical design of the vehicle and the servo-system properties.

The miss distance from target 1 can be written as:

$$\mathbf{L_r = L - L_c} \quad \text{.......} \quad \mathbf{(17)}$$

where:

$L_r$ = residual miss distance.

L = initial miss distance;

$$\mathbf{L = (x/2)\cos\alpha} \quad \text{.......} \quad \mathbf{(18)}$$

$L_c$ = miss compensation distance;

$$\mathbf{L_c = jt^2/2} \quad \text{.......} \quad \mathbf{(19)}$$

j = normal (lateral) acceleration.

$$\mathbf{j = a.g.n} \quad \text{.......} \quad \mathbf{(20)}$$

a = constant =0.5 – 0.7.

g = free fall acceleration =9.8 $m/s^2$.

n = permissible G-Load.

t = time interval from the source resolution moment to the moment when the tracking system passes the base-line.

Thus, the miss compensation distance is given by:

$$\mathbf{L_c = a.g.n.t^2/2} \quad \text{.......} \quad \mathbf{(21)}$$

The time interval, t, can be written as:

$$\mathbf{t = R/v = (x\cos\alpha) / (v\,\theta_r)} \quad \text{.......} \quad \mathbf{(22)}$$

where:

R = distance from the resolution point to the target.

v = tracker closing speed.

x = separation distance between the two targets.

$\theta_r$ = resolution angle.

$\alpha$ = the angle between the direction to the mid-point and the line between the two targets.

Thus,

$$\mathbf{L_c = a.g.n.\ [(x \cos \alpha) / (v\ \theta_r)]^2 / 2} \quad \text{(23)}$$

$$\mathbf{L_r = (x/2) \cos \alpha - a.g.n.[(x \cos \alpha) / (v\ \theta_r)]^2 / 2} \quad \text{(24)}$$

## 7. Optimum Separation Distance for Deception

Maximum miss takes place in the case when a distance between the targets is selected to be optimal. A probable expectation of residual miss under optimal separation is defined by the relation:

$$\mathbf{L_{r\ opt} = (x_{opt}/4) \cos \alpha} \quad \text{(25)}$$

Using equation (25) in equation (24) results:

$(x_{opt}/4) \cos \alpha = (x_{opt}/2) \cos \alpha$ - a.g.n. $[(xopt \cos \alpha) / (v\ \theta_r)]^2 /2$

$$\mathbf{x_{opt} = (v\ \theta_r)^2 / (2.a.g.n. \cos \alpha)} \quad \text{(26)}$$

For a tracking system of $v = 960 m/s$, $\theta_r = 8^0$, $a = 0.5$, $g = 9.8\ m/s^{2,}$ $n = 6$, $\alpha = 15^0$, the optimum separation between the two targets for deception is given by:

$x_{opt} = (960 * 0.14)^2 / (2*0.5*9.8*6* \cos 15) = 318$ m.

If the selected spacing between the two targets is less than the double optimum one, then a residual miss will be given by:

$L_r = (x/2) \cos \alpha$ [1 - a.g.n. $(x \cos \alpha) / (v\ \theta_r)$ 2 ]

$$\mathbf{L_r = (x/2) \cos \alpha\ [1 - x / (2\ x_{opt})]} \quad \text{(27)}$$

and,

$$\mathbf{L_c = (x^2 \cos \alpha) / (4\ x_{opt})} \quad \text{(28)}$$

**Figure (5)** indicates that for the tracking system, the miss distance $L_r$ first increases due to increase of the initial miss, then starts to decreases due to the increase of the time for miss compensation by the servo-system because of the tracking on one target after the resolution point.

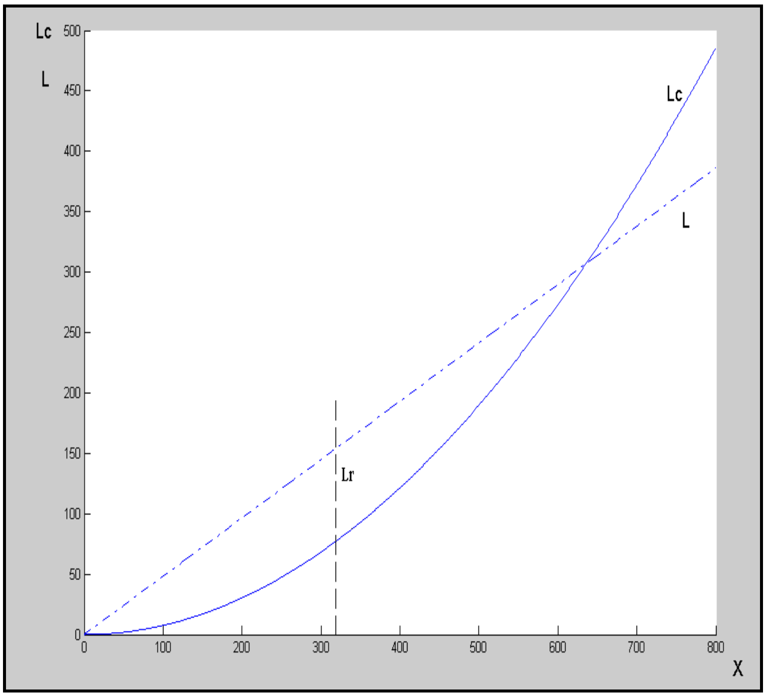


**Figure (5) The relationship between L, Lc, and the separation x**

## 8. Conclusions

This paper introduces an analysis of the capabilities of the deception of monopulse tracking system. This analysis shows that, the deception can be achieved by using two targets separated by certain distance called the optimum distance which gives maximum miss distance from the target that will be tracked after the resolution point. This analysis gives derived and formulated equations which can be used to calculate the required optimum distance for any tracking system and the expected miss distance.

From this analysis, the following points can be calculated:

- The target which has higher amplitude than the other will have more chance to be tracked by the tracker.
- The miss distance between the trajectory path and the target depends on the mechanical design of the vehicle, servo-system proprieties, resolution angle and the geometry.
- The miss distance from the target can be optimized to get a good deception when the separation distance between the two targets has been chosen to be the optimum separation, which can be calculates from the derived formula in this paper.

## 9. References

**1.** M. I., Skolnik, ***"Radar Handbook"***, McGraw-Hill Book Company, 1970.

**2.** M. I., Skolnik, ***"Introduction to Radar System"***, McGraw-Hill Book Company, 1980.

**3.** C., Schleher, ***"Introduction to Electronic Ware fare"***, Artch. House, 1986.

**4.** L. B., Van Brunt, ***"Applied ECM"***, 1978.

**5.** S. J., Asseo, ***"Effect of Monopulse Signal Thresholding on Tracking Multiple Targets"***, IEEE Trans. on Aerospace and Electronic Systems, Vol. Aes-10, No.4, July 1974.

**6.** Lal Chand Godara, ***"Handbook of Antennas in Wireless Communication"***, CRC Press, 2002.

**7.** O., Gasparini, ***"Monopulse Receiver: Influence of the Automatic Gain Control and Servo Circuit in Presence of Amplitude Fluctuation, Glint, and Targets at Low Elevation Angles"***, Selenia Technical Report, No. 74159, 1974.