

## Some New Kinds of Continuous Functions Via Fuzzy Neutrosophic Topological Spaces

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### ABSTRACT

In this paper, we defined fuzzy neutrosophic- $\tau_{0,1}$ continuous, fuzzy neutrosophic- $\tau_{0,2}$ continuous, fuzzy neutrosophic- $\tau_{0,1}$ contra continuous and fuzzy neutrosophic- $\tau_{0,2}$ contra continuous functions. Then, we define the relationship between the define functions and studied functions with their comparative.

### 1- Introduction

The concept of fuzzy sets (FS, for short) was introduced by Zadeh in 1965 [1]. Then the fuzzy set theory are extension by many researchers. Intuitionistic fuzzy sets (IFS, for short) was one of the extension sets by K. Atanassov in 1983 [2,3,4], when fuzzy set give the degree of membership function of an element in the sets. Then, the intuitionistic fuzzy sets give a degree of membership function and a degree of non-membership function. After that, several researches were conducted on the generalizations of the notion of intuitionistic fuzzy sets. The concept of neutrosophy, neutrosophic set and neutrosophic component was F. Smarandache in 1999 [5]. Then the concept of neutrosophic set (NS, for short) and neutrosophic topological space (NTS, for short) define by A. A. Salama and S.A. Alblowi 2012 [7]. In the year 2013 by I. Arockiarani, I. R.Sumathi and J. Martina Jency [8] define the fuzzy neutrosophic set. Next, in the year 2014 by I. Arockiarani and J. Martina Jency [9] define the fuzzy neutrosophic topological space.

The fuzzy neutrosophic sets was define with membership, non-membership and indeterminacy degrees. In the last year, (2017) by Y. Veereswari [10] introduced of fuzzy neutrosophic continuous function.

### 2. Some Basic of Topological Concepts

**Definition 2.1 [8, 10]:** Let  $X$  be a non-empty fixed set. Fuzzy neutrosophic set (FNS, for short),  $\lambda_N$  is an object having the form  $\lambda_N = \{ \langle x, \mu_{\lambda_N}(x), \sigma_{\lambda_N}(x), \nu_{\lambda_N}(x) \rangle : x \in X \}$  where the functions  $\mu_{\lambda_N}, \sigma_{\lambda_N}, \nu_{\lambda_N} : X \rightarrow [0, 1]$  denote the degree of membership function (namely  $\mu_{\lambda_N}(x)$ ), the degree of indeterminacy function (namely  $\sigma_{\lambda_N}(x)$ ) and the degree of non-membership (namely  $\nu_{\lambda_N}(x)$ ) respectively, of each set  $\lambda_N$  we have,  $0 \leq \mu_{\lambda_N}(x) + \sigma_{\lambda_N}(x) + \nu_{\lambda_N}(x) \leq 3$ , for each  $x \in X$ .

**Remark 2.2 [10]:** FNS  $\lambda_N = \{ \langle x, \mu_{\lambda_N}(x), \sigma_{\lambda_N}(x), \nu_{\lambda_N}(x) \rangle : x \in X \}$  can be identified to an ordered triple  $\langle \mu_{\lambda_N}, \sigma_{\lambda_N}, \nu_{\lambda_N} \rangle$  in  $[0, 1]$  on  $X$ .

**Definition 2.3 [10]:** Let  $X$  be a non-empty set and the FNSs  $\lambda_N$  and  $\beta_N$  be in the form  $\lambda_N = \{ \langle x, \mu_{\lambda_N}(x), \sigma_{\lambda_N}(x), \nu_{\lambda_N}(x) \rangle : x \in X \}$  and,  $\beta_N = \{ \langle x, \mu_{\beta_N}(x), \sigma_{\beta_N}(x), \nu_{\beta_N}(x) \rangle : x \in X \}$  on  $X$ . Then:

- i.  $\lambda_N \subseteq \beta_N$  iff  $\mu_{\lambda_N}(x) \leq \mu_{\beta_N}(x), \sigma_{\lambda_N}(x) \leq \sigma_{\beta_N}(x)$  and  $\nu_{\lambda_N}(x) \geq \nu_{\beta_N}(x)$  for all  $x \in X$ ,
- ii.  $\lambda_N = \beta_N$  iff  $\lambda_N \subseteq \beta_N$  and  $\beta_N \subseteq \lambda_N$ ,
- iii.  $1_N - \lambda_N = \{ \langle x, \nu_{\lambda_N}(x), 1 - \sigma_{\lambda_N}(x), \mu_{\lambda_N}(x) \rangle : x \in X \}$ ,
- iv.  $\lambda_N \cup \beta_N = \{ \langle x, \text{Max}(\mu_{\lambda_N}(x), \mu_{\beta_N}(x)), \text{Max}(\sigma_{\lambda_N}(x), \sigma_{\beta_N}(x)), \text{Min}(\nu_{\lambda_N}(x), \nu_{\beta_N}(x)) \rangle : x \in X \}$ ,
- v.  $\lambda_N \cap \beta_N = \{ \langle x, \text{Min}(\mu_{\lambda_N}(x), \mu_{\beta_N}(x)), \text{Max}(\sigma_{\lambda_N}(x), \sigma_{\beta_N}(x)), \text{Max}(\nu_{\lambda_N}(x), \nu_{\beta_N}(x)) \rangle : x \in X \}$ .

$\beta_N = \{ \langle x, \text{Min}(\mu_{\lambda_N}(x), \mu_{\beta_N}(x)), \text{Min}(\sigma_{\lambda_N}(x), \sigma_{\beta_N}(x)), \text{Max}(\nu_{\lambda_N}(x), \nu_{\beta_N}(x)) \rangle : x \in X \}$ ,

vi.  $[ ]_{\lambda_N} = \{ \langle x, \mu_{\lambda_N}(x), \sigma_{\lambda_N}(x), 1 - \mu_{\lambda_N}(x) \rangle : x \in X \}$ ,

vii.  $\langle \rangle_{\lambda_N} = \{ \langle x, 1 - \nu_{\lambda_N}(x), \sigma_{\lambda_N}(x), \nu_{\lambda_N}(x) \rangle : x \in X \}$ ,

viii.  $0_N = \langle x, 0, 0, 1 \rangle$  and  $1_N = \langle x, 1, 1, 0 \rangle$ .

**Definition 2.4 [10]:** Fuzzy neutrosophic topology (FNT, for short) on a non-empty set X is a family  $\tau$  of fuzzy neutrosophic subsets in X satisfying the following axioms.

i.  $0_N, 1_N \in \tau$ ,

ii.  $\lambda_{N1} \cap \lambda_{N2} \in \tau$  for any  $\lambda_{N1}, \lambda_{N2} \in \tau$ ,

iii.  $\cup \lambda_{Nj} \in \tau, \forall \{ \lambda_{Nj} : j \in J \} \subseteq \tau$ .

In this case the pair  $(X, \tau)$  is called fuzzy neutrosophic topological space (FNTPS, for short). The elements of  $\tau$  are called fuzzy neutrosophic open sets (FN-open set, for short). The complement of FN-open set in the FNTPS  $(X, \tau)$  is called fuzzy neutrosophic closed set (FN-closed set, for short).

**Definition 2.5 [9]:** Let  $(X, \tau)$  be FNTPS and  $\lambda_N = \langle x, \mu_{\lambda_N}, \sigma_{\lambda_N}, \nu_{\lambda_N} \rangle$  is FNS in X. Then, the fuzzy neutrosophic-closure (FNCl, for short) and fuzzy neutrosophic-Interior of  $\lambda_N$  (FNInt, for short) are defined by:

$\text{FNCl}(\lambda_N) = \cap \{ \beta_N : \beta_N \text{ is FN-closed set in } X \text{ and } \lambda_N \subseteq \beta_N \}$ ,  $\text{FNInt}(\lambda_N) = \cup \{ \beta_N : \beta_N \text{ is FN-open set in } X \text{ and } \beta_N \subseteq \lambda_N \}$ .

Note that  $\text{FNCl}(\lambda_N)$  is FN-closed set and  $\text{FNInt}(\lambda_N)$  is FN-open set in X. Further,

i.  $\lambda_N$  is FN-closed set in X iff  $\text{FNCl}(\lambda_N) = \lambda_N$ ,

ii.  $\lambda_N$  is FN-open set in X iff  $\text{FNInt}(\lambda_N) = \lambda_N$ .

**Definition 2.6 [10]:** Let  $(X, \tau)$  be FNTPS on X. Then

i.  $\text{FN}\tau_{0,1} = \{ [ ]_{\lambda_N} : \lambda_N \in \tau \}$ ,

ii.  $\text{FN}\tau_{0,2} = \{ \langle \rangle_{\lambda_N} : \lambda_N \in \tau \}$  are FNT on X.

**Definition 2.7 [10]:** If  $\beta_N = \{ \langle y, \mu_{\beta_N}(y), \sigma_{\beta_N}(y), \nu_{\beta_N}(y) \rangle : y \in Y \}$  is FNS in Y. Then, the inverses image of  $\beta_N$  under  $f$ ,  $f^{-1}(\beta_N)$ , for short) is FNS in X defined by  $f^{-1}(\beta_N) = \{ \langle x, f^{-1}(\mu_{\beta_N}(x)), f^{-1}(\sigma_{\beta_N}(x)), f^{-1}(\nu_{\beta_N}(x)) \rangle : x \in X \}$  where,  $f^{-1}(\mu_{\beta_N}(x)) = \mu_{\beta_N}f(x)$ ,  $f^{-1}(\sigma_{\beta_N}(x)) = \sigma_{\beta_N}f(x)$  and  $f^{-1}(\nu_{\beta_N}(x)) = \nu_{\beta_N}f(x)$ .

**Definition 2.8 [10]:** Let  $(X, \tau_x)$  and  $(Y, \tau_y)$  are two FNTPSs. Then a function  $f : (X, \tau_x) \rightarrow (Y, \tau_y)$  is called fuzzy neutrosophic-continuous (FN-con., for short) if the inverses image of every FN-open (FN-closed) set in  $(Y, \tau_y)$  is FN-open (FN-closed) set in  $(X, \tau_x)$ .

**Definition 2.9 [6]:** Let  $(X, \tau_x)$  and  $(Y, \tau_y)$  are two FNTPSs. Then a function  $f : (X, \tau_x) \rightarrow (Y, \tau_y)$  is called fuzzy neutrosophic-contra continuous (FN-ccon., for short) if the inverses image of every FN-open (FN-closed) set in  $(Y, \tau_y)$  is FN-closed (FN-open) set in  $(X, \tau_x)$ .

**Some New Kinds of Continuous Functions Via Fuzzy Neutrosophic Topological Spaces**

Now, we introduced a new concept in fuzzy neutrosophic topological spaces and called it fuzzy neutrosophic- $\tau_{0,1}$ continuous, fuzzy neutrosophic- $\tau_{0,2}$  continuous, fuzzy neutrosophic- $\tau_{0,1}$ contra continuous and fuzzy neutrosophic- $\tau_{0,2}$ contra continuous functions.

**Definition 3.1:** Let  $(X, \text{FN}\tau_{x0,1})$  and  $(Y, \text{FN}\tau_{y0,1})$  are two FNTPSs. Then:

i. A function  $f : (X, \text{FN}\tau_{x0,1}) \rightarrow (Y, \text{FN}\tau_{y0,1})$  is called fuzzy neutrosophic-  $\tau_{0,1}$ continuous (FN- $\tau_{0,1}$ con., for short) if the inverse image of every FN-open (FN-closed) set in  $(Y, \text{FN}\tau_{y0,1})$  is FN-open (FN-closed) set in  $(X, \text{FN}\tau_{x0,1})$ .

ii. A function  $f : (X, \text{FN}\tau_{x0,2}) \rightarrow (Y, \text{FN}\tau_{y0,2})$  is called fuzzy neutrosophic-  $\tau_{0,2}$ continuous (FN- $\tau_{0,2}$ con., for short) if the inverse image of every FN-open (FN-closed) set in  $(Y, \text{FN}\tau_{y0,2})$  is FN-open (FN-closed) set in  $(X, \text{FN}\tau_{x0,2})$ .

**Example 3.2: 1-** Let  $X = Y = \{a, b\}$  define FNSs  $\lambda_N$  in X and  $\beta_N$  in Y as follows:

$\lambda_N = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) \rangle$ . The family,  $\tau_x = \{0_N, 1_N, \lambda_N\}$  is FNT.

And  $\beta_N = \langle y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) \rangle$ . The family,  $\tau_y = \{0_N, 1_N, \beta_N\}$  is FNT.

Define  $f : (X, \tau_x) \rightarrow (Y, \tau_y)$  as follows:  $f(a) = b$  and  $f(b) = a$ .

If,  $\beta_N = \langle y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) \rangle$  is FN-open set in  $\tau_y$ .

Then,  $f^{-1}(\beta_N) = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) \rangle \in \tau_x$ .

So,  $f^{-1}(\beta_N)$  is FN-open set in  $\tau_x$ . Hence,  $f$  is (FN-con.) function.

2- Take, (1) so from  $\tau_x$  we get:

The family,  $\text{FN}\tau_{x0,1} = \{0_N, 1_N, \langle x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.5}) \rangle\}$  is FNT.

And, from  $\tau_y$  we get:

The family,  $\text{FN}\tau_{y0,1} = \{0_N, 1_N, \langle y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.6}) \rangle\}$  is FNT.

Define  $f : (X, \text{FN}\tau_{x0,1}) \rightarrow (Y, \text{FN}\tau_{y0,1})$  as follows:  $f(a) = b$  and  $f(b) = a$ .

Now, let  $\eta_N = \langle y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.6}) \rangle$  is FN-open set in  $\text{FN}\tau_{y0,1}$ .

Then,  $f^{-1}(\eta_N) = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.5}) \rangle \in \text{FN}\tau_{x0,1}$ .

So,  $f^{-1}(\eta_N)$  is FN-open set in  $\text{FN}\tau_{x0,1}$ . Hence,  $f$  is (FN- $\tau_{0,1}$ con.) function.

3- Take, (1) so from  $\tau_x$  we get:

The family,  $\text{FN}\tau_{x0,2} = \{0_N, 1_N, \langle x, (\frac{a}{0.1}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) \rangle\}$  is FNT.

And, from  $\tau_y$  we get:

The family,  $\text{FN}\tau_{y0,2} = \{0_N, 1_N, \langle y, (\frac{a}{0.4}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) \rangle\}$  is FNT.

Define  $f : (X, \text{FN}\tau_{x0,2}) \rightarrow (Y, \text{FN}\tau_{y0,2})$  as follows:  $f(a) = b$  and  $f(b) = a$ .

If,  $\Psi_N = \langle y, (\frac{a}{0.4}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) \rangle$  is FN-open set in  $\text{FN}\tau_{y0,2}$ .

Then,  $f^{-1}(\Psi_N) = \langle x, (\frac{a}{0.1}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) \rangle \in FN\tau_{x0,2}$ .

So,  $f^{-1}(\Psi_N)$  is FN-open set in  $FN\tau_{x0,2}$ . Hence,  $f$  is (FN- $\tau_{0,2}$ con.) function.

**Theorem 3.3:**

Let  $(X, \tau_x), (Y, \tau_y)$  two FNTSs and  $f: (X, \tau_x) \rightarrow (Y, \tau_y)$  is a function.

**i.** If,  $f$  is (FN-con.) function. Then,  $f$  is (FN- $\tau_{0,1}$ con.) function.

**ii.** If,  $f$  is (FN-con.) function. Then,  $f$  is (FN- $\tau_{0,2}$ con.) function.

**Proof:**

**i.** Let  $f$  be (FN-con.) function. Then,

$\beta_N = \{ \langle y, \mu_{\beta_N}(y), \sigma_{\beta_N}(y), \nu_{\beta_N}(y) \rangle : y \in Y \}$  is FN-open set in  $\tau_y$ , so

$f^{-1}(\beta_N) = \{ \langle x, f^{-1}(\mu_{\beta_N}(x)), f^{-1}(\sigma_{\beta_N}(x)), f^{-1}(\nu_{\beta_N}(x)) \rangle : x \in X \}$ , where

$f^{-1}(\mu_{\beta_N}(x)) = \mu_{\beta_N}(f(x)), f^{-1}(\sigma_{\beta_N}(x)) = \sigma_{\beta_N}(f(x))$  and  $f^{-1}(\nu_{\beta_N}(x)) = \nu_{\beta_N}(f(x))$

is FN-open set in  $\tau_x$ . And, by **Definition 2.8** we get:

$\eta_N = \{ \langle y, \mu_{\beta_N}(y), \sigma_{\beta_N}(y), 1-\mu_{\beta_N}(y) \rangle : y \in Y \}$  is FN-open set in

$FN\tau_{y0,1}$ , so  $f^{-1}(\eta_N) = \{ \langle x, f^{-1}(\mu_{\beta_N}(x)), f^{-1}(\sigma_{\beta_N}(x)), f^{-1}(1-\mu_{\beta_N}(x)) \rangle : x \in X \}$

$= \{ \langle x, f^{-1}(\mu_{\beta_N}(x)), f^{-1}(\sigma_{\beta_N}(x)), 1-f^{-1}(\mu_{\beta_N}(x)) \rangle : x \in X \}$  is FN-open

set in  $FN\tau_{x0,1}$ . By **Definition 3.1 (i)**. Hence,  $f$  is (FN- $\tau_{0,1}$ con.) function.

**ii.** Let  $f$  be (FN-con.) function. Then,

$\beta_N = \{ \langle y, \mu_{\beta_N}(y), \sigma_{\beta_N}(y), \nu_{\beta_N}(y) \rangle : y \in Y \}$  is FN-open set in  $\tau_y$ , so

$f^{-1}(\beta_N) = \{ \langle x, f^{-1}(\mu_{\beta_N}(x)), f^{-1}(\sigma_{\beta_N}(x)), f^{-1}(\nu_{\beta_N}(x)) \rangle : x \in X \}$ , where

$f^{-1}(\mu_{\beta_N}(x)) = \mu_{\beta_N}(f(x)), f^{-1}(\sigma_{\beta_N}(x)) = \sigma_{\beta_N}(f(x))$  and  $f^{-1}(\nu_{\beta_N}(x)) = \nu_{\beta_N}(f(x))$

is FN-open set in  $\tau_x$ . And, by **Definition 2.8** we get:

$\Psi_N = \{ \langle y, 1-\nu_{\beta_N}(y), \sigma_{\beta_N}(y), \nu_{\beta_N}(y) \rangle : y \in Y \}$  is FN-open set in  $FN\tau_{y0,2}$ ,

so  $f^{-1}(\Psi_N) = \{ \langle x, f^{-1}(1-\nu_{\beta_N}(x)), f^{-1}(\sigma_{\beta_N}(x)), f^{-1}(\nu_{\beta_N}(x)) \rangle : x \in X \}$

$= \{ \langle x, 1-f^{-1}(\nu_{\beta_N}(x)), f^{-1}(\sigma_{\beta_N}(x)), f^{-1}(\nu_{\beta_N}(x)) \rangle : x \in X \}$  is FN-open set in

$FN\tau_{x0,2}$ . By **Definition 3.1 (ii)**. Hence,  $f$  is (FN- $\tau_{0,2}$ con.) function.

**Remark 3.4:**

The convers of **Theorem 3.3** is not true in general and we can show it by the following example.

**Example 3.5: i.** Let  $X=Y = \{a, b\}$  define FNSs  $\lambda_N$  in  $X$  and  $\beta_N$  in  $Y$  as follows:

$\lambda_N = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.6}) \rangle$ . The family,  $\tau_x = \{0_N, 1_N, \lambda_N\}$  is FNT.

And,  $\beta_N = \langle y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.7}) \rangle$ . The family,  $\tau_y = \{0_N, 1_N, \beta_N\}$  is FNT.

Define  $f: (X, \tau_x) \rightarrow (Y, \tau_y)$  as follows:  $f(a) = b$  and  $f(b) = a$ .

If,  $\beta_N = \langle y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.7}) \rangle$  is FN-open set in  $\tau_y$ .

Then,  $f^{-1}(\beta_N) = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.7}, \frac{b}{0.4}) \rangle \notin \tau_x$ .

Hence,  $f$  is not (FN-con.) function.

But, from  $\tau_x$  we get:

The family,  $FN\tau_{x0,1} = \{0_N, 1_N, \langle x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.5}) \rangle\}$  is FNT.

And, from  $\tau_y$  we get:

The family,  $FN\tau_{y0,1} = \{0_N, 1_N, \langle y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.6}) \rangle\}$  is FNT.

Define  $f: (X, FN\tau_{x0,1}) \rightarrow (Y, FN\tau_{y0,1})$  as follows:  $f(a) = b$  and  $f(b) = a$ .

If,  $\eta_N = \langle y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.6}) \rangle$  is FN-open set in  $FN\tau_{y0,1}$ .

Then,  $f^{-1}(\eta_N) = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.5}) \rangle \in FN\tau_{x0,1}$ .

So,  $f^{-1}(\eta_N)$  is FN-open set in  $FN\tau_{x0,1}$ . Hence,  $f$  is (FN- $\tau_{0,1}$ con.) function.

**ii.** Let  $X = Y = a, b$  define FNSs  $\lambda_N$  in  $X$  and  $\beta_N$  in  $Y$  as follows :

$\lambda_N = \langle x, (\frac{a}{0.1}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) \rangle$ . The family,  $\tau_x = \{0_N, 1_N, \lambda_N\}$  is FNT.

$\beta_N = \langle y, (\frac{a}{0.2}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) \rangle$ . The family,  $\tau_y = \{0_N, 1_N, \beta_N\}$  is FNT.

Define  $f: (X, \tau_x) \rightarrow (Y, \tau_y)$  as follows:  $f(a) = b$  and  $f(b) = a$ .

If,  $\beta_N = \langle y, (\frac{a}{0.2}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) \rangle$  is FN-open set in  $\tau_y$ .

Then,  $f^{-1}(\beta_N) = \langle x, (\frac{a}{0.6}, \frac{b}{0.2}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) \rangle \notin \tau_x$ .

Hence,  $f$  is not (FN-con.) function.

But, from  $\tau_x$  we get:

The family,  $FN\tau_{x0,2} = \{0_N, 1_N, \langle x, (\frac{a}{0.1}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) \rangle\}$  is FNT.

And, from  $\tau_y$  we get:

The family,  $FN\tau_{y0,2} = \{0_N, 1_N, \langle y, (\frac{a}{0.4}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) \rangle\}$  is FNT.

Define  $f: (X, FN\tau_{x0,2}) \rightarrow (Y, FN\tau_{y0,2})$  as follows:  $f(a) = b$  and  $f(b) = a$ .

If,  $\Psi_N = \langle y, (\frac{a}{0.4}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) \rangle$  is FN-open set in  $FN\tau_{y0,2}$ .

Then,  $f^{-1}(\Psi_N) = \langle x, (\frac{a}{0.1}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) \rangle \in FN\tau_{x0,2}$ .

So,  $f^{-1}(\Psi_N)$  is FN-open set in  $FN\tau_{x0,2}$ . Hence,  $f$  is (FN- $\tau_{0,2}$ con.) function.

**Remark 3.6:**

The relation between (FN- $\tau_{0,1}$ con.) and (FN- $\tau_{0,2}$ con.) functions are independent and we can show it by the following example.

**Example 3.7:**

**1-** Take, **Example 3.5 (i)**. Then,  $f$  is (FN- $\tau_{0,1}$ con.) function.

But,  $f$  is not  $(FN-\tau_{0,2}con.)$  function. Since, from  $\tau_x$  we get:

The family,  $FN\tau_{x0,2} = \{0_N, 1_N, \langle x, (\frac{a}{0.7}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.6}) \rangle\}$  is FNT.

And, from  $\tau_y$  we get:

The family,  $FN\tau_{y0,2} = \{0_N, 1_N, \langle y, (\frac{a}{0.6}, \frac{b}{0.3}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.7}) \rangle\}$  is FNT.

Define  $f : (X, FN\tau_{x0,2}) \rightarrow (Y, FN\tau_{y0,2})$  as follows:  $f(a) = b$  and  $f(b) = a$ .

If,  $\Psi_N = \langle y, (\frac{a}{0.6}, \frac{b}{0.3}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.7}) \rangle$  is FN-open set in  $FN\tau_{y0,2}$ .

Then,  $f^{-1}(\Psi_N) = \langle x, (\frac{a}{0.3}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.7}, \frac{b}{0.4}) \rangle \notin FN\tau_{x0,2}$ .

2- Take, **Example 3.5 (ii)**. Then,  $f$  is  $(FN-\tau_{0,2}con.)$  function.

But,  $f$  is not  $(FN-\tau_{0,1}con.)$  function. Since, from  $\tau_x$  we get:

The family,  $FN\tau_{x0,1} = \{0_N, 1_N, \langle x, (\frac{a}{0.1}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.5}) \rangle\}$  is FNT.

And, from  $\tau_y$  we get:

The family,  $FN\tau_{y0,1} = \{0_N, 1_N, \langle y, (\frac{a}{0.2}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.8}, \frac{b}{0.4}) \rangle\}$  is FNT.

Define  $f : (X, FN\tau_{x0,1}) \rightarrow (Y, FN\tau_{y0,1})$  as follows:  $f(a) = b$  and  $f(b) = a$ .

If,  $\eta_N = \langle y, (\frac{a}{0.2}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.8}, \frac{b}{0.4}) \rangle$  is FN-open set in  $FN\tau_{y0,1}$ .

Then,  $f^{-1}(\eta_N) = \langle x, (\frac{a}{0.6}, \frac{b}{0.2}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.8}) \rangle \notin FN\tau_{x0,1}$ .

**Definition 3.8:**

Let  $(X, FN\tau_{x0,1})$  and  $(Y, FN\tau_{y0,1})$  are two FNTSs. Then:

**i.** A function  $f : (X, FN\tau_{x0,1}) \rightarrow (Y, FN\tau_{y0,1})$  is called fuzzy neutrosophic- $\tau_{0,1}$ contra continuous  $(FN-\tau_{0,1}con.,$  for short ) if the inverse image of every FN-open  $(FN-closed)$  set in  $(Y, FN\tau_{y0,1})$  is FN- closed  $(FN-open)$  set in  $(X, FN\tau_{x0,1})$ .

**ii.** A function  $f : (X, FN\tau_{x0,2}) \rightarrow (Y, FN\tau_{y0,2})$  is called fuzzy neutrosophic- $\tau_{0,2}$ contra continuous  $(FN-\tau_{0,2}con.,$  for short ) if the inverse image of every FN-open  $(FN-closed)$  set in  $(Y, FN\tau_{y0,2})$  is FN-closed  $(FN-open)$  set in  $(X, FN\tau_{x0,2})$ .

**Example 3.9: 1-** Let  $X=Y = \{a, b\}$  define FNSs  $\lambda_N$  in  $X$  and  $\beta_N$  in  $Y$  as follows:

$\lambda_N = \langle x, (\frac{a}{0.9}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.5}) \rangle$ . The family,  $\tau_x = \{0_N, 1_N, \lambda_N\}$  is FNT.

Such that,  $1_N-\tau_x = \{1_N, 0_N, \langle x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) \rangle\}$ .

And,  $\beta_N = \langle y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) \rangle$ . The family,  $\tau_y = \{0_N, 1_N, \beta_N\}$  is FNT.

Define  $f : (X, \tau_x) \rightarrow (Y, \tau_y)$  as follows:  $f(a) = b$  and  $f(b) = a$ .

If,  $\beta_N = \langle y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) \rangle$  is FN-open set in  $\tau_y$ .

Then,  $f^{-1}(\beta_N) = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) \rangle \in 1_N-\tau_x$ .

So,  $f^{-1}(\beta_N)$  is FN-closed set in  $\tau_x$ . Hence,  $f$  is  $(FN-ccon.)$  function.

2- Let  $X = Y = \{a, b\}$  define FNSs  $\lambda_N$  in  $X$  and  $\beta_N$  in  $Y$  as follows:

$\lambda_N = \langle x, (\frac{a}{0.4}, \frac{b}{0.2}), (\frac{a}{0.6}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.7}) \rangle$ . The family,  $\tau_x = \{0_N, 1_N, \lambda_N\}$  is FNT.

From  $\tau_x$  we get:

The family,  $FN\tau_{x0,1} = \{0_N, 1_N, \langle x, (\frac{a}{0.4}, \frac{b}{0.2}), (\frac{a}{0.6}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.8}) \rangle\}$  is FNT.

Such that,  $1_N-FN\tau_{x0,1} = \{1_N, 0_N, \langle x, (\frac{a}{0.6}, \frac{b}{0.8}), (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.2}) \rangle\}$ .

And,  $\beta_N = \langle y, (\frac{a}{0.8}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.4}, \frac{b}{0.3}) \rangle$ . The family,  $\tau_y = \{0_N, 1_N, \beta_N\}$  is FNT.

From  $\tau_y$  we get:

The family,  $FN\tau_{y0,1} = \{0_N, 1_N, \langle y, (\frac{a}{0.8}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.2}, \frac{b}{0.4}) \rangle\}$  is FNT.

Define  $f : (X, FN\tau_{x0,1}) \rightarrow (Y, FN\tau_{y0,1})$  as follows:  $f(a) = b$  and  $f(b) = a$ .

If,  $\eta_N = \langle y, (\frac{a}{0.8}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.2}, \frac{b}{0.4}) \rangle$  is FN-open set in  $FN\tau_{y0,1}$ .

Then,  $f^{-1}(\eta_N) = \langle x, (\frac{a}{0.6}, \frac{b}{0.8}), (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.2}) \rangle \in 1_N-FN\tau_{x0,1}$ .

So,  $f^{-1}(\eta_N)$  is FN-closed set in  $FN\tau_{x0,1}$ . Hence,  $f$  is  $(FN-\tau_{0,1}ccon.)$  function.

3- Let  $X = Y = \{a, b\}$  define FNSs  $\lambda_N$  in  $X$  and  $\beta_N$  in  $Y$  as follows:

$\lambda_N = \langle x, (\frac{a}{0.4}, \frac{b}{0.3}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.8}, \frac{b}{0.6}) \rangle$ . The family,  $\tau_x = \{0_N, 1_N, \lambda_N\}$  is FNT.

From  $\tau_x$  we get:

The family  $FN\tau_{x0,2} = \{0_N, 1_N, \langle x, (\frac{a}{0.2}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.8}, \frac{b}{0.6}) \rangle\}$  is FNT.

Such that,  $1_N-FN\tau_{x0,2} = \{1_N, 0_N, \langle x, (\frac{a}{0.8}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.2}, \frac{b}{0.4}) \rangle\}$ . And,

$\beta_N = \langle y, (\frac{a}{0.4}, \frac{b}{0.7}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.2}) \rangle$ . The family,  $\tau_y = \{0_N, 1_N, \beta_N\}$  is FNT.

From  $\tau_y$  we get:

The family,  $FN\tau_{y0,2} = \{0_N, 1_N, \langle y, (\frac{a}{0.6}, \frac{b}{0.8}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.2}) \rangle\}$  is FNT.

Define  $f : (X, FN\tau_{x0,2}) \rightarrow (Y, FN\tau_{y0,2})$  as follows:  $f(a) = b$  and  $f(b) = a$ .

If,  $\Psi_N = \langle y, (\frac{a}{0.6}, \frac{b}{0.8}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.2}) \rangle$  is FN-open set in  $FN\tau_{y0,2}$ .

Then,  $f^{-1}(\Psi_N) = \langle x, (\frac{a}{0.8}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.2}, \frac{b}{0.4}) \rangle \in 1_N-FN\tau_{x0,2}$ .

So,  $f^{-1}(\Psi_N)$  is FN-closed set in  $FN\tau_{x0,2}$ . Hence,  $f$  is  $(FN-\tau_{0,2}ccon.)$  function.

**Remark 3.10: i.** The relation between  $(FN-ccon.)$  and  $(FN-\tau_{0,1}ccon.)$  functions are independent.

**ii.** The relation between  $(FN-ccon.)$  and  $(FN-\tau_{0,2}ccon.)$  functions are independent.

**iii.** The relation between  $(FN-\tau_{0,1}ccon.)$  and  $(FN-\tau_{0,2}ccon.)$  functions are independent.

And we can show it by the following example.

**Example 3.11:**

**i. 1-** Take, **Example 3.9 (1).** Then,  $f$  is  $(FN-ccon.)$  function.

But,  $f$  is not  $(FN-\tau_{0,1}ccon.)$  function. Since, from  $\tau_x$  we get:

The family,  $FN\tau_{x0,1} = \{0_N, 1_N, \langle x, (\frac{a}{0.9}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.1}, \frac{b}{0.4}) \rangle\}$  is FNT.

Such that,  $1_N-FN\tau_{x0,1} = \{1_N, 0_N, \langle x, (\frac{a}{0.1}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) \rangle\}$ .

And, from  $\tau_y$  we get:

The family,  $FN\tau_{y0,1} = \{0_N, 1_N, \langle y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.6}) \rangle\}$  is FNT.

Define  $f : (X, FN\tau_{x0,1}) \rightarrow (Y, FN\tau_{y0,1})$  as follows:  $f(a) = b$  and  $f(b) = a$ .

If,  $\eta_N = \langle y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.6}) \rangle$  is FN-open set in  $FN\tau_{y0,1}$ .

Then,  $f^{-1}(\eta_N) = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.5}) \rangle \notin 1_N-FN\tau_{x0,1}$ .

**2-** Take, **Example 3.9 (2).** Then,  $f$  is  $(FN-\tau_{0,1}ccon.)$  function.

But,  $f$  is not  $(FN-ccon.)$  function.

Since,  $1_N-\tau_x = \{1_N, 0_N, \langle x, (\frac{a}{0.5}, \frac{b}{0.7}), (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.2}) \rangle\}$ .

Define,  $f : (X, \tau_x) \rightarrow (Y, \tau_y)$  as follows:  $f(a) = b$  and  $f(b) = a$ .

If,  $\beta_N = \langle y, (\frac{a}{0.8}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.4}, \frac{b}{0.3}) \rangle$  is FN-open set in  $\tau_y$ .

Then,  $f^{-1}(\beta_N) = \langle x, (\frac{a}{0.6}, \frac{b}{0.8}), (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.4}) \rangle \notin 1_N-\tau_x$ .

**ii. 1-** Take,

**Example 3.9 (1).** Then,  $f$  is  $(FN-ccon.)$  function.

But,  $f$  is not  $(FN-\tau_{0,2}ccon.)$  function. Since, from  $\tau_x$  we get:

The family,  $FN\tau_{x0,2} = \{0_N, 1_N, \langle x, (\frac{a}{0.6}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.5}) \rangle\}$  is FNT.

Such that,  $1_N-FN\tau_{x0,2} = \{1_N, 0_N, \langle x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.5}) \rangle\}$ .

And, from  $\tau_y$  we get:

The family,  $FN\tau_{y0,2} = \{0_N, 1_N, \langle y, (\frac{a}{0.4}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) \rangle\}$  is FNT.

Define  $f : (X, FN\tau_{x0,2}) \rightarrow (Y, FN\tau_{y0,2})$  as follows:  $f(a) = b$  and  $f(b) = a$ .

If,  $\Psi_N = \langle y, (\frac{a}{0.4}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) \rangle$  is FN-open set in  $FN\tau_{y0,2}$ .

Then,  $f^{-1}(\Psi_N) = \langle x, (\frac{a}{0.1}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) \rangle \notin 1_N-FN\tau_{x0,2}$ .

**2-** Take, **Example 3.9 (3).** Then,  $f$  is  $(FN-\tau_{0,2}ccon.)$  function.

But,  $f$  is not  $(FN-ccon.)$  function.

Since,  $1_N-\tau_x = \{1_N, 0_N, \langle x, (\frac{a}{0.8}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.3}) \rangle\}$ .

Define  $f : (X, \tau_x) \rightarrow (Y, \tau_y)$  as follows :  $f(a) = b$  and  $f(b) = a$ .

If,  $\beta_N = \langle y, (\frac{a}{0.4}, \frac{b}{0.7}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.2}) \rangle$  is FN-open set in  $\tau_y$ .

Then,  $f^{-1}(\beta_N) = \langle x, (\frac{a}{0.7}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.2}, \frac{b}{0.4}) \rangle \notin 1_N-\tau_x$ .

**iii. 1-**Take, **Example 3.9 (2).** Then,  $f$  is  $(FN-\tau_{0,1}ccon.)$  function.

But,  $f$  is not  $(FN-\tau_{0,2}ccon.)$  function.

Since, from  $\tau_x$  we get:

The family,  $FN\tau_{x0,2} = \{0_N, 1_N, \langle x, (\frac{a}{0.5}, \frac{b}{0.3}), (\frac{a}{0.6}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.7}) \rangle\}$  is FNT.

Such that,  $1_N-FN\tau_{x0,2} = \{1_N, 0_N, \langle x, (\frac{a}{0.5}, \frac{b}{0.7}), (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.3}) \rangle\}$ .

And, from  $\tau_y$  we get:

The family,  $FN\tau_{y0,2} = \{0_N, 1_N, \langle y, (\frac{a}{0.6}, \frac{b}{0.7}), (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.4}, \frac{b}{0.3}) \rangle\}$  is FNT.

Define  $f : (X, FN\tau_{x0,2}) \rightarrow (Y, FN\tau_{y0,2})$  as follows:  $f(a) = b$  and  $f(b) = a$ .

If,  $\Psi_N = \langle y, (\frac{a}{0.6}, \frac{b}{0.7}), (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.4}, \frac{b}{0.3}) \rangle$  is FN-open set in  $FN\tau_{y0,2}$ .

Then,  $f^{-1}(\Psi_N) = \langle x, (\frac{a}{0.7}, \frac{b}{0.6}), (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.4}) \rangle \notin 1_N-FN\tau_{x0,2}$ .

**2-** Take, **Example 3.9 (3).** Then,  $f$  is  $(FN-\tau_{0,2}ccon.)$  function.

But,  $f$  is not  $(FN-\tau_{0,1}ccon.)$  function. Since, from  $\tau_x$  we get:

The family,  $FN\tau_{x0,1} = \{0_N, 1_N, \langle x, (\frac{a}{0.4}, \frac{b}{0.3}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.7}) \rangle\}$  is FNT.

Such that,  $1_N-FN\tau_{x0,1} = \{1_N, 0_N, \langle x, (\frac{a}{0.6}, \frac{b}{0.7}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.3}) \rangle\}$ .

And, from  $\tau_y$  we get:

The family,  $FN\tau_{y0,1} = \{0_N, 1_N, \langle y, (\frac{a}{0.4}, \frac{b}{0.7}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.3}) \rangle\}$  is FNT.

Define  $f : (X, FN\tau_{x0,1}) \rightarrow (Y, FN\tau_{y0,1})$  as follows:  $f(a) = b$  and  $f(b) = a$ .

If,  $\eta_N = \langle y, (\frac{a}{0.4}, \frac{b}{0.7}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.3}) \rangle$  is FN-open set in  $FN\tau_{y0,1}$ .

Then,  $f^{-1}(\eta_N) = \langle x, (\frac{a}{0.7}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.6}) \rangle \notin 1_N-FN\tau_{x0,1}$ .

**Remark 3.12:**

i. The relation between (FN-ccon.) and (FN-con.) are independent.

ii. The relation between (FN- $\tau_{0,1}$ ccon.) and (FN- $\tau_{0,1}$ con.) are independent.

iii. The relation between (FN- $\tau_{0,2}$ ccon.) and (FN- $\tau_{0,2}$ con.) are independent. And we can show it by the following example.

**Example 3.13:**

i. 1- Take, **Example 3.9 (1)**. Then,  $f$  is (FN-ccon.) function.

But,  $f$  is not (FN-con.) function. Since,  $f^{-1}(\beta_N) \notin \tau_x$ .

2- Take, **Example 3.2 (1)**. Then,  $f$  is (FN-con.) function.

But,  $f$  is not (FN-ccon.) function. Since,  $f^{-1}(\beta_N) \notin 1_N - \tau_x$ .

ii. 1-Take, **Example 3.9 (2)**. Then,  $f$  is (FN- $\tau_{0,1}$ ccon.) function.

But,  $f$  is not (FN- $\tau_{0,1}$ con.) function. Since,  $f^{-1}(\eta_N) \notin FN\tau_{x0,1}$ .

2- Take, **Example 3.2 (2)**. Then,  $f$  is (FN- $\tau_{0,1}$ con.) function.

But,  $f$  is not (FN- $\tau_{0,1}$ ccon.) function. Since,  $f^{-1}(\eta_N) \notin 1_N - FN\tau_{x0,1}$ .

iii. 1-Take, **Example 3.9 (3)**. Then,  $f$  is (FN- $\tau_{0,2}$ ccon.) function.

But,  $f$  is not (FN- $\tau_{0,2}$ con.) function. Since,  $f^{-1}(\Psi_N) \notin FN\tau_{x0,2}$ .

2- Take, **Example 3.2 (3)**. Then,  $f$  is (FN- $\tau_{0,2}$ con.) function.

But,  $f$  is not (FN- $\tau_{0,2}$ ccon.) function. Since,  $f^{-1}(\Psi_N) \notin 1_N - FN\tau_{x0,2}$ .

**Definition 3.14:**

Fuzzy neutrosophic subset  $\lambda_N$  of FNTS  $(X, \tau)$  is called fuzzy neutrosophic-clopen set (FN-clopen, for short) set if  $\lambda_N$  is FN-closed set and FN-open set in same time.

**Theorem 3.15: i.** Let  $(X, \tau_x)$  and  $(Y, \tau_y)$  are two FNTSs and  $f: (X, \tau_x) \rightarrow (Y, \tau_y)$  is a function.  $f$  is

(FN-con.) iff  $f$  is (FN-ccon.) whenever, every the invers image of any FNS in  $\tau_y$  is FN-clopen set in  $\tau_x$ .

ii. Let  $(X, FN\tau_{x0,1})$  and  $(Y, FN\tau_{y0,1})$  are two FNTSs and  $f: (X, FN\tau_{x0,1}) \rightarrow (Y, FN\tau_{y0,1})$  is a function.  $f$  is (FN- $\tau_{0,1}$ con.) iff  $f$  is (FN- $\tau_{0,1}$ ccon.) whenever, every the invers image of any FNS in  $FN\tau_{y0,1}$  is FN-clopen set in  $FN\tau_{x0,1}$ .

iii. Let  $(X, FN\tau_{x0,2})$  and  $(Y, FN\tau_{y0,2})$  are two FNTSs and  $f: (X, FN\tau_{x0,2}) \rightarrow (Y, FN\tau_{y0,2})$  is a function.

$f$  is (FN- $\tau_{0,2}$ con.) iff  $f$  is (FN- $\tau_{0,2}$ ccon.) whenever, every the invers image of any FNS in  $FN\tau_{y0,2}$  is FN-clopen set in  $FN\tau_{x0,2}$ .

**Proof: i.** Let  $f$  be (FN-con.) function. If,  $\beta_N$  be FN-open set in  $\tau_y$ .

Then, by **Definition 2.8**  $f^{-1}(\beta_N) = \omega_N \in \tau_x$ .

But,  $\omega_N$  be FN-clopen set in  $\tau_x$ . Therefore,  $f^{-1}(\beta_N) = \omega_N \in 1_N - \tau_x$ .

Hence, by **Definition 2.9**  $f$  is (FN-ccon.) function.

**Conversely;** the proof is direct.

ii. Let  $f$  be (FN- $\tau_{0,1}$ con.) function. If,  $\eta_N$  be FN-open set in  $FN\tau_{y0,1}$ .

Then, by **Definition 3.1(i)**  $f^{-1}(\eta_N) = \omega_N \in FN\tau_{x0,1}$ .

But,  $\omega_N$  be FN-clopen set in  $FN\tau_{x0,1}$ . So,  $f^{-1}(\eta_N) = \omega_N \in 1_N - FN\tau_{x0,1}$ .

Hence, by **Definition 3.8 (i)**  $f$  is (FN- $\tau_{0,1}$ ccon.) function.

**Conversely;** the proof is direct.

iii. Let  $f$  be (FN- $\tau_{0,2}$ con.) function. If,  $\Psi_N$  be FN-open set in  $FN\tau_{y0,2}$ .

Then, by **Definition 3.1(ii)**  $f^{-1}(\Psi_N) = \omega_N \in FN\tau_{x0,2}$ .

But,  $\omega_N$  is FN-clopen set in  $FN\tau_{x0,2}$ . So,  $f^{-1}(\Psi_N) = \omega_N \in 1_N - FN\tau_{x0,2}$ .

Hence, by **Definition 3.8 (ii)**  $f$  is (FN- $\tau_{0,2}$ ccon.) function.

**Conversely;** the proof is direct.

**Remark 3.16:** The next diagram showing the relationship between different functions. But the convers is not true in general.

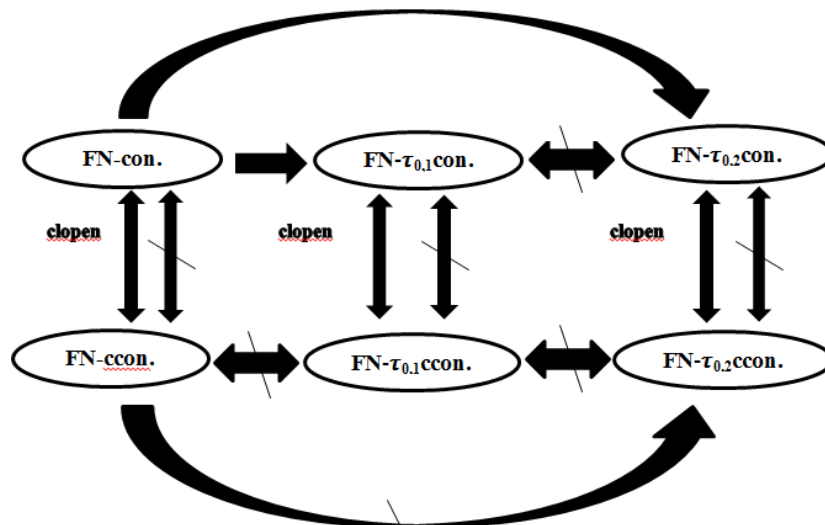


Diagram 3.1

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## بعض الانواع الجديدة من الدوال المستمرة من خلال فضاء تبولوجي نيوتروسوفك المضرب

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### الملخص

في هذا البحث، عرفنا كل من لدوال المستمرة -fuzzy neutrosophic- $\tau_{0,1}$ , fuzzy neutrosophic- $\tau_{0,1}$ contra, fuzzy neutrosophic- $\tau_{0,2}$ contra. ثم وجدنا العلاقات بين الدوال المذكورة والمدروسة مع بعض المقارنات.